DURABLE GOODS AND CONFORMITY

Christopher L. House and Emre Ozdenoren[†]

June 14, 2006

ABSTRACT

Is the variety of products supplied in markets a reflection of the diversity of consumers' preferences? In this paper, we argue that the distribution of durable goods offered in markets tends to be compressed relative to the distribution of consumers' underlying preferences. In particular, there are strong incentives for conformity in markets for durable goods. The reason for conformity is natural: durables (for example houses) are traded and as a result, demand for these goods is influenced by their resale value. Agents may like one product, but purchase another because of resale concerns. We show that in markets for durable goods (1) there is a tendency to conform to the average preference; (2) conformity increases with increases in durability, patience, and the likelihood of trade; and (3) equilibrium conformity is not necessarily optimal. Surprisingly, there tends to be too little conformity in equilibrium.

[†] University of Michigan. The authors gratefully acknowledge the comments of Kai-Uwe Kuhn, Miles Kimball, Greg Lewis, Yusufcan Masatlioglu, Daisuke Nakajima, Steve Salant, Dan Silverman, Lones Smith and Randy Wright.

"Conformity, in terms of size, condition & features, tends to support your home's market value more than anything else." \(^1\)

1. Introduction

Durable goods survive for long periods of time and are often possessed by many different people over the lifetime of the good. Because durables change hands over time, the efficient provision of these goods should reflect the average preferences of many potential owners rather than the preferences of a single individual. The optimal amount of product variety should therefore be compressed relative to the underlying variation in preferences. Put differently, there should be conformity in markets for durable goods. Indeed, because durable goods are often traded, resale concerns influence current demand and thus market forces encourage conformity. In contrast, nondurable goods are consumed by at most a single person and thus there is no incentive to conform.

One market that has received particular attention in this regard is the housing market. New houses often have features and styles that differ only superficially from one house to the next. New housing developments are often derided because they consist merely of "cookie-cutter" houses or "McMansions." These houses are virtually the same – most have cathedral ceilings, walk-in closets, built-in jacuzzis, mud rooms, and so forth. Of course many of these features are desirable but it seems unlikely that preferences are so aligned as to justify such a homogeneous mix of products. The apparent homogeneity among new houses may instead be an efficient market reaction. Rather than catering to individual tastes, builders conform to the average taste anticipating the eventual resale of the house.

We analyze these issues with a matching model in which agents buy and sell a long-lived durable good that must be resold from time-to-time. Although the specific function of the durable is not important for the analysis, we refer to the durable as a house. There are two types of houses in the market and agents differ according to their preferences over the two types. Frictions in the resale market imply that agents are not matched perfectly with others who have the same preferences. Thus, if someone buys an unusual house, he runs the risk that he will not be able to sell it if he needs to move. Resale concerns can be so strong that the individual chooses to purchase a good that he dislikes relative to other

¹Quoted from Accurate Appraisals. See http://www.accurate-appraisal.com/faq.htm.

available goods. When this occurs we say that the individual is conforming to the market.

We show that in equilibrium, there is a tendency to conform to the average preference. Rather than being a knife-edge phenomenon, conformity is the typical outcome in markets for durable goods. Because resale concerns rise with durability and the incidence of trade, there is greater conformity in markets for long-lived durables and for people who trade frequently.

The equilibrium level of conformity in the model is usually not socially optimal. Surprisingly, the model suggests that there is typically too little conformity in durable goods markets. There are two reasons for this inefficiency. First, by conforming, agents reduce search costs. If they have a house that few others want, they will have difficulty selling it if they need to move. Of course the original builder has an incentive to conform to reduce the severity of these search costs. However, the search costs affect both buyers and sellers. Because the original owner only internalizes his own search costs, he has too little incentive to conform. Second, even if the house is sold, it is possible that the house will not be an ideal match for the new owner. The seller typically does not internalize the social costs incurred when the buyer settles for a house that is not ideal for his needs. Because he will likely be matched with someone who has typical preferences, by conforming, the original owner reduces the number of mismatches. Yet, unless the seller captures all of the surplus from the trade, he does not fully internalize this cost and again, there is too little incentive to conform.

The classic example of conformity is perhaps the Keynesian beauty contest. Conformity arises in models of the Keynesian beauty contest because payoffs are assumed to depend both on one's own action and on the average action of the other players. In a sense, our model endogenizes the Keynesian beauty contest. While agents in our model do not care directly about the preferences or actions of other agents, in equilibrium they act as though they do. Because they buy and sell a common set of goods, durability and trade endogenously align the preferences of the agents.

The rest of the paper is organized as follows: Section 2 presents the model. In this section we describe optimal behavior and define and characterize the equilibrium. Section 3 presents the main results of the paper. In this section we show that conformity increases with durability and the incidence of trade. Section 3 also considers the welfare implications of conformity. Section 4 discusses the results and relates our paper to the existing literatures on durables, conformity, and money-search models of liquidity. Section 5 concludes.

2. Model

We consider a continuous-time matching model in which agents can own one of two types of a durable good. Although our analysis holds for any durable good, we assume that the good in our model is a house. We denote the two types of houses as type a and b. The houses could differ along many dimensions. For example, type a houses could be "traditional-style" houses while type b could be "modern-style" houses. Alternatively, type a could be a two-story house with a large yard while type b might be a one-story house with a small yard. Every agent must have a house in every period.

We normalize the utility functions so that all consumers get a flow utility of 1 from living in the type a house. Consumers have different tastes for the type b house. Specifically, each consumer has an individual taste parameter z which quantifies their preference for type b houses. For a consumer with taste parameter $z \in \mathbb{R}$ the flow utility from living in a type b house is 1 + z. The flow utility for an agent with a given z and a given house $x \in \{a, b\}$ is thus

$$u_z(x) = \begin{cases} 1 & \text{if } x = a \\ 1+z & \text{if } x = b \end{cases}$$
.

We assume that z is distributed over the population according to a distribution function F.

From time to time agents switch houses. Agents may switch their house for one of two reasons. First, the house may "die." We think of the "death" of a house as capturing mainly normal depreciation but it may also include extreme idiosyncratic events such as fires, severe water damage, and so forth. When this occurs, the agent must build a new house. We refer to this event as the "build shock." We assume that the build shock obeys a Poisson process with an exogenous arrival rate δ . An agent who gets the build shock decides which type of house to build and incurs a building cost c. Since we want to focus on heterogeneity in tastes, we assume that the building cost is the same for all agents and for either type of house.

Second, the agent may be required to trade his house which we refer to as the "trade shock." Agents who get the trade shock must move out of their house and into a new house. We allow agents to differ in the likelihood of receiving the trade shock. Thus, some agents have to move often while others do not. Specifically, we assume that for each agent, the trade shock obeys a Poisson process with an exogenous arrival rate γ . The arrival rate is distributed over the agents according

to the distribution G. The support of G is restricted to the interval $[0, \infty]$. Here $\gamma = 0$ corresponds to an individual who never needs to trade and $\gamma = \infty$ implies that the individual trades continuously. Trade hazards (γ) and preferences (z) are independent.

To better motivate the trade shock, we imagine that each agent lives and works in one of two cities of equal size. Agents who get the trade shock have to move from one city to the other. When this happens the agent first has an opportunity to trade his house. The agent is matched randomly with a trading partner who is moving in the opposite direction. If both agents agree to trade, they simply exchange houses, otherwise the trade is rejected. If the trade fails, the agents are forced to scrap their old houses and build new houses of their choice. Let π be the difference between the build cost c and the scrap value of their old house. It is important to emphasize that agents are not trading because their preferences over houses change. They trade simply because they have to move from one city to the other. Thus, in the trade state, some agents will exchange houses of the same type (e.g., an a for an a) as well as houses of different types.

Because π is only incurred by agents who fail to trade, we refer to π as the "trade penalty." One can alternatively interpret the trade penalty as reflecting other costs of buying and selling a house. For instance, one could think of π as the expected cost of engaging in a protracted search in an environment with the possibility of re-matching.² Under this interpretation π would include the cost of renting while traders search for new houses and would also include the forgone interest on the sale price while a house waits to be sold. Sales commissions, fees and the costs of renovations could also be included in the trade penalty π .

If agents do not get the build shock or the trade shock they simply continue residing in their current house. Agents seek to maximize the discounted sum of flow utilities less costs. The discount rate is r > 0. The next section analyzes the optimal behavior in this model.

2.1. Optimal Policies and Conformity

In this section we analyze the consumer's maximization problem and present our definition of conformity. In the next section we turn to equilibrium. Throughout, we confine our attention to stationary equilibria. In a stationary equilibrium,

²Allowing for agents to re-match greatly complicates the analysis. By ruling out this possibility, we gain tractability because the agent's trade decisions do not depend on the type of house they possess. We return to this issue again in Section 4.

consumers decide which types of houses to build and also which types of houses to accept in trade, taking the strategies of the other agents and the distribution of houses, both of which are time-invariant, as given. The main result in this section (Proposition 1) shows that the optimal policy takes a simple form. For a given trade hazard γ , the optimal policy can be described by three critical values $z_1(\gamma)$, $z_2(\gamma)$, and $z_3(\gamma)$. If an agent's taste parameter z is less than $z_1(\gamma)$, the agent trades a's exclusively in the trading stage and builds a's in the building stage. If z is between $z_1(\gamma)$ and $z_2(\gamma)$ then he builds a's but accepts either a or b in trade. If z is greater than $z_3(\gamma)$, then he trades b's exclusively and builds b's.

We use standard dynamic programming techniques to analyze the agent's optimization problem. A policy (or strategy) for any agent consists of a decision rule in the trade state and a decision rule in the building state. The trade rule specifies whether an agent accepts or rejects a trade once he enters the trade state given the type of house he has. The build rule specifies which type of house the agent builds when he enters the build state.

Let $V\left(x;z,\gamma\right)$ be the value of following an optimal policy for a given agent with taste parameter z and with trade hazard γ who currently owns a house of type $x \in \{a,b\}$. Let $B\left(z,\gamma\right)$ be the continuation value of entering the build state and let $T\left(x;z,\gamma\right)$ be the continuation value of entering the trading state when the agent has a type x house. Because we are focusing on the behavior of a single agent, we suppress the arguments z and γ in the following discussion. The value function satisfies

$$rV(x) = u(x) + \delta [B - V(x)] + \gamma [T(x) - V(x)]. \tag{1}$$

The continuation value of receiving the build shock is simply

$$B = \max \{V(a), V(b)\} - c. \tag{2}$$

The payoff to entering the trade stage T(x) is more involved. In the trade stage, agents observe each other's houses but do not observe their taste parameters or their trade hazards. They then simultaneously choose to either accept or reject the trade. If they both agree to trade, they swap houses. If either disagrees, the trade is rejected. In this case, both traders pay the trade penalty π and get new houses of their choice. (Recall that when a trade is rejected, the traders scrap their current houses and build new ones; π is the build cost less the scrap value).

To compute T(x), consider an agent who receives the trade shock and currently possesses a type x house. Suppose he is matched with someone with a type y

house. If the trade occurs, the agent gets the type y house. His payoff in this case is simply V(y). If either one rejects the trade then he selects a new house of his choice and pays the trade penalty. His payoff in this case is $\max\{V(a),V(b)\}-\pi$. Notice that the agent's trade decision is only relevant if his trading partner chooses to accept the trade. We assume that agents accept trades whenever $V(y) > \max\{V(a),V(b)\}-\pi$ and reject otherwise.³ Notice also that neither V(y) nor $\max\{V(a),V(b)\}-\pi$ depend on x. Thus, trade decisions are independent of the type of house the agent possesses when he enters the trade stage.

It is easy to show that if an agent builds type $x \in \{a, b\}$ then he also accepts x in trade. To see this, first note that because the agent chooses to build x, it must be the case that $V(x) = \max\{V(a), V(b)\}$. As a result, he also chooses x whenever he gets the trade shock and the trade is rejected. Thus, if he is offered x in trade, he gets V(x) if he accepts the offer and $V(x) - \pi$ if he declines the offer. We present this observation as a Lemma. All proofs are in the appendix.

Lemma 1. If an agent builds type $x \in \{a, b\}$, then he also accepts x in trade.

Because an agent's trading decisions are independent of his trading partner's taste parameter, trade hazard and are also independent of the house he owns, there are only three relevant trading rules to consider: (1) accept a only; (2) accept b only; or (3) accept either a or b. We denote these trading rules simply as a, b, and ab. We refer to agents who accept only a particular type of house (either a or b but not both) as exclusive traders. Agents who play ab and thus accept both types are said to be inclusive traders.

Our aim is to express the value functions $V\left(x\right)$, $T\left(x\right)$ and B in terms of the policies of the other agents. To do this, we first define $\lambda\left(y,\tau\right)$ as the probability of being matched with someone who has a type y house and who follows trading rule $\tau\in\{a,b,ab\}$. For example, $\lambda\left(a,ab\right)$ is the probability of meeting someone who possesses a type a house and follows trading rule ab. Agents who accept type $x\in\{a,b\}$ houses in trade either follow the exclusive trading rule x or the inclusive trading rule ab. The probability of meeting an agent who accepts a type x house in trade is therefore $\sum_{\tau\in\{x,ab\}}\sum_{y\in\{a,b\}}\lambda\left(y,\tau\right)$. We can now write the expected

³This assumption rules out the trivial and uninteresting equilibrium in which agents always reject every trade.

value of entering the trading state with a type x house as

$$T(x) = \sum_{\tau \in \{x, ab\}} \sum_{y \in \{a, b\}} (\lambda(y, \tau) \max\{V(y), \max\{V(a), V(b)\} - \pi\})$$

$$+ \left(1 - \sum_{\tau \in \{x, ab\}} \sum_{y \in \{a, b\}} \lambda(y, \tau)\right) (\max\{V(a), V(b)\} - \pi)$$
(3)

Because we can express the continuation values T(x) and B in terms of the value function V(x) and the matching probabilities $\lambda(y,\tau)$, equation (1) implicitly defines the value function itself solely in terms of the underlying matching probabilities. Given any set of values V(a) and V(b), and fixed matching probabilities, equation (3) implies associated values T(a) and T(b) and equation (2) implies an associated value B. Equation (1) then implies a new set of values $\hat{V}(a)$, $\hat{V}(b)$. It is straightforward to show that this mapping satisfies Blackwell's sufficient conditions for a contraction mapping and thus has a unique fixed point. We summarize this in the following Lemma:

Lemma 2. Given matching probabilities $\lambda(x,\tau)$ for $x \in \{a,b\}$ and $\tau \in \{a,b,ab\}$ with $\sum_{\tau \in \{a,b,ab\}} \sum_{y \in \{a,b\}} \lambda(y,\tau) = 1$ there exist unique values V(a), V(b), T(a), T(b), and B satisfying (1), (2), and (3).

An optimal policy in our model consists of a building rule (whether to build a or b) and a trading rule (whether to accept a, b or both). Because the continuation values in Lemma 2 are fixed numbers, the optimal policy is stationary.

We now characterize the optimal policy. We first note that there are only four relevant policies. To see this note that if you follow the trade rule a (an exclusive trading rule) then by Lemma 1 you must build a type a house in the build stage. Similarly, if you are an exclusive b trader, you build type b. Thus, without loss of generality we can confine our attention to four possible policies: build a and follow trade rule a; build a and follow trade rule ab; build a and follow trade rule ab or build a and follow trade rule ab. We can write these policies compactly as ab0 where ab1 and ab2 and ab3.

Agents with different taste parameters z and different trade hazards γ choose different policies. Intuitively, for a fixed γ , agents with sufficiently negative z's (who strongly dislike type b houses) build type a houses and accept only type a houses in trade. Similarly, agents with sufficiently positive z's (who have strong preferences for b houses) build type b and accept only type b in trade. Agents with

intermediate values of z do not have strong preferences for either type and thus accept either a or b in trade. The following proposition formalizes this intuition.

Proposition 1. Given non-negative $\lambda(a, a)$, $\lambda(b, b)$, $\lambda(a, ab)$, and $\lambda(b, ab)$ summing to 1, define $z_1(\gamma)$, $z_2(\gamma)$, and $z_3(\gamma)$ as follows:

$$z_{1}(\gamma) = -\pi \left\{ r + \delta + \gamma \left[1 - \lambda \left(a, a \right) \right] \right\}$$
$$z_{2}(\gamma) = \gamma \pi \left[\lambda \left(a, a \right) - \lambda \left(b, b \right) \right]$$
$$z_{3}(\gamma) = \pi \left\{ r + \delta + \gamma \left[1 - \lambda \left(b, b \right) \right] \right\}$$

Then, for an agent with taste parameter z and trade hazard γ , if $z \leq z_1(\gamma)$ then (a, a) is an optimal strategy; if $z_1(\gamma) \leq z \leq z_2(\gamma)$, (a, ab) is an optimal strategy; if $z_2(\gamma) \leq z \leq z_3(\gamma)$, (b, ab) is an optimal strategy and if $z_3(\gamma) \leq z$, (b, b) is an optimal strategy.

Notice that the cutoffs depend only on the number of exclusive traders of each type. The precise distribution of tastes of other traders is not relevant once $\lambda\left(a,a\right)$ and $\lambda\left(b,b\right)$ are given. In particular, the average taste parameter is unimportant. The building decisions of the other agents are also irrelevant. Whether the inclusive traders build more a's or more b's has no bearing on an individual's decision. It is important to understand the economic intuition behind this proposition. Because the logic is identical for the b cutoff $(z_3\left(\gamma\right))$ we focus on the determination of the a cutoff $(z_1\left(\gamma\right))$.

Consider an agent with a taste parameter z less than zero who optimally follows an (a,a) policy. Suppose that he receives the trade shock and is matched with someone who has a b and considers a one-shot deviation from his optimal policy. Specifically, suppose that he decides to accept the b house but then revert to the trade rule a for subsequent trade shocks. There are costs and benefits to this deviation. The benefit is that he will avoid the immediate trade penalty π . There are two costs. First, he will reside in a house other than his preferred type for some time. This expected loss is $z/(r+\delta+\gamma)$ (recall that z<0). Second, he may encounter someone who follows an exclusive a trading rule while he still has the b house. In this case he pays the trade penalty π .⁴ This expected cost is $\pi\gamma\lambda$ $(a,a)/(r+\delta+\gamma)$. The discount rate $(r+\delta+\gamma)$ reflects both the agents impatience and the likelihood of moving out of the type b house (which occurs

⁴Note that he would not experience this trade penalty if he did not deviate from the (a, a) strategy.

if the house dies or if he gets another trade shock). There are no additional costs because if the agent is matched with any other type of trader, then the trade penalties are the same as if he continued to follow the (a, a) policy. With probability $\lambda(b, ab) + \lambda(b, b)$ he declines the trade (which he would do under the (a, a) policy) and with probability $\lambda(a, ab)$ he accepts the trade (which he would do under the (a, a) policy). Thus, we can write the net benefit of this deviation as

$$\pi + \frac{z}{r + \delta + \gamma} - \pi \frac{\gamma \lambda (a, a)}{r + \delta + \gamma} \le 0 \tag{4}$$

The inequality follows because we assumed that it was optimal to follow (a, a). If the agent is indifferent between (a, a) and (a, ab), this expression would hold with equality. Rearranging this expression shows that the expression is zero only if $z = z_1(\gamma)$ as given in Proposition 1.⁵

Note that the final component of (4) depends on $\lambda(a, a)$. If $\lambda(a, a)$ is high then many people turn down b's in the trade stage and it is more costly to follow the (a, ab) policy. If an agent believes that many people decline b's in trade then he will decline b's as well. This feedback effect introduces the possibility of multiple equilibria – a possibility we consider in the next section.

Given $\lambda(a, a)$, $z_1(\gamma)$ is closer to zero the closer r, δ and γ are to 0. Thus, given z, an agent is more likely to be exclusive if the object is very durable (low δ), if the agent is very patient (low r), or if it is unlikely that the agent will trade (low γ). Naturally, if the object is very durable and you are not likely to trade it, and you care a lot about the future, then you don't want to get stuck with the wrong house.

The $z_2(\gamma)$ cutoffs are of special interest because they determine the equilibrium number of houses of each type built. If $z_2(\gamma) > 0$ for instance, there may be people who prefer b houses but build a's when they get the build shock (i.e., for whom $0 < z < z_2(\gamma)$). If an agent builds a house other than the type dictated by his taste parameter z, then we say the agent is *conforming* to the market. This discussion motivates our definition of conformity.

Definition 1. If $z_2(\gamma) \neq 0$ for some γ then we say that there is conformity in the market. If $z_2(\gamma) > 0$ for all γ then the market conforms on type a houses, and if

⁵Alternatively, the agent could consider a deviation in which he accepts b and continues to follow the policy (a, ab) until he again aquires a type a house. It is easy to show that this deviation results in the same cutoff.

 $z_2(\gamma) < 0$ for all γ then the market conforms on type b houses. If $z_2(\gamma) = 0$ for all γ then we say that there is no conformity.⁶

Like the trading cutoffs $z_1(\gamma)$ and $z_3(\gamma)$ the building cutoffs $z_2(\gamma)$ are determined by the number of extreme traders. In particular, $z_2(\gamma)$ are determined by the difference $\lambda(a,a) - \lambda(b,b)$. If $\lambda(a,a) > \lambda(b,b)$ then $z_2(\gamma) > 0$ for all γ . Only if $\lambda(a,a)$ and $\lambda(b,b)$ are exactly equal will $z_2(\gamma) = 0$.

To summarize, the optimal policy — what types should an agent accept in trade and what type should the agent build — depends on the behavior of agents with extreme preferences for each types of house rather than the behavior of the typical agent. Specifically, whether or not an agent conforms depends on the relative number of exclusive traders of a and b houses. If many people accept only a houses then there is a strong incentive to reject b houses in trade and to build a houses even if you enjoy living in the b house more than the a house. The dependence on other traders is greater if the durable is very long-lived, if trade is likely, and if the agents are very patient.

We now turn to the solution of the model in which we define and characterize the equilibrium.

2.2. Equilibrium

To find an equilibrium we solve a fixed point problem. Given the perceived matching probabilities $\lambda(a,a)$, $\lambda(b,b)$, $\lambda(a,ab)$, and $\lambda(b,ab)$, agents follow optimal policies. The matching probabilities are, in turn, implied by the policies. This leads to a mapping from perceived probabilities to implied probabilities. An equilibrium is a fixed point of this mapping.

The first step of the mapping, from matching probabilities to optimal policies is described in Proposition 1. The second step of the mapping takes the optimal cutoffs $z_1(\gamma)$, $z_2(\gamma)$, and $z_3(\gamma)$ and derives the implied matching probabilities. Given $z_1(\gamma)$, $z_2(\gamma)$, and $z_3(\gamma)$, computing $\lambda(a, a)$ and $\lambda(b, b)$ is relatively straightforward. Specifically, $\lambda(a, a)$ and $\lambda(b, b)$ are simply the numbers of people who follow policies (a, a) and (b, b).

Unfortunately, the numbers of people who follow policy (a, ab) and policy (b, ab) are not necessarily equal to $\lambda(a, ab)$ and $\lambda(b, ab)$. While some of the people in $\lambda(a, ab)$ follow policy (a, ab), others follow policy (b, ab) and have traded for a

⁶Note that conformity requires $z_2(\gamma)$ to be either strictly positive or strictly negative for all γ . Alternatively, we could require that only the average $z_2(\gamma)$ be positive or negative. Because all $z_2(\gamma)$ have the same sign, these two definitions are equivalent.

type a house in the past. To clarify this distinction we introduce the following notation: Let $P_s(x)$ denote the number of people who follow policy s and hold a type x house where $x \in \{a, b\}$ and $s \in \{(a, a), (b, b), (a, ab), (b, ab)\}$. Using this notation, $\lambda(a, ab) = P_{a,ab}(a) + P_{b,ab}(a)$ and $\lambda(b, ab) = P_{b,ab}(b) + P_{a,ab}(b)$. The complication arises because the number of people who follow policy (a, ab), which is $P_{a,ab}(a) + P_{a,ab}(b)$ is different than $\lambda(a, ab) = P_{a,ab}(a) + P_{b,ab}(a)$, the number of people who follow an inclusive trading rule but possess an a.

The reason that these numbers differ is that often one type of house is rejected more often than the other type. This effectively causes a difference between the distribution of houses for sale and the distribution of houses built. In fact, there are three potentially distinct distributions of houses in our model: the distribution of houses built, the distribution of houses in existence and the distribution of houses for sale on the market. Because the houses for sale are a random selection from the existing housing stock and because houses do not remain on the market after their initial match, the distribution of houses on the market is the same as the distribution of houses in existence. When a trade is rejected however, agents effectively transform their current house into the type of house they want (at the cost π). If one house is rejected more frequently than the other, then the distribution of houses in existence will differ systematically from the distribution of houses built.⁸

Given the cutoff functions $z_1(\gamma)$, $z_2(\gamma)$, and $z_3(\gamma)$, one can compute $P_s(x)$ for all s and x and thus one can solve for the implied $\lambda(a, ab)$ and $\lambda(b, ab)$. This computation is cumbersome however and, as it turns out, only $\lambda(a, a)$ and $\lambda(b, b)$ are necessary for the characterization and analysis of the equilibrium. The following lemma establishes that, given the cutoff functions, there is a unique set of implied matching probabilities.

Lemma 3. Let $z_1(\gamma)$, $z_2(\gamma)$ and $z_3(\gamma)$ be given. The implied steady state matching probabilities $\lambda(a, a)$ and $\lambda(b, b)$ are given by

$$\lambda\left(a,a\right) = \int_{0}^{\infty} F\left(z_{1}\left(\gamma\right)\right) dG\left(\gamma\right) \text{ and } \lambda\left(b,b\right) = \int_{0}^{\infty} \left[1 - F\left(z_{3}\right)\right] dG\left(\gamma\right).$$

⁷Agents who follow exclusive trading rules always hold the same type of house that they build. As a result $P_{a,a}(b) = P_{b,b}(a) = 0$ so $P_{a,a}(a) = \lambda(a,a)$ and $P_{b,b}(b) = \lambda(b,b)$.

⁸In the real world, houses that are difficult to sell remain vacant for some time, which causes the distribution of houses built to differ from the distribution of houses for sale.

⁹The proof of this lemma provides explicit formulas for the implied matching probabilities.

Furthermore, there exist unique nonnegative numbers $P_{a,ab}(a)$, $P_{a,ab}(b)$, $P_{b,ab}(a)$ and $P_{b,ab}(b)$ such that the implied steady state matching probabilities $\lambda(a,ab)$ and $\lambda(b,ab)$ are

$$\lambda\left(a,ab\right) = P_{a,ab}\left(a\right) + P_{b,ab}\left(a\right) \text{ and } \lambda\left(b,ab\right) = P_{a,ab}\left(b\right) + P_{b,ab}\left(b\right).$$

With Lemma 3 we have completed the description of the fixed point mapping necessary to construct an equilibrium. Below we present a formal definition of a steady state equilibrium.

Definition 2. A steady state equilibrium consists of four non-negative numbers $\lambda(a, a)$, $\lambda(b, b)$, $\lambda(a, ab)$, and $\lambda(b, ab)$ summing to one, and three cutoff functions $z_1(\gamma)$, $z_2(\gamma)$ and $z_3(\gamma)$ such that

- 1. Given $\lambda(a, a)$, $\lambda(b, b)$, $\lambda(a, ab)$, and $\lambda(b, ab)$, Proposition 1 implies the cutoff functions $z_1(\gamma)$, $z_2(\gamma)$ and $z_3(\gamma)$.
- 2. Given $z_1(\gamma)$, $z_2(\gamma)$ and $z_3(\gamma)$, Lemma 3 implies $\lambda(a, a)$, $\lambda(b, b)$, $\lambda(a, ab)$, and $\lambda(b, ab)$.

To prove that an equilibrium exists, recall that the cutoff functions for the exclusive a and b traders $(z_1(\gamma))$ and $z_3(\gamma)$ are each governed only by the number of exclusive a and b traders respectively. More precisely, $z_1(\gamma)$ is completely determined once $\lambda(a, a)$ is given and $\lambda(a, a)$ depends only on the cutoff function $z_1(\gamma)$. Similarly $z_3(\gamma)$ depends only on $\lambda(b, b)$ and vice versa. As a result, we can analyze the determination of these cutoffs separately. Define two mappings $L_{a,a}:[0,1] \to [0,1]$ and $L_{b,b}:[0,1] \to [0,1]$ as

$$L_{a,a}\left(\lambda\left(a,a\right)\right) = \int_{0}^{\infty} F\left(-\pi\left\{r + \delta + \gamma\left[1 - \lambda\left(a,a\right)\right]\right\}\right) dG\left(\gamma\right) \in [0,1]$$
 (5)

$$L_{b,b}(\lambda(b,b)) = \int_{0}^{\infty} [1 - F(\pi\{r + \delta + \gamma[1 - \lambda(b,b)]\})] dG(\gamma) \in [0,1].$$
 (6)

While these mappings may not be continuous (for instance if F had mass points at certain z's), they are both increasing functions on a compact set which implies (by Tarski's fixed point theorem) that each has at least one fixed point.

Lemma 4. The mappings $L_{a,a}$ and $L_{b,b}$ defined by (5) and (6) each have at least one fixed point.

Any combination of fixed points of these mappings correspond to equilibrium values of $\lambda(a,a)$ and $\lambda(b,b)$. To complete the construction of an equilibrium, given any fixed points $\lambda(a,a)$ and $\lambda(b,b)$, define the functions $z_1(\gamma), z_2(\gamma)$ and $z_3(\gamma)$ as in Proposition 1. Then $\lambda(a,ab)$ and $\lambda(b,ab)$ are given uniquely by Lemma 3. The resulting matching probabilities $\lambda(a,a), \lambda(b,b), \lambda(a,ab)$, and $\lambda(b,ab)$, and cutoff functions $z_1(\gamma), z_2(\gamma)$ and $z_3(\gamma)$ satisfy the definition of a steady state equilibrium. This establishes the following proposition:

Proposition 2. Given any distribution of types F and distribution of trade hazards G, there exists at least one steady state equilibrium.

We now present two examples. Example 1 illustrates a case in which there is a unique equilibrium and the market conforms to the average taste. Example 2 shows that conformity may arise due to the multiplicity of equilibria even when the distribution is symmetric around zero, so that the average consumer is indifferent between the two types. In both examples we consider the case in which all agents have a common trade hazard γ .

Example 1: Suppose that F is uniform on the interval $[-q + \mu, q + \mu]$.¹⁰ Figure 1 shows the fixed point mappings $L_{a,a}$ and $L_{b,b}$ for two different values of the mean taste μ . The light dashed line corresponds to $\mu = 0$. In this case, the distribution is symmetric around zero and thus $L_{a,a} = L_{b,b}$. In equilibrium $\lambda(a,a) = \lambda(b,b)$, $z_2 = 0$ and there is no conformity. The dark lines correspond to $\mu > 0$. Because F has shifted to the right, $L_{b,b}$ has shifted up while $L_{a,a}$ has shifted down. In the new equilibrium $\lambda(a,a) < \lambda(b,b)$ and $z_2 < 0$ so the market conforms on b.

Example 2: Suppose that F is symmetric about 0 but not uniform. Symmetry implies that the mappings $L_{a,a}$ and $L_{b,b}$ are identical. Figure 2 shows an example with three fixed points: $\lambda_1^* < \lambda_2^* < \lambda_3^*$. By setting $\lambda(a, a) = \lambda_i^*$ and $\lambda(b, b) = \lambda_j^*$ with i, j = 1 or 2 we can construct nine possible equilibria. Three of these are non-conforming equilibria (when $\lambda(a, a) = \lambda(b, b)$) while the other six are conforming equilibria (when $\lambda(a, a) \neq \lambda(b, b)$).

Clearly conformity is a generic property of equilibrium in our model. Non-conforming equilibria only occur in knife-edge cases in which $\lambda\left(a,a\right)=\lambda\left(b,b\right)$ while conforming equilibria occur in all other cases. In the next section we show that markets for highly durable goods in which trade is common have a strong tendency to conform.

¹⁰The figure is drawn under the assumption that the distribution is wide in the sense that $-q + \mu < -\pi (r + \delta + \gamma)$.

3. Comparative Statics and Welfare

In this section we analyze the relationship between conformity and the underlying parameters of the model. We pay particular attention to durability and the frequency of trade. We then discuss the welfare properties of the equilibrium.

3.1. Comparative Statics

In this section we use comparative statics to highlight several features of the equilibrium. Specifically, we consider how variations in the durability of the good and the subjective time discount factor affect the equilibrium. The frequency of trade also matters both for individual behavior and for the equilibrium. We consider variations in the individual trade hazards holding the aggregate distribution of trade hazards fixed. We also consider shifts in the distribution of trade hazards itself. In addition to providing insights into how the model functions, the comparative statics have empirical implications which we discuss in Section 4.

To facilitate the analysis, we place restrictions on the distribution F to rule out multiple equilibria. The following assumption provides sufficient conditions for a unique equilibrium.

Assumption 1. F has a density function f which is symmetric about the mean μ , and quasi-concave with $f(\mu) < \frac{1}{\pi \bar{\gamma}}$ where $\bar{\gamma}$ is the mean trade hazard.

Under assumption 1, we can now present the following proposition.

Proposition 3. If F satisfies Assumption 1 then

- 1. The equilibrium is unique.
- 2. The market conforms to the mean taste whenever $\mu \neq 0$ (i.e., the market conforms on a if $\mu < 0$ and conforms on b if $\mu > 0$).
- 3. If $\mu \neq 0$, an increase in durability (lower δ) or patience (lower r) causes conformity to increase (i.e., $z_2(\gamma)$ increases for all γ if $\mu < 0$ and decreases for all γ if $\mu > 0$).
- 4. Agents with a greater likelihood of trade conform more.

Part 1 of the proposition (uniqueness) follows from the bound on f in Assumption 1 which ensures that the fixed point mappings $L_{a,a}$ and $L_{b,b}$ never have

slopes greater than 1. Part 2 of the proposition demonstrates that conformity is a generic property within this class of distributions.

The third result in the proposition demonstrates that there is more conformity for goods that are more durable and when consumers care more about the future. Trading for a good that you don't prefer is costly if the good is expected to survive for a long time and if you care about the future. Consequently, if the good is more durable or if people are more patient, more agents follow exclusive trading strategies. Our distributional assumptions guarantee that the increase in exclusive trading is greatest for the good that the market conforms to. Because conformity depends on the relative number of exclusive traders, conformity increases as the good becomes more durable or as people become more patient.

The final part of the proposition says that agents who trade frequently have a greater incentive to conform than agents who trade infrequently. If an agent finds himself in the trade stage often, he will reduce the incidence of the trade penalty by conforming. Agents who trade very rarely can indulge in goods that satisfy their idiosyncratic tastes with impunity.

The last result suggests that regions with high turnover (i.e., where everyone has a high trade hazard) may also be regions of greater conformity. The following proposition shows that this is not necessarily true. While increases in the overall likelihood of trade increases conformity for some agents, it may or may not increase conformity for all agents.

To formalize this, we consider a rightward shift of the distribution G. Specifically, write each agent's trade hazard as $(\gamma + \theta)$ where $\theta \geq 0$ is common to all agents and γ is the agent's idiosyncratic trade hazard. We can then associate a marginal increase in the overall trade frequency with a marginal increase in θ at $\theta = 0$. Differentiating the $z_2(\gamma)$ cutoff with respect to θ and evaluating at $\theta = 0$ gives the following expression:

$$\left. \frac{\partial z_{2}\left(\gamma\right)}{\partial \theta} \right|_{\theta=0} = \underbrace{\frac{z_{2}\left(\gamma\right)}{\gamma}}_{\text{"Exposure Effect"}} + \underbrace{\gamma\pi \left[\left. \frac{\partial \lambda\left(a,a\right)}{\partial \theta} \right|_{\theta=0} - \left. \frac{\partial \lambda\left(b,b\right)}{\partial \theta} \right|_{\theta=0} \right]}_{\text{"Composition Effect"}}.$$

The change in $z_2(\gamma)$ depends on two terms. The first term always has the same sign as $z_2(\gamma)$ and thus always serves to increase conformity. Using our expression for $z_2(\gamma)$ we can rewrite this term as $\pi[\lambda(a,a) - \lambda(b,b)]$ which is independent of γ . This term captures the change in $z_2(\gamma)$ from an increase in the trade hazard holding the number of exclusive traders $(\lambda(a,a))$ and $\lambda(b,b)$ fixed. Because the agent needs to trade more frequently, he encounters these exclusive traders more

often and thus chooses to conform more. We refer to this term as the "exposure effect."

The second term, the "composition effect," captures the effects on $z_2(\gamma)$ holding a given agent's trade hazard fixed but considering variations in the number of exclusive traders caused by the shift in the distribution G. It turns out that the composition effect can be either positive or negative depending on the distributions F and G. Notice however, that the importance of composition effect depends positively on γ . Thus, for agents with sufficiently small trade hazards, the exposure effect dominates and conformity increases. For agents with high trade hazards, the composition effect dominates and thus they may or may not conform more. If the composition effect is the same sign as $z_2(\gamma)$ then conformity increases with the general level of trade. While the intuition for this result is natural, it seems to stand in stark contrast to free-market folklore – rather than encouraging market diversity, increased trade in durable goods encourages market conformity.

We summarize this result in the following proposition.

Proposition 4. Assume that agents trade hazards are written as $(\gamma + \theta)$ as described above (so $\theta = 0$ corresponds to the original equilibrium) and consider an F which satisfies Assumption 1. Then, for a marginal increase in θ at $\theta = 0$, there exists $\hat{\gamma} \in (0, \infty]$ such that conformity increases for all agents with $\gamma < \hat{\gamma}$ and decreases for all agents with $\gamma > \hat{\gamma}$.¹¹

It is easy to show that similar arguments hold for changes in the trade penalty π . There are two effects, an exposure effect and a composition effect. As above, the exposure effect always causes conformity to increase while the composition effect may cause conformity to increase or decrease.¹²

3.2. Welfare

The previous section shows that in markets for durable goods, conformity is the rule rather than the exception. In this section, we consider the welfare implications of conformity. While conformity has an obvious cost – people live in houses that they don't prefer – it also has benefits. First, conformity lowers search costs. Naturally, social welfare is lower when houses are vacant or when people are

¹¹The Proposition allows for $\hat{\gamma} = \infty$ in which case conformity increases for all agents.

¹²Our model assumes that π is common to all agents. If we allowed for heterogeneity in trade penalties we would obtain a result similar to Proposition 4 for trade penalties.

forced to conduct protracted searches for a good match. Second, when houses are exchanged, the new owner may not be an ideal match for the house. The potential mismatch of preferences and allocations is another cost to society that conformity mitigates. Thus, a social planner may desire some conformity in these markets.

Below, we explore the welfare implications of conformity in our model. To facilitate the exposition, we simplify the model first by restricting attention to situations in which all agents have the same trade hazard and all are perfectly patient (i.e., r=0). This second assumption allows us to restrict our attention to the steady state flow of total welfare.¹³ We then present simple examples that clearly illustrate the costs and benefits of conformity. Surprisingly, the examples suggest that often there is too little conformity in equilibrium. The first two examples isolate the two benefits that come from conformity. In each case, the benefits to conformity outweigh the costs. We then present an example that demonstrates that there can be too much conformity. This happens when the two benefits work in opposite directions.

We assume that a social planner values the flow utility of all agents equally and seeks to maximize the steady state flow of utility in each period. The following Lemma provides an expression for the flow of social welfare in the steady state when all agents have a common trade hazard. Because there is a single trade hazard, we can consider a single set of cutoffs z_1 , z_2 , and z_3 .

Lemma 5. For any given cutoffs z_1 , z_2 and z_3 , let $P_{a,ab}(a)$, $P_{b,ab}(a)$, $P_{a,ab}(b)$ and $P_{b,ab}(b)$ be given as in Lemma 3 and define

$$\psi_{a,ab} = \left(\frac{P_{a,ab}\left(a\right)}{F\left(z_{2}\right) - F\left(z_{1}\right)}\right) \text{ and } \psi_{b,ab} = \left(\frac{P_{b,ab}\left(b\right)}{F\left(z_{3}\right) - F\left(z_{2}\right)}\right).$$

Then, the flow of social welfare is

$$W(z_{1}, z_{2}, z_{3}) = \left\{1 - \pi\gamma \left[\lambda(b, b) + \lambda(b, ab)\right]\right\} F(z_{1})$$

$$+ \int_{z_{1}}^{z_{2}} \left\{\psi_{a, ab} \left[1 - \pi\gamma\lambda(b, b)\right] + \left(1 - \psi_{a, ab}\right) \left[1 + z - \pi\gamma\lambda(a, a)\right]\right\} dF(z)$$

$$+ \int_{z_{2}}^{z_{3}} \left\{\left(1 - \psi_{b, ab}\right) \left[1 - \pi\gamma\lambda(b, b)\right] + \psi_{b, ab} \left[1 + z - \pi\gamma\lambda(a, a)\right]\right\} dF(z)$$

$$+ \int_{z_{3}}^{\infty} \left\{1 + z - \pi\gamma \left[\lambda(a, a) + \lambda(a, ab)\right]\right\} dF(z) - \delta c.$$

¹³All of the conclusions in this section hold for r > 0 provided that r is sufficiently small.

Each term in the welfare function has a natural economic interpretation. The first term is the flow of welfare to the (a,a) traders. Since they are exclusive traders, they always hold type a houses and thus they all receive a flow utility of 1. In addition, they sometimes incur the trade penalty by matching with (b,b) and (b,ab) traders. The second term is the flow of welfare for agents who follow the (a,ab) policy. These individuals sometimes hold a houses but sometimes hold b's. The fraction of time these agents hold the a house is $\psi_{a,ab}$. In this case, their flow utility is 1 less the trade hazard of meeting an exclusive b trader. The remaining fraction of time $(1-\psi_{a,ab})$ they hold b houses. In this case, their flow utility is 1+z less the trade hazard of meeting an exclusive a trader. The remaining terms have analogous interpretations. Notice that all agents are equally likely to experience the depreciation shock so all welfare flows are reduced by δc .

We now consider the welfare implications of increasing or decreasing conformity. Conformity can increase welfare through two separate channels. First, increased conformity can reduce the incidence of the trade penalty. Second, increased conformity means that the average type resides in their preferred house more often. The following two examples illustrate these effects.

One Inclusive and One Exclusive Type One of the costs associated with insufficient conformity is that trades are declined too often. This happens because agents internalize only their own trade penalties and not those of their trading partners. For agents at the z_2 cutoff, the benefits to living in their preferred houses if they choose not to conform, is exactly offset by the benefits of reducing the incidence of the trade penalty if they choose to conform. Because the house they build will be rejected by one group of exclusive traders, these agents impose a negative trade externality on one group of exclusive traders and a positive trade externality on the other. Since there are more exclusive traders on the conforming side of the market, social welfare would rise if the marginal agents conformed.

The following example isolates this effect. Suppose there are two types of agents. Call these types "medium", and "high". The high type has a strong preference for b while the medium type has a weak preference for a. We write their z's as z^M and z^H respectively. We choose z^H to ensure that the high types play an exclusive trading strategy in equilibrium (i.e., they play (b,b)).¹⁴ Depending on z^M , the medium agents may or may not conform. The number of medium agents is α and the number of high agents is $(1-\alpha)$. In equilibrium, $\lambda(b,b) = (1-\alpha)$

¹⁴If $z^H > \pi (r + \delta + \gamma)$, then the high types will play (b, b) in any equilibrium.

and there is conformity on b. Specifically, the building cutoff is

$$z_2 = -\gamma \pi \left(1 - \alpha\right) < 0$$

In the market equilibrium, the medium types decide to follow strategy (a, ab) if $z^M < z_2$ and follow (b, ab) otherwise. To determine whether increased conformity is socially desirable, we compute social welfare when the medium types play strategy (a, ab) and compare it to welfare when they play (b, ab).

If the medium agents follow strategy (a, ab), then $F(z_2) - F(z_1) = \alpha$ and the medium types never trade with the high types. In this case, $P_{a,ab}(a) = \alpha$ and $P_{a,ab}(b) = P_{b,ab}(a) = P_{b,ab}(b) = 0$. Using Lemma 5, welfare is

$$W_{a,ab} = 1 + z^{H} (1 - \alpha) - 2\pi \gamma \alpha (1 - \alpha) - \delta c$$

where the subscript a, ab indicates that the medium types play (a, ab). If the medium agents follow (b, ab) then similar calculations give welfare as

$$W_{b,ab} = 1 + \alpha z^M + (1 - \alpha) z^H - \delta c.$$

Strategy (a, ab) is socially preferable to (b, ab) if $W_{a,ab} \geq W_{b,ab}$. It is straightforward to show that this requires

$$-2\pi\gamma\left(1-\alpha\right) \ge z^{M}$$

Recalling that in equilibrium $z_2 = -\gamma \pi (1 - \alpha)$ the social optimum requires

$$2z_2 > z^M$$

Put differently, the socially optimal policy calls for twice as much conformity as in the market equilibrium.

The reason for conforming in this case is to avoid the trade penalty. If the medium agents are close to the equilibrium z_2 then they are balancing the benefits of having the type of house they prefer with the costs of incurring the trade penalty more often. Because they do not care about the trade penalty for the other agents, they are too reluctant to conform to the majority type. The social planner internalizes both trade penalties every time a trade is declined and thus sets a cutoff that is twice the equilibrium cutoff.

Two inclusive types This example highlights another source of inefficiency caused by too little conformity. If agents are willing to trade then conforming implies that your trading partners will live in their preferred houses more often. Because agents only care about the houses they live in, conformity can again be inefficiently low.

Suppose there are two types "low" and "high". We assume that $-\pi (r + \delta) < z^L < 0$ and $0 < z^H < \pi (r + \delta)$. This assumption ensures that each type plays an inclusive strategy. Since these are the only types, no agents play exclusive strategies $(\lambda (a,a) = \lambda (b,b) = 0)$ and as a result $z_2 = 0$ and thus in the market equilibrium there is no conformity. As before, we assume there are α low types and $(1-\alpha)$ high types. We contrast the social welfare when both types play inclusive strategies with the cases in which the types conform either on a or on b.

If the two types conform on a then the flow of social welfare is simply

$$W_{(a,a),(a,a)} = 1 - \delta c,$$

where the subscript indicates that both types play (a, a). Likewise, if they conform on b, the flow of welfare is

$$W_{(b,b),(b,b)} = 1 + z^{L}\alpha + z^{H}(1 - \alpha) - \delta c.$$

The difference $W_{(b,b),(b,b)} - W_{(a,a),(a,a)}$ is positive if and only if the mean taste $\mu = z^L \alpha + z^H (1 - \alpha)$ is positive.

Computing the flow of welfare for the inclusive cases is more involved because the agents sometimes have a house other than the type they build. After some algebra, we find the flow of social welfare for the inclusive strategy to be

$$W_{(a,ab)(b,ab)} = 1 + \alpha \left(\frac{\gamma}{\gamma + \delta} (1 - \alpha) \right) z^{L} + (1 - \alpha) \left(1 - \frac{\gamma}{\gamma + \delta} \alpha \right) z^{H} - \delta c.$$

Comparing the inclusive non-conforming cases with the conforming equilibrium shows that $W_{(a,a),(a,a)}$ is preferred to $W_{(a,ab)(b,ab)}$ if

$$-\alpha \left(\frac{\gamma}{\gamma + \delta} (1 - \alpha)\right) z^{L} > (1 - \alpha) \left(1 - \frac{\gamma}{\gamma + \delta} \alpha\right) z^{H}.$$

Likewise, conforming on b is preferred to not conforming if

$$\left(\frac{\gamma}{\gamma+\delta}\left(1-\alpha\right)\right)z^{H} > -\left(1-\frac{\gamma}{\gamma+\delta}\left(1-\alpha\right)\right)z^{L}.$$

Because each type plays an inclusive strategy, $z_2 = 0$ and there is no conformity in equilibrium. Yet conformity may be socially preferable. Specifically, when either trades are very frequent, or when depreciation is very low, one of the two inequalities above must hold and thus conformity would be socially desirable. The intuition for this result is clear. If γ is very high then an individual agent will not reside in a given house for very long and thus should make a building decision that fits with the preferences of the average agent. If δ is very low then any given house will survive for a very long time and will therefore be occupied in equilibrium by the average agent.

The Possibility of Too Much Conformity The previous two examples suggest that there is in general too little conformity in equilibrium. Individuals do not sufficiently internalize trade costs when they decline trades nor do they sufficiently internalize the costs that arise because someone "settles" for a house that isn't a good match. Both forces seem to lead to too little conformity. It is however, possible that a situation could arise in which these forces work in opposite directions and cause too much conformity.

To demonstrate this possibility, consider a hybrid example with three types: low, medium and high. Denote these types as z^L , z^M , and z^H . Assume that z^H is high enough to ensure that they follow (b,b) while z^L is chosen to ensure that they follow (a,ab). We consider the optimal choice for the middle type. Assume that there are μ middle agents, $(1-\mu)\alpha$ low agents and $(1-\mu)(1-\alpha)$ high agents.

If the middle types follow strategy (a, ab) then none of them ever get a b house (the only b builders follow exclusive strategies). If the middle types follow strategy (b, ab) then, as in the second example, the low and middle types will occasionally hold type a and type b houses. One can show that the difference between the two welfare flows $W_{b,ab}$ and $W_{a,ab}$ is

$$W_{b,ab} - W_{a,ab} = \mu \psi_{b,ab} \left[z^{M} + \pi \gamma (1 - \mu) (1 - \alpha) \right] + \pi \gamma (1 - \mu) (1 - \alpha) \left[2 (1 - \psi_{a,ab}) (1 - \mu) \alpha + \mu \psi_{b,ab} \right] + (1 - \mu) \alpha (1 - \psi_{a,ab}) z^{L}.$$
(7)

The first line summarizes the costs and benefits that accrue to the middle agents if they adopt the b, ab policy instead of the a, ab policy. In this case, they reside in the b house for a fraction of time $\psi_{b,ab}$. Because there are $(1 - \mu)(1 - \alpha)$ exclusive

¹⁵If either γ approaches infinity, or δ approaches zero, the inequalities say that conforming on a is preferable if $\mu < 0$ and conforming on b is preferable if $\mu > 0$.

b traders, when the middle agents hold the b houses, they reduce the incidence of the trade penalty by $\pi \gamma (1 - \mu) (1 - \alpha)$. At the same time, holding the b house adds z^M to their flow utility. If these were the only costs and benefits of adopting one policy or another then the critical $z^M = -\pi \gamma (1 - \mu) (1 - \alpha)$ which is simply the build cutoff z_2 in this example.

The second and third lines of equation (7) represent external costs and benefits not internalized by the middle types. The second line reflects the reduction in trade penalties experienced by both the high and low types. The high types now have a probability $\mu\psi_{b,ab}$ of encountering a middle type who holds a b house in the trade stage in which case there will be no trade penalty. The high types also encounter low types who sometimes possess b houses in the trade stage. The number of low types in this state is $(1 - \mu) \alpha (1 - \psi_{a,ab})$. The scalar 2 reflects the fact that two trade penalties are avoided in this case (one for the low type and one for the high type).

The equilibrium z_2 cutoff is $-\pi\gamma(1-\mu)(1-\alpha) < 0$. If $z^M \in [z_2,0]$ they will conform on b. If the last two lines of (7) are negative however, the socially optimal z_2 cutoff is higher than the equilibrium z_2 cutoff. If z^M is greater than the equilibrium z_2 but less than the socially optimal z_2 there is too much conformity in equilibrium.¹⁶

A Numerical Illustration The examples above starkly illustrate the forces that cause conformity to be inefficiently low in equilibrium. Those examples consider simple cases with at most three types of agents. Here we consider the equilibrium and optimal level of conformity for a continuum of agents when the taste parameter is distributed normally. We maintain the assumption of a common trade hazard. Unlike the examples above, the equilibrium here is much too complicated to solve analytically. Instead we numerically solve for the equilibrium and calculate the optimal level of conformity. This example again demonstrates that there is often too little conformity in equilibrium.

Figure 3 calculates the optimal choice for all of the cutoffs simultaneously for several different means of the distribution F(z). As mentioned above, we assume that the distribution of tastes F is normal with unit variance and with mean μ . The other parameters are r = 0.02, $\delta = 0.05$, $\gamma = 0.10$, and $\pi = 5$ (the building

¹⁶To complete this argument, we need to show that there are parameter values for which the sum of the last two lines in equation (7) is negative while still maintaining $z^L > z_1$ so that the low types follow an inclusive trading strategy. It is straightforward to show that for given γ , μ , π , δ , r there exists an α sufficiently close to 1 for which these conditions are satisfied.

cost c matters neither for equilibrium nor for welfare comparisons). The discount rate and depreciation rate are roughly in line with their real-world counterparts. The trade hazard rate implies that people move roughly once every ten years. The figure plots the optimal cutoffs and the equilibrium cutoffs for different means of the distribution F. Figure 3 shows that for $\mu > 0$ there is conformity in equilibrium $(z_2 < 0)$. Not surprisingly, the equilibrium level of conformity rises with the mean. Notice that the optimal level of conformity rises even faster than the equilibrium level of conformity. Also, as the mean rises, the cutoffs for the exclusive a and b traders both fall. More individuals prefer the b houses and thus there is more and more pressure to decline the a houses in trade.

4. Discussion and Related Literature

There are several noteworthy features of the model. Perhaps the most glaring simplification in the model is the assumed trading mechanism. Traders are restricted to simple swaps which rules out bargaining with prices. Moreover, traders only get a single match, and if this match does not result in trade they pay a penalty and leave the trade stage with the house that they desire. This trading mechanism greatly simplifies the model by making the agents' trading rules independent of the house they possess when they enter the trade stage. While these assumptions are convenient they are also highly stylized. Nevertheless, we argue that the insights that emerge will still hold under more realistic assumptions.

First, consider extending the model to allow for price negotiations. There are many trade mechanisms that one could consider (take-it-or-leave-it offers, double auctions etc.). As long as the agents' preferences (their taste parameters) are private information, none of these mechanisms is ex-post efficient (Myerson and Satterthwaite (1983)). Put differently, some trades with positive surplus will not occur. Thus, while these mechanisms will improve efficiency, they do not fully eliminate the two inefficiencies in the model: Trades will still be rejected too often, and too many people will occupy houses that they do not prefer. Even if we allowed for efficient trading mechanisms (which would require observable taste parameters), there would still be too little conformity. For instance, under the Nash bargaining rule, agents receive only half of the surplus in the trade stage. Because they internalize only a portion of the surplus in the trade stage, they make inefficient decisions in the build stage and thus there is still too little conformity. We should emphasize that while price negotiations may improve efficiency, this typically implies more conformity rather than less.

Allowing agents to continue searching after the initial trade is rejected would also leave our basic results intact. Agents would still have an incentive to conform and the level of conformity and the willingness to trade would still be inefficient. In fact, allowing for repeated matches when agents need to sell suggests that there may be additional reasons to conform. By conforming, agents reduce their expected sales costs by reducing their time on the market. We could capture this effect by assuming that the trade penalties were type-specific. If the trade penalties vary inversely with the average willingness to accept that type of house in trade then the incentive to conform is increased.

It is difficult to devise policies that would fully resolve all of the potential inefficiencies in the model. Providing the correct trade incentives, for instance, would likely require taxes or subsidies that are specific to the agents intrinsic preferences. Unlike the incentive to trade, the efficient level of conformity could be achieved with simple government policies. Because our model suggests that diversity in durable goods markets should be discouraged, a tax on unusual houses would improve efficiency.

The model also generates empirical implications that could conceivably be tested against real world data. The model predicts that individuals that are likely to move should conform and purchase "typical" houses. Tenured professors for example should live in houses with more "character" compared to untenured professors who should own typical houses (casual observation suggests that this is indeed the case). Another implication is that, all else equal, there should be more conformity for goods with lower depreciation rates.

Our paper is closely related with three separate lines of research. The first is the literature on conformity itself. Perhaps the best-known paper in this literature is Bernheim (1994). In that paper, agents care about both the action they take and their perceived type which in turn depends on their action. Since the agents all want to be perceived to be a single common type, their actions reflect both their own ideal action and their desire to be perceived to be of a certain status. When agents care strongly about status, those with intermediate preferences all choose the same action (they conform). Agents with extreme preferences choose not to conform yet their choices are still distorted towards the common desired type. The main difference between our environment and Bernheim's is that the desire to conform in our model arises endogenously through resale concerns while in Bernheim's model, agents conform because their preferences place weight on public perceptions of their type.¹⁷

¹⁷There are other important concepts of conformity that have been addressed in the literature.

The second line of research our paper relates to is the literature on durable goods in matching models. Two papers are particularly noteworthy. Wheaton (1990) considers a search model of housing with two types of occupants (families and singles) and two types of houses (large and small). The focus of his paper is on the optimal level of search intensity. He shows that unless the searcher captures all of the surplus of a trade, there will be too little search effort in the model. Our results on the inefficiency of conformity echo Wheaton's finding. Smith (1997) considers a matching model with many types of agents with idiosyncratic tastes and many types of perfectly durable goods. Smith uses his model to demonstrate the "risk-increasing" nature of trades. In both Smith (1997) and Wheaton (1990), the supply of durable goods is exogenous. Conformity, as we have defined it, deals with the determination of the stock of durables given an underlying distribution of idiosyncratic preferences. As a result, neither Smith nor Wheaton address conformity.¹⁸

Finally, our paper is related to the literature on liquidity and matching models. (See among others Kiyotaki and Wright (1989) and (1993)). In matching models of money, fiat money has value because it provides liquidity. Similarly, in our model, the equilibrium value of a good reflects both its intrinsic utility to the agent and the liquidity it provides. Unlike our model however, in the money-search literature, there is typically an exogenous double-coincidence of wants problem. For instance, in money-search models, typically, agents cannot produce the good they want to consume. If they could, there would be no double-coincidence problem and money would play no role. In our model agents can produce the good they want to consume so there is no exogenous double coincidence problem. Indeed, if depreciation rates are very high, the goods are essentially non-durable and there is no conformity – agents simply produce the good they prefer. The source

Social norms can be rationalized as equilibria in repeated games. Prominent papers include Akerlof (1980), Kandori (1992), and Okuno-Fujiwara and Postlewaite (1995). In these models agents adhere to social conventions to avoid punishment by other players in the future. Also related is the literature on optimal product diversity. (See among others Spence (1976), Dixit and Stiglitz (1977), Mankiw and Whinston (1987), and Tirole (1993) chapter 7). In these models, consumers have a taste for variety and imperfectly competitive firms make entry decisions. Typically, the equilibrium amount of entry implies a less than optimal level of product diversity. Our framework highlights the opposite concern. Agents in our model care about resale which endogenously generates a preference against diversity.

¹⁸There is also a literature on the provision and resale of durable goods in market settings (see Waldman (2003) for a summary). Following Akerlof (1970), much of this literature focuses on how adverse selection problems affect the provision and resale of durables. See Hendel and Lizzeri (1999), (2002), and House and Leahy (2004)) for recent contributions.

of the double-coincidence problem in our framework is durability itself. Durable goods survive over long periods of time and may be consumed by many different agents. Thus, unless markets are very efficient at matching people, durability endogenously creates a double-coincidence problem.

5. Conclusion

We have shown that a consumer's demand for a durable good is governed not only by his individual preferences but also by the preferences of other market participants. This interdependence of preferences arises in markets for durables because of the inevitable resale of durable goods. If a majority of the people who buy durables desire goods with certain features, then the original owners will choose to buy goods with these features even if they do not like them. The incentive to conform to the majority taste is strongest for long-lived durable goods and for people who trade frequently. For non-durable goods (goods with a very high depreciation rate) or for durables that are never traded, there is no incentive to conform.

There are two features which lead to conformity in our model. First, because there is a chance that agents will have to sell their house, they care about its resale value. The lower the depreciation rate is, and the more likely it is that they will have to enter the resale market, the more they care about the resale value. In the model, the resale value of the home is determined simply by the likelihood that it will be accepted in trade. Second, frictions in the resale market (due to matching) generate the possibility that the house will be purchased by someone with different preferences from the current owner. As a result, the resale value depends on the average preferences of the buyers in the resale market.

In equilibrium, the degree of conformity is typically suboptimal. Surprisingly, there is typically too little conformity. That is, a social planner would prefer that agents think about the majority more than they actually do. By not conforming, agents impose two negative externalities on members of the majority. First, people with the majority taste incur greater search costs because of product diversity. Second, some members of the majority with moderate preferences settle for the type they do not prefer. Inefficiently low conformity arises because the original builders do not fully internalize these costs.

References

- Akerlof, George. 1970. "The Market for 'Lemons': Quality Uncertainty and the Market Mechanism." Quarterly Journal of Economics 84, August, pp.488-500.
- Akerlof, George. 1980. "A Theory of Social Custom, of Which Unemployment May be One Consequence." Quarterly Journal of Economics 94(4), June, pp. 749-775.
- Bernheim, B. Douglas. 1994. "A Theory of Conformity." *Journal of Political Economy* 102(5), October, pp. 841-877.
- Dixit, Avinash and Stiglitz, Joseph. 1977. "Monopolistic Competition and Optimum Product Diversity." *American Economic Review* 67(3), June, pp. 297-308.
- Hendel, Igal and Lizzeri, Alessandro. 1999. "Adverse Selection in Durable Goods Markets." *American Economic Review.* 89, December, pp. 1097-1115.
- Hendel, Igal, and Lizzeri, Alessandro. "The Role of Leasing under Adverse Selection." *Journal of Political Economy* 89 (February 2002): 1097-1115.
- House, Christopher L. and Leahy, John V. 2004. "An sS Model with Adverse Selection." Journal of Political Economy. 112(3), pp. 581-613.
- Kandori, Michihiro. 1992. "Social Norms and Community Enforcement." Review of Economic Studies 59(1), January, pp. 63-80.
- Kiyotaki, Nobuhiro and Wright, Randall. 1989. "On Money as a Medium of Exchange." *Journal of Political Economy*, 97(4), pp. 927-954.
- Kiyotaki, Nobuhiro and Wright, Randall. 1993. "A Search Theoretic Approach to Monetary Economics." American Economic Review, 83(1), pp. 63-77.
- Mankiw, N. Gregory and Whinston, Michael. 1986. "Free Entry and Social Efficiency." Rand Journal of Economics 17(1), Spring, pp. 48-58.
- Okuno-Fujiwara, Masahiro and Postlewaite, Andrew. 1995. "Social Norms and Random Matching Games." Games and Economic Behavior 9, pp. 79-109.

- Smith, Lones. 1997. "A Model of Exchange where Beauty is in the Eye of the Beholder." working paper, University of Michigan.
- Spence, Michael. 1976. "Product Selection, Fixed Costs, and Monopolistic Competition." Review of Economic Studies 43(2), June, pp. 217-235.
- Tirole, Jean. The Theory of Industrial Organization. The MIT Press. Cambridge, Massachusetts. 1993.
- Waldman, Michael. 2003. "Durable Goods Theory for Real World Markets." Journal of Economic Perspectives 17(1), Winter, pp. 131-154.
- Wheaton, William. 1990. "Vacancy, Search, and Prices in a Housing Market Matching Model." *Journal of Political Economy* 98(6), December, pp. 1270-1292.

Appendix: Proofs of the Propositions

Lemma 1 If an agent builds type x, then he accepts type x in trade.

Proof. The proof is trivial and is identical for a and b. If the agent builds type a then $V(a) \geq V(b)$. Then if he is offered a in trade, he gets V(a) if he accepts the offer and $V(a) - \pi$ if he declines the offer.

Lemma 2 Given matching probabilities $\lambda(x,\tau)$ for $x \in \{a,b\}$ and $\tau \in \{a,b,ab\}$ with $\sum_{\tau \in \{a,b,ab\}} \sum_{y \in \{a,b\}} \lambda(y,\tau) = 1$ there exist unique values V(a), V(b), T(a), T(b), and B satisfying (1), (2), and (3).

Proof. The proof follows from standard dynamic programming arguments.

Lemma 3 Let $z_1(\gamma)$, $z_2(\gamma)$ and $z_3(\gamma)$ be given. The implied steady state matching probabilities $\lambda(a, a)$ and $\lambda(b, b)$ are given by

$$\lambda\left(a,a\right)=\int_{0}^{\infty}F\left(z_{1}\left(\gamma\right)\right)dG\left(\gamma\right)\ and\ \lambda\left(b,b\right)=\int_{0}^{\infty}\left[1-F\left(z_{3}\right)\right]dG\left(\gamma\right).$$

Furthermore, there exist unique nonnegative numbers $P_{a,ab}(a)$, $P_{a,ab}(b)$, $P_{b,ab}(a)$ and $P_{b,ab}(b)$ such that the implied steady state matching probabilities $\lambda(a,ab)$ and $\lambda(b,ab)$ are

$$\lambda\left(a,ab\right) = P_{a,ab}\left(a\right) + P_{b,ab}\left(a\right) \text{ and } \lambda\left(b,ab\right) = P_{a,ab}\left(b\right) + P_{b,ab}\left(b\right).$$

Proof. Fix $z_1(\gamma)$, $z_2(\gamma)$, $z_3(\gamma)$. We will characterize $P_s(x)$ for $s \in \{(a, a), (b, b), (a, ab), (b, ab)\}$, and $x \in \{a, b\}$. Consider agents with the same γ who follow a common policy s. If these agents follow an inclusive trading policy, they occasionally switch from holding an a house to a b house and vice versa. In a stationary equilibrium, the number who switch from holding a to b equals the number who switch from holding b to a. Define $p_s(x, \gamma)$ as the number of people who follow policy $s \in \{(a, a), (b, b), (a, ab), (b, ab)\}$, possess a type $x \in \{a, b\}$ house and have matching hazard γ . Since there is no outflow for people who follow (a, a) or (b, b) we confine attention to policies (a, ab) and (b, ab).

Consider $p_{a,ab}\left(a,\gamma\right)$. To leave this group, an agent must be matched with someone who is willing to accept an a in trade but has a b. There are $P_{a,ab}\left(b\right)+P_{b,ab}\left(b\right)$ such agents. The outflow is $\gamma p_{a,ab}\left(a,\gamma\right)\left[P_{a,ab}\left(b\right)+P_{b,ab}\left(b\right)\right]$. The inflow consists of everyone in $p_{a,ab}\left(b,\gamma\right)$ who is either matched with someone who accepts b but has a or who matches with an exclusive a trader or whose house dies (in the latter two cases, the agent loses their b and builds a). The inflow is $p_{a,ab}\left(b,\gamma\right)\left[\gamma\left(P_{a,a}\left(a\right)+P_{a,ab}\left(a\right)+P_{b,ab}\left(a\right)\right)+\delta\right]$. Equating the inflow and the outflow gives

$$\gamma p_{a,ab}(a,\gamma) [P_{a,ab}(b) + P_{b,ab}(b)] = p_{a,ab}(b,\gamma) [\gamma (P_{a,a}(a) + P_{a,ab}(a) + P_{b,ab}(a)) + \delta].$$

Let $x(\gamma) = F(z_2(\gamma)) - F(z_1(\gamma))$. Because $p_{a,ab}(a,\gamma) + p_{a,ab}(b,\gamma) = x(\gamma)$, we can eliminate $p_{a,ab}(b,\gamma)$ and write the above equation as

$$\gamma p_{a,ab}\left(a,\gamma\right)\left[P_{a,ab}\left(b\right)+P_{b,ab}\left(b\right)\right]=\left(x\left(\gamma\right)-p_{a,ab}\left(a,\gamma\right)\right)\left[\gamma\left(P_{a,a}\left(a\right)+P_{a,ab}\left(a\right)+P_{b,ab}\left(a\right)\right)+\delta\right]$$

which gives

$$p_{a,ab}(a,\gamma) = \frac{x(\gamma)(P_{a,a}(a) + P_{a,ab}(a) + P_{b,ab}(a))}{[1 - P_{b,b}(b)] + \frac{\delta}{\gamma}} + \frac{x(\gamma)\frac{\delta}{\gamma}}{[1 - P_{b,b}(b)] + \frac{\delta}{\gamma}}$$

To find $P_{a,ab}(a)$ we integrate over γ to get

$$P_{a,ab}\left(a\right) = \left[\left(P_{a,a}\left(a\right) + P_{a,ab}\left(a\right) + P_{b,ab}\left(a\right)\right)\right] \int_{0}^{\infty} \left(\frac{x\left(\gamma\right)}{\left[1 - P_{b,b}\left(b\right)\right] + \frac{\delta}{\gamma}}\right) dG\left(\gamma\right) + \int_{0}^{\infty} \left[\frac{x\left(\gamma\right)\frac{\delta}{\gamma}}{\left[1 - P_{b,b}\left(b\right)\right] + \frac{\delta}{\gamma}}\right] dG\left(\gamma\right)$$

which we rewrite compactly as

$$P_{a,ab}(a) = [P_{a,a}(a) + P_{a,ab}(a) + P_{b,ab}(a)] A_1 + A_2.$$
(8)

where $A_1 = \int_0^\infty \left(\frac{x(\gamma)}{[1-P_{b,b}(b)]+\frac{\delta}{\gamma}}\right) dG(\gamma)$ and $A_2 = \int_0^\infty \left[\frac{x(\gamma)\frac{\delta}{\gamma}}{[1-P_{b,b}(b)]+\frac{\delta}{\gamma}}\right] dG(\gamma)$ are known functions of the given cutoffs $z_1(\gamma)$, $z_2(\gamma)$, $z_3(\gamma)$ and the distributions F and G.

Similar steps imply that for agents who follow policy (b, ab) we have

$$P_{b,ab}(b) = [P_{b,b}(b) + P_{a,ab}(b) + P_{b,ab}(b)] B_1 + B_2$$
(9)

where $B_{1}=\int_{0}^{\infty}\frac{y(\gamma)}{\left[1-P_{a,a}(a)\right]+\frac{\delta}{\gamma}}dG\left(\gamma\right),\,B_{2}=\int_{0}^{\infty}\frac{y(\gamma)\frac{\delta}{\gamma}}{\left[1-P_{a,a}(a)\right]+\frac{\delta}{\gamma}}dG\left(\gamma\right)\,\,\mathrm{and}\,\,y\left(\gamma\right)=F\left(z_{3}\left(\gamma\right)\right)-F\left(z_{2}\left(\gamma\right)\right).$

Define X and Y as

$$X = \int_0^\infty x(\gamma) dG(\gamma) = P_{a,ab}(a) + P_{a,ab}(b)$$
(10)

$$Y = \int_0^\infty y(\gamma) dG(\gamma) = P_{b,ab}(a) + P_{b,ab}(b)$$
(11)

and note that like $x(\gamma)$ and $y(\gamma)$, X and Y are known functions of the $z_1(\gamma)$, $z_2(\gamma)$, $z_3(\gamma)$ and F and G. We can now solve for $P_{a,ab}(a)$ and $P_{b,ab}(b)$ by substituting (10) and (11) into (8) and (9).

$$P_{a,ab}(a)[1 - A_1] = [P_{a,a}(a) + Y - P_{b,ab}(b)]A_1 + A_2$$

$$P_{b,ab}(b)[1 - B_1] = [P_{b,b}(b) + X - P_{a,ab}(a)]B_1 + B_2$$

It can be shown that the solution implies

$$P_{b,ab}(b) = \left(\frac{1 - A_1}{1 - A_1 - B_1}\right) \left(B_1 \lambda(b, b) + B_2\right); \ P_{b,ab}(a) = \left(\frac{B_1}{1 - A_1 - B_1}\right) \left(A_1 \lambda(a, a) + A_2\right)$$

$$P_{a,ab}(a) = \left(\frac{1 - B_1}{1 - A_1 - B_1}\right) \left(A_1 \lambda(a, a) + A_2\right); \ P_{a,ab}(b) = \left(\frac{A_1}{1 - A_1 - B_1}\right) \left(B_1 \lambda(b, b) + B_2\right)$$

The implied steady state matching probabilities $\lambda(a, ab)$ and $\lambda(b, ab)$ are

$$\lambda\left(a,ab\right) = P_{a,ab}\left(a\right) + P_{b,ab}\left(a\right) \text{ and } \lambda\left(b,ab\right) = P_{a,ab}\left(b\right) + P_{b,ab}\left(b\right).$$

It remains is to show that $1 - A_1 - B_1$ is positive. To see this,

$$1 - A_1 - B_1 = 1 - \int_0^\infty \left(\frac{x\left(\gamma\right)}{1 - P_{b,b}\left(b\right) + \frac{\delta}{\gamma}} + \frac{y\left(\gamma\right)}{1 - P_{a,a}\left(a\right) + \frac{\delta}{\gamma}} \right) dG\left(\gamma\right) > 1 - \int_0^\infty \left(\frac{x\left(\gamma\right)}{1 - P_{b,b}\left(b\right)} + \frac{y\left(\gamma\right)}{1 - P_{a,a}\left(a\right)} \right) dG\left(\gamma\right)$$

Suppose w.l.o.g. $\lambda(a, a) > \lambda(b, b)$ then,

$$1 - A_1 - B_1 > 1 - \frac{1}{1 - \lambda\left(a, a\right)} \left[\int_0^\infty \left(x\left(\gamma\right) + y\left(\gamma\right)\right) dG\left(\gamma\right) \right] = 1 - \frac{\int_0^\infty \left(x\left(\gamma\right) + y\left(\gamma\right)\right) dG\left(\gamma\right)}{\lambda\left(b, b\right) + \int_0^\infty \left(x\left(\gamma\right) + y\left(\gamma\right)\right) dG\left(\gamma\right)} > 0$$

This completes the proof. ■

Lemma 4 The mappings $L_{a,a}$ and $L_{b,b}$ defined by (5) and (6) each have at least one fixed point.

Proof. Consider λ and λ' both in [0,1] with $\lambda' \geq \lambda$. It is straight forward to show that $L_{a,a}(\lambda') \geq L_{a,a}(\lambda)$. Existence then follows by Tarski's fixed point theorem. The proof for $L_{b,b}$ is identical.

Lemma 5 For any given cutoffs z_1 , z_2 and z_3 , let $P_{a,ab}(a)$, $P_{b,ab}(a)$, $P_{a,ab}(b)$ and $P_{b,ab}(b)$ be given as in Lemma 3 and define

$$\psi_{a,ab} = \left(\frac{P_{a,ab}\left(a\right)}{F\left(z_{2}\right) - F\left(z_{1}\right)}\right) \ and \ \psi_{b,ab} = \left(\frac{P_{b,ab}\left(b\right)}{F\left(z_{3}\right) - F\left(z_{2}\right)}\right).$$

Then, the flow of social welfare is

$$\begin{split} W\left(z_{1},z_{2},z_{3}\right) &= \left\{1 - \pi\gamma\left[\lambda\left(b,b\right) + \lambda\left(b,ab\right)\right]\right\}F\left(z_{1}\right) \\ &+ \int_{z_{1}}^{z_{2}} \left\{\psi_{a,ab}\left[1 - \pi\gamma\lambda\left(b,b\right)\right] + \left(1 - \psi_{a,ab}\right)\left[1 + z - \pi\gamma\lambda\left(a,a\right)\right]\right\}dF\left(z\right) \\ &+ \int_{z_{2}}^{z_{3}} \left\{\left(1 - \psi_{b,ab}\right)\left[1 - \pi\gamma\lambda\left(b,b\right)\right] + \psi_{b,ab}\left[1 + z - \pi\gamma\lambda\left(a,a\right)\right]\right\}dF\left(z\right) \\ &+ \int_{z_{2}}^{\infty} \left\{1 + z - \pi\gamma\left[\lambda\left(a,a\right) + \lambda\left(a,ab\right)\right]\right\}dF\left(z\right) - \delta c. \end{split}$$

Proof. The probability that an agent who follows strategy (a, a) incurs the trade penalty is

$$\gamma \left[\lambda \left(b,b\right) +\lambda \left(b,ab\right) \right]$$

and the probability for a (b, b) agent is

$$\gamma \left[\lambda \left(a,a\right) +\lambda \left(a,ab\right) \right]$$

The flow utility for the (a, a)'s is

$$\int_{-\infty}^{z_1} \left\{ 1 - \delta c - \pi \gamma \left[\lambda \left(b, b \right) + \lambda \left(b, ab \right) \right] \right\} dF(z)$$

and for the (b, b)'s is

$$\int_{z_{a}}^{\infty} \left\{ 1 + z - \delta c - \pi \gamma \left[\lambda \left(a, a \right) + \lambda \left(a, a b \right) \right] \right\} dF(z)$$

The agents who follow inclusive strategies are somewhat more involved. Begin by considering the flow utility of a particular agent of type z. Take any $z \in [z_1, z_2]$. These agents play (a, ab). If the agent has a in inventory the probability of incurring the trade penalty is $\gamma \lambda (b, b)$, if they have b in inventory the probability of incurring the trade penalty is, $\gamma \lambda (a, a)$. The probability of incurring the building cost (the same in either case) is δ . The fraction of (a, ab) agents that hold the house they build (a) is

$$\frac{P_{a,ab}\left(a\right)}{F\left(z_{2}\right)-F\left(z_{1}\right)}.$$

The flow utility from the agents with this z is thus

$$\left(\frac{P_{a,ab}\left(a\right)}{F\left(z_{2}\right)-F\left(z_{1}\right)}\right)\left[1-\pi\gamma\lambda\left(b,b\right)\right]+\left(1-\frac{P_{a,ab}\left(a\right)}{F\left(z_{2}\right)-F\left(z_{1}\right)}\right)\left[1+z-\pi\gamma\lambda\left(a,a\right)\right]-\delta c.$$

The flow utility for the entire group of (a, ab) agents is

$$\int_{z_{1}}^{z_{2}}\left\{\left(\frac{P_{a,ab}\left(a\right)}{F\left(z_{2}\right)-F\left(z_{1}\right)}\right)\left[1-\pi\gamma\lambda\left(b,b\right)\right]+\left(1-\frac{P_{a,ab}\left(a\right)}{F\left(z_{2}\right)-F\left(z_{1}\right)}\right)\left[1+z-\pi\gamma\lambda\left(a,a\right)\right]-\delta c\right\}dF\left(z\right)$$

Similar arguments give the flow utility for the (b, ab) traders as

$$\int_{z_{2}}^{z_{3}} \left\{ \left(1 - \frac{P_{b,ab}\left(b\right)}{F\left(z_{3}\right) - F\left(z_{2}\right)}\right) \left[1 - \pi\gamma\lambda\left(b,b\right)\right] + \left(\frac{P_{b,ab}\left(b\right)}{F\left(z_{3}\right) - F\left(z_{2}\right)}\right) \left[1 + z - \pi\gamma\lambda\left(a,a\right)\right] - \delta c \right\} dF\left(z\right)$$

Total welfare (W) is the sum of the flow utility of the four types which can be rearranged to get

$$W = 1 + \left(\frac{P_{a,ab}\left(b\right)}{F\left(z_{2}\right) - F\left(z_{1}\right)}\right) \int_{z_{1}}^{z_{2}} z dF\left(z\right) + \frac{P_{b,ab}\left(b\right)}{F\left(z_{3}\right) - F\left(z_{2}\right)} \int_{z_{2}}^{z_{3}} z dF\left(z\right) + \int_{z_{3}}^{\infty} z dF\left(z\right) - \delta c -2\pi\gamma \left\{F\left(z_{1}\right)\left[1 - F\left(z_{3}\right)\right] + \left[P_{a,ab}\left(b\right) + P_{b,ab}\left(b\right)\right]F\left(z_{1}\right) + \left[P_{a,ab}\left(a\right) + P_{b,ab}\left(a\right)\right]\left[1 - F\left(z_{3}\right)\right]\right\}$$

This completes the proof.

Lemma 6 If a density function f(z) with mean μ satisfies the following conditions:

- 1. Symmetry (S): for any x, $f(\mu + x) = f(\mu x)$, and $F(\mu + x) = 1 F(\mu x)$.
- 2. Quasi-Concave (QC): for any fixed x, the set $\{y: f(y) \geq f(x)\}$ is convex

then for any z < z', $F(z) \ge 1 - F(z') \Leftrightarrow f(z) \ge f(z')$.

Proof. Define x and x' implicitly as $z = \mu + x$ and $z' = \mu + x'$. Since z' > z, x' > x. Clearly z is either greater than or less than the mean μ .

1. Suppose $z \leq \mu$ (so that $x \leq 0$) then,

- (a) $F(z) \ge 1 F(z') \Rightarrow F(\mu + x) \ge 1 F(\mu + x') \Rightarrow$ (by $S) \Rightarrow 1 F(\mu x) \ge 1 F(\mu + x') \Rightarrow$ $F(\mu + x') \ge F(\mu x) \Rightarrow x' > -x \ge 0$ and $0 \ge x > -x'$. We now have $f(z') = f(\mu + x') =$ $f(\mu x') \le f(y)$ for all $y \in [\mu x', \mu + x']$ (by QC). Since $z = \mu + x > \mu x'$ and $z < z' = \mu + x'$, z is in this interval. Thus $f(z) \ge f(z')$.
- (b) $F(z) \le 1 F(z') \Rightarrow F(\mu + x) \le 1 F(\mu + x') \Rightarrow$ (by $S) \Rightarrow F(\mu + x) \le F(\mu x') \Rightarrow x' < -x \Rightarrow x < -x' \Rightarrow f(z) = f(\mu + x) = f(\mu x) \le f(y)$ for all $y \in [\mu + x, \mu x]$ (by QC) (recall that $x \le 0$). Since $z' = \mu + x' < \mu x$ and $z' > z = \mu + x, z'$ is in this interval. Thus $f(z') \ge f(z)$.

Thus if $z \le \mu$, $F(z) \ge 1 - F(z') \Leftrightarrow f(z) \ge f(z')$.

2. Suppose that $z \ge \mu$ so that $z' \ge \mu$. Because $z' > z \ge \mu$ and F is symmetric, $F(z) \ge \frac{1}{2} \ge 1 - F(z')$. Moreover, $f(y) \ge f(z')$ for all $y \in [\mu - x', \mu + x']$. Since $z \in [\mu, z'] \subset [\mu - x', \mu + x']$ we must have $f(z) \ge f(z')$.

Thus
$$z \ge \mu \Rightarrow F(z) \ge 1 - F(z')$$
 and $f(z) \ge f(z')$.

This completes the proof. ■

Proposition 1 Given non-negative $\lambda(a,a)$, $\lambda(b,b)$, $\lambda(a,ab)$, and $\lambda(b,ab)$ summing to 1, define $z_1(\gamma)$, $z_2(\gamma)$, and $z_3(\gamma)$ as follows:

$$z_{1}(\gamma) = -\pi \left\{ r + \delta + \gamma \left[1 - \lambda \left(a, a \right) \right] \right\}$$
$$z_{2}(\gamma) = \gamma \pi \left[\lambda \left(a, a \right) - \lambda \left(b, b \right) \right]$$
$$z_{3}(\gamma) = \pi \left\{ r + \delta + \gamma \left[1 - \lambda \left(b, b \right) \right] \right\}$$

Then, for an agent with taste parameter z and trade hazard γ , if $z \leq z_1(\gamma)$ then (a, a) is an optimal strategy; if $z_1(\gamma) \leq z \leq z_2(\gamma)$, (a, ab) is an optimal strategy; if $z_2(\gamma) \leq z \leq z_3(\gamma)$, (b, ab) is an optimal strategy and if $z_3(\gamma) \leq z$, (b, b) is an optimal strategy.

Proof. We focus on the continuation value when an agent receives the build shock. At this point, an agent chooses the type of house to build and formulates a contingency plan. Our strategy for the proof is to calculate the building continuation value for each possible strategy. Agents must follow one of the following four strategies $\{a, a\}$, $\{a, ab\}$, $\{b, ab\}$, and $\{b, b\}$. The optimal strategy is the one with the highest value in the building stage. By construction, the dynamic programming problem is dynamically consistent. Thus if it is optimal to follow a given strategy in one state, it is optimal to follow it in any other state.

Suppose that an agent with parameters z, γ follows policy $\{a, a\}$. We write $v_{a,a}$ (.) and $\tau_{a,a}$ (.) to denote the value of following this strategy and the expected value of receiving the trade shock. Note these strategies may not be optimal (i.e. $v_{a,a}(x) \leq V(x)$ and $\tau_{a,a}(x) \leq T(x)$). We consider each policy in turn.

1. Strategy $\{a, a\}$. If he follows $\{a, a\}$ then he always rejects b in the trade stage and $\tau_{a,a}(b)$ is irrelevant. The trade value $\tau_{a,a}(a)$ is

$$\tau_{a,a}(a) = v_{a,a}(a) - \pi \left[\lambda \left(b, ab \right) + \lambda \left(b, b \right) \right]$$

The consumption stage value is then,

$$v_{a,a}(a) = \frac{1 - \delta c - \gamma \pi \left[\lambda \left(b, ab\right) + \lambda \left(b, b\right)\right]}{r}$$
(12)

2. Strategy $\{a, ab\}$. Because the agent accepts either a or b in trade, we must calculate the trade value of a and b. Similarly, we must consider the consumption value of a and b. The trade value of possessing a is

$$\tau_{a,ab}(a) = v_{a,ab}(a) + \lambda(b,ab) [v_{a,ab}(b) - v_{a,ab}(a)] - \pi\lambda(b,b)$$

The trade value of possessing b is

$$\tau_{a,ab}(b) = v_{a,ab}(a) + [\lambda(b,b) + \lambda(b,ab)][v_{a,ab}(b) - v_{a,ab}(a)] - \pi\lambda(a,a)$$

The consumption value for a satisfies

$$rv_{a,ab}(a) = 1 - \delta c + \gamma \left\{ \lambda \left(b, ab \right) \left[v_{a,ab}(b) - v_{a,ab}(a) \right] - \pi \lambda \left(b, b \right) \right\}$$

$$(13)$$

while the consumption value for b satisfies

$$rv_{a,ab}(b) = 1 + z + \delta [v_{a,ab}(a) - v_{a,ab}(b) - c] + \gamma \{v_{a,ab}(a) - v_{a,ab}(b) + [\lambda (b,b) + \lambda (b,ab)] [v_{a,ab}(b) - v_{a,ab}(a)] - \pi \lambda (a,a)\}$$

or,

$$rv_{a,ab}(b) = 1 + z - \delta c - \delta \left[v_{a,ab}(b) - v_{a,ab}(a) \right] + \gamma \left\{ \left[\lambda (b,b) + \lambda (b,ab) \right] \left[v_{a,ab}(b) - v_{a,ab}(a) \right] - \pi \lambda (a,a) \right\} - \gamma \left[v_{a,ab}(b) - v_{a,ab}(a) \right]$$

Subtracting (13) gives

$$v_{a,ab}(b) - v_{a,ab}(a) = \frac{z + \gamma \pi \left[\lambda(b,b) - \lambda(a,a)\right]}{r + \delta + \gamma \left[1 - \lambda(b,b)\right]}$$

Plugging this back into (13) we get the consumption value $v_{a,ab}(a)$.

$$rv_{a,ab}\left(a\right) = 1 + \gamma \left\{ \lambda \left(b,ab\right) \left[\frac{z + \gamma \pi \left[\lambda \left(b,b\right) - \lambda \left(a,a\right)\right]}{r + \delta + \gamma \left[1 - \lambda \left(b,b\right)\right]} \right] - \pi \lambda \left(b,b\right) \right\} - \delta c.$$

To compare $\{a, ab\}$ and $\{a, a\}$, consider an agent who receives the build shock. If he follows $\{a, ab\}$, his payoff is $v_{a,ab}(a) - c$ while if he follows $\{a, a\}$ his payoff is $v_{a,a}(a) - c$. The agent prefers $\{a, ab\}$ to $\{a, a\}$ if $v_{a,ab}(a) > v_{a,a}(a) \Leftrightarrow$

$$1 + \gamma \left\{ \lambda \left(b, ab \right) \left[\frac{z + \gamma \pi \left[\lambda \left(b, b \right) - \lambda \left(a, a \right) \right]}{r + \delta + \gamma \left[1 - \lambda \left(b, b \right) \right]} \right] - \pi \lambda \left(b, b \right) \right\} - \delta c > 1 - \gamma \pi \left[\lambda \left(b, ab \right) + \lambda \left(b, b \right) \right] - \delta c$$

or if

$$z > -\pi \left\{ r + \delta + \gamma \left[1 - \lambda \left(a, a \right) \right] \right\} \equiv z_1 \left(\gamma \right)$$

Any agent with parameters z, γ with $z > z_1(\gamma)$ will prefer $\{a, ab\}$ to $\{a, a\}$

3. Strategy $\{b, ab\}$. Following the argument above we find the trade value of a is

$$\tau_{b,ab}\left(a\right) = v_{b,ab}\left(b\right) + \left[\lambda\left(a,a\right) + \lambda\left(a,ab\right)\right] \left[v_{b,ab}\left(a\right) - v_{b,ab}\left(b\right)\right] - \pi\lambda\left(b,b\right)$$

while the trade value of possessing b is

$$\tau_{b,ab}\left(b\right) = v_{b,ab}\left(b\right) + \lambda\left(a,ab\right)\left[v_{b,ab}\left(a\right) - v_{b,ab}\left(b\right)\right] - \pi\lambda\left(a,a\right)$$

The consumption value for a satisfies

$$rv_{b,ab}(a) = 1 + \delta [v_{b,ab}(b) - v_{b,ab}(a) - c] + \gamma (v_{b,ab}(b) - v_{b,ab}(a) - [\lambda (a, a) + \lambda (a, ab)] [v_{b,ab}(b) - v_{b,ab}(a)] - \pi \lambda (b, b))$$

and the consumption value for b satisfies

$$rv_{b,ab}(b) = 1 + z - \delta c + \gamma \left[-\lambda (a, ab) \left[v_{b,ab}(b) - v_{b,ab}(a) \right] - \pi \lambda (a, a) \right]$$

The implied difference is

$$v_{b,ab}(b) - v_{b,ab}(a) = \frac{z + \gamma \pi \left[\lambda(b,b) - \lambda(a,a)\right]}{r + \delta + \gamma - \gamma \lambda(a,a)}$$

We now solve for $v_{b,ab}(b)$ (for this strategy, the continuation value in the build stage is $v_{b,ab}(b) - c$).

$$rv_{b,ab}\left(b\right) = 1 + z + \gamma \left[-\lambda\left(a,ab\right) \left[\frac{z + \gamma \pi \left[\lambda\left(b,b\right) - \lambda\left(a,a\right)\right]}{r + \delta + \gamma \left[1 - \lambda\left(a,a\right)\right]}\right] - \pi \lambda\left(a,a\right)\right] - \delta c$$

As before, we compare the continuation value for an agent who received the build shock for $\{b, ab\}$ and $\{a, ab\}$. The agent prefers $\{b, ab\}$ to $\{a, ab\}$ if $v_{b,ab}(b) \ge v_{a,ab}(a) \iff$

$$\begin{aligned} 1 + z + \gamma \left[-\lambda \left(a, ab \right) \left[\frac{z + \gamma \pi \left[\lambda \left(b, b \right) - \lambda \left(a, a \right) \right]}{r + \delta + \gamma \left[1 - \lambda \left(a, a \right) \right]} \right] - \pi \lambda \left(a, a \right) \right] - \delta c \\ \ge & 1 + \gamma \left\{ \lambda \left(b, ab \right) \left[\frac{z + \gamma \pi \left[\lambda \left(b, b \right) - \lambda \left(a, a \right) \right]}{r + \delta + \gamma \left[1 - \lambda \left(b, b \right) \right]} \right] - \pi \lambda \left(b, b \right) \right\} - \delta c \end{aligned}$$

This expression can be rewritten as

$$\left(z+\gamma\pi\left[\lambda\left(b,b\right)-\lambda\left(a,a\right)\right]\right)\left\{1-\frac{\gamma\lambda\left(a,ab\right)}{r+\delta+\gamma\left[1-\lambda\left(a,a\right)\right]}-\frac{\gamma\lambda\left(b,ab\right)}{r+\delta+\gamma\left[1-\lambda\left(b,b\right)\right]}\right\}\geq0$$

or

$$(z + \gamma \pi [\lambda (b, b) - \lambda (a, a)]) \Omega \ge 0$$

where

$$\Omega \equiv 1 - \frac{\gamma \lambda \left(a, ab\right)}{r + \delta + \gamma \left[1 - \lambda \left(a, a\right)\right]} - \frac{\gamma \lambda \left(b, ab\right)}{r + \delta + \gamma \left[1 - \lambda \left(b, b\right)\right]}$$

We now show that $\Omega > 0$. Without loss of generality assume that $\lambda(b, b) \geq \lambda(a, a)$. Then,

$$\Omega \ge 1 - \gamma \left[\frac{\lambda(b, ab) + \lambda(a, ab)}{r + \delta + \gamma \left[1 - \lambda(b, b) \right]} \right].$$

 $\Omega > 0$ if

$$r + \delta + \gamma \left[1 - \lambda (b, b) - \lambda (b, ab) - \lambda (a, ab)\right] > 0$$

which is satisfied since $\lambda(b, ab) + \lambda(a, ab) + \lambda(a, a) + \lambda(b, b) = 1$. This implies that $\Omega > 0$. Because $\Omega > 0$, the agents prefers $\{b, ab\}$ to $\{a, ab\}$ whenever

$$z \ge \gamma \pi \left[\lambda (a, a) - \lambda (b, b) \right] \equiv z_2 (\gamma).$$

Agents (z, γ) with $z > z_2(\gamma)$ prefer $\{b, ab\}$ to $\{a, ab\}$.

4. Strategy $\{b,b\}$. An agent who follows $\{b,b\}$ always rejects a so $\tau_{b,b}(a)$ is irrelevant. $\tau_{b,b}(b)$ is

$$\tau_{b,b}(b) = v_{b,b}(b) - \pi \left[\lambda (a, ab) + \lambda (a, a) \right]$$

The consumption value is

$$v_{b,b}\left(b\right) = \frac{1 + z - \delta c - \gamma \pi \left[\lambda\left(a, ab\right) + \lambda\left(a, a\right)\right]}{r}$$

As before, consider an agent who receives the build shock. If he follows $\{b, ab\}$, his payoff is $v_{b,ab}(a) - c$; if he follows $\{b, b\}$, his payoff is $v_{b,b}(a) - c$. The agent prefers $\{b, b\}$ to $\{b, ab\}$ if $v_{b,b}(b) > v_{b,ab}(b) \Leftrightarrow$

$$1 + z - \delta c - \gamma \pi \left[\lambda \left(a, ab \right) + \lambda \left(a, a \right) \right] > 1 + z + \gamma \left[-\lambda \left(a, ab \right) \left[\frac{z + \gamma \pi \left[\lambda \left(b, b \right) - \lambda \left(a, a \right) \right]}{r + \delta + \gamma \left[1 - \lambda \left(a, a \right) \right]} \right] - \pi \lambda \left(a, a \right) \right] - \delta c$$

$$z > \pi \left[r + \delta + \gamma \left[1 - \lambda \left(b, b \right) \right] \right] \equiv z_3 \left(\gamma \right)$$

Any agent with $z > z_3(\gamma)$ will follow $\{b, b\}$ rather than $\{b, ab\}$.

Since $z_1(\gamma) < z_2(\gamma) < z_3(\gamma)$ any agent with $z < z_1(\gamma)$ prefers $\{a,a\}$ to all of the other strategies. To see this note that such an agent prefers $\{a,a\}$ to $\{a,ab\}$ by case one above. However, by case two, he also prefers $\{a,ab\}$ to $\{b,ab\}$ and by case three prefers $\{b,ab\}$ to $\{b,b\}$. Thus $\{a,a\}$ is optimal for this agent. Similar arguments imply that for $z_1(\gamma) < z < z_2(\gamma)$ the optimal strategy is $\{a,ab\}$; for $z_2(\gamma) < z < z_3(\gamma)$ the optimal strategy is $\{b,ab\}$ and for $z > z_3(\gamma)$ the optimal strategy is $\{b,b\}$.

Proposition 2 Given any distribution of types F and distribution of trade hazards G, there exists at least one steady state equilibrium.

Proof. By Lemma 4 $L_{a,a}$ and $L_{b,b}$ have at least one fixed point. Let $\lambda(a,a)$, and $\lambda(b,b)$ be fixed points of $L_{a,a}$ and $L_{b,b}$. We construct an equilibrium as follows:

Use $\lambda(a, a)$, and $\lambda(b, b)$ to construct $z_1(\gamma)$, $z_2(\gamma)$, and $z_3(\gamma)$ from the definitions in 1. With $z_1(\gamma)$, $z_2(\gamma)$, and $z_3(\gamma)$ compute $\lambda(a, a)$, $\lambda(b, b)$, $\lambda(a, ab)$, and $\lambda(b, ab)$ in accordance with Lemma 3. By construction $\lambda(a, a)$, $\lambda(b, b)$, $\lambda(a, ab)$, and $\lambda(b, ab)$ are equilibrium matching probabilities.

Proposition 3 If F satisfies Assumption 1 then

- 1. The equilibrium is unique.
- 2. There is conformity whenever $\mu \neq 0$ and the market conforms to the mean taste (the market conforms on a if $\mu < 0$ and conforms on b if $\mu > 0$).
- 3. If $\mu \neq 0$, an increase in durability (lower δ) or patience (lower r) causes conformity to increase (i.e., $z_2(\gamma)$ increases if $\mu < 0$ and decreases if $\mu > 0$ for all γ).
- 4. All else equal, agents with a greater likelihood of trade conform more.

Proof.

1. By proposition 2, $\lambda(a,a)$ is a fixed point of the mapping $L_{a,a}$,

$$\lambda\left(a,a\right) = L_{a,a}\left(\lambda\left(a,a\right)\right) = \int_{0}^{\infty} F\left(-\pi\left\{r + \delta + \gamma\left[1 - \lambda\left(a,a\right)\right]\right\}\right) dG\left(\gamma\right)$$

By assumption F has a density and thus we can calculate the derivative of $L_{a,a}$. This derivative is

$$0 \leq \frac{\partial L_{a,a}\left(\lambda\left(a,a\right)\right)}{\partial \lambda\left(a,a\right)} = \int_{0}^{\infty} f\left(z_{1}\left(\gamma\right)\right) \pi \gamma dG\left(\gamma\right) < f\left(\mu\right) \pi \int_{0}^{\infty} \gamma dG\left(\gamma\right) = f\left(\mu\right) \pi \bar{\gamma} < 1.$$

Where the second inequality follows from the assumption which guarantees $f(z_1(\gamma)) \leq f(\mu)$. The last inequality follows from the assumption. Thus there can be at most one fixed point. Similar arguments hold for $L_{b,b}$. Since existence of at least one equilibrium is guaranteed by proposition 2, the equilibrium is unique.

2. Because f is symmetric about its mean, if $\mu = 0$ then F(-x) = 1 - F(x) and thus the unique equilibrium must have $\lambda(a, a) = \lambda(b, b)$ and $z_2(\gamma) = 0$ for all γ (no conformity). Let λ^* be the equilibrium $\lambda^* = \lambda(a, a) = \lambda(b, b)$ associated with $\mu = 0$. Note that for any $l \geq \lambda^*$ we must have $L_{a,a}(l) \leq l$ (since the derivative of $L_{a,a}$ is less than 1 by part 1) and for any $l \leq \lambda^*$ we must have $L_{b,b}(l) \geq l$ by the same reasoning.

Consider $\mu > 0$ (the argument for $\mu < 0$ is identical). For any given $l \in [0,1]$, and for all γ ,

$$F(-\pi \{r + \delta + \gamma [1 - l]\}; \mu > 0) < F(-\pi \{r + \delta + \gamma [1 - l]\}; \mu = 0)$$

and

$$1 - F\left(\pi\left\{r + \delta + \gamma\left[1 - l\right]\right\}; \mu > 0\right) > 1 - F\left(\pi\left\{r + \delta + \gamma\left[1 - l\right]\right\}; \mu = 0\right)$$

Therefore, integrating over all γ we have

$$L_{a,a}(l)|_{\mu>0} < L_{a,a}(l)|_{\mu=0}$$

$$L_{b,b}(l)|_{\mu>0} > L_{b,b}(l)|_{\mu=0}$$

When $\mu > 0$, $l \in [\lambda^*, 1]$ cannot be a fixed point of $L_{a,a}$ since $L_{a,a}(l)|_{\mu>0} < L_{a,a}(l)|_{\mu=0} \le l$ for $l \in [\lambda^*, 1]$. Similarly, $l \in [0, \lambda^*]$ cannot be a fixed point of $L_{b,b}$. Because the equilibrium is unique, we conclude that for $\mu > 0$, the equilibrium satisfies $\lambda(b, b) > \lambda^* > \lambda(a, a)$. This implies that $z_2(\gamma) < 0$ for all γ so the market conforms on b.

3. If $\mu > 0$ then $\int_0^\infty \left[1 - F\left(z_3\left(\gamma\right)\right)\right] dG\left(\gamma\right) = \lambda\left(b, b\right) > \lambda\left(a, a\right) = \int_0^\infty F\left(z_1\left(\gamma\right)\right) dG\left(\gamma\right)$ by part (2). This implies that $z_2\left(\gamma\right) < 0$. Differentiating gives

$$\frac{\partial z_2\left(\gamma\right)}{\partial \delta} = \pi \gamma \left[\frac{\partial \lambda\left(a,a\right)}{\partial \delta} - \frac{\partial \lambda\left(b,b\right)}{\partial \delta} \right]$$

$$\frac{\partial \lambda \left(a,a \right)}{\partial \delta} = -\frac{\pi \int f \left(z_{1} \left(\gamma \right) \right) dG \left(\gamma \right)}{1 - \pi \int f \left(z_{1} \left(\gamma \right) \right) \gamma dG \left(\gamma \right)} \text{ and } \frac{\partial \lambda \left(b,b \right)}{\partial \delta} = -\frac{\pi \int f \left(z_{3} \left(\gamma \right) \right) dG \left(\gamma \right)}{1 - \pi \int f \left(z_{3} \left(\gamma \right) \right) \gamma dG \left(\gamma \right)}$$

Lemma 6 implies $f(z_3(\gamma)) \geq f(z_1(\gamma))$ for all γ . Moreover, Assumption 1 guarantees that $1 - \pi \int f(z_1(\gamma)) \gamma dG(\gamma) > 0$ and $1 - \pi \int f(z_3(\gamma)) \gamma dG(\gamma) > 0$. Thus

$$\frac{\pi \int f\left(z_{3}\left(\gamma\right)\right) dG\left(\gamma\right)}{1 - \pi \int f\left(z_{3}\left(\gamma\right)\right) \gamma dG\left(\gamma\right)} > \frac{\pi \int f\left(z_{1}\left(\gamma\right)\right) dG\left(\gamma\right)}{1 - \pi \int f\left(z_{1}\left(\gamma\right)\right) \gamma dG\left(\gamma\right)}$$

which implies,

$$\frac{\partial z_{2}\left(\gamma\right)}{\partial \delta} = \pi \gamma \left[-\frac{\pi \int f\left(z_{1}\left(\gamma\right)\right) dG\left(\gamma\right)}{1 - \pi \int f\left(z_{1}\left(\gamma\right)\right) \gamma dG\left(\gamma\right)} + \frac{\pi \int f\left(z_{3}\left(\gamma\right)\right) dG\left(\gamma\right)}{1 - \pi \int f\left(z_{3}\left(\gamma\right)\right) \gamma dG\left(\gamma\right)} \right] > 0$$

An increase in durability implies a reduction in $z_2(\gamma)$ for all γ . The proof for r is identical.

4. The proof follows immediately by observing that $z_2(\gamma) = \pi \gamma [\lambda(a, a) - \lambda(b, b)]$.

This completes the proof. ■

Proposition 4 Assume that agents trade hazards are writen as $(\gamma + \theta)$ as described above (so $\theta = 0$ corresponds to the original equilibrium) and consider an F which satisfies Assumption 1. Then, for a marginal increase in θ at $\theta = 0$, there exists $\hat{\gamma} \in (0, \infty]$ such that conformity increases for all agents with $\gamma < \hat{\gamma}$ and decreases for all agents with $\gamma > \hat{\gamma}$. (Note that this allows for $\hat{\gamma} = \infty$ in which case conformity increases for all agents).

Proof. First differentiate $\lambda(a, a)$ and $\lambda(b, b)$ with respect to θ and evaluate at $\theta = 0$ to obtain:

$$\left. \frac{\partial \lambda \left(a, a \right)}{\partial \theta} \right|_{\theta = 0} \ = \ \frac{-\pi \left(1 - \lambda \left(a, a \right) \right) \int f \left(z_1 \left(\gamma, 0 \right) \right) dG \left(\gamma \right)}{1 - \int f \left(z_1 \left(\gamma, 0 \right) \right) \gamma dG \left(\gamma \right)}, \\ \left. \frac{\partial \lambda \left(b, b \right)}{\partial \theta} \right|_{\theta = 0} \ = \ \frac{-\pi \left(1 - \lambda \left(b, b \right) \right) \int f \left(z_3 \left(\gamma, 0 \right) \right) dG \left(\gamma \right)}{1 - \int f \left(z_3 \left(\gamma, 0 \right) \right) \gamma dG \left(\gamma \right)}.$$

The conformity cutoff $z_2(\gamma)$ is

$$z_2(\gamma) = (\gamma + \theta) \pi [\lambda(a, a) - \lambda(b, b)].$$

Differentiating z_2 with respect to θ and evaluating at $\theta = 0$ we obtain:

$$\left. \frac{\partial z_{2}\left(\gamma\right)}{\partial \theta} \right|_{\theta=0} = \frac{z_{2}\left(\gamma\right)}{\gamma} + \gamma \pi \left[\left. \frac{\partial \lambda\left(a,a\right)}{\partial \theta} \right|_{\theta=0} - \left. \frac{\partial \lambda\left(b,b\right)}{\partial \theta} \right|_{\theta=0} \right].$$

Suppose $\mu > 0$ ($\mu < 0$ is symmetric.) In this case $z_2(\gamma) < 0$. If the term in square brackets is negative then $\frac{\partial z_2(\gamma)}{\partial \theta}\Big|_{\theta=0} < 0$ for all γ so we set $\hat{\gamma} = \infty$. If the term is positive then let $\hat{\gamma}$ be

$$\hat{\gamma} = \frac{-\left[\lambda\left(a, a\right) - \lambda\left(b, b\right)\right]}{\left[\left.\frac{\partial \lambda\left(a, a\right)}{\partial \theta}\right|_{\theta = 0} - \left.\frac{\partial \lambda\left(b, b\right)}{\partial \theta}\right|_{\theta = 0}\right]} > 0.$$

By construction $\frac{\partial z_2(\gamma)}{\partial \theta}\Big|_{\theta=0} < 0$ for $\gamma < \hat{\gamma}$ and $\frac{\partial z_2(\gamma)}{\partial \theta}\Big|_{\theta=0} > 0$ for $\gamma > \hat{\gamma}$ which proves the result.

FIGURE 1: UNIQUE EQUILIBRIA WITH A UNIFORM DISTRIBUTION

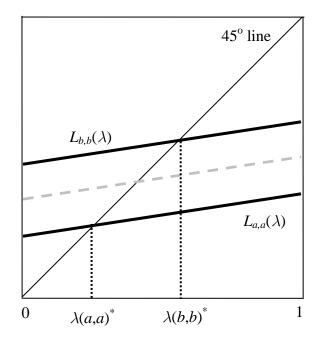


FIGURE 2: MULTIPLE EQUILIBRIA WITH A SYMMETRIC DISTRIBUTION

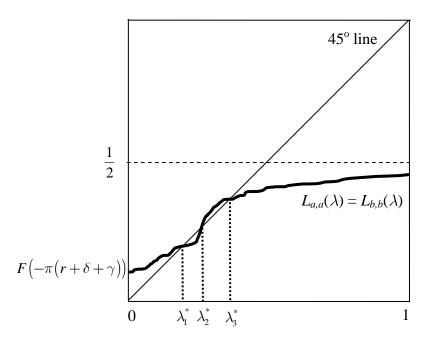
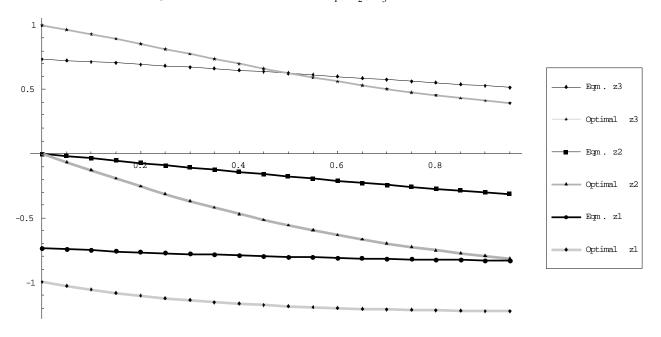


Figure 3: Equilibrium and Optimal z_1 , z_2 , z_3 for Different Means.



The figure shows the equilibrium cutoffs (the solid dark lines) and the optimal cutoffs (the light shaded lines) for different means of F(z). In each case, F is normal with a unit variance. The cutoffs are plotted on the vertical axis while the mean is on the horizontal axis.