Which Inequality? The Inequality of Resources Versus the Inequality of Rewards *

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Abstract

We consider a tournament arising out of a matching market. On one side of the market, aspirants who are differentiated in ability choose a level of output. Under assortative matching, higher output leads to a better match. Thus, rather than a "winner-take-all" tournament, there are multiple ranked rewards. That is, the participant with the highest output gets highest reward (the best match), the second placed competitor gets second-highest reward and so on. In this model, equilibrium strategies and payoffs depend on the distributions of types of the competitors and the distribution of rewards. An increase in the inequality of competitors' *resources* tends to reduce output and can make all competitors better off. However, an increase in the dispersion of the *rewards* increases output and can make all worse off. We use these results to investigate the effect of inequality on individual welfare.

Keywords: inequality, resources, rewards, relative position, games, tournaments, matching.

JEL codes: D63, D62, J41, D31.

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1 Introduction

Perhaps, there is no other economic debate older than that over inequality. Since Plato and Aristotle, a multitude of thinkers have pondered over the variation in human conditions and questioned its justification. As Sen (1980) puts it, most argue for equality of something, but without much consensus on what should be equalised. There is even debate over what is meant by equality and inequality (see Sen (1980), Phelps Brown (1988), Roemer (1996), Lamont (2003) and many others). However, what all these approaches have in common is that equality is a moral question. There may be some distributions of income or of wealth or some methods of distribution of resources which are simply unfair. In contrast, in economics, the second fundamental welfare theorem seemed to separate these moral issues from the mainstream of economic analysis, the study of efficiency. More recently, it has been suggested that people have "social preferences, so that they care about what others' receive as well as their own income or consumption. Differing formulations have been proposed by Frank (1985), Fehr and Schmidt (1999) and Charness and Rabin (2002) amongst many.

Here, we take a purely economic approach and examine a model where individuals care only about their own consumption, yet inequality has a direct effect on market outcomes. We imagine a society where individuals differ in terms of ability but there is formal equality of opportunity. That is, "there is no legal bar to access to education, to all positions and jobs, and that all hiring is meritocratic" (Roemer, 1996, p. 163) and where individuals are rewarded according to their productivity (Lamont, 2003). We model this award system as a tournament, with each individual being rewarded according to his relative achievement and a given distribution of rewards that could represent cash prizes, places at a prestigious university, an attractive job, social esteem or any combination of these. We find that the Nash equilibrium choices of output and the resulting equilibrium utilities depend on both the initial distribution of endowments and the distribution of rewards. Therefore, there is no need to appeal to any notion of justice for equality to matter. It matters because it affects the job one gets, the wage one is paid and the amount of leisure one takes.

Thus, both the distribution of endowments and the distribution of rewards affect individual choices and equilibrium utility. However, we find that changes in the inequality of resources have an equal but opposite effect to changes in the inequality of rewards. An increase in the equality of competitors' abilities raises the return to output as it is easier to overtake other competitors. This leads to higher output but lower equilibrium utility at every level of resources. However, an increase in the equality of the rewards implies there is less difference between a high prize and a low one. This tends to lower equilibrium output and raises equilibrium utility.

To our knowledge, the existing merit- or effort-based theories of justice assume that those who work more, or have greater merit, would be rewarded more. However, in the existing literature on distributive justice, there seems to be little discussion of the reward schedules, or, how exactly does a given amount of merit or output translates into a reward. In effect, it seems to have been assumed that talent is absolute: with equality of opportunity, individuals will get their "just" reward. Yet it is equally reasonable to expect that an individual with a given level of talent could obtain very different rewards in two different societies. The return to talent is determined by the distribution of opportunities and rewards, which varies across countries and across time. For example, it seems that the dispersion of rewards has increased in recent years, particularly in the US.[1] Furthermore, one would expect the return to talent to be affected by its overall supply. Or, in other words, an individual's reward will also depend on the distribution of resources in her society. We will consider a model of imperfect competition in a stylized labor market where the return to talent is relative - it depends on the talents of one's competitors.

Why should we assume that success is driven by relative performance? One reason is empirical. There is now a significant body of research that suggests that significant indicators such as job satisfaction (Brown et al., (2004)), health (Marmot et al. (1991), Marmot (2004)) and overall happiness (Easterlin (1974)) is strongly determined by relative position. Another reason is that the relative approach is more general. We also show that the case where each individual is rewarded according to his absolute ability is a special case of the model we consider. This also clarifies how our work differs from that of Cole, Mailath and Postlewaite (1992, 1995) who pioneered the analysis of this form of tournament but who concentrated on this special case.

The existing literature on relative position and relative concerns has tended to assume that if relative position matters, this implies a distaste for inequality. In particular, Frank (1999, 2000) has argued forcefully that greater inequality exacerbates wasteful social competition. Our earlier work in this area (Hopkins and Kornienko (2004a, 2004c)) showed that this argument has a gap in its logic. In fact, in a model with relative concerns an increase in equality of resources can make everyone worse off. Here, we generalize these results and show that it is an increase in the inequality of rewards that worsens wasteful competition.

Equally, Dworkin (1981), for example, has argued for equality of resources for reasons of distributive justice. Crucial to this argument is the fact that overall resources are made up of many different things, innate talent, the quality of education one is given, parental investment and inherited wealth. Thus, to assure equality of opportunity, where success depends on talent alone, one needs to compensate for inequality in other forms of resources. Our work for the present abstracts away from these issues by assuming that resources are equal to ability or talent. We leave the analysis of the case where resources are a composite of ability and wealth to future research.

This paper concentrates on the analysis of exogenous shifts in the distribution of rewards. It therefore differs from Moldovanu and Sela (2001) who consider what would the optimal distribution of prizes be from the perspective of a contest designer who aimed to generate the maximum effort from contestants. We feel that our approach is

[1] See Frank and Cook (1996) for many empirical examples and some hypotheses about causes.

more appropriate for the matching markets we consider, as, for example, the distribution of job offers in labour market is not under the control of any single planner. Certainly, Cole, Mailath and Postlewaite (1992, 1995, 1998) and Fernandez and Galí (1999) who consider matching tournaments also assume exogenous distributions of characteristics on both sides of the market. However, their focus of interest is not the effect of changes in those distributions but a comparison of the efficiency of different matching institutions.

Inequality has increased considerably in the past few decades, in the UK and in particular in the USA. The level of this change has caused some commentators, for example Frank (1999) and Krugman (2002), to question its effects and causes. Frank (1999, 2000) in particular worries that the highly visible expenditure on luxury items by the very rich is likely to provoke what amounts to an arms race in conspicuous consumption. However, our analysis, in Hopkins and Kornienko (2004a, 2004c) and in this paper, suggests that a bigger worry should be the increase in the dispersion of potential rewards.

2 The Model

In this section, we develop our model of a tournament, where a large population of contestants compete in a matching market. We have in mind three prime examples. The first is students competing for places at college. The second is a market for jobs. For example, students in the final year of graduate school seek faculty positions at universities. The third is the marriage market. In all cases, we make the simplifying assumption that both sides of the market have a common ordering over potential matches. That is, in the job market for example, all students have a consensus over which would be the best job to get, what would be the second best and so on. Equally, schools all agree who is the best candidate on the market and who is second best and so on. Clearly, both sides of the market are making important decisions. However, in this work we concentrate on the decisions on output made by the competing candidates. Indeed, we can also consider the special case of competitive situations such as sports tournaments where in effect there is a single judge who matches each aspirant with one of a range of possible rewards.

The model can be considered as a static version of the model introduced by Cole, Mailath and Postlewaite (1992). However, we generalise their model to allow for different distributions of characteristics on the two sides of the market. This will allow both for a richer model and for comparative statics analysis of the effect of changes in those distributions.

There are two populations of agent: aspirants and partners. They are differentiated in quality with an aspirant's type being z with z distributed on $[\underline{z}, \bar{z}]$ with $\underline{z} \geq 0$ according to the distribution $G(z)$. The distribution $G(z)$ is twice differentiable with density $g(z)$. Responders are also differentiated in their attribute s which has the twice differentiable distribution function $H(s)$ on $[\underline{s}, \bar{s}]$ and density $h(s)$ (in the case of a sports tournament $H(s)$ is just the distribution of reward money). The aspirants will compete amongst themselves to match with the partners. In particular, aspirants must choose a visible output level x, from the continuous interval $[0, z]$. An aspirant's type z has the general interpretation as her total resources, with possible specific interpretations being wealth or a productivity parameter that determines maximum potential output.[2] After the choice of output, matching will take place, with one aspirant matching with each partner. As we will see, stable matching will be positive and assortative. That is, aspirants with high x will match with partners with high s.

We now consider preferences. Partners have an attribute s that is attractive to aspirants. In the context of the academic job market, s could be interpreted as prestige or reputation of a university, in the marriage market, s could be a measure of attractiveness to the opposite sex. In sports tournaments, it is simply the value of a cash prize. For the aspirants, we assume that each has the same utility function

$$U(x, y, s) = V(x, z - x)s \tag{1}$$

That is, an aspirant's type z is her endowment of productivity, that can be employed in production x or leisure $y = z - x$. One can consider V as a standard utility function which is increasing in x, consumption, and strictly increasing in $z-x$, leisure. We assume that V is positive, quasiconcave and twice continuously differentiable with $V_{ij} \geq 0$ and $V_{ii} \leq 0$. There is then additional utility arising from consumption of s. However, no trade is possible, it is not possible, for example, to buy s with x. Rather to obtain s, an aspirant must match with a partner. Once matched, we assume in our initial model that utility is non-transferable between partners, that is, s is neither excludable nor divisible. That is, this is matching with non-transferable utility (NTU).[3] Applicants in their choice of match care solely about the value of s in a partner.

An aspirant's observable action x represents the production of an asset useful to the partners. For example, the wealth an agent has produced may make him attractive as a potential spouse, or a worker's investment in human capital may make her an attractive hire. We assume that partners' preferences are captured by a strictly increasing utility function $W(x)$. So that, in comparing two aspirants, any partner would strictly prefer the one with higher output x.

Following Cole, Mailath and Postlewaite (1992, 1998), a matching is a function $\phi : [0, 1] \to [0, 1] \cup \{\emptyset\}$ that is measure-preserving and one-to-one on $\phi([0, 1])$, where $\phi(i) = j \in [0, 1]$ is i's match and $\phi(i) = \emptyset$ indicates that i is not matched. That is, for all measurable subsets $A \subset [0, 1]$, $\phi^{-1}(A)$ is measurable and $\lambda(\phi^{-1}(A)) = \lambda(A)$, where λ denotes Lebesgue measure. A matching is stable if there does not exist $i \neq i' \in [0, 1]$ such that $\phi(i')P_i\phi(i)$ and $iP_{\phi(i')}i'$, with both preferences holding strictly.

The first condition is the equivalent in a continuum of requiring exactly one aspirant

[2] For example, suppose all aspirants are endowed with the same amount of time that can be used for production or leisure. Then, let z be productivity per hour and an aspirant devoting a proportion x/z of time to production will have output x.

[3] For some analysis of the transferable utility case, see Hopkins (2005).

being matched to one partner. The second is the stability condition standard in most matching problems, that requires that matches made are not subject to unravelling in the sense that it should not be possible to find an aspirant and a partner who would prefer to match with each other in place of their current matches. The theoretical exercise here is to find conditions for when this is the case.

In this context, an equilibrium will be a strategy $x(z)$ for the aspirants and an associated matching scheme that is stable given observable output and the strategy $x(z)$. We call such an equilibrium symmetric if all aspirants use the same strategy, that is, the same mapping $x(z)$ from type to output. Let us aggregate all the output decisions of the aspirants into a distribution summarised by a distribution function $F(x)$.

Suppose for the moment that the equilibrium strategy $x(z)$ is differentiable and strictly increasing (we will go on to show that such an equilibrium exists). Note, first, that this would imply that in equilibrium an agent of type z_i who produces $x(z_i)$ would have a position in the distribution of output $F(x(z_i))$ equal to his rank $G(z_i)$ in the distribution of resources. This enables the partners to infer which aspirant is in fact the most able. This in turn allows the matches to be made through the following assortative matching mechanism so that aspirants with high (respectively low) x are matched with partners with high (respectively low) s. More specifically, an aspirant's rank in level of output determines the rank of his match. That is, an aspirant making a choice x_i will achieve a match of value $s_i = H^{-1}(F(x_i))$ or $F(x_i) = H(s_i)$. Then, we can show that the assortative scheme outlined above is stable. That is, we can find no aspirant and partner who would both prefer each other in place of their current match.[4]

Lemma 1 *Suppose the utility of partners is strictly increasing in x. Suppose all aspirants adopt a symmetric strictly increasing strategy $x(z)$, then the assortative matching, such for an aspirant of type z_i for any $z_i \in [\underline{z}, \bar{z}]$, with output $x_i = x(z_i)$ her match is of type s_i, where*

$$G(z_i) = F(x_i) = \phi(F(x_i)) = H(s_i), \tag{2}$$

is stable.

We now derive a symmetric equilibrium strategy for the aspirants. Suppose all agents adopt a strictly increasing differentiable strategy $x(z)$. Then the equilibrium relationship (2) implies that we can define the function $S(z) = H^{-1}(G(z))$, which gives the equilibrium match of an aspirant of type z, so that $S : [\underline{z}, \bar{z}] \to [\underline{s}, \bar{s}]$. Thus, the marginal return to z in terms of the equilibrium match is

$$S'(z) = \frac{g(z)}{h(H^{-1}(G(z)))}. \tag{3}$$

[4]Results of this type go back to Becker (1973). See Cole, Mailath and Postlewaite (1995), Fernandez and Galí (1999) for a tournament approach similar to that employed here. Eeckhout (2000), Legros and Newman (2004) show that assortative matching is the *only* stable matching scheme.

This also implies a reduced form utility:

$$U(x, y, s) = V(x(z), z - x(z))S(z) = V(x(z), z - x(z))H^{-1}(G(z)).$$

That is, the tournament with assortative matching implies that each individual's payoffs are increasing in her rank $G(z)$ in the distribution of aspirants. It therefore might appear to an outside observer that the individual had some form of social preferences where she cares about her relative position, similar to those analysed by Frank (1985) and Hopkins and Kornienko (2004a). The matching tournament therefore gives a strategic basis to such models.

Suppose now one agent produces $x(\hat{z})$ in place of her equilibrium choice $x(z)$ and then chooses $\hat{z}$ to maximise her payoff. Her utility will be, given the utility function (1),

$$V(x(\hat{z}), z - x(\hat{z}))S(\hat{z}) \tag{4}$$

The first order condition for an optimum will be

$$(V_1(\cdot) - V_2(\cdot))\, S(\hat{z})x'(\hat{z}) + V(\cdot)S'(\hat{z}) = 0 \tag{5}$$

In symmetric equilibrium, it must be that $\hat{z} = z$ which gives us the following differential equation,

$$x'(z) = \frac{S'(z)}{S(z)} \frac{V(x, z - x)}{V_2(x, z - x) - V_1(x, z - x)} = \frac{S'(z)}{S(z)}\psi(x, z). \tag{6}$$

This differential equation will give us our equilibrium strategy, in combination with the boundary condition we now derive.

Lemma 2 *In a symmetric equilibrium in continuous strictly increasing strategies, if* $\underline{s} = 0$*, then* $\lim_{z \to \underline{z}} x(z) = \underline{z}$*, but if* $\underline{s} > 0$*, then* $x(\underline{z}) = x_c(\underline{z})$ *where* $x_c(z)$ *maximises* $V(x, z - x)$*.*

That is, if $\underline{s}$, the lowest prize or reward is zero, low ability aspirants exhaust their entire surplus. But if $\underline{s} > 0$, then $x(\underline{z}) = x_c(\underline{z})$. The lowest ranked aspirant acts as though matching considerations did not matter. This reflects the equilibrium competitive response to the expectation that one is going to come last. If the resulting reward is zero, then low ability candidates are desperate. If even lowest reward is positive, then there is no point producing positive output.

We can also show that there is only one symmetric equilibrium. To do this we need to consider the possibility of, for example, all aspirants choosing the same x. Then, they would all have rank 1, but could not all be matched with the top partner. Our assumption is that in such cases, since the partners cannot distinguish between them, the tying aspirants are uniformly randomly matched with partners of equivalent rank.

Assumption: Tie-Breaking. If there is a mass point at some $\hat{x}$, so that $F(\hat{x}) \neq F^-(\hat{x})$,[5] then the mass of aspirants $F(\hat{x}) - F^-(\hat{x})$ are uniformly randomly matched

[5] $F^-(\hat{x})$ is the left limit of $F(x)$ and is defined as $\lim_{x \to \hat{x}_-} F(x)$.

with the mass of partners $H(s_1) - H(s_0)$ where $F(\hat{x}) = H(s_1)$ and $F^-(\hat{x}) = H(s_0)$. Note that the expected match for an aspirant choosing $\hat{x}$ which we write as $S(\hat{x})$ must satisfy $s_0 < S(\hat{x}) < s_1$.

This assumption will be useful in establishing that in equilibrium, aspirants will in fact not choose the same level of output, as it gives aspirants an explicit incentive to break such ties. An aspirant choosing a fraction more would be able to attract the best partner, of type s_1, for sure.

Proposition 1 *The unique solution to the differential equation (6) on* $(\underline{z}, \bar{z}]$ *together with the boundary conditions,* $x(\underline{z}) = \underline{z}$ *for* $\underline{s} = 0$ *and* $x(\underline{z}) = x_c(\underline{z})$ *for* $\underline{s} > 0$*, and the assortative matching scheme (2) constitute the (essentially) unique symmetric equilibrium to the tournament matching game.*

The equilibrium described is only "essentially unique" as when $\underline{s} = 0$, an agent of type $\underline{z}$ is indifferent over all possible outputs $[0, \underline{z}]$. Therefore, there are other equilibria that only differ in the behaviour ascribed to this lowest rank aspirant. The essence of the proof is to show that any symmetric increasing equilibrium strategy $x(z)$ is continuous and then differentiable. Hence, it must constitute a solution to the differential equation (6), which has a unique solution on $(\underline{z}, \bar{z}]$.

We now give one case where a closed form solution for the equilibrium strategy is possible.

Example 1 *The "simple" model: suppose that* x *has no value to the aspirants only to the partners, that is* $V(x, z - x) = z - x$*. This implies that* $x_c(z) = 0$*. This is also technically convenient as in this case the differential equation (6) has the closed form solution*

$$x(z) = \frac{\int_{\underline{z}}^{z} t dS(t)}{S(z)} = z - \frac{\underline{z}\ \underline{s} + \int_{\underline{z}}^{z} S(t) dt}{S(z)} \tag{7}$$

Note that this matching tournament is isomorphic to an aspirant participating in a standard first price auction with $S(z)$ *taking the place of the probability of winning in the auction (the analogy being particularly strong when* $\underline{s} = 0$ *and* $\bar{s} = 1$*).*

The effect of the value of the lowest match $\underline{s}$ as noted in Lemma 2 can be large. For example, two different equilibrium strategies for the special case (7) are illustrated in Figure 1. Both assume that both z and s are uniformly distributed. The difference is that for one the minimum reward $\underline{s}$ (equivalently the prize for last place) is strictly positive and in the other it is zero. One can see that the output levels of low-ability candidates are quite different in the two situations. Nonetheless, the different equilibrium strategies both separate out the different candidates, but each involve a different level of output. From the point of view of the candidates, they are Pareto ranked. We go on to discuss welfare in the next section.

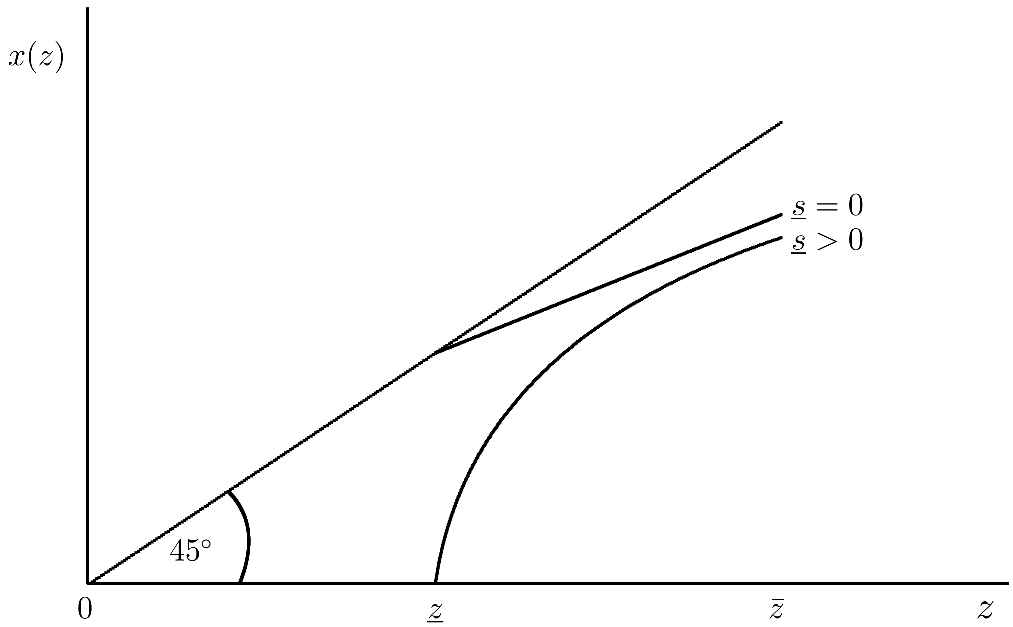

Figure 1: Two Sample Solutions of the "Simple" Model

3 Comparative Statics

We will now consider the effect on equilibrium utility and strategies of changes in the distribution of resources $G(z)$ and changes in the distribution of rewards $H(s)$. We saw in Section 2 that equilibrium behaviour depends on the matching function S which is jointly determined by G and H. Thus, as the distribution of resources G or the distribution of rewards H (or both) change, so does the matching function S. Thus, a change in either distribution of resources or rewards (or both) translates into a change in equilibrium choice of output $x^*(z)$ and, thus, into a change in individual welfare, $U^*(z) = V(x^*(z), z - x^*(z))S(z)$. What we are interested in is what effects would a change in the level of inequality have on individual welfare?

We use stochastic dominance to order different distributions. Second order stochastic dominance (or, equivalently, generalized Lorenz dominance) has become a standard way in which to rank income distributions in terms of inequality. Informally, a distribution $F_A(t)$ second order stochastically dominates another distribution $F_B(t)$ if the distribution F_A is less dispersed than F_B (and its mean is no lower). Formally, F_A second order stochastically dominates F_B if and only if

$$\int_{\underline{t}}^{t} F_A(\omega)d\omega \leq \int_{\underline{t}}^{t} F_B(\omega)d\omega \tag{8}$$

for $t \in (\underline{t}, \bar{t})$. Some of our results will also concern first-order stochastic dominance. One says one distribution F_A is stochastically higher or stochastically dominates another distribution F_B if $F_A(t) \leq F_B(t)$ for all t.

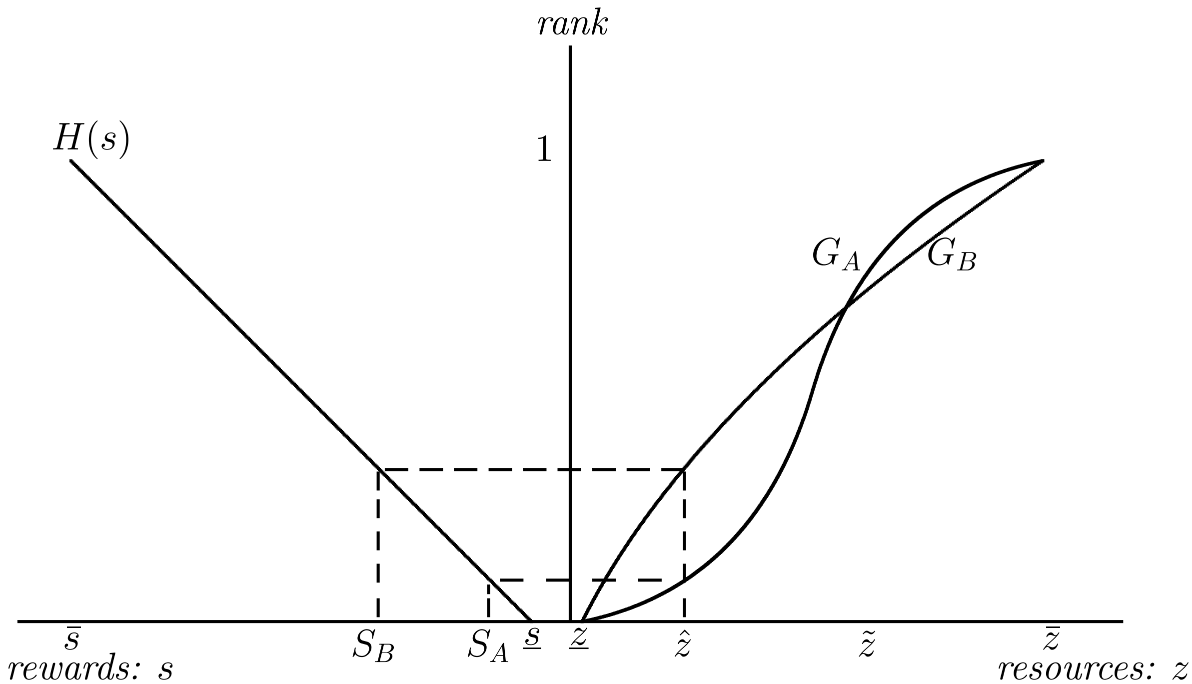

Figure 2: Regime G: an aspirant with fixed low resources $\hat{z}$ has a match S_A under the more equal distribution of resources G_A that is worse than the match S_B under the less equal distribution of resources G_B.

In what follows we assume two societies A and B that are identical apart from having either different distributions of resources or different distributions of rewards, but not both. That is, we will consider two regimes.

Regime G: Different Resources In regime G, we assume that the societies have identical distributions of rewards, i.e. $H_A = H_B = H$, but differ in the distributions of resources, i.e. $G_A \neq G_B$. We also assume that G_A and G_B have the same support $[\underline{z}, \bar{z}]$. Different resources imply that the two societies have different matching functions, i.e. $S_A^G(z) = H^{-1}(G_A(z))$ and $S_B^G(z) = H^{-1}(G_B(z))$.

Regime H: Different Rewards In regime H, we assume that the societies have identical distributions of resources, i.e. $G_A = G_B = G$, but differ in the distributions of rewards, i.e. $H_A \neq H_B$. We again assume that H_A and H_B have the same support $[\underline{s}, \bar{s}]$. Again, different rewards imply that the two societies have also different matching functions, i.e. $S_A^H(z) = H_A^{-1}(G(z))$ and $S_B^H(z) = H_B^{-1}(G(z))$.

We now can state our main result regarding the effects of changes in the distributions of ability and rewards on the individual welfare. Define $U^*(z) = V(x^*(z), z - x^*(z))S(z)$, that is $U^*(z)$ is equilibrium utility. We first show that an increase in equality amongst competitors reduces the utility of the weakest competitors. In contrast, a similar decrease in the dispersion of the rewards has an opposite effect.

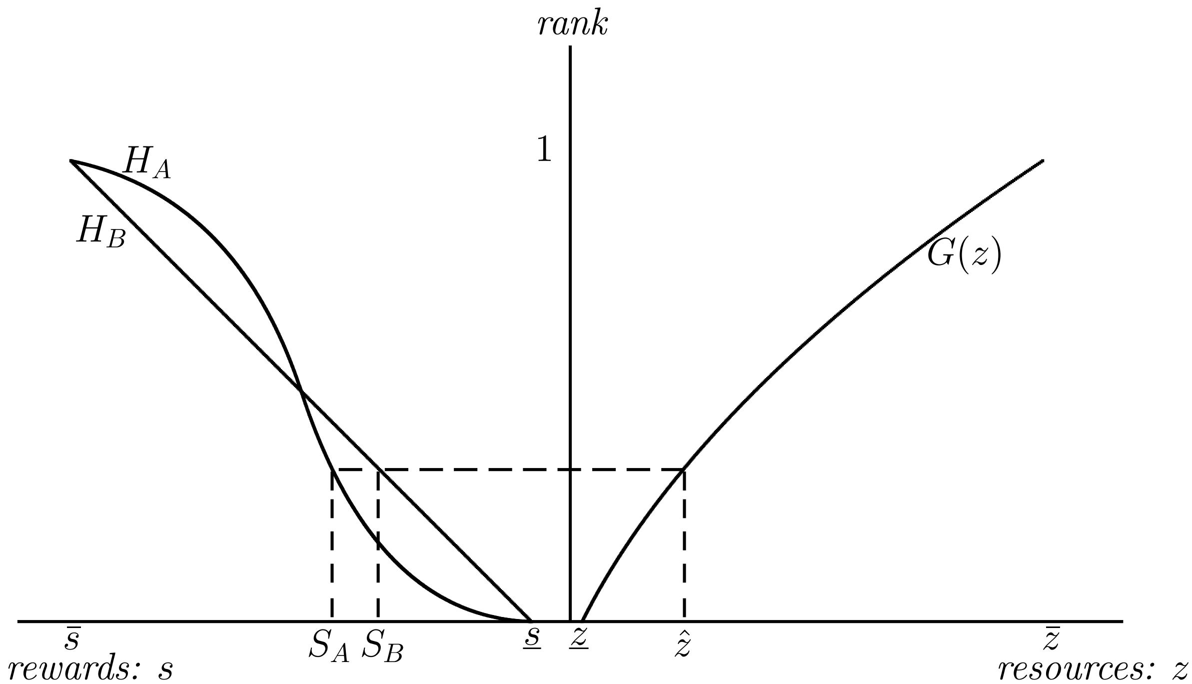

Figure 3: Regime H: an aspirant with fixed low resources $\hat{z}$ has a match S_A under the more equal distribution of rewards H_A that is better than the match S_B under the less equal distribution of rewards H_B.

Proposition 2 (*i*) *Suppose that, in regime G, G_A second order stochastically dominates G_B and that there are a finite number of points in $[\underline{z}, \bar{z}]$ where $G_A(z)$ equals $G_B(z)$. Denote the first crossing as $\tilde{z}_G$. Then, $U_A^*(z) \leq U_B^*(z)$ for all $z \in [\underline{z}, \tilde{z}_G]$.*

(*ii*) *Suppose that, in regime H, H_A second order stochastically dominates H_B and that there are a finite number of points in $[\underline{s}, \bar{s}]$ where $H_A(s)$ equals $H_B(s)$. Denote the first crossing as $\tilde{s}$, and denote $\tilde{z}_H = S^{-1}(\tilde{s}) = G^{-1}(H_A(\tilde{s})) = G^{-1}(H_B(\tilde{s}))$. Then, $U_A^*(z) \geq U_B^*(z)$ for all $z \in [\underline{z}, \tilde{z}_H]$.*

That is, for those "less endowed" (that is, for those whose endowment is sufficiently close to $\underline{z}$, or $z \leq \tilde{z} = \min\{\tilde{z}^G, \tilde{z}^H\}$), a more equal distribution of resources leads to lower individual welfare, while, conversely, a similar decrease in inequality of rewards results in an increase in individual welfare. The principal reason for this is that an individual on a low level of resources has worse matching prospects with a more equal distribution of resources, but better matching prospects with a more equal distribution of rewards. These two points are illustrated in Figure 2 and Figure 3 respectively.

That inequality can be bad may be surprising. But it is important to note *which inequality* we talk about. As we show above, it is *inequality of rewards* which makes the less endowed worse off. In contrast, *inequality of resources* is not so bad for the relatively disadvantaged. The effect of greater equality on those with high resources can go either way. However, for some examples, it can be the same as for the disadvantaged.

Example 2 *In the simple model (see Example 1), the equilibrium utility can be explicitly calculated from the equilibrium strategy (7) as*

$$U^*(z) = (z - x(z))S(z) = \int_{\underline{z}}^{z} S(t)dt + \underline{s}\ \underline{z}. \tag{9}$$

Clearly, greater "equality" in S in the sense of second order stochastic dominance given the definition (8) leads to a reduction in utility at every level of z in $(\underline{z}, z]$. For example, when the distribution of rewards becomes more dispersed, utility for the competitors can be lower almost everywhere.

We now turn to the effect of a increase in society's resources and an increase in society's rewards in the sense of the first order stochastic dominance. It turns out that all we have to do is to extend Proposition 2 slightly so that $\tilde{z}^G$ is now equal to $\bar{z}$ and $\tilde{s}$ is now equal to $\bar{s}$, which gives us the following.

Corollory 1 *Suppose that, in regime G, G_A first order stochastically dominates G_B, while in regime H, H_A first order stochastically dominates H_B. Then, for all $z \in [\underline{z}, \bar{z}]$, $U_A^*(z) \leq U_B^*(z)$ in regime G, but $U_A^*(z) \geq U_B^*(z)$ in regime H.*

In other words, the different types of "richness" of the society have quite the opposite effects on the equilibrium welfare. That is, if a society becomes richer in terms of resources, utility falls at every level of resources, but if a society becomes richer in terms of rewards, utility rises at every level.[6] Note that in the higher distribution of resources, G_A, utility is lower at each resource level. But equally the fact that G_A first order stochastically dominates G_B means that a positive mass of the population will have higher resources under G_A than G_B, a rise that may or may not be enough to offset the fall in utility at each resource level.

The above results concern the relationship between inequality and individual welfare. Overall welfare assessments, however, depend largely on what exactly we take x to be. If, for example, this tournament was run internally at a firm, the whole motivation would be to generate a high level of output. In general, a higher level of x represents an increased transfer from aspirants to partners. The results of Peters (2004) suggest that, as aspirants do not take this benefit into account, the amount of output may be inefficiently low.

This brings us to another question, that is, what is the relationship between inequality and incentives? As the equilibrium choice of output $x(z)$ depends on the incentives that are provided by the equilibrium matching function $S(z)$, a change in either the

[6]This observation may help to explain why, as Easterlin (1974) noticed, economic growth did not bring an increase in happiness in the postwar period. Economic growth tends to increase both resources and rewards, which, as we show here, have opposing effects on individual welfare. In order to judge the overall effects of economic growth on welfare, a dynamic model is needed, but this is beyond the scope of this paper.

distribution of resources $G(z)$ and the distribution of rewards $H(s)$, or both, will typically result in a change in the equilibrium output $x(z)$. In particular, an increase in the equality of resources makes it easier to overtake other competitors. This tends to lead to higher output. However, an increase in the equality of the rewards reduces the difference between a high prize and a low one. Thus, incentives for producing output are lower. To summarise, a more equal distribution of resources, or a more unequal distribution of resources produce an environment that is more competitive.

Thus, as we show below, different types of inequality have opposing effects on the individual output, but the comparison is far more complicated and less clear cut than in the case of equilibrium utility. Specifically, the standard stochastic dominance orders used above are not sufficient to obtain clean comparative statics in the games we consider. Thus we employ the following refinements of first and second order stochastic dominance.

Definition (MLR): The two distributions F_A, F_B satisfy the Monotone Likelihood Ratio (MLR) order and we write $F_A \succ_{MLR} F_B$ if the likelihood ratio $f_A(t)/f_B(t)$ is strictly increasing on $(\underline{t}, \bar{t})$.

The monotone probability ratio order implies that the ratio $f_A(t)/f_B(t)$ is strictly increasing over $(\underline{t}, \bar{t})$. The MLR order implies that F_A (first order) stochastically dominates F_B. Moreover, the average value in society A is greater than that in society B, or $\mu_A = \int t dF_A > \int t dF_B = \mu_B$. We also employ a strengthening of second order stochastic dominance analogous to the monotone likelihood ratio order.

Definition (ULR): Two distributions F_A, F_B satisfy the Unimodal Likelihood Ratio (ULR) order and we write $F_A \succ_{ULR} F_B$ if $\mu_A \geq \mu_B$ and the ratio of their density functions $f_A(t)/f_B(t)$ is unimodal. That is, it is strictly increasing for $t < \hat{t}$ and it is strictly decreasing for $t > \hat{t}$ for some $\hat{t} \in (\underline{t}, \bar{t}]$.

In simple terms, if a distribution $F_A \succ_{ULR} F_B$, then F_A is more equal and less dispersed than F_B. More precisely, it can be shown that if $F_A \succ_{ULR} F_B$, then F_A also second order stochastically dominates F_B (Ramos et al. (2000)). This in turn implies that if $F_A \succ_{ULR} F_B$ and the means are in fact equal, then F_B is a mean preserving spread of F_A. If $F_A \succ_{ULR} F_B$ then the ratio f_A/f_B will have a unique maximum on $(\underline{t}, \bar{t}]$. Let $\hat{t}$ be that value of t that maximizes the ratio. It is also possible to show that F_A and F_B will cross once at $\tilde{z} \in (\underline{z}, \bar{z}]$ with $\tilde{t} \geq \hat{t}$. If $\hat{t} = \bar{t}$, then the above condition reduces to the monotone likelihood ratio order, or in other words, the monotone likelihood ratio order implies the unimodal likelihood ratio order.

To obtain comparative statics results on solutions to the differential equation we will make use of the ratio $P(z) = S_A(z)/S_B(z)$. The important point is that where the ratio $P(z)$ is increasing then $S'_A(z)/S_A(z) \geq S'_B(z)/S_B(z)$ and therefore from the differential equation (6), all other things equal, x_A, output in society A, will be growing more rapidly than x_B. This in turn limits the number of times the two solutions, x_A and x_B can cross.

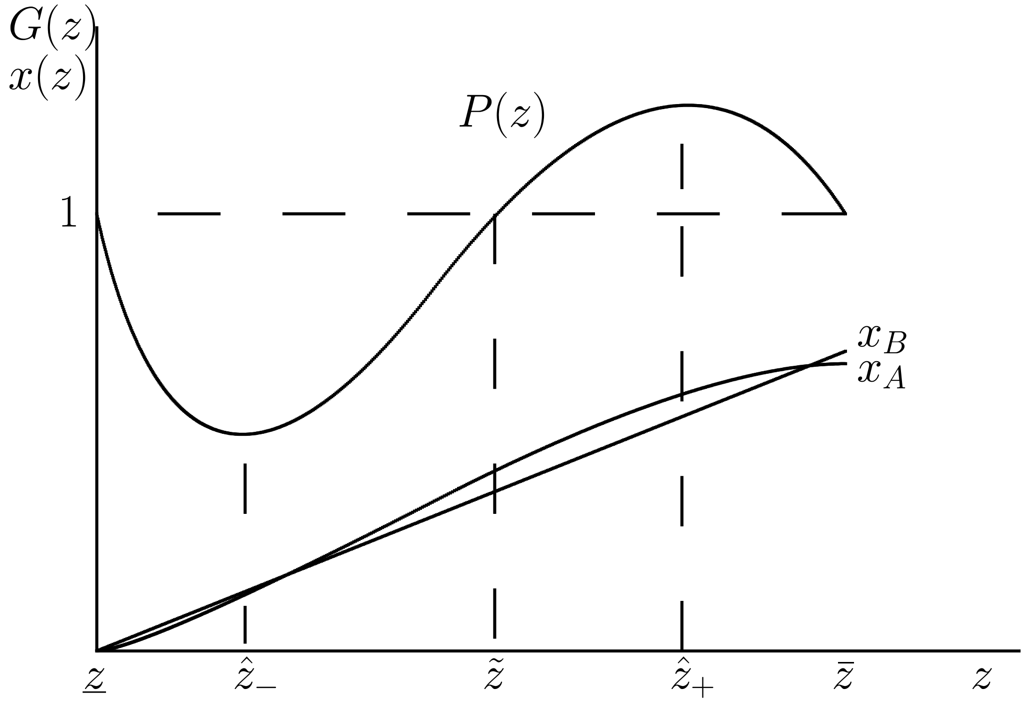

Figure 4: Comparative Statics from an Increase in Equality of Resources or an Increase in Dispersion of Rewards for the case where $\underline{s} > 0$.

As Lemma 4 in the Appendix shows, if the distribution F_A dominates F_B in the sense of the ULR order then $P(z)$ has at most two extremal points, $\hat{z}_-$, where the ratio is at minimum, and $\hat{z}_+$, where the ratio is at maximum, with $\underline{z} \le \hat{z}_- < \tilde{z} < \hat{z} < \hat{z}_+ < \bar{z}$.[7] This is illustrated in Figure 4.

Proposition 3 *Suppose $x_A(z)$ and $x_B(z)$ are the equilibrium choices of output for matching functions S_A and S_B, respectively. Suppose that, in regime G, $G_A(z) \succ_{ULR} G_B(z)$, and suppose further that $H(s)$ is a uniform distribution. Also, suppose that, in regime H, $H_B(z) \succ_{ULR} H_A(z)$ and that $G(z)$ is a uniform distribution. Then, in a regime $j = G, H$*
(a) if $\underline{s} = 0$, $x_A(z)$ crosses $x_B(z)$ at most once. Moreover, $x_A(z) > x_B(z)$ for all $z \in (\underline{z}, \hat{z}^j_+]$, with a possible crossing on $(\hat{z}^j_+, \bar{z}]$;
(b) if $\underline{s} > 0$, $x_A(z)$ crosses $x_B(z)$ at most twice. Moreover, $x_A(z) < x_B(z)$ for all $z \in (\underline{z}, \hat{z}^j_-]$, with a crossing in $(\hat{z}^j_-, \tilde{z}^j)$ so that $x_A(z) > x_B(z)$ for all $z \in [\tilde{z}^j, \hat{z}^j_+]$, with a possible crossing on $(\hat{z}^j_+, \bar{z}]$.

Figure 4 also illustrates the comparative static result (for the case where $\underline{s} > 0$). When $\underline{s}$, the lowest prize, is strictly positive, the aspirants with a low level of resources actually produce less in a more competitive environment (more equal resources or more

[7] Note that $\underline{z} = \hat{z}_-$ if and only if $\underline{s} = 0$. That is, if $\underline{s} = 0$, then $P(z)$ is increasing on $(\underline{z}, \tilde{z})$.

dispersed rewards). Intuitively, if a positive reward is guaranteed, then rather than compete for higher rewards in a difficult environment the weak just fall back on the minimum reward. However, if the lowest possible reward $\underline{s}$ is zero, those at the bottom of the resource distribution will produce more when resources are more equal, but less when the rewards are more equal. This is because they are now desperate to avoid last place. Some results are robust to whether or not the lowest prize is zero. Those with a middle level of resources always produce more, and those with a high level of resources may produce less (the case illustrated in Figure 4) or may produce more.[8]

4 Discussion and Conclusions

This paper offers two innovations. First, it offers a reason why inequality may matter even without any concern for social justice and in the absence of social preferences. This is because when there is interpersonal competition for employment and educational opportunities, inequality has a direct impact on equilibrium effort and equilibrium utility. Second, it makes a new distinction between different kinds of inequality. Equality of resources, the initial distribution of endowments, and equality of rewards, the rewards to success in society, have opposite effects. Greater equality of resources increases the degree of social competition, greater equality of rewards reduces it.

Our findings may also offer a clue of why the socialist system of the former Soviet Union has been unsuccessful. As there was little inherited wealth, one can argue that the distribution of endowments in the former Soviet Union was largely equal to the distribution of talents. As our analysis suggest, the relatively more equal pay scheme of the former Soviet Union resulted in those at the bottom of the distribution of talents being happier. However, those with greatest talents may be less happy. Even in the absence of corruption, these features of a pay-equalizing society entail grim long term prospects as those who are most able have little incentive to exert work effort, and would wish to move to a country with a more unequal pay distribution (thus implying a brain drain).

As we demonstrated in this paper, the relationship between inequality and individual welfare is complex. The gains and losses to greater equality are not uniform across society, and differ according to whether we consider equality of resources or of rewards. We would also like to emphasise that the fact that this work approaches inequality outside the framework of distributive justice, does not mean that moral considerations are irrelevant to the issue of inequality. Rather we hope, this paper, as well as our other related works (Hopkins and Kornienko (2004a, 2004c)), may help the understanding of how inequality interacts with economic decisions, and thereby, aid the understanding of these wider aspects as well.

[8] Maloney and McCormick (2000) find that in data from foot races with monetary rewards a greater dispersion of prize money does indeed lead to faster times.

Appendix

Proof of Lemma 1: In a symmetric equilibrium with a strictly increasing strategy $x(z)$, for an agent of type z_i we have $F(x(z_i)) = \Pr[x(z_i) < x(z)] = \Pr[x^{-1}(x(z_i)) < z] = G(z_i)$. Then the matching that assigns an agent with output x_i to an partner of type $s_i = H^{-1}(F(x_i)) = H^{-1}(G(z_i))$ is clearly stable as while any aspirant with rank $F(x_i)$ would prefer a match with any partner with $s > H^{-1}(F(x_i))$, such a partner would prefer her current match whose x would be greater than $\hat{x}$. ∎

Proof of Lemma 2: In a symmetric equilibrium, an individual with ability $\underline{z}$ has rank 0 and utility $U(\underline{z}, x) = V(x, \underline{z} - x)H^{-1}(0) = V(x, \underline{z} - x)\underline{s}$. If $\underline{s}$ is zero, then her equilibrium utility is 0. The only way she can increase her utility would be to raise her rank. Thus, in equilibrium the expenditure of those with slightly greater income must be such that the poorest agent is unable to increase her rank by increasing expenditure on x. That is, $\lim_{z \to \underline{z}^+} x(z) \geq \underline{z}$. But, as necessarily, $x(z) \leq z$ for all z, we have $\lim_{z \to \underline{z}^+} x(z) = \underline{z}$. Consequently, the match assigned to any $x \leq \underline{z}$ will be $s = \underline{s} = 0$, and so agents with resource $\underline{z}$ are indifferent between any level of x between 0 and $\underline{z}$. Alternatively, suppose $\underline{s}$ is positive. Then $U(\underline{z}, x) = V(x, \underline{z} - x)\underline{s}$ and does not depend on the agent's rank. Therefore, in equilibrium her choice must maximize $V(x, \underline{z} - x)$. That is, she must choose $x_c(\underline{z})$, or there would be a profitable deviation. ∎

Proof of Proposition 1: Proposition 1 in Hopkins and Kornienko (2004a) establishes a similar result for a game where a continuum of players, when all adopt the same strictly increasing strategy, have utility of the form $V(x, z - x)F(x)$. Here, assuming a strictly increasing strategy and assortative matching, aspirants have the (reduced form) utility $V(x, z - x)H^{-1}(F(x))$. It is relatively easy to establish that the proof that any strictly increasing equilibrium strategy must be continuous and differentiable holds also in this case. ∎

Let $x(z)$ be an arbitrary function, giving rise to an arbitrary distribution of actions $F(x)$. Suppose that we match assortatively with aspirants tieing on x being randomly assigned to partners. We show that this is the only stable matching.

Suppose there is another matching $\tilde{S}(x)$, such that a set of aspirants X with positive measure are matched differently than under the positive assortive matching $S(x)$. Then, there must exist $\hat{x} \in X$, $\tilde{S}(\hat{x}) > F(\hat{x})$, that is, there must be a positive measure of aspirants who are matched strictly higher than under $S(x)$. For this matching to be stable, all aspirants with output higher than $\hat{x}$ must be matched with partners whose s is greater than $\tilde{S}(\hat{x})$. If not, then partner $s = \tilde{S}(\hat{x})$ could propose a match with aspirant of type $\tilde{x}$ where $\tilde{x} > \hat{x}$ and the aspirant $\tilde{x}$ would find it acceptable. But the measure $\lambda(x \geq \hat{x}) > \lambda(s \geq \tilde{S}(\hat{x}))$, which is only possible if $\tilde{S}$ is not measure-preserving.

Given assortative matching, a symmetric equilibrium strategy must be strictly increasing. Suppose that it is not strictly increasing so that $\breve{x} = x(z_0) = x(z_1)$ for some $z_0 < z_1$. Then, $F(\breve{x}) > F^{-}(\breve{x})$, that is, there is a mass point in the distribution of consumption at $\breve{x}$. But then $S(\breve{x}) < F(\breve{x}) \leq S(\breve{x} + \epsilon)$ for all $\epsilon > 0$. That is, any small increase in x will lead to a discrete increase in the match. But V is continuous in x, so there must exist an increase in x sufficiently small that the consequent increase in status will be greater than any decrease in direct utility V. That is, there is a profitable deviation, which must be feasible for an agent with income z_1 as $z_1 > z_0 \geq \breve{x}$. Hence, a symmetric equilibrium strategy must be strictly increasing. ∎

Proof of Proposition 2. *(i)* As $G_A \succ_{SOSD} G_B$, for all $z \in [\underline{z}, \tilde{z}_G]$, we have that $G_A(z) \leq G_B(z)$. Thus, $S_A^G(z) \leq S_B^G(z)$ for all $z \in [\underline{z}, \tilde{z}_G]$. The rest follows from the Proposition 2 of Hopkins and Kornienko (2004a).

(ii) As $H_A \succ_{SOSD} H_B$, for all $s \in [\underline{s}, \tilde{s}]$, we have that $H_A(s) \leq H_B(s)$. Thus, $S_A^H(z) \leq S_B^H(z)$ for all $z \in [\underline{z}, \tilde{z}_G]$, where $\tilde{z}_H = S^{-1}(\tilde{s}) = G^{-1}(H_A(\tilde{s})) = G^{-1}(H_B(\tilde{s}))$. The rest follows from the Proposition 2 of Hopkins and Kornienko (2004a). ∎

Proof of Proposition 3. In what follows we consider differential equations of the form

$$x'(z) = \psi(x(z), z)k(z), \quad x(\underline{z}) = \underline{x}, \tag{10}$$

where $\psi(\cdot)$ and $k(\cdot)$ are positive, differentiable functions, with $\psi_1 \leq 0$. Suppose there is some exogenous shift in the function $k(z)$, with the the initial function now labelled $k_B(z)$ and the new $k_A(z)$. Then, the following result limits the changes that can occur to the solution of the corresponding differential equation.

Lemma 3 *(Proposition 1 in Hopkins and Kornienko (2004b)) Let $x_A(z)$ and $x_B(z)$ denote the solutions to the differential equation (12) for functions $k_A(z)$ and $k_B(z)$, respectively, with $x_A(\underline{z}) = x_B(\underline{z})$. Then if $k_A(z) - k_B(z) > 0$ on some interval $(\underline{z}, \underline{z} + \delta), \delta > 0$, then $x_A(z) - x_B(z) > 0$ on this interval as well. Moreover, if $x_A(z)$ and $x_B(z)$ do cross for $z \geq \underline{z} + \delta$, then at every point of crossing $x_A(z)$ crosses $x_B(z)$ from above (below) only if $k_A(z) - k_B(z) < (>)0$ at the point of crossing.*

Notice that the differential equation (6) for the behaviour of equilibrium output $x(z)$ fits the pattern given in (12) with $k(z) = S'(z)/S(z)$. Thus, what we need to establish is the behavior of $P(z) = S_A(z)/S_B(z)$. The rest follows either from the above Lemma 3, or from the Proposition 4 of Hopkins and Kornienko (2004a). We first establish two lemmas.

Lemma 4 *If $G_A(z) \succ_{ULR} G_B(z)$ and $H(s)$ is a uniform distribution then for all $\underline{s} \geq 0$, $P(z)$ has two extremes, a minimum at $\hat{z}_-$ and a maximum at $\hat{z}_+$, such that $\underline{z} \leq \hat{z}_- < \tilde{z} < \hat{z}_+ < \bar{z}$.*

Proof: When $H(s)$ is uniform, the inequality $\partial P(z)/\partial z > 0$ is equivalent to the following conditions

$$\frac{S_A'(z)}{S_A(z)} > \frac{S_B'(z)}{S_B(z)} \Leftrightarrow L_g(z) > P(z) \tag{11}$$

where $L_g(z) = g_A(z)/g_B(z)$. First, if $\underline{s} = 0$, then by Proposition 1 of Hopkins and Kornienko (2004b), if $G_A \succ_{ULR} G_B$ then $P(z)$ is unimodal with a unique maximum in $(\hat{z}, \bar{z}]$. That is, the above Lemma holds with the first extreme at the lower bound, that is, $\underline{z} = \hat{z}_-$. If $\underline{s} > 0$, note that $P(\underline{z}) = P(\tilde{z}) = P(\bar{z}) = 1$. Therefore, there is at least one extreme point for $P(z, \underline{s})$ on each of the two intervals $(\underline{z}, \tilde{z})$ and $(\tilde{z}, \bar{z})$. Note that $\partial P(z)/\partial z = 0$ if and only if $P(z, \underline{s}) = L(z)$. That is, $P(z)$ and $L_g(z)$ cross at the turning points of $P(z)$. Now since $L_g(z)$ is increasing on $(\underline{z}, \hat{z})$, at any crossing point $L_g(z)$ must cross $P(z)$ from below and so there can only be one crossing point on $(\underline{z}, \hat{z})$. Equally there can be only one extreme for $P(z)$ on $(\hat{z}, \underline{z})$ and the result follows. ∎

Lemma 5 *Suppose $H_B \succ_{MLR} H_A$ then $P(z) = S_A(z)/S_B(z)$ has two extremes, a minimum at $\hat{z}_-$ and a maximum at $\hat{z}_+$, such that $\underline{z} \leq \hat{z}_- < \tilde{z} < \hat{z}_+ < \bar{z}$.*

Proof: When $G(z)$ is uniform, $P'(z) > 0$ if and only if $h_B(z)/h_A(z) = L_h(z) > P(z)$, so that $P'(z) = 0$ only when $L_h(z) = P(z)$. As $H_B \succ_{ULR} H_A$, $L_h(z)$ is increasing on $(\underline{z}, \hat{z})$ and decreasing on $(\hat{z}, \bar{z})$. The result follows from a similar proof to that of Lemma 4 above. ∎

Having established the properties of $P(z)$ in the two cases, the results on the behaviour of equilibrium output $x(z)$ then follow from Proposition 3. The differential equation (6) fits the pattern given in (12) with $k(z) = S'(z)/S(z)$. ∎

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