Evaluation of Archived Code with Perturbation Checks and Alternatives.

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Replications merely check whether the results reported by authors are independently verifiable, not whether they are reliable /robust /stable. There can still be specification and interpretation errors in theory, the model and/or the software. Vinod and Ullah (1981) suggested perturbing the data beyond the available digits to evaluate the stability of model results. This paper proposes a simple algorithm to create random perturbations to be used to check "perturbation robustness" of a model, its software and interpretations. We illustrate our proposal with replication examples from the *Journal of Money Credit and Banking* (JMCB) and *Journal of Applied Econometrics* archives. Mandatory posting of data +code creates a free rider problem. If journals allow some parts of the code to be hidden in a black box, it may also hide shortcuts and dishonesty. If authors placing parts of their code in such black boxes are asked to post a perturbation robustness measure α_p developed here, this can prevent some of the abuses.

1. Introduction

Economics journals are focused on publishing "new and different" results, but some do not pay adequate attention to whether the published results are also numerically "reliable and robust," at the same time. Beaton, Rubin and Barone (1976) suggested adding a uniform random number between −0.5 and 0.499 in the "digit after the published digit" to perform a perturbation and found one that it changes the size of one of the regression coefficients quite drastically (from –232 to +237). Since economic data are subject to measurement errors, Vinod and Ullah (1981, pp. 126-129) also discuss similar examples with near-collinear data where a similar perturbation reveals general unreliability of regression coefficients. Vinod (1982) argued that measurement errors can be used solve the "under-identification" problem in simultaneous equation models, by using their variance to remove the singularity of a matrix, which cannot otherwise be inverted. Modified regression estimators which are perturbation robust are called "enduring regressions" in Vinod (1982b). Back in 1980's, a routine study of perturbation was computationally too demanding for general use. Vinod (1990) and Vinod and Basu (1990) are concerned with tools for dealing with fuzzy economic data.

Harrison and Vinod (1992) proposed a "completely randomized factorial design" which allows for all possible combinations of *parameters* to be perturbed to study the sensitivity of applied general equilibrium (AGE) models. An appendix to Vinod (2004) uses the maximum entropy density to construct computerized ensembles for time series inference, which resemble perturbed data for a new "dependent data bootstrap." Santangelo (2004) shows how to assess the extent of measurement errors in many economic series from a review of the sequence of past data revisions. We shall see that in our modern computing environment it is a simple matter to evaluate the effect of measurement errors (represented as perturbations to the observed data) on econometric results. In short, this paper claims that what was once a tedious brute force method, deserves a fresh look.

McCullough and Vinod (1999, 2003, 2003b, 2004, 2004b) in a series of papers and Vinod (2001) have argued that if economics is to be a true science, we have to be willing to let the data + code for our published empirical results readily available for replication by other researchers. The availability of the Internet has made this requirement feasible and many journals including *American Economic Review* are requiring their authors to make data +code available to other researchers for replication exercises.

Some anonymous referees of McCullough and Vinod (1999, 2003) had raised the point that since economic data are subject to considerable measurement error anyway, it might not be important to worry about the numerical accuracy of results. We convinced them that numerical accuracy and replicability are important initial steps; partly because when numerical errors are confounded with measurement errors the uncertainty associated with the substantive economic conclusions only increases. Measurement errors cannot justify numerically defective algorithms or faulty software and economists should start demanding accurate and verifiable software products.

Definition of $100(1-\alpha_p)\%$ Perturbation Robustness
Assume that we know the number *r* of reliable digits associated with each variable in all ranges and sub ranges of available data. We create J sets of perturbed data by randomly making small changes to the trailing digits beyond the reliable *r* digits. Now any conclusion based on numerical results in a paper is said to be $100(1-\alpha_p)\%$ perturbation robust if it is reversed only in proportion α_p (e.g. α_p =0.01) of the J perturbed data sets.

Now I describe the perturbation in general terms. Beaton, et al's (1976) adding a uniform random number between −0.5 and 0.499 in the "digit after the published digit" assumes that all the published digits are accurate. In light of Santangelo (2004), many published digits are clearly inaccurate and hence their assumption is not warranted.

For the purpose of simplicity let us initially assume that all n observations of the data for variable y has L digits to the left of the decimal and R digits to the right. For example, if the mean of y is 123.4567, we have L=3 and R=4. For the purpose of perturbation of y we need prior knowledge regarding the reliable and unreliable digits. Assume that one knows that r digits are reliable, which means that u (=L+R-r) trailing digits are unreliable. In the current example if r=4, we have u=3, implying that the trailing digits 567 are unreliable. We interpret the unreliability to mean that instead of the trailing digits 567 we could have observed any (random) combination of digits 0 to 9 placed in a sequence of 3 digits. Define:

$$A=(10)^{r-L}. \tag{1}$$

In the example where y =123.4567, L=3, and if we know that there are r=4 reliable digits, (1) implies that A=10.

Next, we construct a uniform random sequence of length n between 0 and 1, with elements $U_t \in [0,1]$. Recall that we want a transformed random variable denoted by $U_t{}^* \in$

[−0.5, 0.499], where the uniform has to be re-centered with the new range =0.499–0.5 = 0.999 instead of unity. Since a change of the range is analogous to a change of scale, let us **define:** $U_t^* = (U_t - 0.5)/0.999$

Let floor(x) denote the result equal to the largest integer(s) not greater than the corresponding element(s) of x. The perturbed sequence is obtained by the formula:

$$y_{pt} = (\text{floor}(y_t{*}A) + U_t^*)/\, A, \quad \text{for } t=1, 2, .., n. \tag{2}$$

In the example where y =123.4567, L=3, r=4, and A=10, if it so happens that $U_t^*=0.3 \in [-0.5, 0.499]$, then $y_{pt} = (1234 + 0.3)/10$ or $y_{pt}=123.43$. Thus equation (2) suggests first replacing the unreliable digits 567 by 000, and ultimately by 300.

Instead of (2), the numerical work in this paper assumes that we know that there are r reliable digits *after rounding*. Accordingly, we use:

$$y_{pt} = (\text{round}(y_t{*}A, s) + U_t^*)/\, A, \quad \text{for } t=1, 2, .., n, \tag{3}$$

where round (x, s) rounds the values of x to s places to the right of the decimal. The R software uses the IEEE standard called "go to the even digit" in its rounding. For example, round (0.5) =0, and round (−1.5)= −2. By contrast, in the GAUSS software, round (0.5) =1 and round (−1.5) = −1. Since both GAUSS and R use s=0 as default, and since we need to work with y vectors we use:

$$y_{pj} = (\text{round}(y{*}A) + U_t^*)/\, A \text{ for } j=1,2, .., J, \tag{4}$$

where j-th perturbation of the y vector is denoted by y_{pj}. Each new choice of the seed for the uniform random number yields a new set of n perturbed data values for y. If we know that r (reliable digits) is distinct within different sub ranges of y, the value of A in (4) will have to be distinct for each sub range. For example, in long time series some older data may have fewer reliable digits than newer ones. Similarly, in a cross section, some smaller entities (countries, corporations) may well have a smaller r than larger ones. In any case, we claim that one can carefully construct a large number (J=999, say) of randomly perturbed data sets for each variable in a data set.

Armed with J sets of data, we can re-estimate the model J times and record the J values of estimated coefficients, their standard errors, etc. When data change, it is natural that results also change. According to our definition, perturbation- robustness holds if the results do not change so much that (policy) conclusions are reversed. One way to state the result is to indicate the percentage of results that lead to conclusion reversals. For most problems we want less than 1% reversals.

Vinod and Ullah (1981, Ch. 5) use the 'singular value decomposition' of the matrix X of regressors to define the condition number $K^\# = s_{max}/s_{min}$, where we denote the largest and smallest singular values by suitable subscripts. There are some numerical mathematics results available for studying the effect of a large $K^\#$ on least squares forecasts and coefficients. Unfortunately they are applicable to linear least squares in a very general setting (for all possible values of X) in the form of upper bounds on worst-case scenarios. Since such results do not necessarily apply to the problem at hand, econometricians have mostly ignored them. Let us conclude our general description of perturbation as a tool for checking robustness with the following claim. Modern computing environment makes it rather easy to estimate the perturbation-robustness coefficient $100(1-\alpha_p)$ for any computing algorithm, however complicated, by repeated estimation.

This paper evaluates the robustness of some archived code with perturbation checks based on replacing each data variable by a perturbed set J times and evaluating the proportion α_p of times (if any) the conclusion is reversed. These checks represent a method for addressing the issue of measurement errors in conjunction with numerical accuracy by updating the ideas from 1980's mentioned above in the newer context of Journal sponsored archives for replication of economic results. A routine use of perturbations is fairly simple and gives a new confidence to economic policy makers that reported results are reliable and robust, since the perturbation is designed to quickly reveal any hidden and undue sensitivity of substantive economic conclusions to the presence of certain known ranges of measurement errors.

Journal Policy Implications:

Let us discuss some practical policy issues faced by the Economics profession through our journal editors to understand why some journals do not yet require all authors to make available data +code for all articles. At some journals, such as the *Journal of Applied Econometrics*, the posting of code remains optional. Are the editors worried about the 'free rider' problem? Intellectual work should be compensated and create wealth, which cannot happen if all code is free including highly specialized software. The editors might feel that the Economics profession needs to retain financial incentives for those trying to create /improve software codes while making it user-friendly. Another issue is: How much is a fair compensation for software? Economists, have a natural preference to let free markets rule. Thus we need to address two specific problems: (i) how to prevent price gouging by commercial code writers, and (ii) how to prevent replicators abusing commercial code for other research problems without paying for it.

One way to partly solve the price-gouging problem might be for journals to require the results to be replicable on at least *two* software platforms and/or requiring that the posted code be 'good and clear.' Let us define good and clear code as one having detailed comments and descriptions of all possible notations and explicit mention of intermediate steps and motivations behind the steps. The explanations could be right in the code or in a supplementary document. A possible solution to the free rider abuse problem might be to charge a modest fee for use of the code and somehow compensate the developer of the code. Since Journals are likely to be averse to keeping track of moneys for third parties, the payments would have to be a pre-defined proportion of the annual subscription fee for the Journal and made directly to software developers.

Another way to prevent the free rider abuse is to permit authors to convert proprietary parts of the code into a so-called 'executable' software tools, which accept the data and output from the non-proprietary code and produces the results of some hidden steps eventually leading to the published results in a journal. Let us refer to the executable as a black box (BB), which denies the non-paying user any access to the inner workings of the proprietary software. If we permit BB's of code there remains a chance that a dishonest researcher will use them to hide material flaws and/ or permit self-serving shortcuts. The following are some of the solutions to this practical problem: (1) Journals should allow BB's very sparingly and only for a few steps in obtaining the result. (2) A Journal representative (referee) should see the inner workings of BB's, where the representative cannot keep a copy of the contents. (3) Journals can have authors sign a document expressing the following sentiment: The author accepts sanctions for violation of professional ethics if the hidden portions of the posted code deliberately hide material flaws. (4) Journals can place the hidden code inside the BB's in a separate archive, which goes public after a lapse of some (say 36) months.

A perturbation robustness study proposed in this paper suggests an additional tool, which will automatically unravel at least some of the material flaws or shortcuts. Hence Journals could also require that authors who hide some of their code in BB's must publish the α_p values associated with their work. This will impose an additional hoop through which a dishonest BB's will have to jump. Note that the BB will have to yield distinct, similar and plausible results, which support the conclusions of the published paper in approximately $100(1-\alpha_p)$% cases, even when an independent researcher uses arbitrary seeds for randomized perturbations.

I believe that with these safeguards, all serious journals can begin to impose mandatory public archiving of data + code for all authors and encourage easy replication for the good of the profession. The many advantages of such archiving are discussed in Anderson, Greene, McCullough and Vinod, being presented in the same session as this one at the 2004 annual American Economics Association meetings.

The remaining paper deals with my experience with examples available at the archives of the *Journal of Money Credit and Banking* (JMCB) and the *Journal of Applied Econometrics*. Section 2 considers sensitivity to perturbation of Braun and Evans' (1998) model using GMM software to study seasonal effects (such as those of Christmas season) with those predicted by a model. Section 3 considers perturbation of break point analysis by Bai and Perron (2000, 2003) recently replicated by Zeleis and Kleiber (2004). Beaton et al (1976) report an example of collinear economic data where miniscule perturbation beyond the last published digit changes the size of one of the regression coefficients quite drastically (from –232 to +237). Vinod's (1982b) enduring regression, similar to ridge regression, is designed to obtain perturbation-robust estimators. These references suggest that achieving perturbation-robust results will need modifications to algorithms for at least two kinds of estimators: (i) Nonlinear algorithms sensitive to starting values, similar to the complex maximum likelihood in McCullough and Vinod (2003, 2004). (ii) Algorithms involving near-collinear matrix inversions similar to Beaton, et al (1976). A discussion of such situations is left beyond the scope of this paper.

2. Perturbation of Braun-Evans Seasonal model

Braun and Evans (1998) discuss the seasonality in 'Solow residuals' and other variables in the context of the (Christmas) season. The data on output, consumption, investment, government, capital stock, labor hours, and real wage are used. The results of GMM estimations are reported in their paper. The GAUSS code is archived at the website of the *Journal of Money Credit and Banking*. Our first task is to see if we can replicate the Braun-Evans' results reported in the Journal.

Our Table 1 reports the descriptive statistics for the data provided by the authors along with short descriptions of all variables. The per capita data are computed by using civilian population, 16 years and older. Since these data are obtained from fairly reliable governmental sources, it is decided to round to the first digit to the right of the decimal point while using eq. (4) with the uniform random variable U_t* defined above. This explains our choice of r, the number of reliable digits assumed for the purpose of our perturbation study, reported in the first column of Table 1 in brackets.

Column 1 in Table 2 reports the numbers from Braus-Evans' Table 3 on page 321. For each variable we report the results for the winter, spring, summer and fall seasons sequentially, one below the other. The seasons are explicitly indicated only for the first "Solow Residual" variable. For brevity, we do not explicitly mention season names for all remaining variables (y, cp, dk, k, n, and Productivity), since we follow the identical order of season names for all these variables.

Note that we use the same data but a newer version of GAUSS (v. 4.0) compared to the older version in Braun and Evans (1998). Our results regarding seasonal factors (coefficients) in the second column of Table 2 often do not agree with their Table 3 reproduced in our first column. It is obvious that there are several sign reversals (identified by the symbol Sgn) and /or several intuitively large magnitude differences (identified by the symbol Mgn). We suspect that GAUSS software has undergone many improvements since Vinod (2000) criticism, and most of the Sgn and Mgn disagreements can be attributed to my use of the newer version. Since coefficients disagree so much, it is futile to expect the standard errors to agree over different versions of GAUSS.

Now we turn to perturbation robustness, which is our main focus here. The perturbed data are constructed as follows. The data for y has four main digits to the left of the decimal with the mean 5267.184. Thus L=4 and we have assumed (See Table 1) that there are r=5 reliable digits in these data. Hence we are ready to apply equations (1) and (4) with J=999. The remaining columns of Table 2 provide the results of our perturbation study implemented with the newer version of GAUSS but using the same software commands as in Braun and Evans. Table 3 reports results of our study of perturbation robustness of the estimates of standard errors based on Braun-Evans' GAUSS code implemented on version 4.0.

Before creating our J perturbed versions, we must first store original unperturbed series. For example, we define "yorig" as the original y data series. For replicability, as in

McCullough and Vinod (1999, 2003), we recommend using seeded random number generation. In GAUSS software, we create seeded uniform random numbers by the procedure called "rndus." Since we create j=1, 2, . . ,999 versions of the original data, the seed needs to be updated for every such version. This updating is achieved simply by adding j to the original seed.

Hence using our equations (3) and (4) with A=10, the following GAUSS commands are used for creating the perturbed y variable:

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seed=23+j; rnd2= (rndus(rows(y),1,seed) –0.05)/0.99;
y=(round(yorig*10)+ rnd2)/10;
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In the above code we first multiply the data by 10, round, adjust and rescale the uniform and then divide by 10. It is obvious that if the measurement errors arise only for the third digit to the right of the decimal and beyond, one would first multiply by 100, round, adjust and then eventually divide by 100.

For each j we rerun the software provided by the author and collect the results in a table for each seasonal effect. There are J=999 sets of coefficients for each season for each variable. It is impossible to report them all, but one can summarize them by considering the basic descriptive statistics over the J sets and report them, as we do in Table 2 for seasonal factors. Table 3 reports the standard errors associated with the seasonal factors in Table 2. The numbers reported are rounded to 4 digits, so that the standard deviation of zero simply means that it is too small to show up in the designated number of digits of accuracy. The reported zero standard deviations in these tables do not imply that all 999 perturbed values are identical. It is interesting to note that the average over J perturbed data sets reported in both Tables 2 and 3 are close to the value based on Braun and Evans's software and reported in the second column of our table (not the ones reported in their article). This suggests that the substantive conclusions regarding seasonal patterns by these authors based on our newer calculations in the second column are almost 100% perturbation robust for all variables studied in a univariate fashion (α_p=0)

3. Perturbation of Break Point Analysis by Bai and Perrron

Bai and Perron (2000, 2003) proposed a dynamic programming algorithm in combination with the least squares to study pure and partial structural change models using GAUSS code which is available on the Journal's website. This code is used by Zeleis and Kleiber (2004) to illustrate the use of their R software called "strucchange" for the same econometric problem. The "case study" by Zeleis-Kleiber indicates why it is desirable to perform replication studies with more than one software platform for reliable econometric research. For example, they discovered some numerical problems with GAUSS, beyond those in Vinod (2000), in the log of the cumulative distribution function of the normal density. They also discovered some errors in the original coding by Bai and Perron. It is gratifying to note that Aptech Inc., the owner of GAUSS, is fixing their "lncdfn," and that Professor Perron has also posted a modified version of his code in response to Zeleis-Kleiber at (econ.bu.edu/perron/code.html).

This is a good example of cooperative progress toward more reliable econometrics, which could not have happened without the data+code archives and the patience showed by Zeleis-Kleiber in translating the original GAUSS code to R.

Our first task here is to replicate the results by Zeleis-Kleiber. This was readily done in R. Next, we use equation (4) of our Section 1 and apply it to the Bai-Perron model to evaluate the effect of perturbation in the data on the estimation of structural break points. US ex-post real interest rates are defined as the 3-month Treasury bill rate deflated by the consumer price index (CPI) inflation rate on quarterly data from 1961Q1 to 1986Q3. The illustration has only one regressor, which is a column of ones. Bai and Perron allow up to 5 structural breaks and a trimming of $\varepsilon = 0.15$, in the sense that each segment has at least 15 observations. It allows for both heteroscedasticity and autocorrelation among errors across segments. The optimal estimates of dates when structural breaks are found to have taken place during 1966Q4, 1972Q3 and 1980Q3. These three break dates lead to four time intervals, leading to four sets of separate coefficient estimates.

The four sets of coefficient estimates are given along the first row of our Table 4 marked original unperturbed data. We perturb these numbers by assuming that A=10 in (4) and estimate the model J=999 times. We then find the mean and standard deviation of the estimated coefficient from perturbed data over the 999 realizations. All comparable results are reported in Table 4, side by side. We report the basic descriptive statistics computed over the 999 realizations. It is interesting that break points occur at exactly the same quarters in all 999 perturbations. The means over the 999 realizations for coefficient estimates are extremely close to the original unperturbed data values. The ranges and standard deviations are all small, suggesting results close to the original numbers. The algorithm appears to be very stable and results are almost 100% perturbation robust for all coefficients.

4. Final Remarks

This paper suggests a simple numerical algorithm, which can be used to create systematically perturbed data. The algorithm randomly modifies trailing digits in the available data beyond r digits, which are known to be reliable. We define a concept of $100(1-\alpha_p)$ % perturbation robustness and propose that any published numerical results in journal archives can be checked for perturbation robustness by authors themselves or others trying to replicate their results and perhaps report their experience in journals similar to the *Indian Journal of Economics and Business*. I propose that journals should routinely require authors to provide data + code and perhaps also require posting of perturbation robustness α_p values, before publishing an article. I discuss how α_p can help detect dishonesty in proprietary software code made available in the form of software black boxes, without revealing its contents.

Our experience with respect to perturbation robustness study of two (univariate) archived results is encouraging. Although the JMCB article based on an earlier version of GAUSS could not be replicated on a newer version, it seems near 100% perturbation robust in comparison with the results based on a newer version. The “case study” by Zeleis-

Kleiber indicates why it is desirable to perform replication studies with more than one software platform for reliable econometric research, providing a strong vote in favor of data + code archiving. Their sophisticated code for analysis of structural breaks in economic data is implemented in R on US real interest rates. Again we find near 100% perturbation robustness. Of course, in light of the literature cited above, we should not expect α_p 0 for multivariate and nonlinear models subject to collinearity. In general, it is useful to subject one's data to perturbations and estimate α_p. This activity is similar to a stress test applied to engineering models, designed to reveal hidden limitations in a model, data, code and interpretations.

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Table 1:	Descriptive	Statistics		
Variable and [r=number of reliable digits]	Mean	Std.dev.	Minimum	Maximum
y=output (per capita GDP). [5]	5267.184	413.5599	4249.852	6169.74
cp=consumption (per capita nondurables plus services) expenditures. [5]	2928.226	334.2955	2247.077	3682.16
dk=investment (sum of fixed investment plus durable consumption expenditures,	1277.017	131.3226	1002.618	1589.943

per capita. [5]				
k=capital stock (from the flow of investment expenditures using quarterly depreciation rate of 2.5% and initial value of 1963 [6]	43691.23	6676.478	28668.74	51437.02
n=labor hours (nonagricultural employment times average hours per week times 13 weeks per quarter, per capita. [4]	248.7084	7.9429	226.707	266.767
rr=annualized interest 3 month Treasury Bills (not seasonally adjusted) [4]	976.9805	1533.39	3.5745	7254.685

Table 2

Table 3 of Braun and Evans (p.321)	Original Seasonal Factor Using GAUSS v. 4	Error in the Mean over 999	Standard Deviation	Minimum	Maximum
Solow Residual, Winter -7.439	Mgn -6.3483	0.00001	0.0001	-6.3488	-6.348
Spring, 3.470	3.4597	-0.00001	0.0001	3.4593	3.46
Summer, -0.654	Sgn 0.6716	0.00001	0.0001	0.6714	0.672
Fall, 5.463	Mgn 3.248	-0.00003	0.0001	3.2477	3.2483
y, -6.485	-6.54	0.00002	0.0002	-6.5409	-6.5394
2.884	2.8149	-0.00001	0.0002	2.8142	2.8155
-0.299	Sgn 0.4234	-0.00001	0.0002	0.423	0.424
4.740	4.73	-0.00003	0.0002	4.7294	4.7306
cp,-6.728	-8.7019	0.00008	0.0005	-8.703	-8.7006
3.459	Mgn 5.4922	0.00001	0.0005	5.4908	5.4936
-1.276	Sgn, Mgn 0.0808	-0.00006	0.0005	0.0797	0.0821
5.385	Mgn 3.7447	-0.00009	0.0005	3.7436	3.7466
dk, -5.620	Sgn, Mgn 0.5961	0	0	0.5961	0.5962
0.817	Mgn 0.3481	0	0	0.3481	0.3482
3.182	Mgn 0.4983	0	0	0.4983	0.4984
2.461	Mgn 0.4758	0	0	0.4758	0.4758
k, 0.301	Sgn -3.1192	0.00011	0.003	-3.1264	-3.1106
0.141	Mgn 2.4725	0.00014	0.0028	2.4663	2.4807
0.158	Mgn 0.7197	0.00002	0.0028	0.7114	0.7262
0.240	0.2623	-0.00069	0.0029	0.2535	0.27
n, -1.675	Mgn -3.2291	-0.0001	0.003	-3.2379	-3.2219
0.477	Mgn 0.9872	-0.00015	0.0028	0.9786	0.9933
0.178	Sgn,Mgn -0.048	-0.00001	0.0028	-0.0547	-0.0395

1.021	Mgn 2.9856	0.00066	0.0029	2.9782	2.9942
Labor productivity -4.810	Sgn Mgn 2.756	-0.00033	0.0187	2.7029	2.7988
2.407	SgnMgn -11.133	-0.01926	0.0298	-11.1761	-11.042
-0.477	Sgn Mgn 9.1045	0.00443	0.0349	9.0226	9.1762
3.719	SgnMgn 18.8515	-0.0001	0.0243	18.7859	18.9061
Change in Real rate,2.812	SgnMgn -5.9869	-0.00009	0.003	-5.9958	-5.9796
0.822	Mgn 2.2228	-0.00015	0.0028	2.2141	2.2289
0.397	Mgn 0.0208	-0.00001	0.0028	0.0141	0.0294
-4.031	Sgn 4.0865	0.00065	0.0029	4.0792	4.0949

Table 3

Comparison of Standard Errors of Seasonal Factors

Original SE	Mean over 999	Std.Dev.	Minimum	Maximum
0.3003	0.3003	0	0.3003	0.3004
0.3278	0.3278	0	0.3277	0.3279
0.3015	0.3015	0	0.3015	0.3016
0.3245	0.3245	0	0.3244	0.3246
0.2084	0.2084	0.0001	0.2082	0.2086
0.2352	0.2352	0.0001	0.235	0.2354
0.111	0.111	0.0001	0.1109	0.1111
0.1819	0.1819	0.0001	0.1818	0.182
1.0917	1.0917	0.0001	1.0913	1.092
0.8998	0.8998	0.0001	0.8994	0.9002
0.6594	0.6595	0.0001	0.6591	0.6598
1.1212	1.1212	0.0001	1.1209	1.1215
0.115	0.115	0	0.115	0.115
0.0972	0.0972	0	0.0972	0.0972
0.1029	0.1029	0	0.1029	0.1029
0.0856	0.0856	0	0.0856	0.0856
0.1561	0.1561	0.0006	0.1548	0.1579
0.165	0.1651	0.0006	0.1635	0.1667
0.1273	0.1275	0.0006	0.1252	0.1292
0.1555	0.1556	0.0006	0.154	0.1572
0.2638	0.2638	0.0006	0.2626	0.2655
0.2949	0.295	0.0006	0.2936	0.2965
0.3458	0.346	0.0007	0.344	0.3472
0.3347	0.3348	0.0006	0.3334	0.3363

8.0959	8.096	0.0021	8.0916	8.1
9.0388	9.0383	0.0026	9.0321	9.0454
7.3269	7.3269	0.0101	7.3034	7.3498
11.9456	11.9459	0.0056	11.9326	11.9574
0.4023	0.4023	0.0006	0.4011	0.4041
0.3989	0.399	0.0006	0.3974	0.4004
0.4871	0.4873	0.0007	0.4853	0.4885
0.4299	0.43	0.0006	0.4287	0.4315

Table 4

Data Range	61Q1-66Q4	67Q1-7Q3	72Q4-80Q3	80Q4-86Q3
Original un perturbed data	1.823	0.8661	-1.796	5.643
Minimum	1.796	0.8391	-1.822	5.617
1st Quartile	1.817	0.8609	-1.8	5.638
Median	1.825	0.8652	-1.797	5.642
Mean	1.823	0.8663	-1.796	5.643
Standard Deviation	0.0082	0.0084	0.0073	0.0088
3rd Quartile	1.829	0.8739	-1.791	5.65
Maximum	1.85	0.8957	-1.772	5.671