Beyond the Diversification Cone: A Neo-Heckscher-Ohlin Model of Trade with Endogenous Specialization

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Abstract: We propose a Neo-Heckscher-Ohlin model of trade that combines comparative endowment advantage, comparative technological advantage, international capital mobility, and trade-specific transaction costs. Unlike competing characterizations, our model is the first of its kind to treat specialization in production endogenously using an inframarginal general equilibrium setting. The results are startling! They suggest that production within the diversification cone – a key assumption of Heckscher-Ohlin theory that is required for its core propositions (such as factor price equalization) to hold – may only prevail on the razor's edge, or under exceptional circumstances. In addition, our findings nominate a mechanism by which improvements in transaction efficiency facilitate international trade thereby stimulating cross-country division of labor. Contrary to other generalizations of the Heckscher-Ohlin (such as the various derivatives of the Kemp-Jones model of trade), our model does not assume a purely Ricardian character: comparative endowment advantage may determine the pattern of trade even in the presence of opposing technological differences, as long as total factor productivity coefficients adjusted for transaction efficiency and factor intensity do not confer unambiguous comparative (technological) advantage. Still, "efficiency intensity"-adjusted comparative technological advantage supersedes factor endowments in determining the flow of trade.

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The Heckscher-Ohlin (HO) theorem[1] arguably represents the most celebrated relatively recent contribution of positive trade theory. It's core proposition, that relative factor endowments determine the pattern of trade, appeals to the most elemental aspects of economic intuition. However, it rarely emerges in the data[2].

The reasons for the dubious empirical relevance of the theorem are, of course, well known: in its simplest form, the HO relies on a host of heroic assumptions, few of which are expected to consistently characterize prevailing economic circumstances. Beyond the issue of dimensionality[3], features of the original formulation of the HO

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[1] Ohlin (1933).

[2] According to the majority of relevant studies, the HO is generally found to correctly predict trade flows about 50% of the time, a success rate which, according to one theorist, "…is matched by a coin toss" (Trefler 1995, p. 1029). Still, despite its poor performance, the HO theorem has retained its dominance. A comprehensive epistemological explanation of this observation extends beyond the scope of this paper. However, we submit that any such explanation is likely to reflect, at least in some measure, Bowen *et al.*'s (1987) crudely succinct observation that: "The Heckscher-Ohlin model does poorly, but we do not have anything that does better." (p. 805).

[3] We have often encountered in the literature the assertion that the HO *fails* in formulations other than the classic 2 country, 2 good, 2 input $(2 \times 2 \times 2)$ setup. This is not so. Certainly, when the model incorporates more goods than factors, the direction

model[4] that represent likely candidates for the most prominent offenders, and the usual suspects in the empirical failure of the model's predictions, include the conditions that: (i) production technology is identical across different countries (see Trefler, 1993; Maskus and Webster, 1999; as well as Hakura, 2001); (ii) factors of production are not traded internationally (Brecher and Choudhri, 1993); (iii) transport costs, as well as other trade-specific transaction costs, are non-existent (Melvin, 1968, p. 1265; and Davis and Weinstein, 2001); and (iv) preferences play a peripheral role in the analysis (Trefler 1995, pp. 1038-1041).

Inevitably, theoretical efforts to rehabilitate the HO framework have concentrated on reformulations that relax some of its most restricting, and potentially distorting, assumptions. Notable *generalized HO models* deriving from such research include those proposed by Mundell (1957); Kemp (1966); Jones (1967); Purvis (1972); Batra and Casas (1976); Brecher and Diaz-Alejandro (1977); Ferguson (1978); Markusen and Melvin (1979); Bhagwati and Brecher (1980); Davis (1995); Norman and Venables (1995); Neary (1995); and Ruffin (1988, 2001), to name a few.

It is important to note that studies in this area generally employ fairly distinct frameworks of analysis, which often manifest the particular motivations of any given project. As a result, ensuing modeling approaches depart on a number of levels, including the particular combination of standard HO assumptions that are being relinquished, the closure of the general equilibrium model that is implemented, the production characteristics that are considered in the analysis, and so on. Yet, despite

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of trade in the case of any particular good is indeterminate. However, the model still predicts that relative factor endowments are reflected in the net factor content of trade.

[4] In what follows we refer to the original, 2 country; 2 good; 2 input; $2\times 2\times 2$, setup as the *standard HO model*.

any such differences, research in this area is characterized by a remarkable methodological convergence in at least three important respects.

First, at least in relation to the standard HO assumptions outlined above, it is important to note that none of the proposed models tackles all four of these conditions simultaneously. As a result, ensuing implementations provide a fairly limited perspective of the extent to which synergies in departures from assumptions (i)-(iv) outlined above interact in a non-trivial fashion to produce possible equilibria.

Second, analytical frameworks that have been employed in this area, including the most prominent relevant contributions, have not produced general equilibrium mappings that link the parameters that define relevant economic dimensions, including technology; preferences; transaction costs etc., to the prices that characterize the set of possible equilibria. Hence, while such studies may employ general equilibrium models, they are not fit to investigate the full range of comparative statics of general equilibrium[5]. As a result, beyond a host of fairly rudimentary generalizations, little is known about the direct impact of changes in the vector of relevant parameters that define the economic environment on trade flows. This shortcoming, which is uniform across studies in this area, derives from the fact

[5] Typically, studies in this area limit their scope of inquiry to *comparative statics of decision* which examine how “exogenous” changes in a selected subset of good or factor prices, impact on the remaining prices of the model – and associated trade flows. A setup that facilitates *comparative statics of* (*general*) *equilibrium* requires that changes in the equilibrium values of *all* endogenous variables, including *all* prices (hence patterns of production and associated trade structures) can be directly explained by changes in the model’s parameters. See Yang (2001, p. 103-106; 161-162) for a relevant discussion.

that the preponderance of relevant research has so far relied on non-parametric functional characterizations of technology and, implicitly, preferences. Presumably, the implied benefit of such an approach is that it facilitates a greater generality of findings. However, as advocated by the well known *everything-is-possible* theorem of Sonnenschein (1972, 1973), Debreu (1974), and Mantel (1974) (henceforth SDM), any such benefits may be rather elusive[6].

Third, in line with the standard HO model, research in this area has consistently treated the issue of specialization in production exogenously. This is particularly puzzling given a long standing, and largely unresolved, debate regarding the conditions that lead global production within the cone of diversification[7] in generalized HO formulations. Interest in this issue began with Jones' classic 1967 paper (motivated by Kemp's well known and influential 1966 contribution). In this paper Jones considered a generalization of the $2 \times 2 \times 2$ formulation of the Heckscher-Ohlin model that, in addition to differential factor endowments, allowed for capital mobility, and technological differences across countries. Using this framework, often

[6] According to SDM's everything-is-possible theorem, in the absence of known *parametric* forms characterizing all functional relationships that prevail in any given general equilibrium model, *no unambiguous comparative statics of general equilibrium may emerge* except the, at least for our purpose - tangential, results that Walras' law holds, and that the excess demand functions are homogeneous of degree zero. For a relevant discussion see Cheng, Sachs, and Yang (2000a, p. 2-4).

[7] The diversification cone (McKenzie, 1955) is characterized by uniform non-specialization in production across all countries that are considered in the model. An excellent discussion of the relevance of this construct in the context of the HO theorem is provided by Jones and Neary (1984, p. 14-21).

referred to as the "Kemp-Jones" model (Ruffin, 1984, p. 270), Jones (1967) advocated that *it is either logically impossible, or extremely unlikely that both countries will be incompletely specialized in production*. This assertion was challenged by Inada and Kemp (1969); Chipman (1971); as well as Uekawa (1971), it was refined and re-issued by Jones and Ruffin (1975), and was subsequently further questioned by Brecher and Feenstra (1983).

In discussing Jones' (1967) original contribution, Inada and Kemp (1969) acutely point out that "It is always difficult to come to grips, in the context of pure theory, with statements about likelihood" (p. 524). This may very well be the case. Still, difficulties that have been encountered in efforts to shed light on what may or may not be possible in relevant frameworks are likely to had been exacerbated by treating the matter of specialization exogenously. The likelihood of specific production patterns – an endogenous outcome – depends solely on the likelihood that the exogenous conditions that lead to such patterns may actually prevail in the first place. Abstracting from a systematic study of the link between cause and effect, translates into frameworks that largely disengage any notion of plausibility from specific production outcomes.

Following the "diversification-specialization" debate of the late 1960's and early 1970's, research in this area continued to treat specialization exogenously while focusing, almost exclusively, on outcomes that prevail *within* the diversification cone. This is exemplified by the fact that most relevant studies either assume non-specialization *ex ante* (see for example Batra and Casas 1976, p. 23; and Neary 1995, p. S7) , or only provide a passing reference to the issue (Ruffin 1988, p. 763; 2001, p. 456). Arguably, a relatively recent contribution by Norman and Venables (1995), may be viewed as an exception. In this study the authors do in fact consider outcomes consistent with a small sample of selected patterns of specialization. Still, even in the case of this contribution, which proposes a HO generalization that incorporates mobile factors *and* technological differences across countries (key model ingredients that led Jones,1967, to conclude that production within the cone of diversification is either logically impossible, or extremely unlikely), the authors dedicate the preponderance of their discussion to outcomes that may only prevail within the cone of diversification. Furthermore, similarly to other studies in this area, Norman and Venables (1995) continue to treat the matter of specialization exogenously.

Insufficient consideration of general equilibrium outcomes that may prevail outside the diversification cone has implicitly trivialized the relevance of treating the pattern of production endogenously. As a result, the literature that produced generalized HO formulations has consistently neglected to endogenize patterns of specialization, that, given appropriate circumstances, may nevertheless prevail in general equilibrium. This is particularly troubling given that it is not always clear whether the key predictions of the Heckscher-Ohlin theorem regarding trade flows hold outside the cone (or, in some instances, even within the cone) in generalized HO formulations.

The importance of this omission has not eluded the literature. As recently argued by Ronald Jones (2002) in a review of analytical frameworks that extend the standard HO model "…such an assumption (of operating within the diversification cone)[8] is fundamentally different from the others because the question of specialization is an

[8] Parenthetical remarks added for clarity.

endogenous one, dependent in large part on the differences in factor endowments, and these differences provide the basic key to Heckscher-Ohlin theory" (p. 346).[9]

Methodology aside, mainstream literature has also argued in favor of the empirical relevance of solutions that may prevail outside the cone of diversification. For example, Bhagwati and Dehejia (1994, pp. 39-45) assert that the frequently observed empirical failure of core propositions of the HO, such as *factor price equalization* (FPE) and the *Stolper-Samuelson* (SS), point at the "extraordinarily demanding" (p. 42) nature of the standard assumptions of the theorem, including the assumption that "…equilibria under autarky and free trade lie in the diversification cone" (p. 44) [10].

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[9] In this context it should also be noted that the *operational definition* of the diversification cone that is commonly employed in research that employs general equilibrium analysis is itself problematic. Following the original characterization of the diversification cone outlined in McKenzie (1955), such models typically articulate their assumption that production occurs within the cone using *a particular configuration of relative prices* (see for example Ruffin, 1988, pg. 763). Hence, the ensuing research effectively treats portion of the price space, typically a key endogenous variable in general equilibrium analysis, exogenously. An alternative specification of the diversification cone, that is perhaps more appropriate in the context of general equilibrium analysis, would rely directly on relative values of the model's exogenous parameters.

[10] It is, of course, well known that while the predictions of the standard HO model regarding trade flows may hold outside the diversification cone, core propositions of the theorem, such as FPE and the SS, do not (Deardorff, 2001). Of course, in

More recently, there has been an overwhelming interest in direct empirical tests of generalizations of the Heckscher-Ohlin that explicitly allow for production patterns that may prevail outside the diversification cone. Notable examples of such research include Davis and Weinstein (2001), Xu (2003), and Schott (2003), to name a few. The findings of these studies are startling! Put succinctly, the empirical evidence provides “strong support for Heckscher-Ohlin specialization” (Schott 2003, p. 686).

Theoretical results provided by Jones (1967), views advocated by Bhagwati and Dehejia (1994) as well as Jones (2002), and empirical findings offered by Davis and Weinstein (2001), Xu (2003), and Schott (2003), ultimately converge to a rather simple point: that not only is there nothing uniquely compelling in equilibria that are consistent with the cone of diversification, but, *at the very least*, there is nothing implausible in outcomes consistent with specialization. Hence, ruling out this feature from the analysis *ex ante*, thereby trivializing the relevance of treating production patterns endogenously, limits the usefulness of ensuing models considerably.

In light of the above we consider that the theoretical integrity of any general equilibrium model of trade requires that possible solutions that may prevail within, as well as those that may prevail outside the cone, are treated endogenously.

It may be difficult to speculate on the sources of an excessive preoccupation with the diversification cone. Perhaps the fact that the predictions of the standard HO model regarding trade flows hold symmetrically both in the absence, as well as in the presence of specialization, has diminished interest in efforts to extend the analysis beyond the cone of diversification even in generalized formulations of the HO model (notwithstanding that the aforementioned symmetry in the standard HO model *does*

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generalized HO models both the conventional trade flow predictions, as well as the core theorems may very well fail both outside, as well as within the cone.

not necessarily extend in the case of generalized HO models). Alternatively, questions raised in relation to Jones' (1967) assertion that (in a HO model with technological differences and capital mobility) production within the cone is improbable, may have perversely rationalized the observed monomania with the diversification cone.

Such considerations may have played some role in this matter. However, we deem that the predominant force responsible for concentrating research effort on the study of outcomes that are consistent with the cone of diversification is likely to have methodological origins. In this context we note that the preponderance of research in this area relies exclusively on a *marginalist* perspective which does not, of course, extend beyond *classical mathematical programming for interior solutions.* Yet, the study of equilibria that exist outside the cone, represented by *corner solutions* in the optimization framework, requires, *in addition to* marginal considerations, a total cost-benefit analysis across possible equilibria. This approach was originally proposed by Coase (1946), formally introduced by Buchanan and Stubblebine (1962), who coined the term *inframarginal analysis*, and further developed by Dixit (1987, 1989), Grossman and Hart (1986), Yang and Borland (1991), Borland and Yang (1992), and Yang (2001). Recent applications of inframarginal analysis to trade theory may be found in Cheng, Liu, and Yang (2000); Cheng, Sachs, and Yang (2000a, 2000b); and Sachs, Yang, and Zhang (2002).

In this article we set out to overcome aforementioned limitations of existing generalized HO frameworks. Toward this end we propose a novel generalization of the Kemp-Jones (Kemp, 1966; Jones, 1967) Neo-Heckscher-Ohlin[11] model of trade that departs from the preponderance of competing frameworks in three important respects: (a) it represents the first theoretical construct to relinquish assumptions (i)-

[11] The term is attributed to Kemp (1966).

(iv) of the standard formulation of the HO simultaneously, (b) it incorporates sufficient parametric structure to facilitate the derivation of explicit mappings between the exogenous parameters that define relevant economic dimensions and the prices that characterize the set of possible equilibria, and, most importantly, (c) it endogenizes the pattern of production. This is accomplished by utilizing an inframarginal perspective that facilitates the partition of the parameter space in a host of relevant parameter value subsets that demarcate the various equilibrium patterns of production and trade.

The resulting framework has explicit advantages over rival models. These include our model's capacity to investigate the full range of comparative statics of general equilibrium[12], as well as its potential to infer the exact manner in which precisely measurable violations of assumptions (i)-(iv) alter the predictions of the standard formulation of the Heckscher-Ohlin theorem of international trade. Perhaps most importantly, our general equilibrium model can address explicitly what Jones (2002) and others consider as one of the most important limitations of the preponderance of existing generalizations of the HO: the *ex ante* requirement, or assumption, that global production may only prevail within the diversification cone. By endogenizing the level of specialization in production, in the context of an inframarginal approach, our model facilitates the study of equilibria that may prevail beyond the cone of diversification, and identifies the precise parameter configurations

[12] Comparative statics of general equilibrium (CSGE) differ from comparative statics of decision (CSD), discussed earlier, in two important respects. First, in the case of interior solutions CSGE refer to *parameter driven* marginal comparative statics. Second, contrary to the usual implementation of CSD, CSGE also include relevant inframarginal comparative statics of general equilibrium.

that may lead to such equilibria. Relevant findings shed light on important dimensions of trade theory that have not yet been sufficiently explored in the relevant literature.

The remainder of this paper is organized as follows. The model, as well as the inframarginal framework of analysis upon which it relies are examined in the following section. Section II explores the ensuing comparative statics of general equilibrium, and concluding remarks are reserved for section III.

I. An Inframarginal Neo-Heckscher-Ohlin Model of Trade

In order to stay close to the standard formulation of the Heckscher-Ohlin theorem, as well as most relevant generalizations of this framework, including Kemp (1966); Jones (1967); Ferguson (1978); and Normal and Venables (1995), our model preserves a selection of common characteristics across these specifications. These include dimensions of order $2\times2\times2$: corresponding to 2 countries; 2 goods; and 2 inputs, as well as the assumptions that perfect competition prevails in both good and factor markets; that factors of production are fully employed; and that technology exhibits constant returns to scale.

However, unlike the original formulation of the HO, we allow (a) total factor productivity to differ across goods and countries; (b) capital to be internationally mobile[13]; (c) preferences to be represented in the analysis; (d) the existence of trade-

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[13] It should be noted that international capital mobility represents a key feature of relevant models in this area including Kemp (1966), Jones (1967), Jones and Ruffin (1975), Purvis (1972), Brecher and Diaz-Alejandro (1977), Ferguson (1978), Markusen and Melvin (1979), Bhagwati and Brecher (1980), and Brecher and Feenstra (1983). Ohlin (1933) provides an extensive discussion of the implications of international factor mobility. See in particular p. 325-372.

specific transaction costs that may assume different magnitudes for traded goods on the one hand, and traded capital, on the other; and, most importantly, (e) the possibility of specialization. Given the findings of Sonnenschein (1972, 1973), Debreu (1974), and Mantel (1974), the model proposed herein would be fit to investigate the full range of comparative statics of general equilibrium only in the presence of sufficient parametric structure. To satisfy relevant requirements we adopt Cobb-Douglas characterizations for both preferences as well as technology.

In the interest of succinct exposition, our analysis of the proposed model proceeds in two stages. In the first stage our model is solved in the case of autarky. Such an exercise facilitates two distinct objectives: it helps isolate the structure of key determinants of comparative advantage, and it illustrates how the inclusion of transaction costs in the analysis increases the spectrum of potentially optimum corner solutions. The autarkic solution is used as a point of departure, as well as reference, in the second stage in which we investigate the host of general equilibrium outcomes that may prevail in the presence of international trade.

A. *An Autarkic Perspective*

In this section we consider the case of two countries that are required to operate in Autarky due to, say, policy-driven impediments to trade. Each country *i* (*i*=1,2) is endowed with labor L_i and capital K_i, which can be used to produce two consumer goods: *x* and *y*. Within each country, factors of production are shared amongst consumers who are assumed *ex ante* identical.

The decision problem of a representative consumer in country *i* is summarized below:

$$\max_{x_i, y_i} U_i = x_i^{\theta} y_i^{1-\theta} \quad s.t.\ \ p_i x_i + y_i = w_i L_i + r_i K_i \tag{1}$$

where p_i is the relative price of good *x*, in terms of good *y*, θ represents a preference coefficient, and w_i and r_i collect the wage rate and rental of capital, respectively.

Assume that the production functions for *x* and *y* in country *i* are:

$$x_i = a_{ix} L_{ix}{}^{\alpha} K_{ix}{}^{1-\alpha}, \quad y_i = a_{iy} L_{iy}{}^{\beta} K_{iy}{}^{1-\beta} \tag{2}$$

where $a_{ih}\ (i = 1, 2; h = x, y)$ represents the total factor productivity coefficient of producing good *h* in country *i*.

Constrained by production technology, the representative firm producing *x* in country *i* maximizes its profit:

$$\max_{L_{ix}, K_{ix}} \pi_{ix} = p_i x_i - w_i L_{ix} - r_i K_{ix} = p a_{ix} L_{ix}{}^{\alpha} K_{ix}{}^{1-\alpha} - w_i L_{ix} - r_i K_{ix} \tag{3}$$

The corresponding decision problem for a firm producing *y* in the same country readily follows.

Using the first order conditions of the decision problems of representative firms producing *x* and *y* in country *i* we obtain:

$$\frac{K_{ix}}{L_{ix}} = \left[p_i \frac{a_{ix}}{a_{iy}} \left(\frac{\alpha}{\beta} \right)^{\beta} \left(\frac{1-\alpha}{1-\beta} \right)^{1-\beta} \right]^{\frac{1}{\alpha-\beta}} \tag{4}$$

$$\frac{K_{iy}}{L_{iy}} = \frac{\alpha(1-\beta)}{\beta(1-\alpha)} \frac{K_{ix}}{L_{ix}} \tag{5}$$

Without loss of generality, we assume $\alpha < \beta$ which is a necessary and sufficient condition that industries *x* and *y* are capital and labor intensive, respectively $\left(\text{i.e. } \frac{K_{ix}}{L_{ix}} > \frac{K_{iy}}{L_{iy}} \right)$.

Solving the general equilibrium problem in autarky requires a simultaneous solution of (4), (5), the first order conditions deriving from (1), and all relevant market clearing conditions for both factors of production and consumer goods. The resulting relative price of *x* that prevails in country *i* corresponds to:

$$p_i = \frac{a_{iy}}{a_{ix}}\left[\frac{L_i}{K_i}\frac{(1-\alpha)\theta+(1-\beta)(1-\theta)}{\alpha\theta+\beta(1-\theta)}\right]^{\beta-\alpha}\frac{\beta^{\beta}(1-\beta)^{1-\beta}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \tag{6}$$

If, similarly with the standard formulation of the HO, we assume that relative total factor productivities are identical in countries 1 and 2 $\left(\text{i.e. } a_{1x}/a_{1y} = a_{2x}/a_{2y}\right)$ then, given (6), $p_1 < p_2$ if and only if $K_1/L_1 > K_2/L_2$. In other words, comparative advantage is solely driven by factor abundance. This is, of course, the main result of the Heckscher-Ohlin theorem.

Alternatively, if the source of technological comparative advantage is not explicitly neutralized, then $p_1 < p_2$ if and only if

$$\left(a_{2y}a_{1x}/a_{2x}a_{1y}\right)^{1/(\beta-\alpha)}\left(K_1L_2/K_2L_1\right) > 1 \tag{7}$$

That is, the extent to which country 1 has an *overall* comparative advantage in the production of a particular good depends on the synergy between the degree of its relevant comparative technological advantage, $\left(a_{2y}a_{1x}/a_{2x}a_{1y}\right)$, and the scale of its comparative endowment advantage, $\left(K_1L_2/K_2L_1\right)$.

Rudimentary considerations based on expressions (6) and (7), such as those outlined above, illustrate that technical differences have the potential to play a firmly symmetric role to factor endowments in determining *net* comparative advantage – a point neglected by the standard HO formulation. More importantly, the preceding analysis illustrates that *in the absence of transaction costs and internationally mobile factors*, cross-country differences in relative prices that are consistent with a state of autarky, or comparable results that derive analogously from the utilization of a marginalist perspective, may very well be employed to infer the pattern of trade (that is, if in autarky $p_1 < p_2$ then, in the presence of the opportunity to engage in international trade, country 1 will export *x* and import *y*). However, in the presence of trade-specific transaction costs and internationally mobile factors autarkic prices lose any relevant informational content. To illustrate this point consider a case in which the very existence of trade-specific transaction costs counters any price advantages that may be driven by differences in technology, and international factor mobility neutralizes any comparative endowment advantage. An important implication that readily follows is that the addition of such elements in the analysis renders autarky a potentially optimum outcome (even in the absence of policy-driven impediments to trade), and enhances the likelihood of a host of other possible corner equilibria.

B. *Trade Structures in General Equilibrium*

The option to engage in international trade compounds the complexity of the framework outlined in the previous section. Aside from the possibility that optimum outcomes may require cross-country transfers of consumer goods or capital, we must also account for trade-specific transportation, policy-driven, and related costs. We incorporate all such outlays in the analysis by assuming that international trade in goods and factors incurs *iceberg* transaction costs – similar to those discussed by Samuelson (1952) and Normal and Venables (1995). Specifically, for each unit of consumer good *x* or *y* (capital) imported by country *i* a fraction $1-k_i$ $\left(1-t_i\right)$ disappears in transit due to costs of international exchange. While, in principle, $0 \leq t_i, k_i \leq 1$, we assume the prevalence of strict inequalities in an effort to facilitate relevant mathematical derivations.

Expanding on the findings of the previous section, it is clear that a comprehensive study of both interior as well as corner solutions compels the use of an inframarginal approach. Typically, the first step of inframarginal analysis requires the explicit identification of all *prime configurations*. These correspond to all possible profiles of production and consumption that may characterize individual agents within countries, and which are consistent with meaningful patterns of exchange (we consider such patterns to include Autarky – a pattern of exchange with zero flows). Subsequently, prime configurations are used to construct possible *market structures* representing feasible combinations of cross-country patters of production, consumption, and trade. In the case of our model, possible market structures and associated configurations are summarized in figure 1. This figure depicts a total of 25 structures, [a]-[c.2.iii], that are divided in five groupings (a), (b), (c1), (c2), (c). Structures that belong to the same grouping are characterized by identical output profiles and consumption choices – but different patterns of exchange in final goods and capital. For example, using a notation that designates production profiles in parentheses – where the first profile corresponds to country 1 – and employs subscripts to denote exports, consider structures $(xy)_{xK}(xy)_y$ and $(xy)_x(xy)_y$ corresponding to items (b.1.i) and (b.1.iii) of figure 1, respectively. In the case of both structures countries 1 and 2 uniformly produce both goods *x* and *y*, while country 1 exports good *x* and country 2 exports good *y*. However, while in structure (b.1.i) country 2 imports capital from country 1, in (b.1.iii) capital is not traded internationally.

In order to identify the precise configurations of the fifteen-dimensional parameter space $(\theta,\ \alpha,\ \beta,\ k_1,\ k_2,\ t_1,\ t_2,\ a_{1x},\ a_{2x},\ a_{1y},\ a_{2y},\ L_1,\ L_2,\ K_1,\ K_2)$ that lead to the various market structures of figure 1, we adopt a methodology recently proposed by Sun *et al.* (2003) and Sun (2003) for the identification of equilibria in the case of models with endogenous specialization. In the context of an inframarginal analysis of the framework developed herein, our implementation of this approach translates into the following series of steps.

First, an appropriate Kuhn-Tucker optimization framework is developed. This must account for the possibility of corner solutions by allowing appropriate combinations of relevant inequality constraints to bind. Subsequently, Kuhn-Tucker conditions are solved simultaneously in order to derive conditions (expressed in terms of relative prices) that demarcate the price vector's space in numerical subsets within which specific patterns of specialization (inherent in relevant prime configurations) maximize the indirect utility function of *ex ante* (but not necessarily, *ex post*) identical agents. The next step entails solving for the *local equilibrium* using marginal analysis in the context of the constraints implicit in any given structure. Finally, the conditions that each *structure-consistent local equilibrium* represents the *general equilibrium* are identified. This is accomplished by solving simultaneously the expressions corresponding to local *equilibrium prices* and the functions involving *relative prices* identified in the first step. These steps are repeated in the case of each of the 25 structures identified in figure 1.

Having provided a general outline of the methodology employed in our analysis, we proceed with the relevant implementation.

A representative consumer residing in country *i* faces the following decision problem:

$$\begin{aligned} &\underset{x_i, x_{ji}, y_i, y_{ji}}{Max} u_i = (x_i + k_i x_{ji})^{\theta} (y_i + k_i y_{ji})^{1-\theta} \\ &\text{s.t.} \quad p_{ix} x_i + p_{jx} x_{ji} + p_{iy} y_i + p_{jy} y_{ji} = w_i L_i + r_i K_i \\ &\qquad x_i \geq 0, x_{ji} \geq 0, y_i \geq 0, y_{ji} \geq 0 \\ &\quad \forall\, i, j = 1, 2; i \neq j \end{aligned} \tag{8}$$

where p_{ih} is the price of good h $\left[\forall h \in (x, y)\right]$ in country *i*; x_{ji} $\left(y_{ji}\right)$ represents the amount of good x $\left(y\right)$ delivered from country *j* to country *i*; and w_i (r_i) corresponds to the wage rate (normalized rental rate of capital, with $r_1 = 1$) in country *i*, respectively. The corresponding lagrangean function is given by:

$$\underset{x_i, x_{ji}, y_i, y_{ji}, \lambda, \boldsymbol{\mu}}{Max} \mathbb{Z}_{ci} = (x_i + k_i x_{ji})^{\theta} (y_i + k_i y_{ji})^{1-\theta} + \lambda (w_i L_i + r_i K_i - p_{ix} x_i - p_{jx} x_{ji} - p_{iy} y_i - p_{jy} y_{ji}) + \mu_1^i x_i + \mu_2^i x_{ji} + \mu_3^i y_i + \mu_4^i y_{ji} \quad (9)$$

where λ and $\boldsymbol{\mu}^i \equiv \left[\mu_m^i \right] \forall m = 1, \ldots, 4$ represent associated multipliers.

The resulting Kuhn-Tucker conditions are:

$$\theta (x_i + k_i x_{ji})^{\theta - 1} (y_i + k_i y_{ji})^{1-\theta} - \lambda p_{ix} + \mu_1^i = 0 \quad (10)$$

$$\theta k_i (x_i + k_i x_{ji})^{\theta - 1} (y_i + k_i y_{ji})^{1-\theta} - \lambda p_{jx} + \mu_2^i = 0 \quad (11)$$

$$(1-\theta)(x_i + k_i x_{ji})^{\theta} (y_i + k_i y_{ji})^{-\theta} - \lambda p_{iy} + \mu_3^i = 0 \quad (12)$$

$$(1-\theta) k_i (x_i + k_i x_{ji})^{\theta} (y_i + k_i y_{ji})^{-\theta} - \lambda p_{jy} + \mu_4^i = 0 \quad (13)$$

$$w_i L_i + r_i K_i - p_{ix} x_i - p_{jx} x_{ji} - p_{iy} y_i - p_{jy} y_{ji} = 0 \quad (14)$$

$$\mu_1^i \geq 0,\ \mu_2^i \geq 0,\ \mu_3^i \geq 0,\ \mu_4^i \geq 0 \quad (15)$$

$$\mu_1^i x_i = 0,\ \mu_2^i x_{ji} = 0,\ \mu_3^i y_i = 0,\ \mu_4^i y_{ji} = 0 \quad (16)$$

All meaningful combinations of zero and positive values that may be assumed by elements of vectors $\boldsymbol{\mu}^i \ \forall i \in (1,2)$ [14] (including all-positive vectors)[15] across the two countries generate a host of possible *n*-dimensional subsets of real values that may represent the price space (corresponding to *n* relevant prices) – each consistent with

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[14] It is important to note that the requirement of positive utility and convex preferences eliminates a host of possible combinations of zero and positive values that may be assumed by elements of $\boldsymbol{\mu}^i \ \forall i \in (1,2)$. For example, both x_i and x_{ji}, or y_i and y_{ji}, may not be equal to zero simultaneously.

[15] An all-zero $\boldsymbol{\mu}^i$ is not possible.

specific combinations of prime configurations corresponding to the various market structures.

By way of an example, consider the case of $\mu_1^1 = 0$, $\mu_2^1 > 0$, $\mu_3^1 = 0$, $\mu_4^1 = 0$, $\mu_1^2 > 0$, $\mu_2^2 = 0$, $\mu_3^2 = 0$, $\mu_4^2 > 0$.

Simultaneous solution of (10) and (11) in the case of country 1, given $\mu_1^1 = 0$ and $\mu_2^1 > 0$, renders

$$\frac{p_{1x}}{p_{2x}} < \frac{1}{k_1} \tag{17}$$

Correspondingly, given $\mu_1^2 > 0$ and $\mu_2^2 = 0$, (10) and (11) in the case of country 2, imply

$$\frac{p_{1x}}{p_{2x}} < k_2 \tag{18}$$

Similar solutions of (12) and (13) in the case of country 1, given $\mu_3^1 = 0$ and $\mu_4^1 = 0$, and in the case of country 2, given $\mu_3^2 = 0$ and $\mu_4^2 > 0$ result in

$$k_2 < \frac{p_{1y}}{p_{2y}} = \frac{1}{k_1} \tag{19}$$

Jointly, (17) - (19) suggest that the optimum choice requires that $x_1 > 0$, $y_1 > 0$, $x_{21} = 0$, $y_{21} > 0$, $x_2 = 0$, $y_2 > 0$, $x_{12} > 0$, $y_{12} = 0$.

Denote the amount of good *h* that is produced in country *i* as h_i^s, $\forall h \in (x, y)$. Given the possibility for international transfers of capital, the decision problem for a representative firm in country *i* assumes the following form:

$$\begin{aligned} \underset{K_{ih}, K_{jih}, L_{ih}, \eta_h^i}{Max} \mathbb{Z}_{fih} &= p_{ih}h_i^s - w_i L_{ih} - r_i K_{ih} - r_j K_{jih} \\ &= p_{ih} a_{ih} L_{ih}{}^{\delta} (K_{ih} + t_i K_{jih})^{1-\delta} - w_i L_{ih} - r_i K_{ih} - r_j K_{jih} + \eta_h^i K_{jih} \\ s.t. \quad & K_{jih} \geq 0 \\ & \delta \equiv \begin{cases} \alpha \text{ if } h_i^s = x_i^s \\ \beta \text{ if } h_i^s = y_i^s \end{cases} \end{aligned} \tag{20}$$

where K_{jih} corresponds to the amount of capital that is exported from country *j* to country *i* where it is employed in the production of final consumer good *h*.

The associated Kuhn-Tucker conditions are:

$$(1-\delta) p_{ih} a_{ih} L_{ih}{}^{\delta} (K_{ih} + t_i K_{jih})^{-\delta} - r_i = 0 \tag{21}$$

$$(1-\delta) t_i p_{ih} a_{ih} L_{ih}{}^{\delta} (K_{ih} + t_i K_{jih})^{-\delta} - r_j + \eta_h^i = 0 \tag{22}$$

$$\delta p_{ih} a_{ih} L_{ih}{}^{\delta-1} (K_{ih} + t_i K_{jih})^{1-\delta} - w_i = 0 \tag{23}$$

$$\eta_h^i \geq 0, \quad \eta_h^i K_{jih} = 0 \tag{24}$$

Assuming $\eta_h^i = 0$, equations (21) and (22) imply that

$$\frac{r_i}{r_j} = \frac{1}{t_i} \tag{25}$$

and the optimum decision requires that $K_{jih} > 0$.

Alternatively, if $\eta_h^i > 0$ then (21) and (22) require that

$$\frac{r_i}{r_j} < \frac{1}{t_i} \tag{26}$$

In this case, the optimum decision entails $K_{jix} = 0$.

Consider now the possibility that $\eta_h^1 > 0$ while $\eta_h^2 = 0$. Given (25), (26), and recalling the normalization $r_1 = 1$, this implies that

$$r_2 = \frac{1}{t_2} > t_1 \tag{27}$$

Hence, conditions (17), (18), (19), and (27) are necessary for structure x_{21}, y_{12}, x_2, $K_{21h} = 0$; K_{12h}, x_{12}, y_{21}, x_1, y_1, $y_2 > 0$ to prevail. This structure is denoted by $\left(xy\right)_{xK}\left(y\right)_y$ and is summarized in figure 1, panel c2.1.i. The corresponding Kuhn-Tucker conditions pertaining to the remaining possible trade structures of figure 1 are derived by considering alternative meaningful combinations of possible values that may be assumed by vector $\mathbf{g} \equiv \left[\boldsymbol{\mu}^1, \eta_h^1, \boldsymbol{\mu}^2, \eta_h^2\right]$.

The second step of the inframarginal methodology, in the context of the framework that was previously outlined, requires that we employ marginal analysis in order to solve for the local equilibrium corresponding to each of the identified structures. Continuing with the example that we have so far utilized, corresponding to structure $\left(xy\right)_{xK}\left(y\right)_y$, we first consider the representative consumer's decision problem in countries 1 and 2. Given the requirements of this structure summarized in (17)-(19), the Kuhn-Tucker consumer framework outlined in expressions (9)-(16) simplifies to

$$\underset{x_1,y_1,y_{21},\lambda}{Max}\ \mathbb{Z}_{c1} = (x_1)^{\theta}(y_1 + k_1 y_{21})^{1-\theta} + \lambda(w_1 L_1 + r_1 K_1 - p_{1x}x_1 - p_{1y}y_1 - p_{2y}y_{21}) \tag{28}$$

$$\underset{x_{12},y_2,\lambda}{Max}\ \mathbb{Z}_{c2} = (k_2 x_{12})^{\theta}(y_2)^{1-\theta} + \lambda(w_2 L_2 + r_2 K_2 - p_{1x}x_{12} - p_{2y}y_{21}) \tag{29}$$

The resulting first order conditions correspond to:

$$\theta(x_1)^{\theta-1}(y_1 + k_1 y_{21})^{1-\theta} = \lambda p_{1x} \tag{30}$$

$$(1-\theta)(x_1)^{\theta}(y_1 + k_1 y_{21})^{-\theta} = \lambda p_{1y} \tag{31}$$

$$(1-\theta)k_1(x_1)^{\theta}(y_1 + k_1 y_{21})^{-\theta} = \lambda_1 p_{2y} \tag{32}$$

$$w_1 L_1 + r_1 K_1 - p_{1x}x_1 - p_{1y}y_1 - p_{2y}y_{21} = 0 \tag{33}$$

$$\theta k_2 (k_2 x_{12})^{\theta-1}(y_2)^{1-\theta} = \lambda p_{1x} \tag{34}$$

$$(1-\theta)(k_2 x_{12})^{\theta-1}(y_2)^{-\theta} = \lambda p_{2y} \tag{35}$$

$$w_2 L_2 + r_2 K_2 - p_{1x} x_{12} - p_{2y} y_{21} = 0 \tag{36}$$

Similarly, given (25)-(27), the decision problem of representative firms producing *x* and/or *y* in countries 1 and 2 that is outlined in (20)-(24), simplifies to

$$\underset{K_{1x},L_{1x}}{Max}\, \mathbb{Z}_{f1x} = p_{1x} a_{1x} L_{1x}{}^{\alpha}(K_{1x})^{1-\alpha} - w_1 L_{1x} - r_1 K_{1x} \tag{37}$$

$$\underset{K_{1y},L_{1y}}{Max}\, \mathbb{Z}_{f1y} = p_{1y} a_{1y} L_{1y}{}^{\beta}(K_{1y})^{1-\beta} - w_1 L_{1y} - r_1 K_{1y} \tag{38}$$

$$\underset{K_{2y},K_{12y},L_{2y}}{Max}\, \mathbb{Z}_{f2y} = p_{2y} a_{2y} L_{2y}{}^{\beta}(K_{2y} + t_2 K_{12y})^{1-\beta} - w_2 L_{2y} - r_2 K_{2y} - r_1 K_{12y} \tag{39}$$

The corresponding first order conditions of the three lagrangean expressions outlined above are:

$$\alpha p_{1x} a_{1x} L_{1x}{}^{\alpha-1}(K_{1x})^{1-\alpha} = w_1 \tag{40}$$

$$(1-\alpha) p_{1x} a_{1x} L_{1x}{}^{\alpha}(K_{1x})^{-\alpha} = r_1 \tag{41}$$

$$\beta p_{1y} a_{1y} L_{1y}{}^{\beta-1}(K_{1y})^{1-\beta} = w_1 \tag{42}$$

$$(1-\beta) p_{1y} a_{1y} L_{1y}{}^{\beta}(K_{1y})^{-\beta} = r_1 \tag{43}$$

$$\beta p_{2y} a_{2y} L_{2y}{}^{\beta-1}(K_{2y} + t_2 K_{12y})^{1-\beta} = w_2 \tag{44}$$

$$(1-\beta) p_{2y} a_{2y} L_{2y}{}^{\beta}(K_{2y} + t_2 K_{12y})^{-\beta} = r_2 \tag{45}$$

$$(1-\beta) t_2 p_{2y} a_{2y} L_{2y}{}^{\beta}(K_{2y} + t_2 K_{12y})^{-\beta} = r_1 \tag{46}$$

Finally, we consider the market clearing conditions relevant to the five decision problems given by (28); (29); (37); (38); (39) that are relevant to the structure at hand. These are:

$$L_{1x} + L_{1y} = L_1 \tag{47}$$

$$L_{2x} + L_{2y} = L_2 \tag{48}$$

$$K_{1x} + K_{1y} + K_{12y} + K_{2y} = K_1 + K_2 \tag{49}$$

Joint solution of equations (30)-(36) and (40)-(49) generates all relevant local general equilibrium results: prices, quantities, etc. These are summarized below[16]:

$$w_1 = F\,\frac{(a_{1y})^{\frac{1}{\beta}} t_2^{\frac{\beta-1}{\beta}} (K_1 + \frac{1}{t_2} K_2)}{(a_{1y})^{\frac{1}{\beta}} t_2^{\frac{\beta-1}{\beta}} L_1 + (k_1 a_{2y})^{\frac{1}{\beta}} L_2} \qquad (50)$$

$$w_2 = F\,\frac{(k_1 a_{2y})^{\frac{1}{\beta}} (K_1 + \frac{1}{t_2} K_2)}{(a_{1y})^{\frac{1}{\beta}} t_2^{\frac{\beta-1}{\beta}} L_1 + (k_1 a_{2y})^{\frac{1}{\beta}} L_2} \qquad (51)$$

$$p_{1x} = \alpha^{-\alpha}(1-\alpha)^{\alpha-1} a_{1x}^{-1} F^{\alpha}\,\frac{(a_{1y})^{\frac{\alpha}{\beta}} t_2^{\frac{\alpha(\beta-1)}{\beta}} (K_1 + \frac{1}{t_2} K_2)^{\alpha}}{\left[(a_{1y})^{\frac{1}{\beta}} t_2^{\frac{\beta-1}{\beta}} L_1 + (k_1 a_{2y})^{\frac{1}{\beta}} L_2\right]^{\alpha}} \qquad (52)$$

$$p_{2x} = \alpha^{-\alpha}(1-\alpha)^{\alpha-1} a_{2x}^{-1} F^{\alpha}\,\frac{(k_1 a_{2y})^{\frac{\alpha}{\beta}} (K_1 + \frac{1}{t_2} K_2)^{\alpha}}{\left[(a_{1y})^{\frac{1}{\beta}} t_2^{\frac{\beta-1}{\beta}} L_1 + (k_1 a_{2y})^{\frac{1}{\beta}} L_2\right]^{\alpha}} \quad \text{(shadow price)} \qquad (53)$$

$$p_{1y} = \beta^{-\beta}(1-\beta)^{\beta-1} a_{1y}^{-1} F^{\beta}\,\frac{(a_{1y}) t_2^{\beta-1} (K_1 + \frac{1}{t_2} K_2)^{\beta}}{\left[(a_{1y})^{\frac{1}{\beta}} t_2^{\frac{\beta-1}{\beta}} L_1 + (k_1 a_{2y})^{\frac{1}{\beta}} L_2\right]^{\beta}} \qquad (54)$$

$$p_{2y} = \beta^{-\beta}(1-\beta)^{\beta-1} a_{2y}^{-1} F^{\beta}\,\frac{(k_1 a_{2y})\ (K_1 + \frac{1}{t_2} K_2)^{\beta}}{\left[(a_{1y})^{\frac{1}{\beta}} t_2^{\frac{\beta-1}{\beta}} L_1 + (k_1 a_{2y})^{\frac{1}{\beta}} L_2\right]^{\beta}} \qquad (55)$$

$$L_{1x} = \frac{\alpha\theta}{\alpha\theta + \beta(1-\theta)}\left[L_1 + \left(\frac{k_1 a_{2y}}{a_{1y}}\right)^{\frac{1}{\beta}} t_2^{\frac{1-\beta}{\beta}} L_2\right] \qquad (56)$$

$$L_{1y} = \frac{\beta(1-\theta)}{\alpha\theta + \beta(1-\theta)} L_1 - \frac{\alpha\theta}{\alpha\theta + \beta(1-\theta)}\left(\frac{k_1 a_{2y}}{a_{1y}}\right)^{\frac{1}{\beta}} t_2^{\frac{1-\beta}{\beta}} L_2 \qquad (57)$$

[16] In the case of domestic prices corresponding to goods not produced by a given country we employ the corresponding local equilibrium shadow prices.

$$K_{1x} = \frac{\alpha\theta}{1-\alpha\theta-\beta(1-\theta)}\left(K_1 + \frac{1}{t_2}K_2\right) \tag{58}$$

$$K_{1y} = \frac{(1-\beta)}{\beta}L_{1y}w_1 \tag{59}$$

$$K_{2y} + t_2 K_{12y} = \frac{(1-\beta)}{\beta}L_2 w_2 \tag{60}$$

$$X_1 = \theta p_{1x}^{-1}(w_1 L_1 + K_1) \tag{61}$$

$$X_2 = \theta p_{1x}^{-1}(w_2 L_2 + \frac{1}{t_2}K_2) \tag{62}$$

$$Y_1 = (1-\theta)\, p_{1y}^{-1}(w_1 L_1 + K_1) \tag{63}$$

$$Y_2 = (1-\theta)\, p_{2y}^{-1}(w_2 L_2 + \frac{1}{t_2}K_2) \tag{64}$$

$$u_1 = X_1^{\theta} Y_1^{1-\theta}, \quad u_2 = X_2^{\theta} Y_2^{1-\theta} \tag{65}$$

where $F = \frac{\alpha\theta+\beta(1-\theta)}{1-\alpha\theta-\beta(1-\theta)}$, and X_i and Y_i corresponds to the total demand for goods *x* and *y* by consumers residing in country *i*, respectively. Total demand includes quantities that may have been purchased from foreign markets. Specifically, $X_1 = x_1$, $Y_1 = y_1 + k_1 y_{21}$, $X_2 = x_{12}k_2$, and $Y_2 = y_2$.

Subsequently, equilibrium prices given by (52)-(55) are inserted in constraints (17)-(19). The final step of this approach entails using the resulting expressions, in conjunction with (27), as well as endowment relevant inequalities that compare individual industry factor usage with economy-wide endowments (i.e. $L_{1x}, L_{1y} < L_1$, $K_{1x} + K_{1y} < K_1$, $L_{2y} = L_2, K_{2x}, K_{2y} < K_2$) to infer the subset of values of the fifteen dimensional parameter space within which structure $(xy)_{xK}(y)_y$ represents the general equilibrium outcome. This parameter value subset is defined by the following expressions:

$$\beta > \alpha \tag{66}$$

$$\left(\frac{t_1}{t_2}\right)^{\beta-\alpha} < \left(k_1 k_2\right)^{\alpha+\beta} \tag{67}$$

$$\left(\frac{a_{1y}}{a_{2y}}\right)^{\alpha}\left(\frac{a_{2x}}{a_{1x}}\right)^{\beta} < t_1^{\beta-\alpha} k_1^{\alpha} k_2^{\beta} \tag{68}$$

$$\frac{L_1}{L_2} > c_1 = \frac{\alpha\theta}{\beta(1-\theta)}\left(\frac{k_1 a_{2y}}{a_{1y}}\right)^{\frac{1}{\beta}} t_1^{\frac{\beta-1}{\beta}} \tag{69}$$

$$\frac{K_1}{K_2} > d_2 = \frac{\beta\left(\frac{a_{1y}}{k_1 a_{2y}}\right)^{\frac{1}{\beta}} t_2^{\frac{\beta-1}{\beta}} \frac{L_1}{L_2} + \beta - (1-\beta)F}{t_2(1-\beta)F} \tag{70}$$

Implementation of the procedure outlined above in the case of the remaining 24 structures of figure 1 generates a comprehensive demarcation of the fifteen dimensional parameter space, given by $(\theta,\ \alpha,\ \beta,\ k_1,\ k_2,\ t_1,\ t_2,\ a_{1x},\ a_{2x},\ a_{1y},\ a_{2y},$ $L_1,\ L_2,\ K_1,\ K_2)$, in a host of *exclusive* parameter value subsets. Within each such parameter value subset the local equilibrium of the corresponding structure represents the general equilibrium solution of the economy described herein.

II. Inframarginal Comparative Statics of General Equilibrium

Figure 2 reports all *non-empty* parameter value subsets together with their associated market structures corresponding to the 25 general equilibrium models that are examined in this study. Each parameter value subset is defined in terms of relative population size $\left(L_1/L_2\right)$, relative capital endowment $\left(K_1/K_2\right)$, relative tastes

$\left(\theta/(1-\theta)\right)$, the degree of exogenous (technology driven) comparative advantage in producing goods *x* and *y* across the two countries $\left(a_{1x}/a_{2x}, a_{1y}/a_{2y}\right)$, labor elasticities $\left(\alpha, \beta\right)$, and, finally, trade-specific transaction efficiency coefficients for consumption goods and capital $\left(k_1, k_2, t_1, t_2\right)$.

The analytical framework developed herein is fit to examine all relevant comparative statics of general equilibrium. Small changes of prices and quantities in response to changes in parameter values within each structure – corresponding to marginal, or interior, comparative statics – may be easily investigated using equations such as those given by (50)-(65) in the case of structure $\left(xy\right)_{xK}\left(y\right)_y$. Alternatively, our framework may be used to study a host of inframarginal comparative statics relevant to the specification of the adopted model. These relate to inframarginal changes, or discontinuous jumps, of trade-production patterns across structures, as parameters reach certain critical values – or as parameter values shift between parameter value subsets that demarcate the relevant structures. In what follows we concentrate on the latter set of results.

Casual inspection of figure 2 suggests that transaction efficiency coefficients feature prominently in all conditions, including all overarching conditions[17], corresponding to the various structures, or sets of structures. This emphasizes the key role of transaction costs in preventing international equalization of prices. It readily follows that, as predicted in section I.A, if such costs are significant, autarky may represent a potentially optimum outcome even in the absence of policy-driven restrictions to trade. To illustrate this result – a useful point of reference – in the

[17] Overarching conditions correspond to expressions that lead the hierarchical structures of the various panels of figure 2.

context of the results outlined in figure 2, consider a rather extreme scenario in which the vector of transaction efficiency coefficients corresponding to imports of consumption goods, given by $\mathbf{k} \equiv [k_1, k_2]$, converges to the positive orthant of the immediate neighborhood of zero. In this case the following inequality necessarily holds for all reasonable values that may be assumed by all remaining parameters in our model:

$$t_2^{\alpha-\beta} k_1^{\alpha} k_2^{\beta} < \left(\frac{a_{1y}}{a_{2y}}\right)^{\alpha} \left(\frac{a_{2x}}{a_{1x}}\right)^{\beta} < t_2^{\alpha-\beta} \left(\frac{1}{k_1}\right)^{\beta} \left(\frac{1}{k_2}\right)^{\alpha} \tag{71}$$

Hence, as illustrated in figure 2d, under such circumstances autarky is bound to prevail.

It is important to note that fragments of all overarching conditions of figure 2, such as (71), that incorporate measures of transaction costs associated with trade in final consumption goods are "weighted" by transaction efficiency coefficients corresponding to trade in capital - our mobile factor. This is rather intuitive as our results aim to capture the extent to which the synergy between $\mathbf{k} \equiv [k_1, k_2]$ and $\mathbf{t} \equiv [t_1, t_2]$ has the *potential* to facilitate substitution of traded goods for traded capital. By way of clarification consider that, given differential factor endowments across countries – but other things equal, as **k** decreases, it is natural to expect that demand for foreign, internationally mobile, factors of production increases. However, the precise extent to which such decreases in **k** will lead to substitution of traded goods for traded factors will depend on the relative values of factor-specific transaction efficiency coefficients $\mathbf{t} \equiv [t_1, t_2]$. Still, there are certain notional boundaries beyond which values assumed by **t** cease to "regulate" the degree to which changes in **k** have the potential to lead to such substitutions. To illustrate this point return to the example discussed above in which **k** converges to zero, and assume that country 1 is capital abundant. It may be noted from expression (71) that, under such extreme circumstances, even significant increases in middle-range values assumed by t_2 [18] may not dislodge the economy from a state of autarky. The reason for this is, of course, readily evident: demand for foreign factors is not a sufficient condition for factor trade to prevail. The realization of such trade also requires that payment dispensed to the exporter of factors, in the form of final goods, is not fully consumed by relevant transaction costs – that is, the relevant payment does not disappear "in transit".

Arguments regarding the *potential* for factor trade to substitute for consumer good trade notwithstanding, our model suggests that the relationship between trade in factors and trade in goods may very well be complementary. Relevant examples in figure 2 abound – and include structures such as $(x)_{xK}(y)_y$ (see figure 2, panel 2a) in which a country may export simultaneously both the capital intensive commodity, as well as raw capital. This result stands in stark contrast with the findings of generalizations of the Heckscher-Ohlin that assume identical production technology across countries, such as Mundell (1957), but is entirely consistent with relevant generalizations such as those of Purvis (1972) and Markusen (1983) that, similarly to the model adopted herein, allow for comparative technological advantage.

The relevance of differential technologies in enabling complementarities of the nature discussed above is clarified in subsequent discussion that examines our model's determinants of trade in final commodities. At this juncture we concentrate on the determinant of trade in capital that, perhaps predictably, appears to rely solely

[18] The parameter value subsets of figure 2 require condition (67) to hold. In the context of this constraint, assume that as $\mathbf{k} \to \mathbf{0}'$, $t_1 \to 0$ and $0 \ll t_2 \leq 1$.

on cross-country relative abundance of capital. Specifically, if the difference in capital endowment between the two countries is sufficiently small (i.e. $d_1 < K_1/K_2 < d_2$, $d_3 < K_1/K_2 < d_4$, $d_5 < K_1/K_2 < d_6$, $d_7 < K_1/K_2 < d_2$, $d_9 < K_1/K_2 < d_4$, $d_{11} < K_1/K_2 < d_6$, $d_7 < K_1/K_2 < d_8$, $d_9 < K_1/K_2 < d_{10}$, $d_{11} < K_1/K_2 < d_{12}$), then capital is not traded. However, if this difference is sufficiently large (i.e. $K_1/K_2 < d_1$, d_3, d_5, d_7, d_9, d_{11}; $K_1/K_2 > d_2$, d_4, d_6, d_8, d_{10}, d_{12}), then the country with the larger endowment will export capital. This result is summarized in the following proposition:

PROPOSITION 1: *Trade flow in capital is determined by relative endowment size of capital. The decision to export capital is inframarginal and discontinuous in the sense that country i will export capital to country j only when its relative capital endowment exceeds a critical value expressed in terms of the remaining parameters of the model.*

Our earlier discussion regarding conditions that may lead to autarky sets the scene for the implementation of a dynamic perspective that may be employed in efforts to investigate the evolution of production patterns and associated trade networks over time. Consider for example a scenario in which, at a given point in time, transaction costs relevant to trade in final consumption goods are sufficiently high so that the following expression of figure 2b holds, and autarky prevails.

$$t_1^{\beta-\alpha} k_1^{\alpha} k_2^{\beta} < \left(\frac{a_{1y}}{a_{2y}}\right)^{\alpha} \left(\frac{a_{2x}}{a_{1x}}\right)^{\beta} < t_1^{\beta-\alpha} \left(\frac{1}{k_1}\right)^{\beta} \left(\frac{1}{k_2}\right)^{\alpha} \tag{72}$$

Now assume that, over time, changes in, say, prevailing institutional circumstances lead to improvements in transaction efficiency, that generate a sufficiently large increase in the value of elements of **k** so that:

$$\left(\frac{a_{1y}}{a_{2y}}\right)^{\alpha}\left(\frac{a_{2x}}{a_{1x}}\right)^{\beta} < t_1^{\beta-\alpha}k_1^{\alpha}k_2^{\beta}; \quad \frac{L_1}{L_2} > c_1 = \frac{\alpha\theta}{\beta(1-\theta)}\left(\frac{k_1 a_{2y}}{a_{1y}}\right)^{\frac{1}{\beta}} t_1^{\frac{\beta-1}{\beta}} \tag{73}$$

As it may be noted from figure 2a, possible general equilibrium structures that may prevail under (73) are: $(xy)_x(y)_{yK}$, $(xy)_x(y)_y$, and $(xy)_{xK}(y)_y$. Consider now, a further increase in transaction efficiency over time that leads to:

$$\begin{gathered}\left(\frac{a_{1y}}{a_{2y}}\right)^{\alpha}\left(\frac{a_{2x}}{a_{1x}}\right)^{\beta} < t_1^{\beta-\alpha}k_1^{\alpha}k_2^{\beta}; \\ c_2 = \frac{\alpha\theta}{\beta(1-\theta)}\left(\frac{a_{2x}}{k_2 a_{1x}}\right)^{\frac{1}{\alpha}} t_1^{\frac{1-\alpha}{\alpha}} < \frac{L_1}{L_2} < c_1 = \frac{\alpha\theta}{\beta(1-\theta)}\left(\frac{k_1 a_{2y}}{a_{1y}}\right)^{\frac{1}{\beta}} t_1^{\frac{\beta-1}{\beta}}\end{gathered} \tag{74}$$

In this case possible trade structures correspond to: $(x)_x(y)_{yK}$, $(x)_x(y)_y$, and $(x)_{xK}(y)_y$. A host of similar examples may be observed in the context of the remaining inframarginal comparative statics of figure 2.

Reflection on these results renders an important conclusion. It suggests that, other things equal, subsequent waves of sufficiently large increases in transaction efficiency have the potential to cause the general equilibrium to jump from autarky to partial division of labor [see structures c2.1.i, c2.1.ii, and c2.1.iii of figure 1 consistent with (73)], and from partial division of labor to complete division of labor [see structures c.1.i, c.1.ii, and c.1.iii of figure 1 consistent with (74)]. Hence, transaction efficiency has the potential to play an instrumental role in determining the size of the market, and the size of the market, in turn, determines the equilibrium level of the division of labor. This finding partly addresses Hendrik Houthakker's (1956) call for a formal treatment of Adam Smith's (1776) well known conjecture that the division of labor is limited by the extent of the market, and that the extent of the market is determined by trading efficiency (see in particular chapters 1, 2 and 3) [19]. The only exception to the nominated regularity relates to advances in transaction efficiency that may lead to movements from partial division of labor toward the structures corresponding to complete diversification of figures 2c and 2e. As it may be readily noted from the relevant conditions, this is due to fact that, given a sufficiently low transaction cost efficiency coefficient corresponding to trade in capital, an increase in transaction efficiency corresponding to consumer goods will encourage substitution from trade in capital, to an expansive trade in both final goods that – given relevant parameter value subsets – is consistent with complete diversification. These findings are summarized below:

PROPOSITION 2: *With the exception of possible movements 'into' the diversification cone, sufficiently large improvements in transaction efficiency corresponding to trade in final consumer goods will lead to increases in cross country levels of division of labor*.

[19] It is interesting to note that, in our model, an increase in individual transaction efficiency coefficients can lead to the seemingly paradoxical result of a larger *realized* total transaction cost. This possibility arises from the fact that an increase in transaction efficiency can generate a discontinuous "jump" of the prevalent general equilibrium structure from one that is defined by a low level of division of labor and relative self-sustenance, to another that is characterized by a comparatively high level of division of labor and extensive trade – thus, increasing the total number of transactions. This result may accommodate empirical findings by Wallis and North (1986) suggesting that as transaction efficiency in the U.S. improved over time, the income share of transaction costs increased.

Findings outlined in this proposition provide a tangible mechanism that has the potential to set in motion the evolution of trade networks which is an area that has recently attracted considerable interest in the literature[20]. In addition, these findings illustrate the relevance of treating specialization endogenously – an objective implicitly trivialized in the literature by an excessive preoccupation with production patterns that are consistent with complete diversification[21].

Such interest in the diversification cone has been remarkably resilient, – and this despite an ongoing debate regarding the plausibility of diversified production in the context of models with the key characteristics of the framework developed herein: $2 \times 2 \times 2$ dimensions, different technologies and factor endowments across countries, and internationally mobile capital. On one side of this debate, Ronal Jones employed such a model to advocate that production within the cone *is either logically impossible, or extremely unlikely* (See Jones, 1967; Jones and Ruffin, 1975; and Ruffin, 1984, p. 269). On the other, the retractors have vigorously contested this claim (see Inada and Kemp, 1969; Chipman, 1971; Uekawa, 1971; and Brecher and Feenstra, 1983). Yet, curiously, the relevant literature has not produced models that treat the matter of specialization endogenously.

The analytical framework developed herein eradicates this omission. Hence, contrary to competing characterizations, our model is fit to generate necessary conditions *that only involve exogenous parameters* for the prevalence of global production within the diversification cone, thereby addressing explicitly the plausibility of this outcome in a general equilibrium setting.

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[20] See Gereffi (1999) for a relevant discussion.

[21] For a relevant discussion see Jones (2002).

Using information provided in figure 2, the host of independently necessary, conditions, or sets of conditions, for the prevalence of a global equilibrium within the cone of diversification are outlined below:

$$\left(\frac{a_{1y}}{a_{2y}}\right)^{\alpha}\left(\frac{a_{2x}}{a_{1x}}\right)^{\beta}=t_1^{\beta-\alpha}k_1^{\alpha}k_2^{\beta} \tag{75}$$

$$\left(\frac{a_{1y}}{a_{2y}}\right)^{\alpha}\left(\frac{a_{2x}}{a_{1x}}\right)^{\beta}=t_1^{\beta-\alpha}\left(\frac{1}{k_1}\right)^{\beta}\left(\frac{1}{k_2}\right)^{\alpha} \tag{76}$$

$$\begin{gathered} t_1^{\beta-\alpha}\left(\frac{1}{k_1}\right)^{\beta}\left(\frac{1}{k_2}\right)^{\alpha}<\left(\frac{a_{1y}}{a_{2y}}\right)^{\alpha}\left(\frac{a_{2x}}{a_{1x}}\right)^{\beta}<t_2^{\alpha-\beta}k_1^{\alpha}k_2^{\beta}; \\ c_6<\frac{L_1}{L_2}<c_5; \quad d_9<\frac{K_1}{K_2}<d_4 \end{gathered} \tag{77}$$

$$\left(\frac{a_{1y}}{a_{2y}}\right)^{\alpha}\left(\frac{a_{2x}}{a_{1x}}\right)^{\beta}=t_2^{\alpha-\beta}k_1^{\alpha}k_2^{\beta} \tag{78}$$

$$\left(\frac{a_{1y}}{a_{2y}}\right)^{\alpha}\left(\frac{a_{2x}}{a_{1x}}\right)^{\beta}=t_2^{\alpha-\beta}\left(\frac{1}{k_1}\right)^{\beta}\left(\frac{1}{k_2}\right)^{\alpha} \tag{79}$$

$$\begin{gathered} \left(\frac{a_{1y}}{a_{2y}}\right)^{\alpha}\left(\frac{a_{2x}}{a_{1x}}\right)^{\beta}>t_2^{\alpha-\beta}\left(\frac{1}{k_1}\right)^{\beta}\left(\frac{1}{k_2}\right)^{\alpha}; \\ c_4<\frac{L_1}{L_2}<c_3; \quad d_9<\frac{K_1}{K_2}<d_{10} \end{gathered} \tag{80}$$

Given, $c_1=\frac{\alpha\theta}{\beta(1-\theta)}\left(\frac{k_1a_{2y}}{a_{1y}}\right)^{\frac{1}{\beta}}t_1^{\frac{\beta-1}{\beta}}$, $c_2=\frac{\alpha\theta}{\beta(1-\theta)}\left(\frac{a_{2x}}{k_2a_{1x}}\right)^{\frac{1}{\alpha}}t_1^{\frac{1-\alpha}{\alpha}}$,

$c_3=\frac{\beta(1-\theta)}{\alpha\theta}\left(\frac{k_1a_{2x}}{a_{1x}}\right)^{\frac{1}{\alpha}}t_2^{\frac{\alpha-1}{\alpha}}$, $c_4=\frac{\beta(1-\theta)}{\alpha\theta}\left(\frac{a_{1y}}{k_1a_{2y}}\right)^{\frac{1}{\beta}}t_2^{\frac{1-\beta}{\beta}}$, $c_5=\max\{c_1,c_3\}$,

$c_6=\min\{c_2,c_4\}$, $d_4=\frac{(1-\alpha)\theta}{t_2(1-\theta)(1-\beta)}$, $d_9=\frac{(1-\theta)(1-\beta)t_1}{(1-\alpha)\theta}$, and $d_{10}=\frac{(1-\theta)(1-\beta)}{t_2(1-\alpha)\theta}$.

Out of the six possibilities corresponding to conditions (75)-(80), four, (75);

(76); (78); and (79), represent exceptional, *razor edge*, cases that are unlikely to prevail for any prolonged period of time, if at all. In addition, the remaining two conditions, given by (77) and (80), represent *middle path* solutions in the hierarchical framework of figure 2. As such, realization of these conditions not only imposes restrictions on the degree of relative comparative technological advantage across the two countries, but also requires that L_1/L_2 and K_1/K_2 *simultaneously* assume values within the boundaries given by either (c_6, c_5) and (d_9, d_4), or (c_4, c_3) and (d_9, d_{10}), respectively. In an effort to illustrate the relative "width" of *middle path* boundaries as well as the nature of *razor edge* restrictions, we undertake two common assumptions in this line of research (see Jones 1967, Inada and Kemp 1969, Purvis 1972, Ferguson 1978, and Brecher and Feenstra 1983): that trade specific transaction costs are nonexistent (i.e. $k_1, k_2, t_1, t_2 \to 1$), and that technology *is not* identical across the two countries. Under these assumptions, prevalence of conditions (75), (76), (77), (78), and (79) is *logically impossible*, and the diversification cone may only prevail under the host of middle path restrictions outlined in (80)[22].

This is a startling result! Put simply, it suggests that the general equilibrium structures that prevail within the cone of diversification are rather unlikely to prevail. For this reason, far from representing the most relevant structures deserving the near-

[22] If we further assume that, other things equal, consumers are indifferent between an increase in their consumption of *x* or *y* (i.e. $\theta = 0.5$), and, though $\beta > \alpha$, $\alpha \to 0.5^-$ and $\beta \to 0.5^+$, then even if we ignore restrictions nominated by (80) pertaining to cross-country relative labor endowments, and those that involve total factor productivity coefficients, production within the diversification cone is impossible as it would require $1 < K_1/K_2 < 1$.

exclusive attention of studies that generalize the HO, solutions that prevail within the cone are exceptional, and hence of relatively little interest. From an intuitive point of view this result should not be surprising. Indeed, a mere superficial overview of the global economy suggests that specialization in traded commodities is, rather than the exception, an almost ubiquitous feature of cross-country production – a result echoed by the empirical findings of Davis and Weinstein (2001), as well as Schott (2003).

Still, our results contradict diametrically the findings of certain theoretical studies in this area, including Inada and Kemp (1969), Chipman (1971), Uekawa (1971), and Brecher and Feenstra (1983). Such differences largely derive from the fact that, *contrary to these authors*, we have adopted a parametric inframarginal methodology that facilitates the implementation of a general equilibrium approach in determining the optimum pattern of production. For this reason, our model accounts for elements or dimensions that have not received sufficient attention in such studies. For example, contrary to Brecher and Feenstra (1983) who confine their investigation to "…'small'…differences in technology between countries" (p. 335), our model facilitates the study of all relevant possibilities. And as we show, differences in technology play a key role in determining the likelihood of diversified production (indeed, as it may be noted from figures 2b and 2d, in the absence of transaction costs and technological differences, production within the diversification cone is guaranteed!).

Also, contrary to the preponderance of the remaining studies, such as, say, Inada and Kemp (1969), who rely on the *existence* of the "Chipman flat"[23] to argue for the

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[23] Given convex technologies and internationally mobile capital, Chipman (1971) has shown that the global production possibilities frontier incorporates a flat section (the "Chipman flat") where cross-country production may be diversified.

likelihood of diversification, our model extends the analysis to whether a global equilibrium may actually prevail on the *flat* – an outcome that depends not only on the flat's existence, but also on the relative values of a host of relevant exogenous variables. Furthermore, unlike such studies that rely on a *static* notion of the "Chipman flat", our model determines endogenously its location, size, and slope – characteristics that depend not only on the relative size of cross country total-factor productivity coefficients, but also on the global distribution of labor endowments – our immobile factor. Hence, in the context of the inframarginal comparative statics outlined in figure 2, a "movement" into the cone (that may derive from, say, relative factor endowments "jumping" discontinuously across the parameter value subsets that demarcate the various patterns of specialization) would signify a convergence of two distinct sets of conditions: those that render production within the cone technically possible, and those that ensure that diversified production is globally optimum.

Turning our attention to the determinants of trade flows we note that the most striking result of figure 2 pertains to the patterns of trade deriving from the overarching conditions of panels 2a and 2e given by:

$$\left(\frac{a_{1y}}{a_{2y}}\right)^{\alpha}\left(\frac{a_{2x}}{a_{1x}}\right)^{\beta} < t_1^{\beta-\alpha}k_1^{\alpha}k_2^{\beta} \tag{81}$$

$$\left(\frac{a_{1y}}{a_{2y}}\right)^{\alpha}\left(\frac{a_{2x}}{a_{1x}}\right)^{\beta} > t_2^{\alpha-\beta}\left(\frac{1}{k_1}\right)^{\beta}\left(\frac{1}{k_2}\right)^{\alpha} \tag{82}$$

As it may be noted from the relevant panels, given (81), country 1 will export *x* and import *y*; and, given (82), country 1 will export *y* and import *x* *irrespectively* of the particular profile of specialization, and distribution of relative factor endowments. That is, cross-country technological differences determine the pattern of trade. Dividing both sides of these inequalities by the expressions that appear on the right of the inequality sign in each case, will transform the expression on the left of the sign, originally given by $\left(a_{1y}/a_{2y}\right)^{\alpha}\left(a_{2x}/a_{1x}\right)^{\beta}$, from an index of *factor intensity-adjusted comparative technological advantage* to a corresponding index of *factor intensity transaction efficiency-adjusted comparative technological advantage* (henceforth "intensity-efficiency-adjusted comparative technological advantage"). In this light the relevant finding is outlined in the following proposition.

PROPOSITION 3: *When intensity-efficiency-adjusted total factor productivity coefficients confer clear comparative (technological) advantage, relative factor endowments are irrelevant for the purpose of predicting trade flows. Trade flows are solely determined by intensity-efficiency-adjusted comparative technological advantage.*

In an effort to shed light on the implications of this proposition, we consider the possibility that transaction efficiency coefficients converge to the positive orthant of the immediate neighborhood of unity (i.e. $\mathbf{k},\mathbf{t}\rightarrow[1,1]'$) in the presence of differences in technology and, possibly, but not necessarily, differences in factor endowments across countries. Given these assumptions, conditions (81) and (82) would necessarily hold, and the value of the right hand side of these inequalities would correspond to unity. Hence, the modified edition of (81) corresponds to $\left(a_{1y}/a_{2y}\right)^{\alpha}\left(a_{2x}/a_{1x}\right)^{\beta}<1$ and gives rise to structures outlined in figure 2a that are consistent with a pattern of trade that requires country 1 to export *x* and import *y*. Similarly, the modified edition of (82) corresponds to $\left(a_{1y}/a_{2y}\right)^{\alpha}\left(a_{2x}/a_{1x}\right)^{\beta}>1$ and gives rise to structures outlined in figure 2e associated with a pattern of trade in which country 1 exports *y* and imports *x*.

Under such circumstances proposition 3 suggests that *in the absence of transaction costs*, but in the presence of cross country differences in technology, our model assumes a Ricardian nature. This finding echoes Ferguson's (1978) characterization of the implications of the Kemp-Jones model of trade that, as per the assumption above, does not incorporate transaction costs. In this context, our findings may be viewed as a generalization of the singular relevance of comparative technological advantage for the prediction of trade flows nominated by the Kemp-Jones model of trade, and its many variants.

It is important to note that this generalization transcends the difference between proposition 3, on the one hand, and the reduced edition of this proposition under the assumptions of nonexistent transaction costs and cross country differences in technology, on the other. Specifically, our generalization of the Kemp-Jones findings extends to the host of parameter value subsets that are mutually exclusive with the requirements of proposition 3. Such subsets correspond to circumstances in which intensity-efficiency-adjusted total factor productivity coefficients *do not* confer clear comparative (technological) advantage. In this case, possible structures, other than those consistent with autarky or razor edge conditions, are illustrated in figure 2c.

With regard to the structures corresponding to this panel, it is fairly easy to show that relative factor endowments determine the pattern of trade. And this is so even when comparative technological advantage opposes comparative endowment advantage, as long, of course, as intensity-efficiency-adjusted total factor productivity coefficients do not confer clear comparative (technological) advantage, as per proposition 3 above.

To illustrate this result by way of a numerical example, recall that under our assumption that $\beta > \alpha$ industries *x* and *y* are capital and labor intensive, respectively, and assume that $\beta = 0.6 > \alpha = 0.4$; $t_1 = 0.5$; $t_2 = 0.6$; $k_1 = k_2 = 1$; $\theta = 0.5$; $\frac{a_{1y}}{a_{2y}} = 1$; $\frac{a_{2x}}{a_{1x}} = 1.1$; $\frac{L_1}{L_2} = 5$, and $\frac{K_1}{K_2} = 20$. Hence, $F = \frac{\alpha\theta + \beta(1-\theta)}{1-\alpha\theta-\beta(1-\theta)} = 1$;

$$c_1 = \frac{\alpha\theta}{\beta(1-\theta)}\left(\frac{k_1 a_{2y}}{a_{1y}}\right)^{\frac{1}{\beta}} t_1^{\frac{\beta-1}{\beta}} = 1.06;\ c_3 = \frac{\beta(1-\theta)}{\alpha\theta}\left(\frac{k_1 a_{2x}}{a_{1x}}\right)^{\frac{1}{\alpha}} t_2^{\frac{\alpha-1}{\alpha}} = 4.2;$$

$c_5 = \max\{c_1, c_3\} = 4.2$; and $\left(\frac{t_1}{t_2}\right)^{\beta-\alpha} = 0.96 < k_1^{\alpha+\beta} k_2^{\alpha+\beta} = 1$. Consider now the parameter value subset of figure 2c defined by

$$t_1^{\beta-\alpha}\left(\frac{1}{k_1}\right)^{\beta}\left(\frac{1}{k_2}\right)^{\alpha} = 0.87 < \left(\frac{a_{1y}}{a_{2y}}\right)^{\alpha}\left(\frac{a_{2x}}{a_{1x}}\right)^{\beta} = 1.07 < t_2^{\alpha-\beta} k_1^{\alpha} k_2^{\beta} = 1.11;\ \frac{L_1}{L_2} = 5 > c_5 = 4.2;$$

and
$$\frac{K_1}{K_2} = 20 > d_2 = \frac{\left[\beta\left(\frac{a_{1y}}{k_1 a_{2y}}\right)^{\frac{1}{\beta}} t_2^{\frac{\beta-1}{\beta}} \frac{L_1}{L_2} + \beta - (1-\beta)F\right]}{t_2(1-\beta)F} = 18.40$$

This parameter value subset leads to the general equilibrium market structure given by $(xy)_{xK}(y)_y$. Hence, as expected, comparative endowment advantage may determine the pattern of trade even in the presence of opposing comparative technological advantage. Similar results obtain in the case of the remaining structures of figure 2c. These findings are summarized in the following proposition that may be viewed as a corollary to proposition 3:

PROPOSITION 4: *When intensity-efficiency-adjusted total factor productivity coefficients do not confer clear comparative (technological) advantage, the pattern of trade is determined by comparative endowment advantage (even in the presence of opposing comparative technological advantage).*

Our model's predictions regarding trade flows are not consistent with the standard formulation of the Heckscher-Ohlin that advocates the singular relevance of factor endowments. They are also inconsistent with the purely Ricardian predictions of Neo-Heckscher-Ohlin frameworks such as those that rely on variations of the Kemp-Jones model of trade. Instead, our model suggests that when comparative endowment advantage is in contest with comparative technological advantage, prevalence of either force relies heavily (and via a "first order" relationship) on relative degrees of cross-country transaction efficiency.

III. Concluding Remarks

The generalization of the Heckscher-Ohlin model of trade that is developed in this paper diverges from competing specifications in a number of respects. Most importantly, and unlike the preponderance of such generalizations that may be found in the literature, our model is the first of its kind to treat specialization in production endogenously, while allowing capital factors to be internationally mobile, technology across countries to differ, and trade specific transaction costs to prevail.

A good portion of the ensuing results are both novel and striking. For example, by endogenizing specialization in the context of an inframarginal general equilibrium setting, we are able to generate necessary conditions *that only involve exogenous parameters* for the prevalence of global production within the diversification cone.

Hence, we are able to shed direct light on the long standing debate regarding the plausibility of this outcome – a debate that, as we argue, has been prolonged by the practice of treating the pattern of production exogenously. Our findings suggest that production within the cone may only prevail under very strict, and often *razor edge*, conditions that are generally unlikely to hold for any prolonged period of time, if at all. And this has important implications, as core propositions of the Heckscher-Ohlin theorem, such as factor price equalization, require the assumption of diversified production in order to hold.

Furthermore, our findings nominate a mechanism by which improvement in transaction efficiency determines the size of the market, that, in turn, regulates the equilibrium level of the division of labor. Such a mechanism can be employed in efforts to explain a host of important phenomena, from issues pertaining to the course of economic development, to the process of evolution of trade networks.

Contrary to other generalizations of the Heckscher-Ohlin (such as the various derivatives of the Kemp-Jones model of trade), our model does not assume a purely Ricardian character. That is so as comparative endowment advantage may determine the pattern of trade even in the presence of *opposing* technological differences. However, this can occur only as long as total factor productivity coefficients adjusted for factor intensity and transaction efficiency do not confer unambiguous comparative (technological) advantage. Generalizing the findings of the Kemp-Jones model of trade, we show that intensity-efficiency-adjusted comparative technological advantage supersedes factor endowments in determining the flow of trade.

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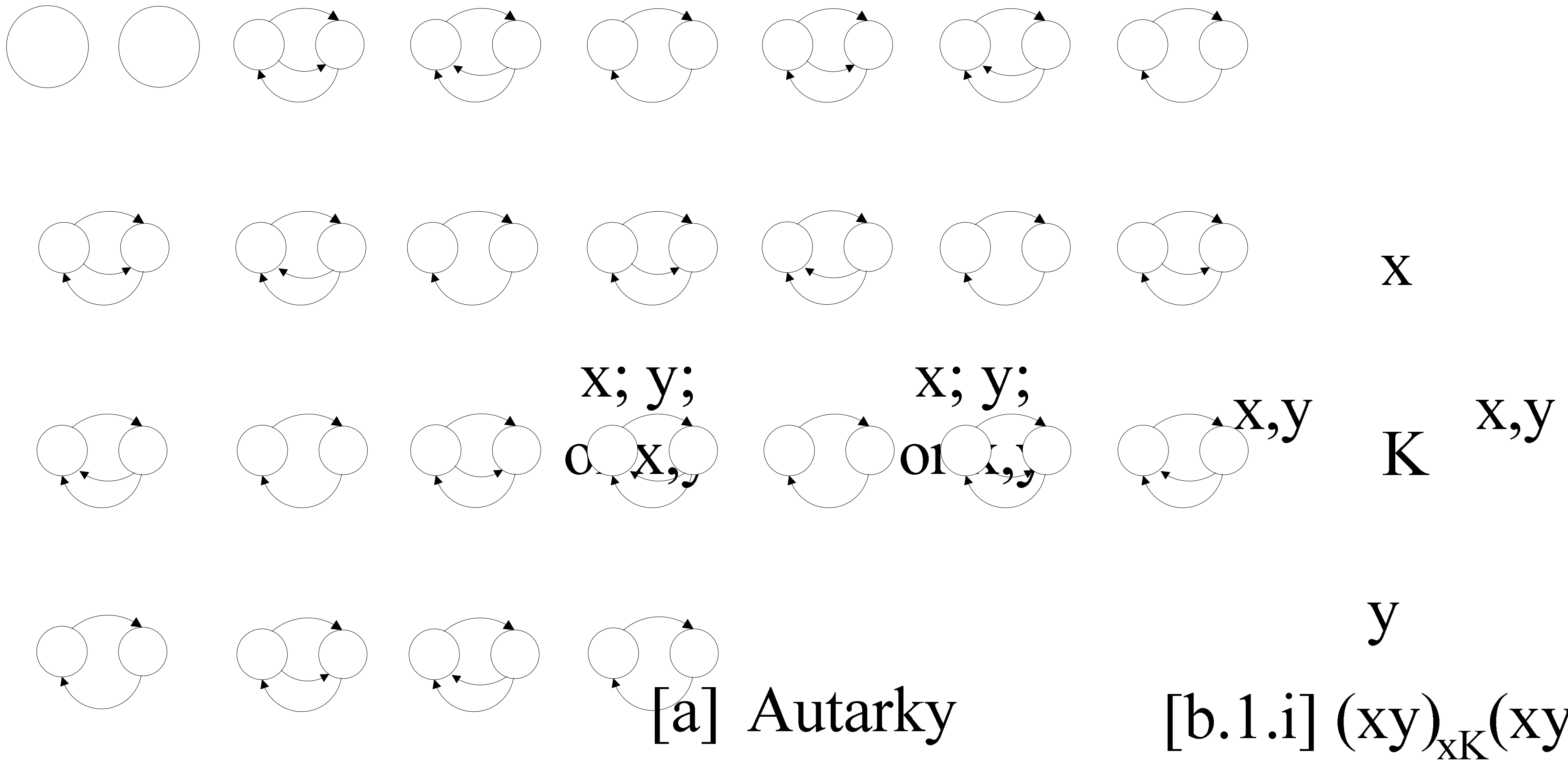
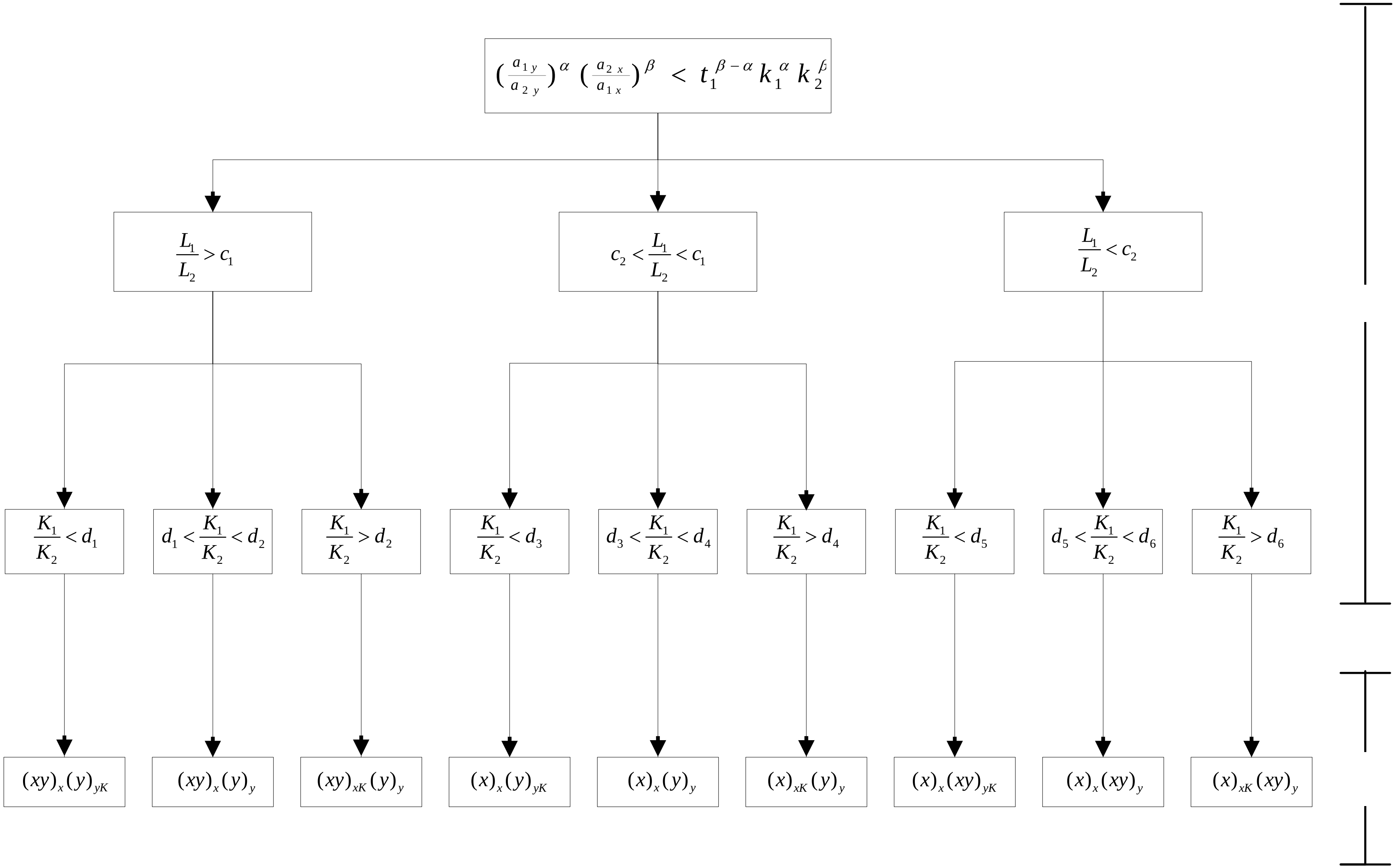


Notes: In each market structure country "1" ("2") is designated by the circle on the left (right). Production profiles are summarized by letters within circles, and patterns of trade by arrows. Labels obey the following conventions: "a" denotes autarky; "b" corresponds to outcomes that prevail within the diversification cone; "cj" and "c" represent complete specialization in country j, and complete specialization in both countries, respectively. "1" ("i") and "2" ("ii") correspond to exports of commodity *x* (capital) by countries 1 and 2, respectively. "iii" denotes structures that do not involve trade in capital.


$(\frac{a_{1y}}{a_{2y}})^{\alpha} (\frac{a_{2x}}{a_{1x}})^{\beta} < t_1^{\beta-\alpha} k_1^{\alpha} k_2^{\beta}$
$\frac{L_1}{L_2} > c_1$
$c_2 < \frac{L_1}{L_2} < c_1$
$\frac{L_1}{L_2} < c_2$
$\frac{K_1}{K_2} < d_1$
$d_1 < \frac{K_1}{K_2} < d_2$
$\frac{K_1}{K_2} > d_2$
$\frac{K_1}{K_2} < d_3$
$d_3 < \frac{K_1}{K_2} < d_4$
$\frac{K_1}{K_2} > d_4$
$\frac{K_1}{K_2} < d_5$
$d_5 < \frac{K_1}{K_2} < d_6$
$\frac{K_1}{K_2} > d_6$
$(xy)_x (y)_{yK}$
$(xy)_x (y)_y$
$(xy)_{xK} (y)_y$
$(x)_x (y)_{yK}$
$(x)_x (y)_y$
$(x)_{xK} (y)_y$
$(x)_x (xy)_{yK}$
$(x)_x (xy)_y$
$(x)_{xK} (xy)_y$

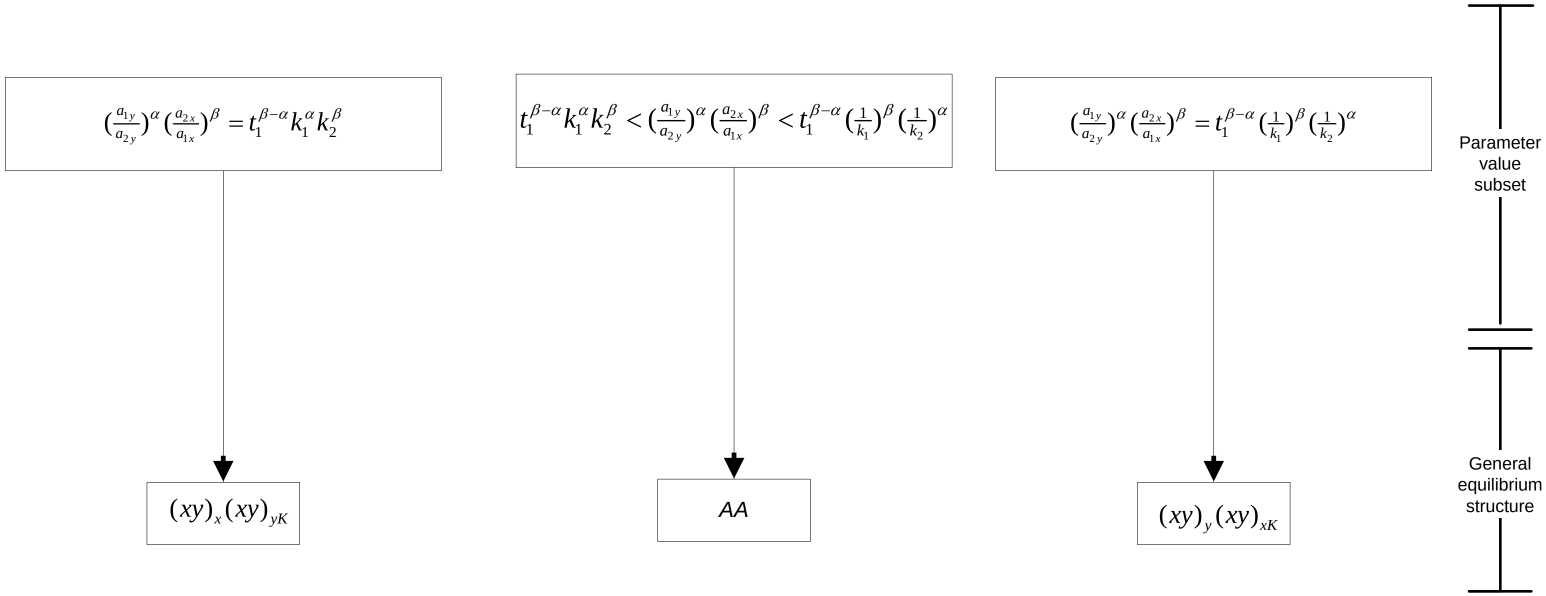
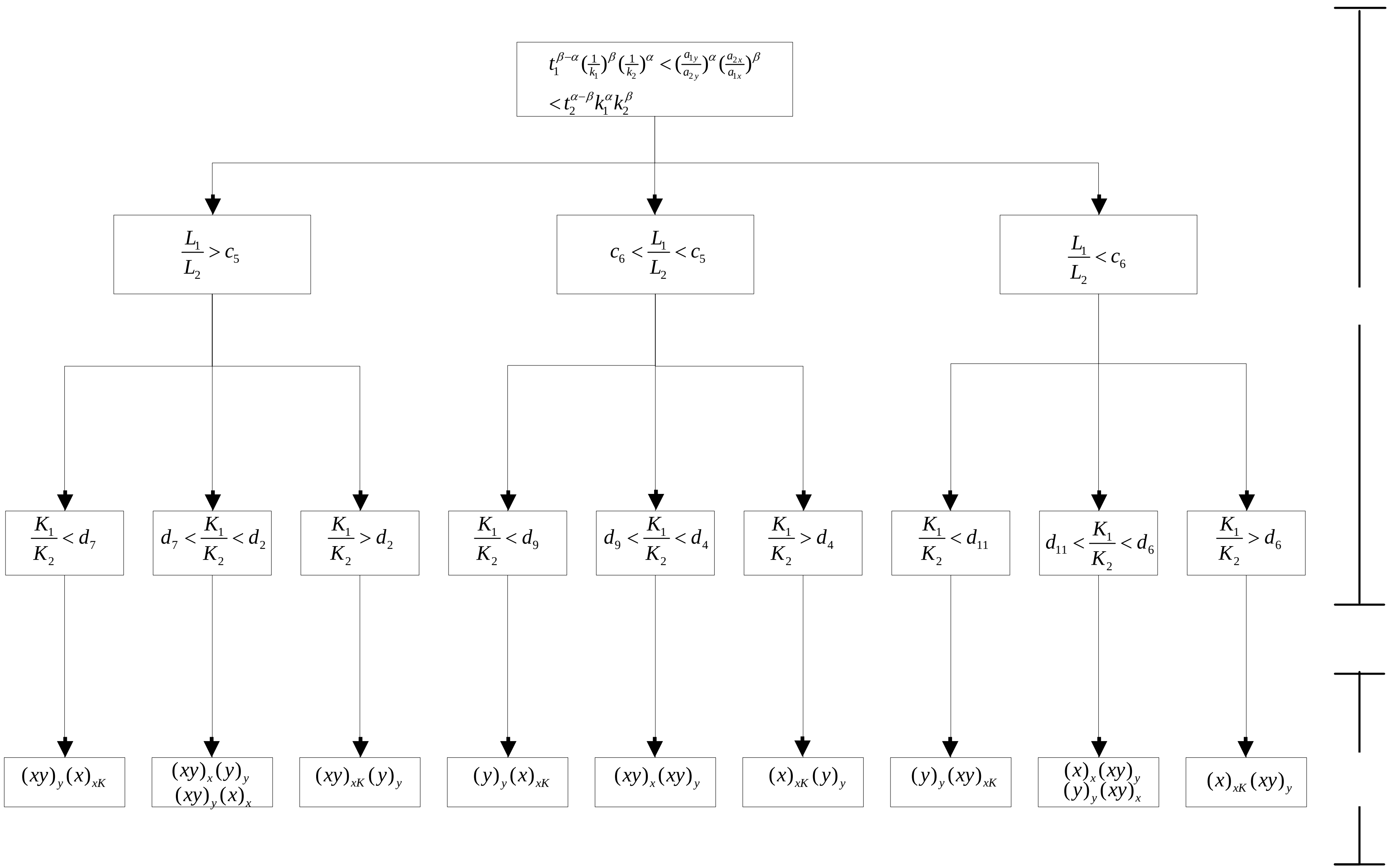


FIGURE 2b. GENERAL EQUILIBRIUM AND ITS INFRAMARGINAL COMPARATIVE STATICS (CONTINUED)



$t_1^{\beta-\alpha}(\frac{1}{k_1})^{\beta}(\frac{1}{k_2})^{\alpha} < (\frac{a_{1y}}{a_{2y}})^{\alpha}(\frac{a_{2x}}{a_{1x}})^{\beta}$
$< t_2^{\alpha-\beta}k_1^{\alpha}k_2^{\beta}$
$\frac{L_1}{L_2} > c_5$
$c_6 < \frac{L_1}{L_2} < c_5$
$\frac{L_1}{L_2} < c_6$
$\frac{K_1}{K_2} < d_7$
$d_7 < \frac{K_1}{K_2} < d_2$
$\frac{K_1}{K_2} > d_2$
$\frac{K_1}{K_2} < d_9$
$d_9 < \frac{K_1}{K_2} < d_4$
$\frac{K_1}{K_2} > d_4$
$\frac{K_1}{K_2} < d_{11}$
$d_{11} < \frac{K_1}{K_2} < d_6$
$\frac{K_1}{K_2} > d_6$
$(xy)_y(x)_{xK}$
$(xy)_x(y)_y$
$(xy)_y(x)_x$
$(xy)_{xK}(y)_y$
$(y)_y(x)_{xK}$
$(xy)_x(xy)_y$
$(x)_{xK}(y)_y$
$(y)_y(xy)_{xK}$
$(x)_x(xy)_y$
$(y)_y(xy)_x$
$(x)_{xK}(xy)_y$

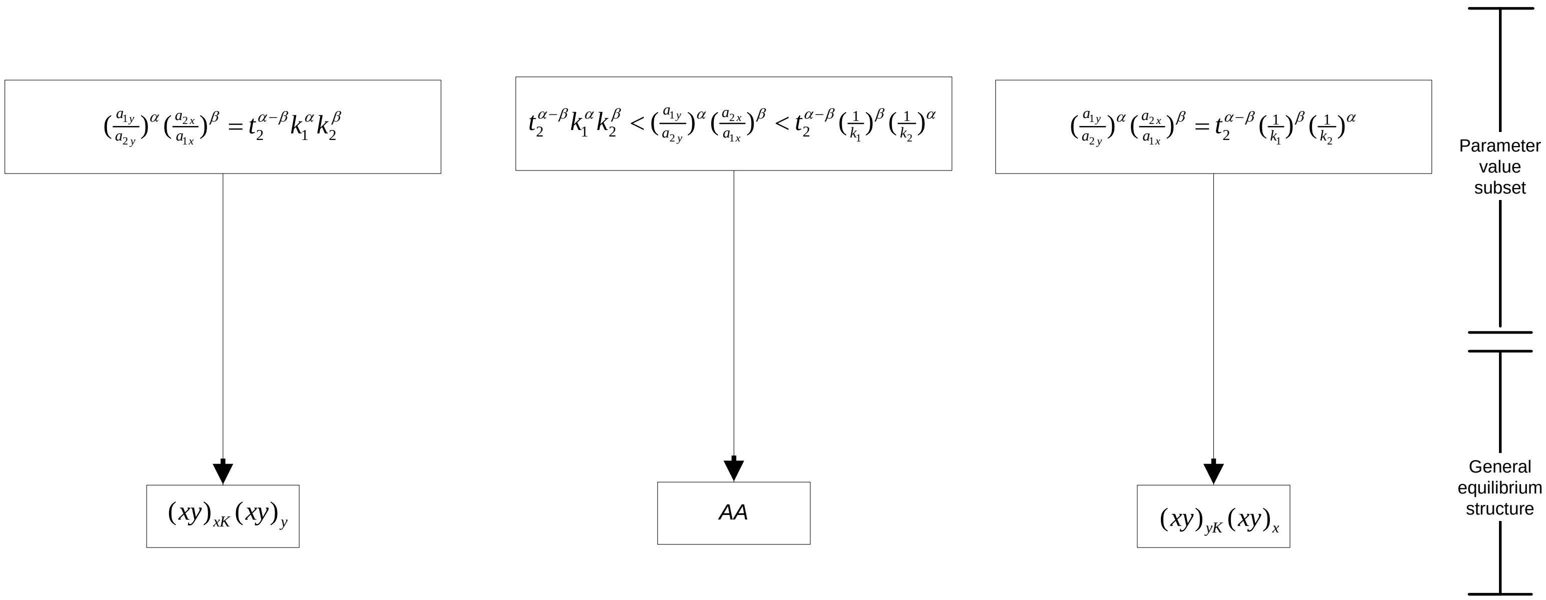
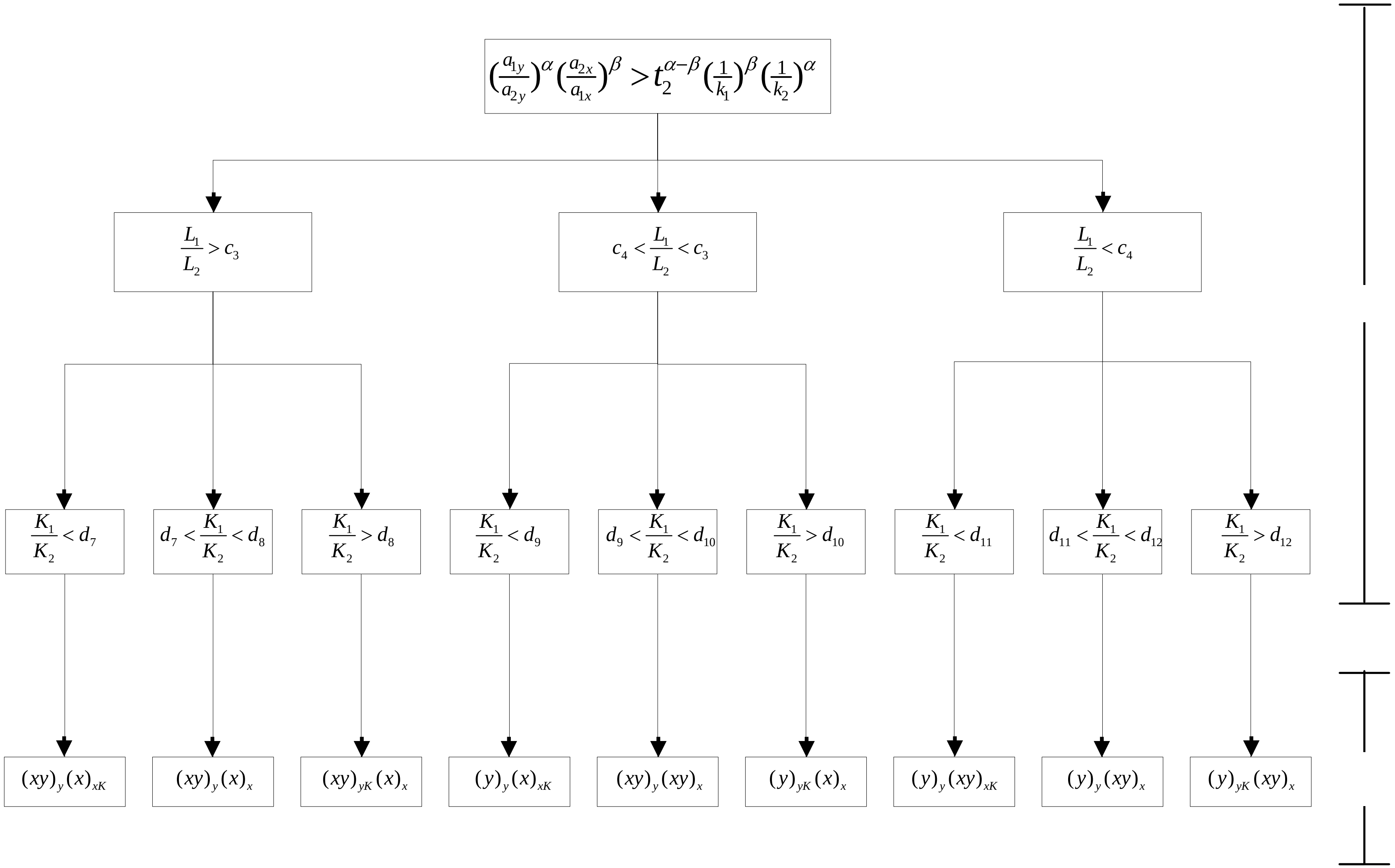


FIGURE 2d. GENERAL EQUILIBRIUM AND ITS INFRAMARGINAL COMPARATIVE STATICS (CONTINUED)


$(\frac{a_{1y}}{a_{2y}})^{\alpha}(\frac{a_{2x}}{a_{1x}})^{\beta} > t_2^{\alpha-\beta}(\frac{1}{k_1})^{\beta}(\frac{1}{k_2})^{\alpha}$
$\frac{L_1}{L_2} > c_3$
$c_4 < \frac{L_1}{L_2} < c_3$
$\frac{L_1}{L_2} < c_4$
$\frac{K_1}{K_2} < d_7$
$d_7 < \frac{K_1}{K_2} < d_8$
$\frac{K_1}{K_2} > d_8$
$\frac{K_1}{K_2} < d_9$
$d_9 < \frac{K_1}{K_2} < d_{10}$
$\frac{K_1}{K_2} > d_{10}$
$\frac{K_1}{K_2} < d_{11}$
$d_{11} < \frac{K_1}{K_2} < d_{12}$
$\frac{K_1}{K_2} > d_{12}$
$(xy)_y(x)_{xK}$
$(xy)_y(x)_x$
$(xy)_{yK}(x)_x$
$(y)_y(x)_{xK}$
$(xy)_y(xy)_x$
$(y)_{yK}(x)_x$
$(y)_y(xy)_{xK}$
$(y)_y(xy)_x$
$(y)_{yK}(xy)_x$

Notes: *AA* corresponds to a state of Autarky. $(xy)_x(y)_{yK}$ represents a structure in which country 1 produces both goods "x" and "y", and exports portion of "x", whereas country 2 produces only good "y", and exports portion of this good as well as portion of its endowment of capital "K". The notation corresponding to the remaining structures readily follows. All structures examined in this figure rely on the assumptions that $\beta > \alpha$, and $\left(\frac{t_1}{t_2}\right)^{\beta-\alpha} < k_1^{\alpha+\beta} k_2^{\alpha+\beta}$. We define $F = \frac{\alpha\theta+\beta(1-\theta)}{1-\alpha\theta-\beta(1-\theta)}$, $c_1 = \frac{\alpha\theta}{\beta(1-\theta)}\left(\frac{k_1 a_{2y}}{a_{1y}}\right)^{\frac{1}{\beta}} t_1^{\frac{\beta-1}{\beta}}$, $c_2 = \frac{\alpha\theta}{\beta(1-\theta)}\left(\frac{a_{2x}}{k_2 a_{1x}}\right)^{\frac{1}{\alpha}} t_1^{\frac{1-\alpha}{\alpha}}$,

$c_3 = \frac{\beta(1-\theta)}{\alpha\theta}\left(\frac{k_1 a_{2x}}{a_{1x}}\right)^{\frac{1}{\alpha}} t_2^{\frac{\alpha-1}{\alpha}}$, $c_4 = \frac{\beta(1-\theta)}{\alpha\theta}\left(\frac{a_{1y}}{k_1 a_{2y}}\right)^{\frac{1}{\beta}} t_2^{\frac{1-\beta}{\beta}}$, $c_5 = \max\{c_1, c_3\}$, $c_6 = \min\{c_2, c_4\}$, $d_1 = \frac{t_1\left[\beta\left(\frac{a_{1y}}{k_1 a_{2y}}\right)^{\frac{1}{\beta}} t_1^{\frac{1-\beta}{\beta}} \frac{L_1}{L_2} + \beta - (1-\beta)F\right]}{(1-\beta)F}$,

$d_2 = \frac{\left[\beta\left(\frac{a_{1y}}{k_1 a_{2y}}\right)^{\frac{1}{\beta}} t_2^{\frac{\beta-1}{\beta}} \frac{L_1}{L_2} + \beta - (1-\beta)F\right]}{t_2(1-\beta)F}$, $d_3 = \frac{(1-\alpha)\theta t_1}{(1-\theta)(1-\beta)}$, $d_4 = \frac{(1-\alpha)\theta}{t_2(1-\theta)(1-\beta)}$, $d_5 = \frac{t_1(1-\alpha)F}{\left[\alpha\left(\frac{a_{2x}}{k_2 a_{1x}}\right)^{\frac{1}{\alpha}} t_1^{\frac{\alpha-1}{\alpha}} \frac{L_2}{L_1} + \alpha - (1-\alpha)F\right]}$,

$d_6 = \frac{(1-\alpha)F}{t_2\left[\alpha\left(\frac{a_{2x}}{k_2 a_{1x}}\right)^{\frac{1}{\alpha}} t_2^{\frac{1-\alpha}{\alpha}} \frac{L_2}{L_1} + \alpha - (1-\alpha)F\right]}$, $d_7 = \frac{t_1\left[\alpha\frac{L_1}{L_2} + \alpha\left(\frac{k_1 a_{2x}}{a_{1x}}\right)^{\frac{1}{\alpha}} t_1^{\frac{1-\alpha}{\alpha}} - (1-\alpha)F\right]}{(1-\alpha)F}$, $d_8 = \frac{\alpha\frac{L_1}{L_2} + \alpha\left(\frac{k_1 a_{2x}}{a_{1x}}\right)^{\frac{1}{\alpha}} t_2^{\frac{\alpha-1}{\alpha}} - (1-\alpha)F}{t_2(1-\alpha)F}$,

$d_9 = \frac{(1-\theta)(1-\beta)t_1}{(1-\alpha)\theta}$, $d_{10} = \frac{(1-\theta)(1-\beta)}{t_2(1-\alpha)\theta}$, $d_{11} = \frac{t_1(1-\beta)F}{\left[\beta\left(\frac{k_2 a_{1y}}{a_{2y}}\right)^{\frac{1}{\beta}} t_1^{\frac{\beta-1}{\beta}} + \beta\frac{L_2}{L_1} - (1-\beta)F\right]}$, $d_{12} = \frac{(1-\beta)F}{t_2\left[\beta\left(\frac{k_2 a_{1y}}{a_{2y}}\right)^{\frac{1}{\beta}} t_2^{\frac{1-\beta}{\beta}} + \beta\frac{L_2}{L_1} - (1-\beta)F\right]}$.