DOES SOCIAL SECURITY PRIVATIZATION PRODUCE EFFICIENCY GAINS?

Shinichi Nishiyama

Congressional Budget Office
Washington, DC
E-mail: shinichi.nishiyama@cbo.gov

Kent Smetters

University of Pennsylvania & NBER
Philadelphia, PA
E-mail: smetters@wharton.upenn.edu

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Abstract

While privatizing Social Security would not produce Pareto efficiency gains or losses in a deterministic economy with labor taxes and inelastic labor supply, the presence of elastic labor supply and uncertainty changes matters considerably. On one hand, privatization could reduce the effective tax rate on labor and labor supply distortions even if all policy instruments are second best. On the other hand, privatization could reduce valuable risk sharing when households face uninsurable wage shocks or uninsurable longevity uncertainty. Determining the overall change in efficiency thus requires the use of simulation analysis.

We investigate these competing effects using a heterogeneous overlapping-generations (OLG) model in which agents with elastic labor supply face idiosyncratic earnings shocks and longevity uncertainty. We find that stylized privatization can have a powerful effect on labor supply incentives: When wage shocks are insurable, a stylized 50-percent privatization can produce new resources of roughly $21,000 for each future household (growth adjusted over time), after all households have been fully compensated for possible transitional losses from reform. However, when wages are not insurable, privatization *reduces* efficiency—by roughly $4,400 per future household—despite the improved labor supply incentives. This loss is mitigated to some degree if privatization is accompanied with increased risk sharing, such as introducing contribution matching to low income households or raising progressivity in the benefit schedule.

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1 Introduction

It has been known for some time that shutting down ("privatizing") a pay-as-you-go Social Security system would simply reallocate resources between generations when all economic variables are fully deterministic, government revenue is financed by labor taxes, and labor supply is inelastic (see, e.g., Breyer, 1989; Geanakoplos, Mitchell and Zeldes, 1998; Murphy and Welch, 1998; Mariger, 1999; Shiller, 1999; Diamond and Orszag, 2003). In particular, no new resources would be created in present value once the "winners" have fully compensated the "losers." As a result, analysis about privatization in this model setting necessarily focuses on distributional fairness and not on the potential for improving Pareto economic efficiency.

The intuition for this zero-sum result is not widely appreciated since other fiscal reforms, such as removing a capital income tax, could produce efficiency gains in a deterministic economy with inelastic labor supply. Like a capital income tax, a pay-as-you-go Social Security program might substantially "crowd out" capital saving (Feldstein, 1974). But, unlike a capital income tax, a pay-as-you-go Social Security system financed by labor income taxes does not directly distort the relative price of consumption over the lifecycle. Shutting down Social Security would increase the returns on saving earned by the young and future generations who, under Social Security, are required to pay off the implicit debt inherited from previous generations who received more in present value than they contributed. But the gains to young and future generations from privatization would come at an equal cost (in present value) to the initial elderly. Compensating the initial elderly for their loses, therefore, would simply undo the privatization experiment without changing relative prices.

Allowing for elastic labor supply and various sources of uncertainty that cannot be easily insured in the private market, however, can change the potential for efficiency gains considerably. In this richer environment, privatization can lead to either efficiency gains or even losses.

1.1 Elastic Labor Supply

With elastic labor supply, privatization could produce large efficiency gains by reducing the effective tax rate on labor supply. Social Security payroll taxes are distortionary because each extra dollar contributed to Social Security does not produce an extra dollar in benefits in present value. Instead, some of the contributions are effectively used to pay off the implicit debt inherited from past generations who received more than they contributed. Moreover, the Social Security program in the U.S. is progressive in the sense that it gives households with a low average index of monthly wages (AIME) a larger Social Security benefit *relative* to the AIME (i.e., a larger "replacement rate"). Contributions by households with higher-than average lifetime earnings, therefore, are effectively used to subsidize the contributions by households with lower-than average lifetime earnings.

If the losers (the initial elderly) from privatization are assumed to be compensated with first-best (lump sum) transfers by levying first-best taxes on the winners (young and future generations), privatization can obviously produce efficiency gains by removing the distortions to labor supply. The assumption of first-best compensation is implicit in the traditional welfare analysis of Hicks (1943, 1944-5) and Harberger (1974), which has been rigorously extended to the overlapping generations environment by Auerbach and Koltlikoff (1987). We further extend the method developed by Auerbach and Kotlikoff to an environment with stochastic wages and uncertain longevity.

A point that is less obvious, however, is that, in a model in which agents are allowed to live *three* or more periods, shutting down Social Security could produce efficiency gains even if the losers were compensated by levying second-best (labor income) taxes on the winners.[1] In particular, in a model in which agents work the first two periods of life and retire the third period, shutting down Social Security would "default" on both the Social Security benefits owed to initial third-period agents *and* the benefits accrued to date by second-period workers. The "default" to third-period agents, of course, does not produce efficiency gains itself since they must be compensated for their losses using second-best taxes levied on workers. But the

[1]The distinction between first-best and second-best compensation is not important for revenue-neutral policy changes such as fundamental tax reform (e.g., Nishiyama and Smetters, 2003). The interpretation of efficiency changes, however, becomes more important for other experiments which are not revenue neutral.

"default" on benefits accrued by second-period workers can produce efficiency gains because this implicit lump sum "wealth levy" is used to finance part of the higher rate of return that they will earn on their second-period contributions after privatization, thereby reducing the effective tax rate on labor (Smetters, 2004).

1.2 Uninsurable Shocks

The progressive nature of the U.S. Social Security system effectively provides insurance against wage shocks.[2] Since these shocks are typically difficult to insure in the private market, privatization could reduce a valuable source of risk sharing unless the privatized system is complementary with some other form of risk sharing such as matching contributions on a progressive basis. We consider a contribution match later in the paper. Social Security also pays benefits until the beneficiary and spouse dies rather than over a fixed number of years. To the extent that longevity uncertainty is difficult to insure in the private market, privatization could also reduce risk sharing by eliminating a valuable source of annuity protection.

1.3 Summary of Net Effects

Determining the overall change in efficiency requires the use of simulation analysis. We investigate these competing effects using a heterogeneous overlapping-generations (OLG) model in which agents with elastic labor supply face idiosyncratic earnings shocks and longevity uncertainty.

In this paper, we explore the welfare implications of a stylized partial privatization of Social Security. In the policy experiment, households redirect 50 percent of their payroll tax into private accounts; traditional Social Security benefits are cut cohort by cohort in a phased-in manner so that the benefits are also eventually cut by 50 percent; and the residual transitional cost is financed by a consumption tax.

[2]The value of Social Security in insuring earnings and longevity shocks that are difficult to insure in the private market has been investigated by İmrohoroğlu, İmrohoroğlu, and Joines (1995) and Conesa and Krueger (1999). İmrohoroĝlu *et al.* focus on steady states and find that the risk sharing benefit is outweighed by the crowding out of capital when the Social Security system is unfunded. Conesa and Krueger consider a voting model that includes the effect of Social Security reform on transitional generations. Consistent with previous deterministic models, they find that the losses from privatization to voting transitional generations might explain why a pay-as-you-go system is politically stable even though the newborns are better off; the loss in risk sharing serves as another reason for the political stability. In our analysis, we find that privatization may not, in fact, produce efficiency gains when the transitional generations are fully compensated.

We find that such a privatization can have a powerful effect on labor supply incentives: When wages shocks are insurable, 50-percent phase-in privatization can produce new resources equal to $21,200 for each future household (growth adjusted over time), after all households have been fully compensated for possible transitional losses from reform. However, when wages are not insurable, 50-percent privatization *reduces* efficiency—by as much as $4,400 per future household—despite the improved labor supply incentives. Interestingly, the efficiency loss is even larger with perfect private annuity markets. This loss is reduced to $3,500 if some risk sharing is introduced into the privatized system by subsidizing wage contributions on a progressive basis and paying for the subsidy by increasing general revenue income taxes in a proportional manner. The loss is also reduced when the benefit schedule is made to be more progressive. Although the main results of this paper reflect the effect of partial privatization, full privatization would probably produce qualitatively similar outcomes.[3]

1.4 Outline

The rest of the paper is laid out as follows: Section 2 describes the model, Section 3 explains the calibration of the model, Section 4 explain the policy experiments of Social Security privatization, and Section 5 concludes the paper. Appendixes show Markov transition matrixes and survival rates, and explain the computational algorithm used in this paper.

2 Model

The model we use to analyze tax reform has three sectors: heterogeneous households with elastic labor supply; a competitive representative firm with constant-returns-to-scale production technology; and a government with a full commitment technology. Like most previous analyses of Social Security reform, our model's pre-reform neoclassical economy is stationary by construction, and so we don't capture the effects that projected demographic changes might have on factor prices in the baseline economy in the closed economy version of our model.[4] We, however, are only interested in comparing the efficiency of gains from

[3]Estimating the welfare impacts of full privatization, however, was difficult because large transfers necessary to compensate transitional generations created instabilities in this model.

[4]We are aware of only a few papers, including De Nardi, İmrohoroğlu and Sargent (1999), Kotlikoff, Smetters and Walliser (2001), and Nishiyama (2004), that attempt to capture the effect of non-stationary demographics on

privatization against the baseline, not examining the implications of demographics on factor prices in the baseline economy.

2.1 The Household Sector

Households are heterogeneous with respect to ages i, working abilities e_i (measured by their hourly wages), beginning-of-period wealth holdings a_i, and average historical earnings b_i that determine their Social Security benefits. Every year, a large number (normalized to unity) of new households of age 20 enter the economy. The population of this economy grows at a constant rate of ν. A household of age i observes an idiosyncratic working ability shock, e_i, at the beginning of each year and chooses its optimal consumption c_i, working hours h_i, and end-of-period wealth holding a_{i+1}, taking the government's policy rule and series of factor prices and the government's policy variables as given.[5] At the end of each year, a fraction of households die based on the mortality tables. The model assumes that no one lives longer than 110 years. For simplicity, all households are assumed to be two-earner married couples of the same age.

2.1.1 The Household's Problem

Let $\mathbf{s}_i$ denote the state of an age i household,

$$\mathbf{s}_i = (i, e_i, a_i, b_i),$$

where $i \in I = \{20, ..., 109\}$ is the household's age, $e_i \in E = [e^{\min}, e^{\max}]$ is its working ability (the hourly wage), $a_i \in A = [a^{\min}, a^{\max}]$ is its beginning-of-period wealth, and $b_i \in B = [b^{\min}, b^{\max}]$ is its average historical earnings for Social Security purposes.[6]

Let $\mathbf{S}_t$ denote the state of the economy at the beginning of year t,

$$\mathbf{S}_t = (x_t(\mathbf{s}_i), W_{LS,t}, W_{G,t}),$$

baseline factor prices.

[5]Because there are no aggregate shocks in the present model, households can perfectly foresee these factor prices and policy variables, using the current distribution of households and the current policy variables. Yet, their own future working ability and mortality are uncertain.

[6]The average historical earnings are used to calculate the Social Security benefits of each household. The variable b_i approximates the average indexed monthly earnings (AIME) multiplied by 12 as of age i.

where $x_t(\mathbf{s}_i)$ is the joint distribution of households with $\mathbf{s}_i \in I \times E \times A \times B$, $W_{LS,t}$ is the beginning-of-period net wealth held by the Lump-Sum Redistribution Authority (LSRA), which is described below, and $W_{G,t}$ is the net wealth of the rest of the government.

Let $\mathbf{\Psi}_t$ denote the government policy schedule known at the beginning of year t,[7]

$$\mathbf{\Psi}_t = \{W_{LS,s+1}, W_{G,s+1}, C_{G,s}, \tau_{I,s}(.), \tau_{P,s}(.), \tau_{C,s}, tr_{SS,s}(\mathbf{s}_i), tr_{LS,s}(\mathbf{s}_i)\}_{s=t}^{\infty},$$

where $C_{G,s}$ is government consumption, $\tau_{I,s}(.)$ is an income tax function, $\tau_{P,s}(.)$ is a payroll tax function for Social Security (OASDI), $\tau_{C,s}$ is a consumption tax rate, $tr_{SS,s}(.)$ is a Social Security benefit function, and $tr_{LS,s}(\mathbf{s}_i)$ is an LSRA wealth redistribution function.

The household's problem is

$$v(\mathbf{s}_i, \mathbf{S}_t; \mathbf{\Psi}_t) = \max_{c_i, h_i} u_i(c_i, h_i) + \hat{\beta}\phi_i E\left[v(\mathbf{s}_{i+1}, \mathbf{S}_{t+1}; \mathbf{\Psi}_{t+1}) | e_i\right] \tag{1}$$

subject to

$$\begin{aligned} a_{i+1} = & \frac{1}{1+\mu}\{w_t e_i h_i + (1+r_t)(a_i + tr_{LS,t}(\mathbf{s}_i)) \\ & - \tau_{I,t}(w_t e_i h_i, r_t(a_i + tr_{LS,t}(\mathbf{s}_i)), tr_{SS,t}(\mathbf{s}_i)) \\ & - \tau_{P,t}(w_t e_i h_i) + tr_{SS,t}(\mathbf{s}_i) - (1+\tau_{C,t})c_i\} \geq a_{i+1,t}^{\min}(\mathbf{s}_i), \\ a_{20} = 0, & \quad a_{i\in\{65,\dots,110\}} \geq 0, \end{aligned} \tag{2}$$

where $u_i(.)$ is a period utility function of an age i household, $\hat{\beta}$ is the growth-adjusted time-preference factor, ϕ_i is the survival rate, w_t is the wage rate per efficiency unit of labor, and r_t is the interest rate (the rate of return to capital).[8] Individual variables of the model are normalized by the steady-state per capita growth rate μ.[9] $a_{i+1,t}^{\min}(\mathbf{s}_i)$ is the state-contingent minimum level of end-of-period wealth that is sustainable, that is, even if the household receives the worst possible shocks in future working abilities. At the beginning of the next

[7] In this model economy, the government does not solve an optimization problem. The government's policy rule is described as a set of tax and spending functions, in which functional forms are possibly time variant, and a financing rule must satisfy an intertemporal budget constraint.

[8] So, $w_t e_i h_i$ is the earnings of a household of age i with working ability e_i in year t.

[9] When the time preference parameter is β and $u(c,h) = \{c^{\alpha}(1-h)^{1-\alpha}\}^{1-\gamma}/(1-\gamma)$, the growth-adjusted time preference factor $\hat{\beta}$ is $\beta(1+\mu)^{\alpha(1-\gamma)}$.

period, the state of the household becomes

$$\mathbf{s}_{i+1} = (i+1, e_{i+1}, a_{i+1} + q_t, b_{i+1}), \tag{3}$$

where q_t denotes accidental bequests that a household receives at the end of the period. In the presence of perfect annuity markets, the household's state in the next period is

$$\mathbf{s}_{i+1} = (i+1, e_{i+1}, a_{i+1}/\phi_i, b_{i+1}), \tag{4}$$

instead. The average historical earnings b_i for Social Security purposes follows

$$b_{i+1} = \begin{cases} 0 & \text{if } i \leq 24 \\ \frac{1}{i-24}\{(i-25)b_i \frac{w_t}{w_{t-1}} + \min(w_t e_i h_i/2, weh_t^{\max})\} & \text{if } 25 \leq i \leq 59 \\ b_i/(1+\mu) & \text{if } i \geq 60 \end{cases}, \tag{5}$$

where $weh_t^{\max}$ is the Old-Age, Survivors, and Disability Insurance (OASDI) tax cap, which is \$80,400 in 2001. For simplicity, the model assumes that the highest 35 years of earnings correspond to those in ages between 25 and 59.[10]

2.1.2 The Measure of Households

Let $x_t(\mathbf{s}_i)$ denote the measure of households, and let $X_t(\mathbf{s}_i)$ be the corresponding cumulative measure. The measure of households is adjusted by the steady-state population growth rate ν. The population of age 20 households is normalized to be unity in the baseline economy on the balanced growth path, that is,

$$\int_E \mathrm{d}X_t(20, e_{20}, 0, 0) = 1.$$

Let $\mathbf{1}_{[a=y]}$ be an indicator function that returns 1 if $a = y$ and 0 if $a \neq y$. Then, the law of motion of the measure of households is, for $i \in I = \{20, ..., 109\}$,

$$x_{t+1}(\mathbf{s}_{i+1}) = \frac{\phi_i}{1+\nu} \int_{E \times A \times B} \mathbf{1}_{[a_{i+1} = a_{i+1}(\mathbf{s}_i, \mathbf{S}_t; \mathbf{\Psi}_t) + q_t]}$$
$$\times \mathbf{1}_{[b_{i+1} = b_{i+1}(w_t e_i h_i(\mathbf{s}_i, \mathbf{S}_t; \mathbf{\Psi}), b_i)]} \pi_{i,i+1}(e_{i+1}|e_i) \mathrm{d}X_t(\mathbf{s}_i),$$

[10] Social Security benefits in the United States are computed on the basis of the highest 35 years of earnings, adding an additional state variable to the model. Earnings before age 60 are wage indexed and earnings after age 60 are price indexed.

where $\pi_{i,i+1}$ denotes the transition probability of working ability from age i to age $i+1$. For simplicity, a working age household is assumed to receive accidental bequests q_t with constant probability η, where q_t is the average wealth left by deceased households and η is the ratio of deceased household to the surviving working-age households. That is,

$$q_t = \frac{\sum_{i=20}^{109}(1-\phi_i)\int_{E\times A\times B} a_{i+1}(\mathbf{s}_i, \mathbf{S}_t; \mathbf{\Psi}_t)\mathrm{d}X_t\left(\mathbf{s}_i\right)}{\sum_{i=20}^{109}(1-\phi_i)\int_{E\times A\times B}\mathrm{d}X_t\left(\mathbf{s}_i\right)},$$

$$\eta = \frac{\sum_{i=20}^{109}(1-\phi_i)\int_{E\times A\times B}\mathrm{d}X_t\left(\mathbf{s}_i\right)}{\sum_{i=20}^{64}\phi_i\int_{E\times A\times B}\mathrm{d}X_t\left(\mathbf{s}_i\right)}. \tag{6}$$

The steady-state condition is

$$\mathbf{S}_{t+1} = \mathbf{S}_t \tag{7}$$

for all t and $\mathbf{s}_i \in I \times E \times A \times B$.

2.2 The Firm

National wealth W_t is the sum of total private wealth, LSRA net wealth $W_{LS,t}$, and the remaining government net wealth $W_{G,t}$. Total labor supply L_t is measured in efficiency units.

$$W_t = \sum_{i=20}^{109}\int_{E\times A\times B} a_i\mathrm{d}X_t\left(\mathbf{s}_i\right) + W_{LS,t} + W_{G,t}, \tag{8}$$

$$L_t = \sum_{i=20}^{109}\int_{E\times A\times B} e_i h_i(\mathbf{s}_i, \mathbf{S}_t; \mathbf{\Psi}_t)\mathrm{d}X_t\left(\mathbf{s}_i\right). \tag{9}$$

There is a perfectly competitive representative firm in this economy. In a closed economy, capital stock is equal to national wealth, that is, $K_t = W_t$, and gross national product Y_t is determined by a constant-returns-to-scale production function,

$$Y_t = F(K_t, L_t).$$

The profit-maximizing condition of the firm is

$$F_K(K_t, L_t) = r_t + \delta, \tag{10}$$

$$F_L(K_t, L_t) = w_t, \tag{11}$$

where δ is the depreciation rate of capital.

In a small open economy, factor prices, r_t^* and w_t^* are fixed at international levels, and domestic capital stock $K_{D,t}$ and labor supply L_t are determined so that the firm's profit maximizing condition satisfies,

$$F_K(K_{D,t}, L_t) = r_t^* + \delta, \tag{12}$$

$$F_L(K_{D,t}, L_t) = w_t^*. \tag{13}$$

Gross domestic product $Y_{D,t}$ is determined by the production function,

$$Y_{D,t} = F(K_{D,t}, L_t),$$

and gross national product Y_t is determined by

$$Y_t = (r_t^* + \delta)\, W_t + w_t^*\, L_t.$$

Net foreign investment is shown by the difference between national wealth and domestic capital stock, that is, $W_t - K_{D,t}$.

2.3 The Government

In the policy experiments reported below, we follow Auerbach and Kotlikoff (1987) by measuring the pure efficiency gains from a policy change using a Lump-Sum Redistribution Authority, but we extend their approach to a heterogeneous-agent OLG model. To see how the LSRA works, suppose that a new policy is announced at the beginning of period 1. The LSRA first makes a lump-sum transfer (tax if negative), $tr_{CV,1}\,(\mathbf{s}_i)$, to *each living* household of age i to bring its expected remaining lifetime utility at state $\mathbf{s}_i$ back to its pre-reform level in the baseline economy. Next, the LSRA makes a lump-sum transfer (or tax), $tr_{CV,t}\,(\mathbf{s}_{20})$, to *each future* household (that is, each newborn household in periods 2, 3, ...) to make it as well off in the baseline economy, conditional on its initial state at age 20. Thus far, however, the net present value of these transfers at the beginning of period 1 across living and future households will generally not sum to zero. So, finally, the LSRA makes an additional lump-sum transfer (tax), Δtr, to each future household so that the net present value across all transfers is zero. For illustrative purposes, we assume that these additional transfers are uniform across future generations on a growth-adjusted basis. The lump-sum transfers made by the LSRA, therefore, are

$$tr_{LS,t}(\mathbf{s}_i) = \begin{cases} tr_{CV,t}(\mathbf{s}_i) & \text{if } t = 1 \\ tr_{CV,t}(\mathbf{s}_i) + \Delta tr & \text{if } t > 1 \text{ and } i = 20 \\ 0 & \text{otherwise} \end{cases}. \tag{14}$$

If $\Delta tr > 0$ then tax reform has produced extra resources after the expected remaining lifetime utility of each household has been restored to its pre-reform level. In this case, we say that tax reform has generated efficiency gains. If, however, $\Delta tr < 0$, then tax reform reduces efficiency. The total net lump-sum transfer to living households at time t, $Tr_{LS,t}$, is

$$Tr_{LS,t} = \sum_{i=20}^{109} \int_{E \times A \times B} tr_{LS,t}(\mathbf{s}_i)\, \mathrm{d}X_t(\mathbf{s}_i). \tag{15}$$

Government tax revenue consists of federal income tax $T_{I,t}$, payroll tax for Social Security (OASDI) $T_{P,t}$, and consumption tax $T_{C,t}$. These revenues are

$$T_{I,t} = \sum_{i=20}^{109} \int_{E \times A \times B} \tau_{I,t}\left(w_t e_i h_i(\mathbf{s}_i, \mathbf{S}_t; \boldsymbol{\Psi}_t), r_t\left(a_i + tr_{R,t}(\mathbf{s}_i)\right), tr_{SS,t}(\mathbf{s}_i)\right) \mathrm{d}X_t(\mathbf{s}_i), \tag{16}$$

$$T_{P,t} = \sum_{i=20}^{109} \int_{E \times A \times B} \tau_{P,t}\left(w_t e_i h_i(\mathbf{s}_i, \mathbf{S}_t; \boldsymbol{\Psi}_t)\right) \mathrm{d}X_t(\mathbf{s}_i), \tag{17}$$

$$T_{C,t} = \sum_{i=20}^{109} \int_{E \times A \times B} \tau_{C,t} c_i(\mathbf{s}_i, \mathbf{S}_t; \boldsymbol{\Psi}_t) \mathrm{d}X_t(\mathbf{s}_i). \tag{18}$$

Social Security (OASDI) benefit expenditure $Tr_{SS,t}$ is

$$Tr_{SS,t} = \sum_{i=20}^{109} \int_{E \times A \times B} tr_{SS,t}(\mathbf{s}_i)\, \mathrm{d}X_t(\mathbf{s}_i). \tag{19}$$

The law of motion of the government wealth (normalized by productivity growth and population growth) is

$$W_{G,t+1} = \frac{1}{(1+\mu)(1+\nu)} \left\{(1 + r_t) W_{G,t} + T_{I,t} + T_{P,t} + T_{C,t} - Tr_{SS,t} - C_{G,t}\right\}, \tag{20}$$

$$W_{LS,t+1} = \frac{1}{(1+\mu)(1+\nu)} (1+r_t)(W_{LS,t} - T_{LS,t}), \tag{21}$$

where $C_{G,t}$ is government consumption.

2.4 Recursive Competitive Equilibrium

Definition **Recursive Competitive Equilibrium (Equilibrium Transition Path):** Let $\mathbf{s}_i = (i, e_i, a_i, b_i)$ be the individual state of households, let $\mathbf{S}_t = (x_t(\mathbf{s}_i), W_{LS,t}, W_{G,t})$ be the state of the economy, and let $\mathbf{\Psi}_t$ be the government policy schedule known at the beginning of year t,

$$\mathbf{\Psi}_t = \{W_{LS,s+1}, W_{G,s+1}, C_{G,s}, \tau_{I,s}(.), \tau_{P,s}(.), \tau_{C,s}, tr_{SS,s}(\mathbf{s}_i), tr_{LS,s}(\mathbf{s}_i)\}_{s=t}^{\infty}.$$

A series of factor prices $\{r_s, w_s\}_{s=t}^{\infty}$, accidental bequests $\{q_s\}_{s=t}^{\infty}$, the policy variables $\{W_{LS,s+1}, W_{G,s+1}, C_{G,s}, \tau_{C,s}, tr_{LS,s}(\mathbf{s}_i)\}_{s=t}^{\infty}$, the parameters of policy functions $\{\varphi_s\}_{s=t}^{\infty}$, the value function of households $\{v(\mathbf{s}_i, \mathbf{S}_s; \mathbf{\Psi}_s)\}_{s=t}^{\infty}$, the decision rule of households

$$\{\mathbf{d}(\mathbf{s}_i, \mathbf{S}_s; \mathbf{\Psi}_s)\}_{s=t}^{\infty} = \{c_i(\mathbf{s}_i, \mathbf{S}_s; \mathbf{\Psi}_s), h_i(\mathbf{s}_i, \mathbf{S}_s; \mathbf{\Psi}_s), a_{i+1}(\mathbf{s}_i, \mathbf{S}_s; \mathbf{\Psi}_s)\}_{s=t}^{\infty},$$

and the measure of households $\{x_s(\mathbf{s}_i)\}_{s=t}^{\infty}$, are in a recursive competitive equilibrium if, in every period $s = t, ..., \infty$, each household solves the utility maximization problem (1)–(5) taking $\mathbf{\Psi}_t$ as given; the firm solves the profit maximization problem, and the capital and labor markets clear, that is, (8)–(13) hold; the government policy schedule satisfies (14)–(21); and the goods market clears.

Definition **Recursive Competitive Equilibrium (Steady State):** Let $\mathbf{s}_i = (i, e_i, a_i, b_i)$ be the individual state of households and let $\mathbf{\Psi}$ be the time-invariant government policy rules,

$$\mathbf{\Psi} = \{W_{LS}, W_G, C_G, \tau_I(.), \tau_P(.), \tau_C, tr_{SS}(\mathbf{s}_i), tr_{LS}(\mathbf{s}_i)\}.$$

Factor prices (r, w), accidental bequests q, the policy variables $(W_{LS}, W_G, C_G, \tau_C, tr_{LS}(\mathbf{s}_i))$, the parameters φ of policy functions, the value function of households $v(\mathbf{s}_i; \mathbf{\Psi})$, the decision rule of households

$$\mathbf{d}(\mathbf{s}_i; \mathbf{\Psi}) = \{c_i(\mathbf{s}_i; \mathbf{\Psi}), h_i(\mathbf{s}_i; \mathbf{\Psi}), a_{i+1}(\mathbf{s}_i; \mathbf{\Psi})\},$$

Table 1: Common Parameters

Coefficient of relative risk aversion	γ	2.0
Capital share of output	θ	0.30
Depreciation rate of capital stock	δ	0.047
Long-term real growth rate	μ	0.018
Population growth rate	ν	0.010
Probability of receiving bequests	η	0.0161
Total factor productivity *	A	0.949

* Total factor productivity is chosen so that $w$equals 1.0.

Table 2: Other Main Parameters

		Representative-Agent Model without Wage Shocks	Heterogeneous-Agent Economy with Wage Shocks			
			without Annuity Markets	with Annuity Markets	Lower Transitory Shocks to	
					1/2	1/5
Time preference parameter *1	β	1.004	0.985	0.992	0.992	1.000
Share parameter for consumption *2	α	0.436	0.466	0.452	0.456	0.450
Income tax adjustment factor *3	φ_I	1.000	0.818	0.834	0.847	0.874
OASDI benefit adjustment factor *4	φ_{SS}	1.232	1.385	1.380	1.388	1.388

*1. The capital-GDP ratio is targeted to be 2.74 (r = 6.25 percent).

*2. The average annual working hours are 3414 per married couple when $h_{\max}$= 8760.

*3. In a heterogeneous-agent economy, the ratio of income tax revenue to GDP is 0.123.

*4. The OASDI budget is assumed to be balanced.

and the measure of households $x(\mathbf{s}_i)$, are in a steady-state recursive competitive equilibrium if, in every period, each household solves the utility maximization problem (1)–(5) taking $\mathbf{\Psi}$ as given; the firm solves the profit maximization problem, and the capital and labor markets clear, that is, (8)–(13) hold; the government policy rules satisfy (14)–(21); the goods market clears; and the measure of households is constant, that is, (7) holds.

3 Calibration

Tables 1 and 2 summarize the key parameters discussed below.

Table 3: Number of People Under 18 Years of Age in a Married Household

Age cohorts	Number of children	Age cohorts	Number of children
20-24	0.642	50-54	0.908
25-29	1.167	55-59	0.562
30-34	1.451	60-64	0.231
35-39	1.755	65-69	0.156
40-44	1.753	70-74	0.055
45-49	1.439	75-plus	0.000

Source: Authors' calculations from the 2001 Survey of Consumer Finances (SCF).

3.1 Households

Utility Function. We use the following Cobb-Douglas utility function that is nested within a time-separable isoelastic functional form, which is compatible with the existence of a steady state:

$$u(c,h) = \frac{\left\{((1+n_i/2)^{-\zeta}\, c)^{\alpha}(h^{\max} - h)^{1-\alpha}\right\}^{1-\gamma}}{1-\gamma}.$$

γ is the coefficient of relative risk aversion, n_i is the number of dependent children at the parents' age i, ζ is the "adult equivalency scale" parameter that is used to convert the consumption by children into their adult equivalent amounts, and $h_i^{\max}$ is the maximum working hours.[11] The coefficient of relative risk aversion is assumed to be 2.0. The number of dependent children by age cohort is calculated using the 2001 Survey of Consumer Finances (SCF) (see Table 3). The consumption adjustment parameter ζ is assumed to be 0.6.[12]

The annual working hours in the model are the sum of the working hours of a husband and a wife. The maximum working hours, $h_i^{\max}$, are set at 8,760, which equals 12 hours per day per person $\times$ 365 days $\times$ two persons.[13] Using a smaller value for maximum hours would reduce the effective labor supply elasticity. The parameter α is chosen so that the average working hours of households between age 20 and age 64 equals 3,414 hours in the initial steady-state economy, the average number of hours supplied by married households in

[11]The growth-adjusted $\hat{\beta}$, therefore, is $\beta(1+\mu)^{\alpha(1-\gamma)}$.

[12]Hence, a married couple with two dependent children must consume about 52 percent (i.e., $2^{0.6} = 1.517$) more than a married couple with no children to attain the same level of utility, ceteris paribus.

[13]The 95th and 99th percentiles of the working hours per married couple of aged 20-64 in the 2001 SCF are 5,280 and 6,375, respectively.

Table 4: Working Abilities of a Household (in U.S. Dollars per Hour)

	Percentile	Age Cohorts					
		20-24	25-29	30-34	35-39	40-44	45-49
e^1	0-20th	3.89	5.47	6.86	6.01	7.43	5.73
e^2	20-40th	8.35	10.11	12.38	12.27	13.90	13.14
e^3	40-60th	10.28	14.04	16.46	16.96	18.76	18.47
e^4	60-80th	12.31	17.30	21.87	22.57	25.79	25.71
e^5	80-90th	17.47	21.58	29.37	30.19	35.37	35.56
e^6	90-95th	22.17	27.21	33.96	46.92	48.30	54.59
e^7	95-99th	29.43	36.60	43.76	81.75	96.44	97.48
e^8	99-100th	42.31	62.29	182.78	327.65	262.03	284.00
	Percentile	Age cohorts					
		50-54	55-59	60-64	65-69	70-74	75-79
e^1	0-20th	5.00	2.42	0.00	0.00	0.00	0.00
e^2	20-40th	13.99	10.65	3.36	0.00	0.00	0.00
e^3	40-60th	20.95	15.60	11.00	1.92	0.00	0.00
e^4	60-80th	29.13	24.60	18.33	11.14	1.77	1.30
e^5	80-90th	40.89	34.75	29.08	19.14	10.93	10.21
e^6	90-95th	54.11	51.62	44.41	29.99	20.66	20.88
e^7	95-99th	91.67	99.24	91.12	56.19	38.26	26.41
e^8	99-100th	282.18	333.58	555.90	244.71	193.00	86.76

Source: Authors' calculations from the 2001 SCF.

the 2001 SCF.

Working Ability. The working ability in this calibration corresponds to the hourly wage (labor income per hour) of each household in the 2001 SCF.[14] The average hourly wage of a married couple (family members #1 and #2 in SCF) used in the calibration is calculated by

$$\text{Hourly Wage} = \frac{\text{Regular and Additional Salaries (\#1 + \#2) + Payroll Taxes / 2}}{\max\{\text{Working Hours (\#1 + \#2), 2080}\}}.$$

We adjusted the salaries in the numerator by adding imputed payroll taxes paid by their employers. The max operator in the denominator is used to adjust the hourly wage for a small fraction of households with large reported salaries but few reported working hours such as executives and the self-employed.

[14]According to Bureau of Labor Statistics data, the average hourly earnings of production workers have increased by 3.8 percent from 2000 to 2001. Since the 2001 SCF wages correspond to year 2000 while our tax function introduced below is calibrated to the year 2001, we multiply the SCF wages shown in Table 4 by 1.038 to convert the hourly wages in 2000 into growth-adjusted wages in 2001.

Table 4 shows the eight discrete levels of working abilities of five-year age cohorts. We use a shape-preserving cubic spline interpolation between each five-year age cohort to obtain the working ability for each age cohort.[15] In the version of our model where we "turn off" the idiosyncratic wage shocks, the hourly wages of the representative household are assumed to be those of the 40–60th percentile households.

Table 4, however, only shows the different potential "wage buckets" by age as well as the proportion of households in each bucket. It does not itself capture the uncertainty over wages. Using the Panel Study of Income Dynamics (PSID), therefore, we estimate Markov transition matrixes that specify the probabilities that a household's wage will move from one wage a different wage the next year. Separate transition matrixes were constructed for four age ranges—20-29, 30-39, 40-49, and 50-59—in order to capture the possibility that the probabilities themselves might change over the lifecycle. (For households aged 60 or older, we used the matrix for ages 50-59.)

For sensitivity analysis in Section 4, the off-diagonal transition probabilities were lowered to one half or one fifth of their original values and the diagonal elements were adjusted to preserve the adding-up properties of the matrix. The appendix reports the matrixes in more detail.

The probability η of receiving bequests each year for a working-age household is calculated to 0.0161 by Equation (6).

Population Growth and Mortality. The population growth rate ν is set to one percent per year, which is consistent with Social Security Administration (2001) long-run estimates. The survival rate ϕ_i at the end of age $i = \{20, ..., 109\}$ are the weighted averages of the male and female survival rates calculated by SSA (Table 4.C6). The survival rates at the end of age 109 are replaced by zero, thereby capping the maximum length of life. See Appendix A for more details.

[15] An alternative approach of estimating eight different wage rates for each age would have relied on too few observations.

3.2 Production

Capital and Private Wealth. Capital K is the sum of private fixed assets and government fixed assets. In 2000, private fixed assets were $21,165 billion, government fixed assets were $5,743 billion, and the public held about $3,410 billion of government debt.[16] Government net wealth, therefore, is set equal to 9.5 percent of total private wealth in the initial steady-state economy in our model. Moreover, the time preference parameter β is chosen in each version of our model so that the capital-GDP ratio in the initial steady state economy is 2.74, the empirical value in 2000.[17]

Production Technology. Production takes the Cobb-Douglas form,

$$F(K_t, L_t) = A_t K_t^{\theta} L_t^{1-\theta}.$$

where, recall, L_t is the sum of working hours in efficiency units. The capital share of output θ is chosen equal to

$$\theta = 1 - \frac{\text{Compensation of Employees } + (1-\theta) \times \text{Proprietors' Income}}{\text{National Income } + \text{ Consumption of Fixed Capital}}.$$

The value of θ in 2000 was 0.30.[18] The annual per-capita growth rate μ is assumed to be 1.8 percent, the average rate between 1869 to 1996 (Barro [1997]). Total factor productivity A is set at 0.949, which normalizes the wage (per efficient labor unit) to unity.

The Depreciation Rate of Fixed Capital. The depreciation rate of fixed capital δ is chosen by the following steady-state condition,

$$\delta = \frac{\text{Total Gross Investment}}{\text{Fixed Capital}} - \mu - \nu.$$

In 2000, private gross fixed investment accounted for 17.2 percent of GDP, and government (federal and state) gross investment accounted for 3.3 percent of GDP.[19] With a capital-

[16]Source: Department of Commerce, Bureau of Economic Analysis.
[17]*Ibid.*
[18]Source: Department of Commerce, Bureau of Economic Analysis. The average of θ in years between 1996 and 2000 is 0.31.
[19]*ibid.*

Table 5: Marginal Individual Income Tax Rates in 2001 (Married Household, Filed Jointly)

Taxable Income			Marginal Income Tax Rate (%)
$0	–	$45,200	$15.0 \times \varphi_I$
$45,200	–	$109,250	$28.0 \times \varphi_I$
$109,250	–	$166,500	$31.0 \times \varphi_I$
$166,500	–	$297,350	$36.0 \times \varphi_I$
$297,350	–		$39.6 \times \varphi_I$

output ratio of 2.74, the ratio of gross investment to fixed capital is 7.5 percent. Subtracting productivity and population growth rates, the annual depreciation rate is 4.7 percent.

3.3 The Government

Income Taxes. Federal income tax and state and local taxes are assumed to be at the level in year 2001 before the passage of the "Economic Growth and Tax Relief Reconciliation Act of 2001" (EGTRRA). Since households in our model are assumed to be married, we use a standard deduction of $7,600. However, following Altig *et al.* (2001), we allow higher income households to itemize deductions when it is more valuable to do so, and we assume that the value of the itemized deduction increases linearly in the Adjusted Gross Income.[20] The additional exemption per dependent person is $2,900 where the number of dependent children is consistent with Table 3. Table 5 shows the statutory marginal tax rates before EGTRRA.[21] As explained before, a household's labor income in this calibration includes the imputed payroll tax paid by its employer. Thus, taxable income is obtained by subtracting the employer portion of payroll tax from labor income.

The standard deduction, the personal exemption, and all tax brackets grow with productivity over time so that there is no real bracket creep; this indexing is also needed for the initial economy to be in steady state. In 2000, the ratio of total individual income tax revenue (not including Social Security and Medicare taxes) to nominal GDP was 0.102 and the ratio of corporate income tax to GDP was 0.021. Each statutory federal income tax rate shown in Table 5, therefore, is multiplied by φ_I so that income tax revenue (including cor-

[20]In particular, the deduction taken by a household is the greater of the standard deduction and $0.0755 \times$AGI, or $\max\{\$7600, 0.0755 \times \text{AGI}\}$.

[21]The key qualitative results reported herein are unaffected if the tax function were instead modeled as net taxes, that is, after substracting transfers indicated in the Statistics of Income.

Table 6: Marginal Payroll Tax Rates in 2001

Taxable Labor Income per Worker			Marginal Tax Rate (%) OASDI	HI
\$0	–	\$80,400	$12.4 \times \varphi_P$	2.9
\$80,400	–		$0.0 \times \varphi_P$	2.9

Note: The same taxes are levied to employers. The payroll tax adjustment factor φ_P is set to 1.0.

Table 7: OASDI Replacement Rates in 2001

AIME (b/12)			Marginal Replacement Rate (%)
\$0	–	\$561	$90.0 \times \varphi_{SS}$
\$561	–	\$3,381	$32.0 \times \varphi_{SS}$
\$3,381	–		$15.0 \times \varphi_{SS}$

Note: The OASDI benefit adjustment factor φ_{SS} is set so that the OASDI is pay-as-you-go in the baseline economies.

porate income tax) totals 12.3 percent of GDP in the initial steady state. The adjustment factor is between 0.82 and 0.87 for heterogeneous-agent economies with idiosyncratic wage shocks and set to 1.0 for a representative-agent economy without wage shocks. Also, since the effective tax rate on capital income is reduced by investment tax incentives, accelerated depreciation and other factors (Auerbach [1996]), the tax function is further adjusted so that the cross-sectional average tax rate on capital income is about 25 percent lower than the average tax on labor income.[22] State and local income taxes are modeled parsimoniously with a 4.0 percent flat tax on income above the deduction and exemption levels used at the federal level.

Social Security. The tax rate levied on both employers and employees for Old-Age, Survivors, and Disability Insurance (OASDI) is 12.4 percent, and the tax rate for Medicare (HI) is 2.9 percent. In 2001, employee compensation above \$80,400 was not taxable for OASDI. (See Table 6.)

Social Security benefits are based on each worker's Average Indexed Monthly Earnings

[22]This relative reduction to the tax rate on capital is commonly used by CBO, and it balances the legal tax preferences given to capital versus the legal tax benefits given to labor, including tax-preferred fringe benefits.

Table 8: Summary of the Design of Each Experiment

Run #	Wage Shocks Operative?	Closed/Open Economy?	Annuity Market Available?	With Government Matching?	Increased Progressivity?	Lower Transitory Shocks?
1	No	Closed	No	No	No	n.a.
2	Yes	Closed	No	No	No	unchanged
3	Yes	Small Open	No	No	No	unchanged
4	Yes	Closed	Yes	No	No	unchanged
5	Yes	Closed	No	Yes	No	unchanged
6	Yes	Closed	No	No	Yes	unchanged
7	Yes	Small Open	No	No	No	1/2
8	Yes	Small Open	No	No	No	1/5

(AIME), $b_i/12$, and the replacement rate schedule in the United States. The replacement rates are 90 percent for the first \$561, 32 percent for amounts between \$561 and \$3,381, and 15 percent for amounts above \$3,381. Social Security, therefore, is progressive in that a worker's benefit relative to AIME—the "replacement rate"—is decreasing in the value of the AIME.

In this calibration, OASDI benefits are adjusted so that total benefit equals total payroll tax revenue for OASDI. The adjustment factor φ_{SS} is between 1.38 and 1.39 in economies with wage shocks and 1.23 in an economy without wage shocks. The benefits received by retired workers accounted for 69.1 percent of total OASDI benefits in December 2000.[23] Considering spousal and survivors' benefits, these adjustment factors are roughly consistent with the U.S. OASDI system.[24]

4 Policy Experiments

We consider a stylized 50-percent phased-in privatization experiment. At the beginning of the first year, the Social Security payroll tax rate is reduced to one half of the baseline economy so that all workers immediately start redirecting half of their payroll tax payments into private saving accounts. Implicity in most previous work, assets in the new private

[23] See Table 5.A1 in Social Security Administration (2001).

[24] Other government programs that are not included in this model could provide some risk sharing in principle. But, preliminary experiments suggested that including them would not have a large impact on the results of this model.

accounts are assumed to be perfectly substitutable with other private assets. In particular, both assets earn the market rate of return, and the produced income is taxed by the same income tax schedule.

Social Security benefits are reduced linearly over time. Households age 66 and older in year 1 receive the current-law (baseline) benefits throughout the rest of their lifetime; households of age 65 in year 1 receive benefits that are 1.25% lower than the current-law level throughout the rest of lifetime; households of age 65 in year 2 receive benefits 2.5 % lower than the current law-level, and so on. Households age 25 or younger in year 1, therefore, receive one half of their traditional Social Security benefits when they turn 65.

During the transition, the Social Security cash-flow deficit is financed by a consumption tax so that the Social Security budget is balanced every year. Since changes to the Social Security system will influence the size of the capital stock and government receipts, the rest of the government budget is balanced by proportional changes in marginal income tax rates.

As summarized in Table 8, this partial privatization plan is evaluated under different combinations of model assumptions. We first consider the representative-agent economy without wage shocks (equivalently, with *insurable* wage shocks) where all households have the wage profile for 40–60th percentile households shown in Table 4 (i.e., lifetime income group e^3). We then turn to a heterogeneous-agent economy with uninsurable wage shocks.

4.1 Representative-Agent Economy without Wage Shocks

As shown in Run 1 in Table 9, partial privatization in the representative-agent economy would increase national wealth by 20.7 percent in the long run compared to the baseline economy. GNP increases by 9.3 percent. Moreover, national wealth and GNP increase throughout the entire transition path. A large part of these increases is caused by the reduction of unfunded liability of the pay-as-you-go Social Security system.

Labor supply also increases over time, by 4.8 percent in the long run, after payroll tax rates are reduced in half. In a closed economy, the increase in the wage rate following the increase in national wealth reinforces the increase in labor supply in the long run. However, since labor supply increases faster than capital during the first decade after reform, the wage rate initially falls before eventually rising, thereby mitigating some of the increase in

Table 9: Percent Change in Selected Macro Variables Relative to Baseline

		Without LSRA			With LSRA		
			National	Labor		National	Labor
Run #	Year t	GNP	Wealth	Supply	GNP	Wealth	Supply
1	1	2.4	0.0	3.5	1.5	0.0	2.2
	10	3.8	6.4	2.7	2.2	2.5	2.1
	20	5.4	10.9	3.1	3.0	4.0	2.6
	40	8.3	17.5	4.6	4.9	6.2	4.3
	Long Run	9.3	20.7	4.8	5.5	6.4	5.1
2	1	1.3	0.0	1.8	0.4	0.0	0.6
	10	2.5	4.3	1.7	0.5	-0.5	1.0
	20	4.0	8.1	2.2	1.2	0.1	1.7
	40	6.7	15.1	3.4	3.0	2.9	3.0
	Long Run	7.8	18.7	3.5	3.8	4.7	3.4
3	1	2.1	0.0	3.0	0.1	0.0	0.1
	10	3.6	7.3	2.0	0.2	-1.8	1.1
	20	5.6	14.5	1.8	1.0	-1.5	2.0
	40	9.3	27.1	1.7	3.0	2.7	3.2
	Long Run	11.5	36.5	0.8	4.3	6.8	3.2
4	1	1.0	0.0	1.5	0.4	0.0	0.6
	10	2.4	4.0	1.7	0.5	-0.6	0.9
	20	3.9	7.8	2.3	1.1	-0.3	1.7
	40	6.1	13.7	3.0	2.6	2.0	2.9
	Long Run	6.7	15.2	3.3	3.3	3.3	3.3

labor supply in the short run. Also, the consumption tax that finances the transition cost of privatization helps mitigate the short-run increase in labor supply.

Despite these positive gains to economic variables, not everyone is better off from privatization. As shown for Run 1 in Table 10, all households alive at the time of the reform (that is, aged 20 or older) are worse off. For example, the age-20 household at the time of policy change *loses* $17,000, as measured by the equivalent variation of an one-time wealth transfer made at the time of the reform. The loss of an age-40 household is about $60,000.

The welfare losses to initial generations are mainly caused by the consumption tax that is needed to finance the transition toward privatization, which especially harms middle-aged households with a large amount of non-Social Security wealth accumulation.[25] In a closed economy, these households are also hurt by the fall in the interest rate, which is 1.0 percent-

[25] Social Security wealth under law and in our model is protected by inflation, including that caused by consumption taxes.

Table 10: Change in Welfare per Household (1,000 dollars in 2001)

		Without LSRA*				With LSRA**
	Age in	Select Productivities				For all
Run #	Year 1	e^1	e^3	e^5	e^8	Productivities
1	79	-	-7.5	-	-	0.0
	60	-	-47.4	-	-	0.0
	40	-	-59.9	-	-	0.0
	20	-	-17.0	-	-	0.0
	0	-	25.8	-	-	21.2
	-20	-	49.2	-	-	21.2
2	79	-4.8	-5.7	-14.7	-79.3	0.0
	60	-27.6	-43.5	-64.4	-361.8	0.0
	40	-18.7	-46.7	-76.4	-368.4	0.0
	20	2.2	-1.5	-5.2	-15.5	0.0
	0	32.8	33.7	36.1	43.4	-4.4
	-20	52.4	56.7	63.5	84.3	-4.4
3	79	-4.8	-5.6	-14.4	-75.7	0.0
	60	-25.8	-40.1	-54.6	-184.8	0.0
	40	-15.3	-37.3	-57.1	-155.5	0.0
	20	1.1	1.6	3.5	15.9	0.0
	0	23.6	29.9	39.2	73.0	-6.6
	-20	39.8	50.1	64.2	112.5	-6.6
4	79	-4.6	-5.5	-13.2	-64.8	0.0
	60	-24.0	-36.5	-52.5	-323.6	0.0
	40	-16.1	-35.9	-57.0	-318.3	0.0
	20	3.4	0.8	-1.0	-4.7	0.0
	0	22.2	22.9	25.3	37.6	-7.8
	-20	31.8	34.3	38.8	57.6	-7.8

* Standard equivalent variations measures.
** Value of Δtr.

age point (100 basis points) lower in the long run.[26]

Each future household gains about $49,200 from privatization, mainly from higher wage rates and from the reduced distortions from payroll taxes. They also pay very little of the transition cost. Higher private wealth and higher wages generate more general revenue from capital income taxes and labor taxes if the tax rates were unchanged. To balance the rest of government budget, therefore, average federal income tax rates are reduced by about 22 percent in the long run, which also benefit future households.

Tables 9 and 10 also show the effect of the partial privatization when the Lump-Sum

[26] For example, the welfare loss of a 40-year old is reduced by half in a small open economy (not shown), where the interest rate does not change.

Redistribution Authority (LSRA) is operative. Since all households alive at the time of the reform would be worse off from privatization, the LSRA transfers exactly enough wealth to them so that their remaining expected lifetime utility returns to pre-reform levels. These transfers and interest costs increase the net wealth of the LSRA to -74 percent of GDP at its peak. This debt owed by the LSRA, in turn, mitigates the increase in national wealth, although the increase is still positive throughout the entire transition path. In the long run, national wealth increases by 6.4 percent relative to the baseline economy while GNP rises by 5.5 percent.

However, the gains to future generations outweigh the losses to current generations in present value. After the LSRA returns the welfare of all households (current and future) to their pre-reform levels, it has a positive level of resources left over, equal to about $21,000 (in 2001 growth-adjusted dollars) per each future household that enters the economy in year 2 and later. In other words, the 50-percent phase-in privatization increases efficiency in our deterministic economy setting.

4.2 Heterogeneous-Agent Economy with Wage Shocks

Run 2 in Table 9 shows the effects of the same stylized privatization experiment in a more realistic economy with uninsurable wage shocks. Social Security's progressive benefit formula, therefore, provides some risk sharing that is unattainable in the private market.

Notice that national wealth now increases by 18.7 percent in the long run relative to the baseline. The increase in wealth is smaller relative to the representative-agent economy (Run 1) because part of private saving in the presence of uncertainty is for precautionary motives; this type of saving is less sensitive to changes in Social Security policy. Labor supply increases by 3.5 percent in the long run, although the consumption tax mitigates some of the increase in the short run. GNP also increases throughout the transition path and is 7.8 percent higher in the long run. The interest rate decreases by 1.0 percentage point in the long run.

Similar to the representative-agent economy, most of the households alive at the time of the reform are worse off because they have to pay higher taxes to finance the transition. Run 2 in Table 10 shows that relatively high wage (and, hence, wealthier) households tend to be hit the hardest. Like before, most future households gain from the lower effective payroll tax rate and from lower income tax rates.

Future households, however, do not gain as much in present value from privatization as current households lose. Hence, after the LSRA returns the welfare of all households to their pre-reform levels, it has a negative level of resources left over, equal to about $4,400 (in 2001 growth-adjusted dollars) per each future household. This result contrasts sharply with the gain of $21,200 in a representative-agent economy without wage shocks.

4.3 Alternative Experiments in a Heterogeneous-Agent Economy

In a Small Open Economy. Run 3 shows the effect of privatization in the setting of a small open economy where the interest rate and the wage rate remain fixed at their pre-reform levels throughout the entire transition path; any changes in the capital-labor ratio are nullified immediately by international capital flows. Since interest rates do not decline, national wealth increases by more than in the closed economy setting. The increase in labor supply is also smaller since the wage rate does not rise. Table 9 shows that GNP increases gradually and by 11.5 percent in the long run.

Table 10 shows that the welfare losses of households alive at the time of the reform tend to be smaller relative to the closed economy, mostly due to the fixed interest rate. The gains to future generations also tends to be higher. So it might appear at first that privatization in an open economy setting would produce relatively less efficiency losses than in the closed economy. This hunch, however, is incorrect. When the LSRA is operative, Table 10 shows that the efficiency losses are actually slightly *larger* in the small open economy, equal to $6,600 (in 2001 growth-adjusted dollars) per each future household. The reason is that the LSRA's cost of borrowing is higher in the small open economy setting since the interest rate does not decrease over time.

Perfect Annuity Markets. Thus far, we have assumed private annuity markets do not exist and so, in addition to sharing wage uncertainty, the Social Security system shares *longevity* uncertainty in a way that cannot be easily replicated in the private market. It would appear at first glance, therefore, that privatization has a better chance of producing efficiency gains if we instead assumed that a private annuity market is available. This intuition, however, turns out to be incorrect.

Run 4 in Table 10 shows that the efficiency losses are actually *larger* with perfect private annuity markets than without (Run 2). In particular, each household loses $7,800, compared to $4,400 shown earlier (Run 2) without an annuity market. As shown in Table 9, privatization *without* a private annuity market leads to a larger increase in national wealth because households increase their precautionary savings as the annuity insurance provided by Social Security is reduced; in contrast, households can rely more on private insurance rather than precautionary savings in Run 4. National wealth increases by 18.7 percent in the long run without an annuity market whereas it increases by only 15.2 percent with an annuity market. The smaller amount of precautionary savings in Run 4 with a private annuity market has four effects that produce larger efficiency losses: (i) the LSRA must borrow at a higher interest rate; (ii) capital income taxes must be higher, thereby enhancing the non-pecuniary externality produced by the tax system; (iii) wages are lower; and (iv) the interest elasticity of saving is higher, increasing the importance of falling interest rates on subsequent saving as national wealth increases.

Contribution Matching. The pre-reform Social Security system is progressive in the sense that it gives households with a lower average index of pre-retirement wages a larger Social Security benefit relative to their pre-retirement wages, that is, a larger "replacement rate." To be sure, the privatized experiment considered above preserves some progressivity by reducing benefits proportionally by half in the long run—the poor still get a relatively larger replacement rate. But a proportional reduction in benefits still reduces progressivity relative to the pre-reform system.

Run 5, therefore, considers privatization with a progressive contribution match. In particular, working households with low levels of labor income receive a match equal to 5 percent of their *earnings*. This matching rate declines linearly to zero as labor income approaches $60,000 of labor income, which is slightly above the median household income in the model economy.[27] As before, the costs for the traditional benefits are financed by the consump-

[27]This matching schedule is equivalent with the marginal labor income tax of -10 percent at $0 of labor income,

Table 11: Percent Change in Selected Macro Variables Relative to Baseline

		Without LSRA			With LSRA		
Run #	Year t	GNP	National Wealth	Labor Supply	GNP	National Wealth	Labor Supply
5	1	0.6	0.0	0.9	-0.3	0.0	-0.4
	10	1.6	3.3	0.9	-0.3	-1.5	0.2
	20	3.1	6.7	1.6	0.3	-1.3	1.0
	40	5.8	13.5	2.7	2.0	1.3	2.3
	Long Run	6.9	17.1	2.9	2.8	3.0	2.7
6	1	0.4	0.0	0.5	-0.7	0.0	-1.0
	10	1.2	2.7	0.6	-0.6	-1.7	-0.2
	20	2.6	5.9	1.2	0.0	-1.6	0.7
	40	5.4	12.7	2.5	1.8	1.0	2.1
	Long Run	6.6	16.4	2.6	2.6	2.8	2.5
7	1	2.7	0.0	3.9	0.8	0.0	1.1
	10	4.6	9.4	2.6	1.1	0.2	1.5
	20	7.0	18.4	2.2	2.0	1.5	2.2
	40	11.3	33.3	1.8	4.2	6.2	3.4
	Long Run	14.1	45.3	0.7	5.6	10.5	3.4
8	1	3.2	0.0	4.5	1.6	0.0	2.3
	10	5.7	11.4	3.2	2.4	3.3	2.1
	20	8.6	22.7	2.5	3.7	6.7	2.4
	40	13.6	40.8	1.9	6.1	12.7	3.3
	Long Run	17.5	57.0	0.5	7.6	17.2	3.4

tion tax so that the Social Security budget is balanced each year. The contribution match, however, is exactly financed each year by increasing the marginal income tax rates proportionally; the key qualitative results would be unchanged if the consumption tax were used instead. As before, income tax rates are changed so that the rest of government budget is balanced throughout the transition path.

As shown in Run 5 in Table 11, privatization with contribution matching increases national wealth, GNP and labor supply throughout the transition path compared to the pre-reform baseline economy. However, the long-run increases in national wealth, labor supply, and GNP are slightly smaller relative to the case without contribution matching (Run 2 in Table 9) since the match must be financed with distorting taxes. In the long run, GNP increases by only 6.9 percent, compared to 7.8 percent without the match.

Table 12 reports the welfare gains for Run 5. Contribution matching tends to improve

0 percent at $30,000, 10 percent at $60,000, and 0 percent for labor income above $60,000.

Table 12: Change in Welfare per Household (1,000 dollars in 2001)

		Without LSRA* Select Productivities				With LSRA** For all
Run #	Age in Year 1	e^1	e^3	e^5	e^8	Productivities
5	79	-5.0	-6.0	-15.6	-85.1	0.0
	60	-28.0	-42.2	-66.4	-455.0	0.0
	40	-13.9	-47.7	-81.4	-417.6	0.0
	20	6.8	2.8	-4.6	-23.4	0.0
	0	36.5	37.1	35.7	34.4	-3.5
	-20	56.4	60.3	63.6	75.6	-3.5
6	79	6.2	4.8	-7.9	-90.9	0.0
	60	-18.3	-27.0	-60.5	-514.1	0.0
	40	-11.6	-45.6	-83.4	-436.7	0.0
	20	5.0	0.6	-7.5	-29.1	0.0
	0	35.0	35.2	33.3	29.4	-2.6
	-20	55.5	59.3	61.9	72.2	-2.6
7	79	-4.2	-5.1	-13.5	-108.8	0.0
	60	-23.3	-40.0	-53.4	-115.8	0.0
	40	-21.8	-43.8	-55.2	-95.3	0.0
	20	-3.9	-2.7	1.9	35.7	0.0
	0	17.9	28.3	44.3	112.3	-7.6
	-20	33.5	49.9	73.0	162.9	-7.6
8	79	-4.8	-6.1	-12.8	-153.4	0.0
	60	-23.0	-42.1	-53.8	-5.4	0.0
	40	-29.4	-49.5	-46.5	45.8	0.0
	20	-8.3	-4.8	7.1	89.9	0.0
	0	13.4	31.5	61.0	205.1	2.2
	-20	28.8	56.2	96.1	276.0	2.2

* Standard equivalent variations measures.

** Value of Δtr.

the welfare of poorer households relative to Run 2 without the match. Whereas the poorest household born in the long run gains $52,400 without the match, they gain $56,400 with the match. The richest households, however, are worse off. They gained $84,300 without the match in the long run but only $75,600 with the match.

With the LSRA, privatization still leads to efficiency losses, equal to about $3,500 (in 2001 growth-adjusted dollars) per future household. But this loss is smaller than the $4,400 loss without the match.

Progressive Benefit Schedule. Run 6 takes a different approach to maintaining some progressivity after privatization. It increases the progressivity of the Social Security benefit that remains after privatization by raising the replacement rate of the lowest wage income bracket from 90 percent to 100 percent while reducing the replacement rate of the highest marginal income bracket from 15 percent to 10 percent. Table 12 shows that this approach is especially effective at protecting the welfare of the poor at the time of reform. The risk sharing properties of progressive benefits are also slightly stronger than contribution matching. Still, privatization reduces efficiency by $2,600 per each future household under the LSRA.

4.4 Alternative Assumptions of Transitory Shocks and Persistence

A key assumption in our model is the size of the transitory working ability shocks and their persistence. Recall that we constructed the age-working ability schedule from the 2001 Survey of Consumer Finances (SCF) and the transition matrices from the 1989-92 Panel Study of Income Dynamics (PSID). To deal with possible measurement errors, our benchmark Markov transition matrixes are calculated after taking three-year moving averages of workers' hourly wages. This *ad hoc* treatment reduces the size of transitory wage shocks in the original data by about one-third.

Floden and Lindé (2001), however, persuasively argue that measurement error in the PSID might be as large as the size of the real fluctuation. Thus, although we have already "smoothed away" some of that error by focusing on three-year moving averages, the transitory shocks in our model might still be too large.

Run 7 shown in Tables 11 and 12 show the economic and welfare effects, respectively, of privatization when the transitory shocks are reduced to only half of their previous values we used in the main calibration. Run 8 goes even further: it reduces the shocks to only *one-fifth* of the values under the main calibration. For computational simplicity, these Runs were performed in the small open economy setting, and so we compare our results with those shown for Run 3.

National wealth increases by a remarkable 45.3 percent in the long run in Run 7, compared to 36.5 percent for Run 3. However, notice that the efficiency losses actually *increase* to $7,600 per future household (relative to a $6,600 loss in Run 3) under the LSRA. This counter-intuitive result can be explained by the fact that a reduction in transitory shocks also increases the *persistence* of any shock. As a result, the effect of any negative shock becomes more permanent, potentially increasing the value of the risk sharing in the former Social Security system.[28]

In the limit, however, the model collapses to one with no wage uncertainty if the transitory shocks are eliminated. In Run 8, the transitory shocks are reduced to one-fifth of their benchmark levels and in this case, privatization actually produces a small efficiency gain equal to about $2,200 per future household. These two policy experiments show that changes in the amount of wage uncertainty have a *non-monotonic* impact on efficiency gains: privatization can produce larger losses as the transitory wage shocks are reduced yet efficiency gains as the transitory shocks are reduced even more.

5 Concluding Remarks

This paper investigated whether a stylized Social Security privatization generates efficiency gains or losses in the presence of an overlapping-generations economy with elastic labor supply and idiosyncratic wage shocks and longevity uncertainty. We found that the privatization of Social Security produces efficiency gains in a representative-agent economy without wage shocks (or, equivalently, if these shocks are insurable). In a heterogeneous-agent economy with idiosyncratic and uninsurable wage shocks, however, the overall efficiency of the economy is reduced by our stylized privatization since the existing Social Security system provides a valuable source of risk sharing through its progressive benefit formula. This result was fairly robust to a wide range of model considerations, including (i) the degree of openness of the economy, (ii) the introduction of various risk-sharing mechanisms after privatization, (iii) allowing the availability of actuarially-fair private annuities and (iv) reducing by half the size of transitory wage shocks. Only in the extreme case in which transitory shocks were reduced to one-fifth of their benchmark values did we find efficiency gains.

One of the possible limitations of our model is that it does not distinguish by various demographic groups, including race and gender. There is some evidence, for example, that black Americans do not live as long as non-blacks, even after controlling for differences in earnings. Blacks are also less likely to be married at the point of retirement and, therefore,

[28]We benefited from a helpful conversation with Dirk Krueger on this point.

less likely to qualify for a spousal benefit.[29] In contrast, women have a higher life expectancy than males, and they might also face a higher effective marginal tax rate on their contributions if they are the household's secondary earner. Incorporating these additional sources of heterogeneity would possibly change the welfare implication of this paper.

[29] See, for example, Gustman and Steinmeier (2001).

Appendices

A Transition Matrixes and Survival Rates

Markov Transition Matrixes. The Markov transition matrixes of working ability are constructed for four age groups—20-29, 30-39, 40-49, and 50-59—from the hourly wages in the PSID individual data 1990, 91, 92, and 93. To reduce the size of transitory shocks, possibly caused by measurement errors, the transition matrix of each age group is calculated with transition probabilities from the average wages between 1989 and 91 to the average wages between of 1990 and 92. For households aged 60 or older, we used the matrix for ages 50-59.

$$\Gamma_{i\in\{20,\ldots,29\}} = \begin{pmatrix} 0.7601 & 0.2101 & 0.0289 & 0.0000 & 0.0009 & 0.0000 & 0.0000 & 0.0000 \\ 0.1919 & 0.6171 & 0.1635 & 0.0221 & 0.0054 & 0.0000 & 0.0000 & 0.0000 \\ 0.0406 & 0.1546 & 0.5976 & 0.1794 & 0.0153 & 0.0068 & 0.0028 & 0.0029 \\ 0.0063 & 0.0181 & 0.1787 & 0.6723 & 0.1128 & 0.0118 & 0.0000 & 0.0000 \\ 0.0018 & 0.0000 & 0.0506 & 0.2524 & 0.5792 & 0.0995 & 0.0165 & 0.0000 \\ 0.0000 & 0.0000 & 0.0244 & 0.0000 & 0.2744 & 0.5719 & 0.1293 & 0.0000 \\ 0.0010 & 0.0000 & 0.0000 & 0.0000 & 0.0372 & 0.1937 & 0.6646 & 0.1035 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.4727 & 0.5273 \end{pmatrix},$$

$$\Gamma_{i\in\{30,\ldots,39\}} = \begin{pmatrix} 0.8537 & 0.1332 & 0.0082 & 0.0049 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.1283 & 0.6837 & 0.1744 & 0.0129 & 0.0007 & 0.0000 & 0.0000 & 0.0000 \\ 0.0175 & 0.1723 & 0.6788 & 0.1269 & 0.0000 & 0.0045 & 0.0000 & 0.0000 \\ 0.0005 & 0.0109 & 0.1314 & 0.7374 & 0.1014 & 0.0144 & 0.0040 & 0.0000 \\ 0.0000 & 0.0000 & 0.0144 & 0.2008 & 0.7016 & 0.0728 & 0.0104 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0460 & 0.1874 & 0.6182 & 0.1484 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0298 & 0.0011 & 0.2008 & 0.7147 & 0.0536 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.2145 & 0.7855 \end{pmatrix},$$

$$\Gamma_{i\in\{40,\ldots,49\}} = \begin{pmatrix} 0.8561 & 0.1371 & 0.0068 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.1301 & 0.7275 & 0.1222 & 0.0145 & 0.0057 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1111 & 0.7145 & 0.1588 & 0.0000 & 0.0156 & 0.0000 & 0.0000 \\ 0.0000 & 0.0179 & 0.1484 & 0.6992 & 0.1221 & 0.0124 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0162 & 0.2548 & 0.6594 & 0.0696 & 0.0000 & 0.0000 \\ 0.0252 & 0.0257 & 0.0000 & 0.0000 & 0.1702 & 0.6744 & 0.1045 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0933 & 0.8366 & 0.0701 \\ 0.1496 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.1308 & 0.7196 \end{pmatrix},$$

$$\Gamma_{i\in\{50,\ldots,78\}} = \begin{pmatrix} 0.8249 & 0.1629 & 0.0000 & 0.0122 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.1552 & 0.6754 & 0.1694 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0200 & 0.0997 & 0.7643 & 0.1052 & 0.0108 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0620 & 0.0663 & 0.8079 & 0.0559 & 0.0079 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.1492 & 0.7194 & 0.1314 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.2942 & 0.6239 & 0.0819 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.1024 & 0.8573 & 0.0403 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.1613 & 0.8387 \end{pmatrix},$$

where $\Gamma_i(j,k) = \pi(e_{i+1} = e^k_{i+1} | e_i = e^j_i)$.

Survival Rates of Households. The survival rates ϕ_i at the end of age $i = \{20, ..., 109\}$ are the weighted averages of males and females and calculated from the period life table (Table 4.C6) in Social Security Administration (2001). The survival rates at the end of age 109 are replaced by zero.

Table 13: Survival Rates in 1998 in the United States (Weighted Average of Males and Females)

Age	Survival Rate	Age	Survival Rate	Age	Survival Rate	Age	Survival Rate	Age	Survival Rate
20	0.999104	40	0.997966	60	0.989315	80	0.937788	100	0.676630
21	0.999057	41	0.997807	61	0.988305	81	0.931527	101	0.658554
22	0.999027	42	0.997640	62	0.987134	82	0.924684	102	0.639355
23	0.999018	43	0.997451	63	0.985773	83	0.917252	103	0.618962
24	0.999023	44	0.997252	64	0.984249	84	0.909150	104	0.597297
25	0.999034	45	0.997027	65	0.982548	85	0.900275	105	0.574281
26	0.999040	46	0.996778	66	0.980759	86	0.890541	106	0.549828
27	0.999033	47	0.996514	67	0.979000	87	0.879882	107	0.523850
28	0.999006	48	0.996237	68	0.977325	88	0.868264	108	0.496251
29	0.998962	49	0.995938	69	0.975647	89	0.855676	109	0.000000
30	0.998911	50	0.995603	70	0.973769	90	0.842119		
31	0.998857	51	0.995222	71	0.971613	91	0.827606		
32	0.998796	52	0.994797	72	0.969264	92	0.812154		
33	0.998727	53	0.994324	73	0.966703	93	0.795784		
34	0.998651	54	0.993795	74	0.963868	94	0.778522		
35	0.998564	55	0.993198	75	0.960661	95	0.761075		
36	0.998466	56	0.992534	76	0.957027	96	0.743640		
37	0.998358	57	0.991818	77	0.952967	97	0.726432		
38	0.998240	58	0.991051	78	0.948449	98	0.709688		
39	0.998111	59	0.990216	79	0.943423	99	0.693653		

Source: Authors' calculations from the Table 4.C6, Social Security Administration (2001).

B The Computation of Equilibria

The algorithm to solve the model for a steady-state equilibrium and an equilibrium transition path is similar to those in Conesa and Krueger (1999), Nishiyama (2002), and Nishiyama and Smetters (2003).[30]

B.1 The Discretization of the State Space

The state of a household is $\mathbf{s}_i = (i, e_i, a_i, b_i) \in I \times E \times A \times B$, where $I = \{20, ..., 109\}$, $E = [e^{\min}, e^{\max}]$, $A = [a^{\min}, a^{\max}]$, and $B = [b^{\min}, b^{\max}]$. To compute an equilibrium, the state space of a household is discretized as $\hat{\mathbf{s}}_i \in I \times \hat{E}_i \times \hat{A} \times \hat{B}$, where $\hat{E}_i = \{e_i^1, e_i^2, ..., e_i^{N_e}\}$,

[30] The authors are grateful to José Víctor Ríos-Rull for his teaching of the computational procedure of heterogeneous agent models.

$\hat{A} = \{a^1, a^2, ..., a^{N_a}\}$, and $\hat{B} = \{b^1, b^2, ..., b^{N_b}\}$. For all these discrete points, the model computes the optimal decision of households, $\mathbf{d}(\hat{\mathbf{s}}_i, \mathbf{S}_t; \mathbf{\Psi}_t) = (c_i(.), h_i(.), a_{i+1}(.)) \in (0, c^{\max}] \times [0, h_i^{\max}] \times A$, the marginal values, $\frac{\partial}{\partial a} v(\hat{\mathbf{s}}_i, \mathbf{S}_t; \mathbf{\Psi}_t)$ and $\frac{\partial}{\partial b} v(\hat{\mathbf{s}}_i, \mathbf{S}_t; \mathbf{\Psi}_t)$, and the values $v(\hat{\mathbf{s}}_i, \mathbf{S}_t; \mathbf{\Psi}_t)$, given the expected factor prices and policy variables.[31]

To find the optimal end-of-period wealth, the model uses the Euler equation and bilinear interpolation (with respect to a and b) of marginal values at the beginning of the next period.[32] In a heterogeneous-agent economy, N_e, N_a, and N_b are 8, 57, and 8, respectively. In a representative-agent economy, the numbers of grid points are 1, 61, and 6, respectively.[33]

B.2 A Steady-State Equilibrium

The algorithm to compute a steady-state equilibrium is as follows. Let $\mathbf{\Psi}$ denote the time-invariant government policy rule $\mathbf{\Psi} = (W_{LS}, W_G, C_G, \tau_I(.), \tau_P(.), \tau_C, tr_{SS}(\hat{\mathbf{s}}_i), tr_{LS}(\hat{\mathbf{s}}_i))$.

1. Set the initial values of factor prices (r^0, w^0), accidental bequests q^0, the policy variables $(W_{LS}^0, W_G^0, C_G^0, \tau_C^0)$, and the parameters φ^0 of policy functions $(\tau_I(.), \tau_P(.), tr_{SS}(\hat{\mathbf{s}}_i))$ if these are determined endogenously.[34]

2. Given $\mathbf{\Omega}^0 = (r^0, w^0, q^0, W_{LS}^0, W_G^0, C_G^0, \tau_C^0, \varphi^0)$, find the decision rule of a household $\mathbf{d}(\hat{\mathbf{s}}_i; \mathbf{\Psi}, \mathbf{\Omega}^0)$ for all $\hat{\mathbf{s}}_i \in I \times \hat{E}_i \times \hat{A} \times \hat{B}$.[35]

   (a) For age $i = 109$, find the decision rule $\mathbf{d}(\hat{\mathbf{s}}_{109}; \mathbf{\Psi}, \mathbf{\Omega}^0)$. Since the survival rate $\phi_{109} = 0$, the end-of-period wealth $a_{i+1}(\hat{\mathbf{s}}_{109}; .) = 0$ for all $\hat{\mathbf{s}}_{109}$. Compute consumption and working hours $(c_i(\hat{\mathbf{s}}_{109}; .), h_i(\hat{\mathbf{s}}_{109}; .))$ and, then, marginal values $\frac{\partial}{\partial a} v(\hat{\mathbf{s}}_{109}; \mathbf{\Psi}, \mathbf{\Omega}^0)$ and values $v(\hat{\mathbf{s}}_{109}; \mathbf{\Psi}, \mathbf{\Omega}^0)$ for all $\hat{\mathbf{s}}_{109}$.[36]

   (b) For age $i = 108, ..., 20$, find the decision rule $\mathbf{d}(\hat{\mathbf{s}}_i; \mathbf{\Psi}, \mathbf{\Omega}^0)$, marginal values $\frac{\partial}{\partial a} v(\hat{\mathbf{s}}_i; \mathbf{\Psi}, \mathbf{\Omega}^0)$, and values $v(\hat{\mathbf{s}}_i; \mathbf{\Psi}, \mathbf{\Omega}^0)$ for all $\hat{\mathbf{s}}_i$, using $\frac{\partial}{\partial a} v(\hat{\mathbf{s}}_{i+1}; \mathbf{\Psi}, \mathbf{\Omega}^0)$ and $\frac{\partial}{\partial b} v(\hat{\mathbf{s}}_{i+1}; \mathbf{\Psi}, \mathbf{\Omega}^0)$ recursively.

      i. Set the initial guess of $a_{i+1}^0(\hat{\mathbf{s}}_i; .)$.

      ii. Given $a_{i+1}^0(\hat{\mathbf{s}}_i; .)$, compute $(c_i(\hat{\mathbf{s}}_i; .), h_i(\hat{\mathbf{s}}_i; .))$, using $\frac{\partial}{\partial b} v(\hat{\mathbf{s}}_{i+1}; \mathbf{\Psi}, \mathbf{\Omega}^0)$. Plug these into the Euler equation with $\frac{\partial}{\partial a} v(\hat{\mathbf{s}}_{i+1}; \mathbf{\Psi}, \mathbf{\Omega}^0)$.

[31] Because the marginal value with respect to historical earnings, $\frac{\partial}{\partial b} v(\hat{\mathbf{s}}_i, \mathbf{S}_t; \mathbf{\Psi}_t)$, is difficult to obtain analytically, it is approximated by $(v(., b^{j+1}, \mathbf{S}_t; \mathbf{\Psi}_t) - v(., b^j, \mathbf{S}_t; \mathbf{\Psi}_t))/(b^{j+1} - b^j)$ where $j = 1, 2, ..., N_b$.

[32] The marginal values with respect to wealth, $\frac{\partial}{\partial a} v(\hat{\mathbf{s}}_i, \mathbf{S}_t; \mathbf{\Psi}_t)$, are used in the Euler equation to obtain optimal savings, the marginal values with respect to historical earnings, $\frac{\partial}{\partial b} v(\hat{\mathbf{s}}_i, \mathbf{S}_t; \mathbf{\Psi}_t)$, are used in the marginal rate of substitution condition of consumption for leisure to obtain optimal working hours, and the values, $v(\hat{\mathbf{s}}_i, \mathbf{S}_t; \mathbf{\Psi}_t)$, are used to calculate welfare changes measured by compensating and equivalent variations in wealth.

[33] The grid points on A and B are not equally spaced. In a heterogeneous-agent economy, $\hat{A}$ ranges from -$271,000 to $33,825,000 (in 2001 growth-adjusted dollars) and $\hat{B}$ ranges from $8,000 to $80,400. In a representative-agent economy, $\hat{A}$ and $\hat{B}$ range from -$333,000 to $1,573,000 and from $8,000 to $80,400, respectively.

[34] Actually, if we find the capital-labor ratio, both r and w are calculated from the given production function and depreciation rate.

[35] In the steady-state economy, the decision rule of a household $\mathbf{d}(\hat{\mathbf{s}}_i; \mathbf{\Psi}, \mathbf{\Omega}^0)$ is not a function of the aggregate state of economy $\hat{\mathbf{S}} = (x(\hat{\mathbf{s}}_i), W_{LS}, W_G)$. The measure of household $x(\hat{\mathbf{s}}_i)$ is determined uniquely by the steady-state condition, and the government's wealth W_G is determined by the policy rule $\mathbf{\Psi}$.

[36] The marginal value with respect to historical earnings, $\frac{\partial}{\partial b} v(\hat{\mathbf{s}}_i; \mathbf{\Psi}, \mathbf{\Omega}^0)$, is zero when $i > 60$ in this paper.

iii. If the Euler error is sufficiently small, then stop. Otherwise, update $a^0_{i+1}(\hat{\mathbf{s}}_i;.)$ and return to Step ii.

3. Find the steady-state measure of households $x(\hat{\mathbf{s}}_i;\mathbf{\Omega}^0)$ using the decision rule obtained in Step 2. This computation is done forward from age 20 to age 109. Repeat this step to iterate q for q^1.

4. Compute new factor prices (r^1,w^1), the policy variables $(W^1_{LS},W^1_G,C^1_G,\tau^1_C)$, and the parameters φ^1 of policy functions.[37]

5. Compare $\mathbf{\Omega}^1=(r^1,w^1,q^1,W^1_{LS},W^1_G,C^1_G,\tau^1_C,\varphi^1)$ with $\mathbf{\Omega}^0$. If the difference is sufficiently small, then stop. Otherwise, update $\mathbf{\Omega}^0$ and return to Step 2.

B.3 An Equilibrium Transition Path

Assume that the economy is in the initial steady state in period 0, and that the new policy schedule (rule) $\mathbf{\Psi}_1$, which was not expected in period 0, is announced at the beginning of period 1, where $\mathbf{\Psi}_1=\{W_{LS,t+1},W_{G,t+1},C_{G,t},\tau_{I,t}\left(.\right),\tau_{P,t}\left(.\right),\tau_{C,t},tr_{SS,t}\left(\hat{\mathbf{s}}_i\right),tr_{R,t}(\hat{\mathbf{s}}_i)\}^{\infty}_{t=1}$. Let $\hat{\mathbf{S}}_1=(x_1(\hat{\mathbf{s}}_i),W_{LS,1},W_{G,1})$ be the state of the economy at the beginning of period 1. The state of the economy $\hat{\mathbf{S}}_1$ is usually equal to that of the initial steady state.

There is no aggregate productivity shocks in this model. So, a time series $\{\mathbf{\Omega}_s\}^T_{s=t}$ is deterministic, and each household perfectly foresees $\{\mathbf{\Omega}_s\}^T_{s=t}$ based on the information $\hat{\mathbf{S}}_t$ in an equilibrium. Since $\hat{\mathbf{S}}_t$ is in a household decision rule only to make the household expect $\{\mathbf{\Omega}_s\}^T_{s=t}$ rationally, in the computation, we can avoid the "curse of dimensionality" by replacing $\mathbf{d}(\hat{\mathbf{s}}_i,\hat{\mathbf{S}}_t;\mathbf{\Psi}_t)$ with $\mathbf{d}(\hat{\mathbf{s}}_i;\mathbf{\Psi}_t,\{\mathbf{\Omega}_s\}^T_{s=t})$. The algorithm to compute a transition path to a new steady-state equilibrium (thereafter, final steady-state equilibrium) is as follows.

1. Choose a sufficiently large number, T, such that the economy is said to reach the new steady state within T periods.[38] Set the initial guess, $\{\mathbf{\Omega}^0_t\}^T_{t=1}$, on factor prices (r^0_t,w^0_t), accidental bequests q^0_t, the policy variables $(W^0_{LS,t+1},W^0_{G,t+1},C^0_{G,t},\tau^0_{C,t})$, and the parameters φ^0_t of policy functions for $t=1,2,...,T$.

2. Given $\mathbf{\Omega}^0_T=(r^0_T,w^0_T,q^0_T,W^0_{LS,T},W^0_{G,T},C^0_{G,T},\tau^0_{C,T},\varphi^0_T)$, find the final steady-state decision rule $\mathbf{d}(\hat{\mathbf{s}}_i;\mathbf{\Psi}_T,\mathbf{\Omega}^0_T)$, marginal values, $\frac{\partial}{\partial a}v(\hat{\mathbf{s}}_i;\mathbf{\Psi}_T,\mathbf{\Omega}^0_T)$, and values $v(\hat{\mathbf{s}}_i;\mathbf{\Psi}_T,\mathbf{\Omega}^0_T)$ for all $\hat{\mathbf{s}}_i\in I\times\hat{E}_i\times\hat{A}\times\hat{B}$. (See the algorithm for a steady-state equilibrium.)

3. For period $t=T-1,T-2,...,1$, based on the guess, $\mathbf{\Omega}^0_t$, find backward the decision rule $\mathbf{d}(\hat{\mathbf{s}}_i;\mathbf{\Psi}_t,\{\mathbf{\Omega}^0_s\}^T_{s=t})$, marginal values $\frac{\partial}{\partial a}v(\hat{\mathbf{s}}_i;\mathbf{\Psi}_t,\{\mathbf{\Omega}^0_s\}^T_{s=t})$, and values $v(\hat{\mathbf{s}}_i;\mathbf{\Psi}_t,\{\mathbf{\Omega}^0_s\}^T_{s=t})$ for all $\hat{\mathbf{s}}_i\in I\times\hat{E}_i\times\hat{A}\times\hat{B}$, using the next period marginal values $\frac{\partial}{\partial a}v(\hat{\mathbf{s}}_{i+1};\mathbf{\Psi}_{t+1},\{\mathbf{\Omega}^0_s\}^T_{s=t+1})$ and values $v(\hat{\mathbf{s}}_{i+1};\mathbf{\Psi}_{t+1},\{\mathbf{\Omega}^0_s\}^T_{s=t+1})$ recursively.

4. For period $t=1,2,...,T-1$, compute forward $\mathbf{\Omega}^1_t=(r^1_t,w^1_t,q^1_t,W^1_{LS,t+1},W^1_{G,t+1},C^1_{G,t},\tau^1_{C,t},\varphi^1_t)$ and the measure of households $x_{t+1}(\hat{\mathbf{s}}_i)$, using the decision rule $\mathbf{d}(\hat{\mathbf{s}}_i;\mathbf{\Psi}_t,$

[37] In this paper, the endogenous policy variables are C_G in baseline economies and τ_C, φ_I, $tr_{LS}(\hat{\mathbf{s}}_i)$, and W_{LS} in policy experiments.

[38] As in Auerbach and Kotlikoff (1987), we found setting T at 150 to be sufficient.

$\{\mathbf{\Omega}_s^0\}_{s=t}^T$) obtained in Step 3 and using the state of economy $\hat{\mathbf{S}}_t = (x_t(\hat{\mathbf{s}}_i), W_{LS,t}, W_{G,t})$ recursively.

5. Compare $\{\mathbf{\Omega}_t^1\}_{t=1}^T$ with $\{\mathbf{\Omega}_t^0\}_{t=1}^T$. If the difference is sufficiently small, then stop. Otherwise, update $\{\mathbf{\Omega}_t^0\}_{t=1}^T$ and return to Step 2. If the final steady-state equilibrium is known, return to Step 3 instead.

B.4 The Lump-Sum Redistribution Authority

When the Lump-Sum Redistribution Authority (LSRA) is assumed, the following computation is added to the iteration process.

1. For period $t = T, T-1, ..., 2$, compute the lump-sum transfers to newborn households $tr_{CV}(\hat{\mathbf{s}}_{20}, \hat{\mathbf{S}}_t; \mathbf{\Psi}_t)$ to make those households as well off as under the pre-reform economy.

   (a) Set the initial value of lump-sum transfers $tr_{CV}(\hat{\mathbf{s}}_{20}, \hat{\mathbf{S}}_t; \mathbf{\Psi}_t)$ to newborn households.

   (b) Given $tr_{CV}(\hat{\mathbf{s}}_{20}, \hat{\mathbf{S}}_t; \mathbf{\Psi}_t)$, find the decision rule of newborn households $\mathbf{d}(\hat{\mathbf{s}}_{20}, \hat{\mathbf{S}}_t; \mathbf{\Psi}_t)$ and values $v(\hat{\mathbf{s}}_{20}, \hat{\mathbf{S}}_t; \mathbf{\Psi}_t)$.

   (c) Find the compensating variation in wealth $\Delta tr_{CV}(\hat{\mathbf{s}}_{20}, \hat{\mathbf{S}}_t; \mathbf{\Psi}_t)$ to make those households indifferent from the baseline economy. (The initial wealth of newborn households is assumed to be zero since they do not receive any bequests.) If the absolute value of $\Delta tr_{CV}(\hat{\mathbf{s}}_{20}, \hat{\mathbf{S}}_t; \mathbf{\Psi}_t)$ is sufficiently small, then go to Step (d). Otherwise, update $tr_{CV}(\hat{\mathbf{s}}_{20}, \hat{\mathbf{S}}_t; \mathbf{\Psi}_t)$ by adding $\Delta tr_{CV}(\hat{\mathbf{s}}_{20}, \hat{\mathbf{S}}_t; \mathbf{\Psi}_t)$ and return to Step (b).

   (d) Set the lump-sum transfers $tr_{LS,t}(\hat{\mathbf{s}}_{20}) = tr_{CV}(\hat{\mathbf{s}}_{20}, \hat{\mathbf{S}}_t; \mathbf{\Psi}_t) + \Delta tr$ where an additional lump-sum transfer Δtr is precalculated, and find the decision rule of newborn households $\mathbf{d}(\hat{\mathbf{s}}_{20}, \hat{\mathbf{S}}_t; .)$.

2. For period $t = 1$, compute the lump-sum transfers to all current households $tr_{CV}(\hat{\mathbf{s}}_i, \hat{\mathbf{S}}_1; \mathbf{\Psi}_1)$ to make those households as much better off as the pre-reform economy. The procedure is similar to Step 1. Set the lump-sum transfers $tr_{LS,1}(\hat{\mathbf{s}}_i) = tr_{CV}(\hat{\mathbf{s}}_i, \hat{\mathbf{S}}_1; \mathbf{\Psi}_1)$.

3. Compute an additional lump-sum transfer Δtr to newborn households so that the net present value of all transfers becomes zero. Compute the LSRA wealth, $\{W_{LS,t}^1\}_{t=1}^T$, which will be used to calculate national wealth. Recompute Δtr and $\{W_{LS,t}^1\}_{t=1}^T$ using new interest rates $\{r_t\}_{t=1}^T$.

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