Informal Insurance in Social Networks

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Abstract

This paper studies informal insurance across networks of individuals. Two characteristics are fundamental to the model developed here: first, informal insurance is a *bilateral* activity, and rarely involves explicit arrangements across several people. Second, insurance is a *social* activity, and transfers are often based on norms. In the model studied here, only directly linked agents make transfers to each other, although they are aware of the (aggregate) transfers each makes to third parties. An insurance scheme for the network as a whole is an equilibrium of several interacting bilateral arrangements. This model serves as a starting point for investigating stable insurance networks, in which all agents actually have private incentives to abide by the ongoing insurance arrangement.

The resulting analysis shows that network links have two distinct and possibly conflicting roles to play. First, they act as conduits for transfers, and in this way this make better insurance possible. Second, they act as conduits for information, and in this sense they affect the severity of punishments for noncompliance with the scheme. A principal task of this paper is to describe how these two forces interact, and in the process characterize stable networks. The concept of "sparse" networks, in which the removal of certain links increases the number of network components, is crucial in this characterization. As corollaries, we find that both "thickly connected" networks (such as the complete graph) and "thinly connected" networks (such as trees) are likely to be stable, whereas intermediate degrees of connectedness jeopardize stability.

Finally, we study in more detail the notion of networks as conduits for transfers, by simply assuming a punishment structure (such as autarky) that is independent of the precise architecture of the tree. This allows us to isolate a *bottleneck effect*: the presence of certain key agents who act as bridges for several transfers. Bottlenecks are captured well in a feature of trees that we call *decomposability*, and we show that all decomposable networks have the same stability properties and that these are the least likely to be stable.

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1. Introduction

Informal risk-sharing arrangements exist in most developing countries – especially in rural areas where credit and insurance markets are scarce. In order to face income fluctuations due to a variety of exogenous factors, villagers enter into mutual insurance schemes, whose institutional details may vary from country to country. These institutions are often intertwined with the social networks of the community. It is now widely recognized that social networks (based on kin, gender or occupation) play a dominant role in people's protection to risk in developing countries, and their neglect is a recurring criticism of development projects and safety net policies.

The objective of this paper is to develop a model of risk-sharing network in order to explain the influence of social networks on informal insurance agreements and to derive endogenously the architecture of self-insurance networks. Most of the theoretical literature on mutual insurance schemes implicitly assumes either that institutions are based at the community level or among two individuals only. (Posner (1980), Kimball (1988), Coate and Ravallion (1993), Kocherlakota (1996), Kletzer and Wright (2000) and Ligon, Thomas and Worrall (2002)). Genicot and Ray (2003) go one step further, and suppose that informal insurance schemes may be formed by subgroups in the community. However, empirical evidence suggests that insurance schemes may in fact be designed at an even more disaggregated level (see Fafchamps (1992), Fafchamps and Lund (2003) and Murgai et al. (2002)). Moreover, actual mapping of insurance networks by de Weerdt (2000) and Dercon and de Weerdt (2000) in Tanzania reveal a complex architecture of risk-sharing networks.[1] In fact, the complexity of networks and the fact that social networks play an important role in mutual risk-sharing arrangements has been long stressed by anthropologists and sociologists (See for instance Clark (1975), Wellman (1992)).

Informal insurance arrangements typically suffer from the absence of enforcement by third parties. An individual agent cannot be forced to participate in the scheme and pay the transfers he is called to make. As a result, stable mutual insurance schemes must be *self-enforcing.* At no point must individuals called upon making a transfer have incentive to deviate and not make the transfer given that they will be punished by some sort of exclusion from the scheme in the future (and possibly other social exclusions). At the community level, the papers mentioned above suppose that a deviating agent is punished by being entirely barred from the scheme, and hence will have to bear all the fluctuations in income after a deviation. Genicot and Ray (2003) allow for group deviations, by which a subgroup of agents can still form a smaller mutual insurance scheme after the deviation, and find (rather unexpectedly) that this changes completely the picture, and that there is an upper bound on the size of the group which can form informal insurance arrangements. In the context of networks, the possibilities of punishment after a deviation are even more varied: an agent could only be punished by those agents to whom he has not transferred money, or by the entire community, or by any subset of agents in between.

In this paper, we study informal insurance networks, and builds a model of risk-sharing which captures two features. First, informal insurance in networks essentially results from a collection of *bilateral* arrangements rather than an explicit agreement across several people. In the model network studied here, only directly linked agents make transfers to each other, though they are aware of the (aggregate) transfers each makes to others. Linked agents have information

[1] A recent literature in development studies the importance of social network for learning and information transmission [see for instance Bertrand, Luttmer and Mullainathan (2000); Bandiera and Rasul (2002); Conley and Udry (2003); Kremer and Miguel (2003); and Dearden, Pritchett and Brown (2004)]. However, it is only recently that household surveys have begun asking individuals to and from whom they are giving and receiving transfers.

only on each other's commitments, but not necessarily on the overall insurance scheme of the community. This assumption implies in particular that agents do not need to know all the transfer made in the entire network. Second, insurance is often based on internalized *norms* regarding mutual help. A bilateral transfer arrangement between two linked agents is viewed as a *bilateral transfer norm* determining the agents' transfer to each other as a function of their identities, the component they belong to, their income realizations and the transfers made to or received from other agents. Based on these bilateral transfer norms between all pairs of agents, we define a *consistent transfer scheme* as a fixed point of the resulting mapping. Examples of transfer norms include *equal sharing*, in which consumptions of linked agents are equated in each state, non-welfarist norms in which agents keep a fraction of their resource (their income plus the transfer received from others) and divide the remaining, and *Nash bargaining*, in which transfers are chosen to maximize the Nash product, given the outside options of each agent.

With this setup as background, the paper then studies the stability of insurance networks, explicitly recognizing the possibility that the lack of commitment may destabilize insurance arrangements among the network. Note that in the theoretical literature on norms within groups of agents or institutions, two different modeling approaches have been used. In the first one, as in this paper, institutions form first and agents abide by the norms of that group, which may make that group unstable (Farrell and Scotchmer [1988], Hoff [1997]). Another possible approach would be to require the norm to preserve the stability of the network or group (such as in Dutta and Ray [1989]). However, evidence on informal insurance in traditional village communities (see Platteau [2004]) suggest that social norms are pervasive and relatively rigid so that traditional reciprocity networks can erode under the pressure of market integration and new opportunities.

We assess the stability of insurance networks under different possible punishment schemes that determine the severance of links to a deviant. The *weakest punishment* is one in which only individuals with respect to whom a deviant did not fulfill her obligations break links with her, and the best (from the deviant's viewpoint) stable subnetwork forms. One can strengthen such schemes by asking that individuals who are connected directly to a deviant and are q links away from her victim, but not via the deviant, to also break links with the deviant, where q can be made progressively larger to capture wider information flows. We call these punishment schemes *level-q punishment* and say that a network is *q-stable* when it is stable for a punishment structure of level-q. Finally, in the *strong punishment* case, a deviating agent is excluded by the entire community, and receives after his deviation his autarchic allocation, bearing all come fluctuations alone.

Our analysis highlights two, possibly conflicting, forces in the relation between the architecture of the network and the stability of insurance schemes: a *transit* effect, a short-term effect that comes from the role of links as channel of transfers, and a *information effect* that determines the capacity of the network to punish its deviant.

In *monotone* transfer norms, bringing additional individuals into the component by linking them to one member increases that member's payoff. Looking at the stability of networks under all monotone transfer norms for high level of discount rate isolates the information effect. All networks are stable under strong punishment. In contrast, for weaker punishment schemes, the density of links (as well as their specific placement in the network) can weaken punishment and has important consequences for network stability. In particular, for high values of the discount factor network composed of trees are the only stable networks. We show tat for intermediate levels of punishment both minimally connected and highly dense graphs are stable while networks that display intermediary density level ten to be unstable. This illustrates clearly the importance of the punishment scheme, and of the architecture of the network, for its stability.

For lower values of the discount rate, the *transit* or bottleneck effects become important for the stability of a network. Assessing the stability of mutual insurance schemes in the context of social networks is a difficult task. We do so assuming a specific risk-sharing norm: equal sharing – by which all agents divide equally income at every state.

As transfers can only flow along the links in the network, in order to reduce agents' incentives to deviate from the insurance scheme, one ought to minimize the amount of transfers going through any particular agent. This is particularly clear in the case of strong punishment, where the continuation vales for any deviation are the same. In this situation, one could determine in any network the "bottleneck" agent as the agent who has the highest short term incentive to deviate, and the enforcement constraint faced by this bottleneck agent defines the stability of the entire network. We show that there is a class of "decomposable" trees (including stars and lines) for which the bottleneck effect is identical, and hence stability conditions are identical. Furthermore, the addition of new links can only relax the bottleneck effect, as new links can be used to reroute transfers at every state. Hence, adding links can only improve the stability of the network, and the complete network is stable for lower values of the discount factor than any other network.

For weaker punishment schemes, a higher density in a network will have an ambiguous effect. On the one hand, it reduces the bottleneck effects, thereby helping stability, but, as seen earlier, it also reduces the potential punishment a deviant would suffer which hurts stability. Under weak punishment, we show that the stability conditions are identical for all fully decomposable trees (a class of trees including stars and lines).

2. Transfer Norms in Insurance Networks

2.1. **Endowments and Preferences.** We consider a community of individuals, all identical except for their positions in a social network (see below). At each date, a state of nature θ (with probability $p(\theta)$) is drawn from some finite set Θ. The state determines a strictly positive endowment y for each agent. Denote by $\mathbf{y}(\theta)$ the vector of income realizations for all agents. Assume symmetry: if $\mathbf{y}$ is the realization at state θ and $\mathbf{y}'$ is a permutation of $\mathbf{y}$, then there is another state θ' with $p(\theta) = p(\theta')$ and $\mathbf{y}' = \mathbf{y}(\theta)$. Finally, assume that every possible inter-individual combination of (a finite set of) outputs has strictly positive probability. [This condition guarantees, in particular, that outputs are not perfectly correlated.]

All agents are endowed with a von Neumann-Morgenstern utility u defined over consumption, which is smooth, increasing and strictly concave. They have a common discount factor $\delta \in (0, 1)$. Individual consumption will not generally equal individual income as agents will make transfers to one another. However, we assume that the good is perishable and that the community as a whole has no access to outside credit, so aggregate consumption cannot exceed aggregate income at any date.

2.2. **Social Networks.** Agents interact in a social network. Formally, this is a graph g — a collection of pairs of agents — with the interpretation that the pair ij belongs to g if they are directly linked. Of course, linkage can mean many things and presently we shall discuss some of the implications, but for now we simply mean that i and j are linked if and only if they can make transfers to each other.

Some graph-theoretic considerations will be useful in what we do, so we briefly recall the relevant concepts. The *complete network* is just the graph in which all conceivable pairs are directly linked, and the *empty network* is the graph in which no pair is linked. A *path* between agents i and j is said to exist if there is a sequence of direct links leading from i to j; in this case, i and j are said to be *connected.* The *distance* between i and j is the number of links along the shortest path between them. The network itself is *connected* if all pairs are connected. A *component* of a network is any maximally connected subgraph of that network, and the *size* of a component is the number of agents in the component. A component will be denoted by the notation d. The set of agents in any graph g (or in any component d) will be denoted by $N(g)$ (or $N(d)$). Let $\aleph_i(g) \equiv \{j|ij \in g\}$ be the set of agents directly linked to i.

A *cycle* is a sequence $i^1, \ldots, i^m$ of $m \geq 3$ distinct agents such that i^k and i^{k+1} are linked for $1 \leq k \leq m-1$ and i^1 and i^m are also linked. A network is *acyclic* if it does not contain any cycle. An acyclic connected network is called a *tree.*

Two special trees will play a prominent role. A *star* is a tree g such that there is one agent i with $ij \in g$ for all other j. Agent i (who is uniquely identified in this way) is the *center* of the star, and all other agents form the *periphery.* A *line* is a tree with the property that for some labelling $1, \ldots, n$ of the agents, i is linked to $i+1$ for all $1 \leq i \leq n-1$.

2.3. **Bilateral Transfer Norms.** The solution concept that we use emphasizes the bilateral nature of transfers (see Introduction). To be sure, such transfers from or to an individual must take into account what her partner is likely to receive from (or give to) third parties. Admittedly, in many situations this is easier said than done, but as a first approximation we assume that the *aggregate* of such transfers between the partner and third parties is indeed observable.

So, given some linked pair ij, the following "transfer-relevant" items are assumed to be commonly observed: incomes $\mathbf{y}_i$ and $\mathbf{y}_j$, and total transfers received from (or made to) third parties by each agent: $\mathbf{z}_i$ and $\mathbf{z}_j$. These variables are in boldface because they are actually mappings from the state θ to realizations, and below we will often write them as such. But it is important to keep in mind that $z_i(\theta)$ and $z_j(\theta)$ — while taken as given by ij — are endogenous mappings which will be pinned down in society-wide equilibrium.

By convention x_{ij} will stand for the transfer, positive or negative, from j to i. A *bilateral transfer scheme* is just a collection of state-contingent transfers across those two individuals, which we write as $\mathbf{x}_{ij}$. A *bilateral transfer norm* between i and j generates state-contingent transfer schemes as a function of various observables. We permit $\mathbf{x}_{ij}$ to depend on individual identities i and j, the component d of the graph in which the pair is embedded,[2] and, of course on $\mathbf{y}_i$, $\mathbf{y}_j$, $\mathbf{z}_i$ and $\mathbf{z}_j$. So in its most general form, we may write a bilateral transfer norm as

$$\mathbf{x}_{ij} = \mathbf{x}(i, j, d, \mathbf{y}_i, \mathbf{y}_j, \mathbf{z}_i, \mathbf{z}_j)$$

where, for each state θ, the value $\mathbf{x}_{ij}(\theta)$ is to be interpreted as the transfer *from j to i.*

As written, a bilateral transfer norm has something to say about all conceivable situations, including those in which negative consumptions might result. This is only for convenience. As we shall see in Proposition 1, under mild restrictions there are always solutions in which all consumptions are positive in all states.

A transfer norm is presumably the expression of some underlying social norm (possibly commonly held across different pairs in the same society). [There are, to be sure, private enforcement

[2]The reason that we permit the transfer norm to depend on the component d to which i and j belong but not on the entire graph g is essentially to economize notation. [Otherwise we would have to worry about other communities when defining payoffs to the members of a particular community.] In addition, two different components are completely insulated from each other in all possible ways so there is no conceptual reason why the norm should include other components.

constraints which might cause such a bilateral scheme to break down, but more on that at a later stage.] It is easy enough to provide several examples.

Equal Sharing. A focal case, one which will receive special attention in this paper, is that of equal-sharing. The transfer between i and j is chosen to equalize consumption across the two agents in each state, so that for all θ, $x_{ij}(\theta)$ solves

$$y_i(\theta) + z_i(\theta) + x_{ij}(\theta) = y_j + z_j(\theta) - x_{ij}(\theta) \tag{1}$$

The equal-sharing norm, apart from its intrinsic interest, belongs to a wider family of norms which we might describe as *multilaterally coherent.* Such bilateral norms have the property that there is some "multilateral" norm — which allocates transfers across larger (connected) groups of individuals depending on their income realizations — with the property that for every bilateral situation they induce,[3] the prescribed transfer is coherent with the bilateral norm for the pair in question.

Transfer norms that allocate to each person a weighted share of consumption (depending perhaps on that person's identity or her income realization) are also multilaterally coherent provided that the relative weights for every pair $\{ij\}$ equal the relative weights arrived at "indirectly" by compounding relative weights along any other path joining i to j.

Welfare Functions. One may also wish to derive transfer norms from bilateral welfare functions. In its most general form, such a welfare function would depend on state-contingent consumptions $\mathbf{c}_i$ and $\mathbf{c}_j$ of the two agents, and it may also depend on other variables, such as the ambient network component and the identity of the agents. These last two items are to be interpreted in most cases as proxies for the utilities achievable by the agents (in the absence of the ij link), which may well be asymmetric. The social norm may wish to take account of such asymmetries. So suppose that for each linked pair ij there is a welfare function

$$W(i, j, d, \mathbf{c}_i, \mathbf{c}_j)$$

defined on i, j, d and nonnegative state-contingent consumption vectors. This is the function that each pair — viewed as a social entity — seeks to maximize, and in doing so, would generate a bilateral transfer norm.

Transfer norms can also be derived from constrained welfare maximization. An example is Nash bargaining. Construct "disagreement points" by supposing that the link between i and j is no longer used. Then i's payoff will depend on some prediction of what will occur in the ambient network $d - ij$. Without entering into the details of how such a prediction is to be made (except to say that a recursive formulation would be natural), define $\underline{v}_i(d - ij)$ and $\underline{v}_j(d - ij)$ to be the expected utilities derived by i and j from this state of affairs. Then the transfer function $\mathbf{x}_{ij}$ is chosen to maximize the Nash product

$$\left[\mathbb{E}u\left(c_i(\theta)\right) - \underline{v}_i(d - ij)\right]\left[\mathbb{E}u\left(c_j(\theta)\right) - \underline{v}_j(d - ij)\right]$$

subject, of course, to the "participation constraint" that there exists some feasible utility vector that dominates $(\underline{v}_i(d - ij), \underline{v}_j(d - ij))$. If not, one supposes that no transfer takes place at all.

The resulting outcome, viewed as a suitable mapping, is a bilateral transfer norm. However, it will generally fail to be continuous (exhibiting a jump around the point where a feasible payoff vector that dominates outside options comes into being). Thus Proposition 1 below on the

[3] This has the obvious meaning: a multilateral norm creates, for every pair i and j and every state, particular values of z_i and z_j by simply adding up the prescribed transfers from (or to) all other individuals.

existence of consistent transfer schemes should not be asserted without qualification for the case of Nash bargaining.

Non-Welfarist Transfer Norms. Finally, bilateral transfer norms subsume a large class of sharing rules which are not easily amenable to a welfarist interpretation. For instance, suppose that each individual has full, unqualified access to some fraction ϵ of the resources at her disposal, and must only share the rest using, say, an even split. The resulting transfer norm would then look like this:

$$x_{ij}(\theta) = (1-\epsilon)\left[\frac{y_i(\theta)+z_i(\theta)+y_j+z_j(\theta)}{2} - (y_i(\theta)+z_i(\theta))\right].$$

This norm belongs to a wider family of norms which display what one might call a *private domain.* Bilateral norms belonging to this class leave individuals with a consumption in each state no lower than some positive fraction of their individual resources in that state. That is, there is $\epsilon \in (0,1)$ such that

$$-(1-\epsilon)e_i \leq \mathbf{x}_{ij} \leq (1-\epsilon)e_j$$

2.4. **Consistent Transfer Schemes.** We are now in a position to describe transfer schemes that satisfy a (society-wide) requirement of "consistency". Let g be a social network. Let $\mathbf{x}$ be a full collection of bilateral transfer schemes for every linked pair $k\ell \in g$; i.e., $\mathbf{x} = \{\mathbf{x}_{k\ell}(\theta)\}_{k\ell\in g}$. For a particular linked pair ij, this collection of bilateral schemes induces third-party transfers conditional on the realization of each state:

$$z_i(\theta) \equiv \sum_{k\neq j:ik\in g} x_{ik}(\theta) \text{ and } z_j(\theta) \equiv \sum_{k\neq i:jk\in g} x_{jk}(\theta) \qquad (2)$$

If the existing bilateral scheme for ij is indeed prescribed by the bilateral transfer norm given the collection $(\mathbf{z}_i, \mathbf{z}_j, \mathbf{y}_i, \mathbf{y}_j)$ (and g of course), and if this is simultaneously true for every linked pair, we will say that $\mathbf{x}$ is a *consistent transfer scheme.*

We formalize this in the following proposition.

PROPOSITION **1.** *Suppose that for every linked pair ij, the bilateral transfer norm is continuous in $\mathbf{z}_i$ and $\mathbf{z}_j$, and that, in any state, the prescribed transfers cannot exceed some exogenous upper bound (say, the total output produced in society in that state). Suppose, moreover, that the norm never prescribes positive transfers from an individual with non-positive consumption to another with positive consumption.*

Then a consistent transfer scheme exists, and exhibits positive consumption for every individual at every state.

Proof. Fix a network g. Denote by $M(\theta)$ the size of the bound in state θ. Let $\mathbf{X}$ be the set of all transfer schemes such that $x_{k\ell}(\theta) \in [-M_\theta, M_\theta]$ for every linked pair $k\ell \in g$. That is $\mathbf{X} \equiv \prod_{\theta\in\Theta}[-M_\theta, M_\theta]^{|g|}$ where $|g|$ is the number of pairs linked in g.

Given a transfer norm $\mathcal{X}$, define the following mapping $\phi_{\mathcal{X}}(\mathbf{x})$ for all $\mathbf{x} \in \mathbf{X}$. Pick $\mathbf{x} \in \mathbf{X}$. For every linked pair $ij \in g$ and θ, construct the agent's net transfer from others $(z_i(\theta), z_j(\theta))$ as in (2). Now for each linked pair $ij \in g$ and for any state θ, build $\mathbf{x}'_{ij}(\theta)$ by setting

$$\mathbf{x}'_{ij} = \mathcal{X}(i, j, d, \mathbf{y}_i, \mathbf{y}_j, \mathbf{z}_i, \mathbf{z}_j)$$

provided that the absolute value does not exceed $M(\theta)$. Otherwise, set $\mathbf{x}'_{ij}$ equal to $M(\theta)$ or to $-M(\theta)$ as the case may be. Let $\phi_{\mathcal{X}}(\mathbf{x}) = \mathbf{x}'$. Note that by construction $x'_{ij}(\theta) \in [-M_\theta, M_\theta]$ for all i,j and θ. So $\phi_{\mathcal{X}}$ is a mapping from $\mathbf{X}$ to $\mathbf{X}$. Moreover, it is a continuous map. Consequently, the existence of a fixed point $\mathbf{x}^*$, $\phi_{\mathcal{X}}(\mathbf{x}^*) = \mathbf{x}^*$ for some $x^* \in \mathbf{X}$ is guaranteed by Brouwer's fixed point theorem.

It remains to prove that any fixed point of $\phi_{\mathcal{X}}$ yields strictly positive consumption to all individuals in all states. Suppose this is false for some fixed point $\mathbf{x}^*$. We know that in any connected component d all consumptions cannot be negative, as aggregate income is positive. Hence, there exists a linked pair of agents $ij \in d$ and a state θ such that $c_i(\theta) \leq 0$ and $c_j(\theta) > 0$. By our assumption, it must be that i is making no transfer to j, and her output $y_i(\theta) > 0$. So she must be making a transfer to some other set of individuals, say K. By the same logic, $c_k(\theta) \leq 0$ for all $k \in K$. Repeating this argument for all individuals in K (and continuing if necessary), we see that ultimately we must encounter a pair of linked individuals — say ℓm — such that ℓ make sa positive transfer to m, $c_\ell(\theta) \leq 0$, and $c_m(\theta) > 0$. This is a contradiction. ■

Some remarks on this proposition are in order. In particular, the reader might wonder about the role of exogenously imposed bounds on transfers. What part do they play and why do we need them? To begin with, observe that for norms that are multilaterally coherent there isn't even a need for an elaborate fixed-point argument. Simply consider the allocation prescribed the multilateral transfer norm (with which all the bilateral norms are consistent). That state-contingent allocation will generate a consistent transfer scheme.

Notice that multilateral consistency is not at all a far-fetched presumption. After all, bilateral norms do not exist in a vacuum. As we have already mentioned, they may reflect some commonly held social values. In that case, multilateral consistency may be a natural property of the entire family of bilateral transfer norms.

In contrast, consider the following example. There are three agents connected to each other in a circle. Assume that players 1 and 2 have a social norm that involves giving player 2 2/3 of their joint endowment. Likewise, players 2 and 3 wish to give player 3 2/3 of their joint endowment, and a symmetric circle is completed by players 3 and 1. Obviously, a serious discrepancy exists here (in particular, the norms aren't multilaterally coherent). One might react to this by saying that there is no consistent transfer scheme, or by stating that — given the lack of "agreement" — each player simply consumes in the end her own endowment. The bounded-transfer model does the latter, creating an "artificial transfer" between each linked pair of players equal to the exogenous upper bound and thereby forcing each player to consume her income endowment. For the results we obtain, either interpretation — nonexistence or autarky — will do.

This discussion appears to suggest that once the family of bilateral transfer norms ceases to be multilaterally coherent, existence is jeopardized (or equivalently, with bounded transfers, the agents fall back on autarky). This is false. For instance, in the case of non-welfarist transfer norms with private domain, it is easy to check that there is always a "bonafide" equilibrium in which no exogenous restrictions on transfers need be imposed. All that is needed is the private domain restriction, while the remainder of the surplus can be divided in any proportion, including those failing to satisfy the requirement of multilateral consistency.

We end with a remark on uniqueness of consistent transfer schemes. In general this is not a property to be had free of charge. But consider the following additional restriction: bilateral norms between i and j depend on $\mathbf{z}_i$ and $\mathbf{z}_j$ only through their *sum* at each state, $z_i(\theta) + z_j(\theta)$ (and in addition may depend on i, j, $y_i(\theta)$, $y_j(\theta)$ and d as before). Moreover, assume that consumptions are *normal*, that is both consumptions in each state are increasing in this sum. Then there is a unique consistent transfer scheme in the sense of *outcomes*: consumption vectors are fully pinned down in every state. For if this were false, then there are two consistent schemes and a state θ such that the induced vectors of consumptions across individuals in that state are distinct. Refer to these vectors as $\{c_k(\theta)\}$ and $\{c'_k(\theta)\}$. Then in some state there must be some linked pair ij such that $c_i(\theta) \leq c'_i(\theta)$ and $c_j(\theta) > c'_j(\theta)$, but this violates normality.

Equal division and other welfarist norms will typically satisfy the above property and will generate unique consistent solutions (in consumption space). Other non-welfarist norms, such as the private domain norms discussed above, do not satisfy the additional restriction. However, different uniqueness arguments can be made for such norms using contraction arguments (we omit a more detailed discussion for the sake of brevity).

For every network, or more precisely, for every connected component d and for every consistent transfer scheme $\mathbf{x}$ associated with that component, denote by $w_i(d, \mathbf{x})$ the expected payoff accruing to member i.

We will follow a harmless convention. Notice that exactly the same (multilateral) outcomes can sometimes be achieved by the use of two different transfer schemes, each consistent. [The equal division norm, for instance, can be supported in a variety of ways.] We will assume that if, along some consistent transfer scheme and some state, only *one* person is a (net) giver and all others are (net) recipients, then the giver uses *all* her direct links to make transfers.

It is easy to see that for every consistent transfer scheme $\mathbf{x}$ lacking this property, there is another $\mathbf{x}'$ that yields exactly the same payoffs and *does* have this property. Indeed, assume that for a consistent transfer scheme $\mathbf{x}$ there is a state θ such that $\sum_\ell x_{i\ell}(\theta) < 0$ for some individual i, $\sum_\ell x_{k\ell}(\theta) > 0$ for all $k \neq i$ and $x_{ij}(\theta) \geq 0$ for some individual j such that $ij \in g$. Let W be the social norm under which $\mathbf{x}$ is consistent. Now, it is possible to construct a transfer scheme $\mathbf{x}'$ such that $\mathbf{x}'(\theta') = \mathbf{x}(\theta')$ for all $\theta' \neq \theta$, $x'_{ji}(\theta) = \sum_\ell x_{j\ell}(\theta)$ and $\sum_\ell x'_{k\ell}(\theta) = \sum_\ell x_{k\ell}(\theta)$ for all k. Hence, $\mathbf{x}$ and $\mathbf{x}'$ yield the same consumptions vectors in each state but in $\mathbf{x}'$ i makes a direct transfer to j of the net value of the transfers he received in $\mathbf{x}$. [Since i is the only individual making a net contribution in state θ, $\mathbf{x}'$ is constructed by finding a set P of indirect paths from i to j, $p \equiv i^0 = i, i^1, \ldots, i^m = j$ $(m > 1)$, along which the transfers are set such that $x'_{i^{n-i}i^n} - x_{i^{n-i}i^n} = c_p$ for all $n \in \{1, m\}$ and for some constant c_p (specific to each path), and $\sum_{p \in P} c_p$.] The transfer scheme $\mathbf{x}'$ is consistent. Assume not, then there exist two individuals k and ℓ, and transfers $\mathbf{x}''_{k\ell}$ such that $W(k, \ell, d, \mathbf{c}''_k, \mathbf{c}''_\ell) > W(k, \ell, d, \mathbf{c}'_k, \mathbf{c}'_\ell)$ where $\mathbf{c}'$ are generated by $\mathbf{x}'$ and $\mathbf{c}''$ are generated by $\mathbf{x}''$ ($\mathbf{x}'$ where $\mathbf{x}''_{k\ell}$ replaces $\mathbf{x}'_{k\ell}$). But then changing $\mathbf{x}_{k\ell}$ in the initial allocation by $\mathbf{x}''_{k\ell} - \mathbf{x}'_{k\ell}$ should improve $W(k, \ell, d, \mathbf{c}_k, \mathbf{c}_\ell)$ and therefore this contradicts the initial optimality of $\mathbf{x}_{k\ell}$.

2.5. **Monotone Norms.** Say that a transfer norm is "monotone" if whenever more individuals are brought into a connected network by being connected to one individual, *this* individual's payoff increase. Intuitively, more individuals create better insurance possibilities, and monotone transfer norm should give some of the extra benefits to the individual serving as a "bridge".

There are two difficulties with defining monotonicity in the context of networks of bilateral insurance. First, it is problematic to make the comparison across two networks in which the second, say, has more individuals but fewer links. This can be easily dealt with by using a less restrictive definition which applies only when one network is a subnetwork of the other (in addition to having less individuals). Formally, suppose that g and g', with $g \subseteq g'$, are two connected components such that $N(g) \subset N(g')$. Suppose, moreover, that $jk \in g'$ only if $jk \in g$ for all $j, k \neq i$. Then, say that the transfer norm is *monotone* if for every pair of consistent transfer schemes $\mathbf{x}$ (for g) and $\mathbf{x}'$ (for g'), $w_i(g', \mathbf{x}') > w_i(g, \mathbf{x})$.

The second difficulty is more conceptual than logical. We are concerned with *bilateral* schemes, but the definitions of monotonicity we have in mind are emphatically multilateral. In short, we are requiring that an *equilibrium* of bilateral schemes — captured by the notion of consistency — exhibit a desirable social property. Therefore, monotonicity embodies more than a purely normative definition; it requires in addition that the equilibrium itself does not "misbehave" as we move across networks.[4]

It is easily seen that the equal division norm is monotone. Whether or not other norms satisfy this property needs to be more thoroughly investigated. Moreover we conjecture that the Nash Bargaining extension described earlier is monotone.

3. Enforcement Constraints and Stability

While a transfer norm, as defined by us, comes from a fairly general class, it is time to emphasize a particular feature (already discussed in the Introduction). These norms are primarily "normative" in that they take little or no account of self-enforcement constraints. But this isn't to say that such constraints do not exist. Each individual may recognize that as a social being she is constrained to abide by the transfer norm in her dealings with j, *provided that she wants to maintain those dealings*. But she may not want to maintain them. It may be that (in some states) the transfer she is called upon to make outweighs the future benefits of maintaining a relationship with j under the transfer norm. If that is the case, something must give, either the norm or the ij link. Our paper takes the point of view that the norm is more durable than the link, and that the link will ultimately fail.

[Generally speaking then, should we conceive of norms as restricted or unrestricted by incentives? This is an important open question that we do not pretend to address in any satisfactory way. Norms may range all the way from the fully idealistic (purely derived from ethical considerations, such as equal-sharing) to the purely pragmatic (wary of all enforcement and participation constraints, with ethical matters only invoked subject to the limits posed by such constraints). In this paper, we take the point of view that norms are not constrained by incentives, but of course we do use such incentive constraints to see if the resulting transfer schemes will or will not survive.]

In a network setting, an agent could choose to renege on some (or all) transfers that she is required to make under a particular bilateral norm. We take such deviance to be tantamount to the permanent severance of the relevant bilateral links. That is, in all that follows we suppose that a subgame equilibrium path is adopted in which the initially mistreated links are permanently cut thereafter. Of course, there may be equilibria in which badly treated partners may resume dealings with the deviant, but in line with the bulk of the literature on repeated-game-theoretic models of informal insurance (see, e.g., Coate and Ravallion (1993), Fafchamps (1996), Ligon, Thomas and Worrall (2001) and Genicot and Ray (2002)) we do not consider such subgame strategies. Individuals who are the direct victims of a deviation are presumed to impose sanctions on the deviant thereafter by not interacting with them.

This much may be clear, but nevertheless the extent of the punishment imposed on a deviant remains ambiguous. What about the rest of society, who were not directly harmed by the deviant? Do they, too, sever links with the deviant?

The answer to this question depends in part on what we are willing to assume about the extent of information flow in the society. In turn, this forces us to ask the question of just what the network links precisely mean. They certainly limit physical transfers, but do they also limit the

[4]One possible source of "misbehavior" is nonuniqueness of consistent schemes for a given network. While this in itself is by no means inconsistent with monotonicity, it makes the concept less intuitive.

flow of information? One possible interpretation is that the network represents a set of physical conduits and physical conduits alone, while information flows freely across all participants and is not constrained in any way by the architecture of the network. In this case the following notion of a punishment may be appropriate:

STRONG PUNISHMENT. Following a deviation, every agent severs its direct link (if any) with the deviant, so that the deviant is thereafter left in autarky.

In models of informal insurance in groups with self-enforcement constraints, this is the commonly adopted punishment structure. But in such scenarios, there are no networks, insurance is fully multilateral, and the event of a deviation is common knowledge among the group as a whole. In a situation in which network effects are under explicit consideration, the opposite presumption may seem more natural:

WEAK PUNISHMENT. Following a deviation, *only* those agents who have been directly mistreated by the deviant sever their links (with the deviant).

In our view, this concept is far more appropriate to the case at hand than strong punishment. In the model we study, insurance is bilateral, and linked agent pairs know very little about the particulars of other dealings (except the aggregate of transfers made to or received from third parties).[5] So it is entirely consistent to impose the restriction that while directly injured parties react, other agents do not, while strong punishments are more appropriate to a multilateral situation in which there are no restrictions on information flows and no network effects.

At the same time, if we take the network structure seriously, not just as a routeway for physical transfers but also for the flow of information, then we can define "intermediate" layers of punishment that are worth investigation in their own right. For instance, if I am an injured party and can communicate with those I am directly linked to, I can tell them about my experience. One might then adopt the equilibrium selection rule that all the individuals I talk to sever direct links (if any) with the deviant.

To be sure, once this door is opened, we might entertain notions in which the news of an individual's mistreatment "radiates outwards" over paths of length that exceed a single link, and all those who hear about the news breaks off direct links (if any) with the original deviant. There are many ways to model such a scenario: we take the simplest route by indexing such punishments by the length of the required path.

LEVEL-q PUNISHMENTS. Following a deviation, all agents who are connected to a victim by a path not exceeding length q (but not via the deviant) sever direct links (if any) with the deviant.

In this definition, q is to be viewed as a nonnegative integer, so that weak punishment may be thought of as a special case in which $q = 0$. In this sense, level-q punishments are quite general.[6]

With the punishment structure in place, we may define q-stable networks. (Sometimes, when there is no danger of ambiguity, we shall simply use the term "stable" instead of q-stable.) We proceed recursively on graphs. For the empty graph $g = \emptyset$, define

$$v_i^*(\emptyset) \equiv \sum_{\theta} p(\theta) u(y_i(\theta))$$

[5]However, for our model to have proper game-theoretic underpinnings — the formalities of which we do not explore here — it will be necessary to assume that the network itself is commonly known.

[6]Of course, one might conceive of still more general punishment structures in which verifiable information decays — perhaps probabilistically — as it radiates along a path, but we avoid these for the sake of simplicity.

to be the expected lifetime utility (normalized) of an agent living in autarky. Such a graph is obviously stable, as no transfers are called for and there is no question of a deviation. And there is only one payoff vector associated with that stable graph: $\mathbf{v}^*(\emptyset) \equiv \{v_i^*(\emptyset)\}$.

Proceeding recursively, consider a network g and suppose that the set of stable subnetworks of g, $\Gamma(g)$, has been defined, along with collections of stable payoff vectors for each of those subnetworks $\mathbf{v}^*(g)$. Now consider g, and pick some consistent transfer scheme $\mathbf{x}$, with attendant payoff vector $\mathbf{v}$. Consider any individual i. For any realization θ, by abiding to the ongoing transfer scheme, i obtains a lifetime (normalized) expected payoff of

$$(1-\delta)u\left(y_i(\theta)+\sum_j x_{ij}(\theta)\right)+\delta v_i.$$

To describe the payoffs following a deviation, we must first adopt a convention that tells us the payoffs that accrue to player i when she finds herself at some network $g' \subset g$. If g' is stable, it is to be expected that i will enjoy a payoff of v_i', where this is the ith component of some stable payoff vector for g'. If g' is not stable, the resulting payoff will be presumably drawn from some stable subnetwork of g' itself. Two often-used devices to pin down the precise outcome in the face of potential multiplicity are "optimistic" and "pessimistic" beliefs (see, e.g., Greenberg (1990)). [7] We assume that if g' is not stable then a subnetwork g'' would form where g'' is the or one of the largest stable subnetworks of g' to which i belongs. In this case, v_i' is the ith component of some stable payoff vector for g''. We do not insist on any particular selection rule at this conceptual stage, but we must take note of the "baseline" graph that player i induces on her deviation. This depends on two things: the set of players who are her "victims", and the value of q that determines the punishment level.

If i deviates by not honoring commitments to a set of neighbors S, a level-q punishment will set the new graph to g', which is obtained by removing from g all direct links to i that are from individuals who are no more than q steps distant from some member of S (but the connecting path should include the original deviant). Thus, the continuation payoff depends on two things: the set of players who are her "victims", and the value of q that determines the punishment level. Formally, the payoff to i is then

$$(1-\delta)u\left(y_i(\theta)+\sum_{j\notin S} x_{ij}(\theta)\right)+\delta v_i(g').$$

where $v_i(g')$ is found by applying the selection criterion as described above.

We may therefore say that a transfer scheme $\mathbf{x}$ is *q-stable* if it is consistent, and if for every player i, every state θ and every set of direct neighbors S of i,

$$(1-\delta)u\left(y_i(\theta)+\sum_{j\notin S\}} x_{ij}(\theta)\right)+\delta v_i(g') \le (1-\delta)u\left(y_i(\theta)+\sum_j x_{ij}(\theta)\right)+\delta v_i. \tag{3}$$

Finally, say that the network g is *q-stable* if it admits a q-stable transfer scheme.

[7] For instance, in the former case, v_i' would be the maximum value of v_i drawn from all stable payoff vectors drawn from any (stable) subgraph of g'.

4. Stable Networks

In this section we attempt to describe the set of q-stable networks. [To be sure, this is a very different concept of stability than Jackson (2001, 2004) and Baya and Goyal (2000)]. Throughout, we concentrate on the case in which discount factors are "sufficiently" close to unity. As discussed in the introduction, two effects that pertain to social networks are critical for their stability: the *transit* effect, a short-term effect, and the *punishment* capacity of networks, a long term effect. Looking at high values of the discount rate allows us to focus on the long-term aspect, that is the punishment values of networks. The transit or "bottleneck" effect will be examined in the next section.

4.1. **Universal Stability.**

Notice that our model accommodates an enormous variety of transfer norms. One difficult and possibly not very insightful project would be to try and classify the set of stable networks for each social norm. It turns out that the question becomes more tractable (and the answer possibly interesting) when we study networks that are "universally stable" in the sense that they are stable over a *class* of transfer norms. Later, we shall describe the class we have in mind.

Formally, fix some hierarchy q of information flow and let $\mathcal{X}$ be any collection of transfer norms. Say that a network g is *universally q-stable for the class $\mathcal{X}$ of norms* if for every norm $\mathbf{x}$ in that class, there exists a threshold discount factor $\bar{\delta} \in (0, 1)$ such that for all $\delta \in (\bar{\delta}, 1)$, g is q-stable.

Thus a universally stable network is simultaneously robust over a full variety of transfer norms. We don't have to know exactly what the norm is. In this sense it may be an appealing concept.[8]

4.2. **Sparse Networks.**

It this section, we formalize a certain notion of a "sparsely connected" network. The following definition is central: for any integer $q \geq 0$, say that a graph g is *q-sparse* if for every linked pair $ij \in g$, the graph formed by removing from g the links to i along all paths of size $m \leq q + 1$ between i and j has strictly more components than g.

Observe that all networks which *only* have trees as their components are 0-sparse (and that this fully describes the set of 0-sparse networks). Note too that if a graph is q-sparse it is q'-sparse for all $q' \geq q$ (more links can be broken under q' than under q). Finally, if g is q-sparse then any component of it must be q-sparse as well.

We will see that sparseness and universal stability are closely connected for a certain class of transfer norms.

4.3. **Universal Stability for Monotone Transfer Norms.**

Proposition **2.** *A network g is universally q-stable under the class of all monotone transfer norms if and only if it is q-sparse.*

Proof. *Sufficiency.* Let $\mathbf{x}$ be any monotone transfer norm. Let g be q-sparse. Pick any player i. Let d be the connected component of g she belongs to. Let v_i be her payoff under some consistent transfer scheme induced by $\mathbf{x}$ for d. By monotonicity, there exists $\epsilon > 0$ such that for

[8] Notice, however, that in our definition of universal stability the threshold discount factor is permitted to depend on the particular member chosen from the class. If the class is finite this is not a problem.

any subcomponent d' of d (to which i also belongs), her payoff $v_i(d)$ under any consistent transfer scheme satisfies $v_i \geq v_i(d') + \epsilon$.

Because g is q-sparse, so is the component d. It follows that *any* deviation by i, followed by a level-q punishment, will land i in some strictly smaller component. [This is because the punishment effectively removes all links to i on paths between i and j of length $q+1$ or less, and so must break d up into smaller components.] Hence, our definition of ϵ ensures that no matter what selection rule is used to forecast continuation payoffs v_i' thereafter, our definition of ϵ ensures that

$$v_i - v_i' \geq \epsilon.$$

Define M to be maximum aggregate output and $m = min_{\{i,\theta\}} c_i(\theta)$ to be the minimum individual consumption [$m > 0$ under any consistent transfer scheme]. For any state θ, we see that for δ close enough to 1,

$$\delta[v_i - v_i'] \geq \epsilon/2 \geq (1-\delta)[u(M) - u(m)] \geq (1-\delta)[u\left(y_i(\theta) + \sum_{j \notin S} x_{ij}(\theta)\right) - u\left(y_i(\theta) + \sum_{j} x_{ij}(\theta)\right)],$$

which means that (3) is satisfied. Hence g is universally q-stable for the class of all monotone transfer norms.

Necessity. Consider the equal division norm 1, where

$$x_{ij}(\theta) = \frac{e_j(\theta) - e_i(\theta)}{2}$$

This norm equalizes consumption across every pair of linked individuals in every state. Therefore in any connected component, consumption must be *fully* equalized across all individuals in every state. This has two implications:

(a) An individual obtains the same payoff in any two connected networks g and g' as long as $N(g) = N(g')$. [The equal division of *total* output is completely unaffected as long as both graphs are connected.]

(b) The equal-division norm is monotone. [As long as all individuals are symmetric, and outputs are imperfectly correlated but divided equally, adding more individuals increases the welfare of each individual.]

Now suppose that g is universally q-stable for the class of all monotone transfer norms. Then, by part (b) above, g must be q-stable for the equal-division norm. Now suppose, contrary to our assertion, that g is not q-sparse. Then there is some component d of g (possibly g itself, of course) and some linked pair ij such that even if all links to i along paths of length $q+1$ or less are removed between i and j, the resulting graph d' is connected as well.

Consider the situation in which i has received the highest income realization while all other individuals received the lowest income (this event has positive probability). Using the convention that all links are used in this state, $x_{ij} < 0$. We look at i's incentive not to renege on x_{ij} and the resulting network g'. Following such deviation, i would end up in d' if d' is stable. If d' is not stable then the resulting network $d'' \subset d'$ is connected. This is because when a network is unstable the or one of the largest stable subnetworks form and there exists a subnetwork of the same size as d since, using part [1] of the proof, we know that a tree (0-sparse) of that size is stable. Finally, by (a), d', d'' and d yield the same vector of payoffs such that $v_i(g') = v_i(g)$ for any payoff selection rule.

g	minimal sparsity	1-stable
a.	0	$\surd$
b.	6	$\times$
c.	2	$\times$
d.	2	$\times$
e.	1	$\surd$

Table 1. Sparsity and Stability.

Hence, i's incentive constraint (3) not to renege on x_{ij},

$$(1-\delta)u\left(y_i(\theta)+\sum_{k\neq j}x_{ik}(\theta)\right)+\delta v_i(g')\leq(1-\delta)u\left(y_i(\theta)+\sum_{j}x_{ij}(\theta)\right)+\delta v_i,$$

is clearly violated and this completes the proof of necessity. ■

5. Discussion

5.1. **Strong and Weak Punishment.**

It is particularly interesting to contrast the universal stability of networks across different punishment schemes. For values of δ close to 1, all network are stable under strong punishment. In contrast, for weaker punishment schemes, the density of links weakens punishments and has important consequences for network stability. In particular, under weak punishment only trees are stable for high discount rates.

Corollary **1.** *Under weak punishment, a network g is universally stable for all monotone transfer norms if and only if it has only trees as components. While under strong punishment, all networks are universally stable for all monotone transfer norms.*

Proof. Networks that have only trees as components are the only network that are 0-sparse. Since stability under weak punishment corresponds to q-stability for $q=0$, the proof of the fist part of the corollary is a direct application of Proposition 2.

Take any network g. There exists $\overline{q}$ such that g is q-sparse for $q\geq\overline{q}$. It follows from Proposition 2 that g is universally q-stable under monotone transfer norms for $q\geq\overline{q}$. That is, the inequality (3) where $v_i(g')$ is determined by level q punishment for $q\geq\overline{q}$ holds for every player i, every state θ and every set of direct neighbors S of i. Under strong punishment, any individual receives a payoff $v_i^*(\emptyset)$ following a deviation. To be sure, $v_i(g')\geq v_i^*(\emptyset)$ for all i and g'. Hence, (3) holds under strong punishment too and g is stable. ■

5.2. **Density and q-Stability.**

It is important to realize that the *sparsity* and the *density* of a network are two very different concepts. While 0-sparse networks are just trees (or more precisely, graphs which only have trees as components), q-sparse networks for $q\geq 1$ are more complicated. This is illustrated in Table 1 and Figure 1. The second column in Table 1 shows the lowest sparsity of the networks represented in Figure 1. For instance, the complete network is always q-sparse when $q\geq 1$, but a circle of size n is q-sparse only for $q\geq n-1$.

Consider monotone transfer norms. We saw that for weak punishment, only networks composed of trees are universally stable. Now the question is: what happens when we increase q? As the level of punishment rises above the weak level, for instance to $q=1$, what happens to universal stability? It follows from Proposition 2 that very well-connected graphs as well as tree-like

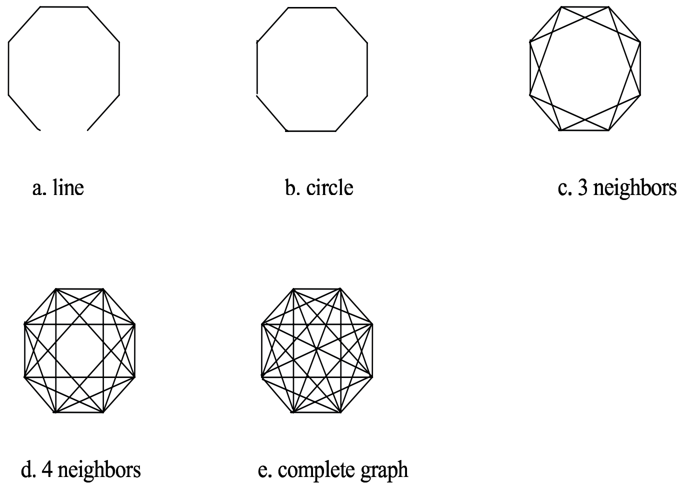

FIGURE 1.

graphs stay stable; but intermediate graphs become unstable. As the final row of Table 1 shows, only the line and the complete graph are stable.

Finally, what happens as q rises? Using the minimum sparsity reported in Table 1, we see that as q reaches 2 the three and four neighbor networks become universally stable, while the circle needs a level of punishment of at least 7 to be stable.

We can use simulations to show that the U-shaped relationship between density and universal q-stability for intermediate q is quite general. Consider a community of ten individuals. We generate random networks of size 10 for each possible number of links in this community and assess the networks' universal q-stability for different values of q. Figure 2 maps the proportion of universally q-stable networks on network density. Each curves correspond to a different level of punishment q ranging from 0 to 9. Naturally, the stronger the punishment the larger the proportion of stable graph, such that higher curves correspond to larger q's. To be sure, networks of intermediate density tend to be unstable.

Consider a densely connected network. Our result suggests that if, following some exogenous shocks, some links are broken, it could destabilize the network and create an unraveling of the existing relationships, resulting in a parsely connected graph.

5.3. **Other Punishment Schemes.** Our q-level punishment captures two important features of communication within networks: it is generally limited and it depends on the graph itself.

One could imagine pushing these ideas further and have a punishment consistent the following *communication protocol*: information is transmitted only via individuals who know the person concerned. The idea is that before relaying to a person information on x, one would first ask to the person if she knows x. If she does, then the information is transmitted but otherwise not. In our model, knowing the person is taken to mean having a link with the person. We can define a new punishment structure using this particular communication protocol.

INFORMATION SHARING. Information on a deviation is transmitted from an agent to another if and only if they are linked to each other and to the deviant. Following a deviation, all informed agents sever direct links with the deviant.

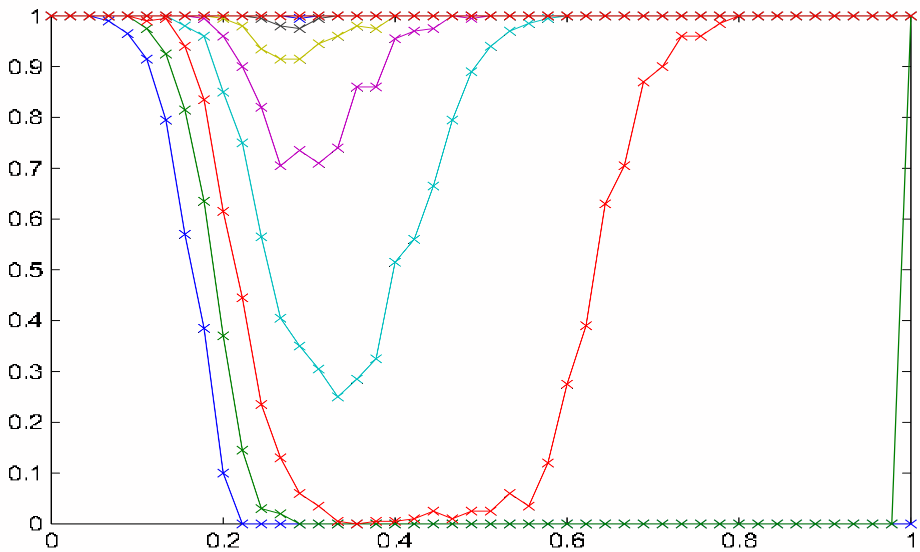

FIGURE 2. STABILITY & DENSITY

Formally, say that i is *strongly connected* to k via j if there exists a sequence of individuals $\{i^0 = j, i^1, ...i^n\}$ such that $i^m i^{m+1} \in g$ for all $m = \{1, ...n-1\}$ and $i^{mk} \in g$ for all $m = 1, ...n$.

Using arguments similar to Proposition 2, it is is easy to show that a network g is *universally stable under information sharing* for the class of all monotone transfer norms if and only if for every linked pair $ij \in g$, the graph formed by removing from g the links between i and all individuals strongly connected to i via j has strictly more components than g.

This punishment structure is interesting as it is related to the *clustering coefficient* defined in the social network literature on "small world" (see Wasserman and Faust (1994), and Watts and Strogatz (1998)). Recall that $\aleph_i$ is the neighbor set of i (the set of individuals with direct links to i). The *clustering coefficient* for i in g, given that $|\aleph_i| \geq 2$, is the actual number of links among $\aleph_i(i)$ divided by the maximum number of links among i's neighbor set $|\aleph_i| * |(\aleph_i - 1)|/2$. Intuitively, highly clustered networks are more likely to be stable under our information sharing protocol. Note that, in a different model, Vega-Redondo (2003) uses numerical simulations to examine the importance of the architecture of the network: its density and cohesiveness, to transmit information on deviant in repeated Prisoner's Dilemma games.

We illustrate this using a specific example and simulations. Figure 3 pictures three different graphs with 8 individuals having 3 links each. The clustering coefficient of each individual is indicated next to her node. In the first network, the *wheel*, each individual has a clustering coefficient of 0. It follows that, for high values of δ, this graph won't be stable. Next, consider the *neighbor* network. If individual i in the Figure defects on a transfer to individual j, where both have clustering coefficients of $2/3$, then our information protocol would isolate and effectively punish i. In contrast, if it is individual k (whose clustering coefficient is only $1/3$) who defects on i, we see that our information protocol will be unable to punish him. Hence, graph b. is also unstable. Finally, look at the *cliques*. Here individual are clustered in two cliques of four individuals. Their clustering coefficient is 1 and therefore it is possible to fully isolate any deviant. Hence, the graph is universally stable for monotone norms.

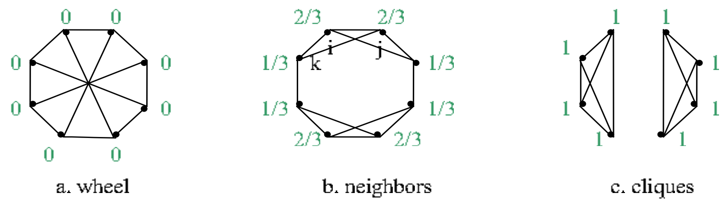

FIGURE 3.

Using simulations, we can simulate a large number of network of size 10 for each possible density level, and plot their stability under our information protocol on their clustering coefficient. Figure 4 reports the results. The crosses represent the proportion of universally stable graph under our information protocol for different clustering coefficients, while the asterisks are the proportion of universally q-stable for $q = 2$. Except for the empty graph, the relationship between stability and clustering is clearly positive, especially under our information protocol.

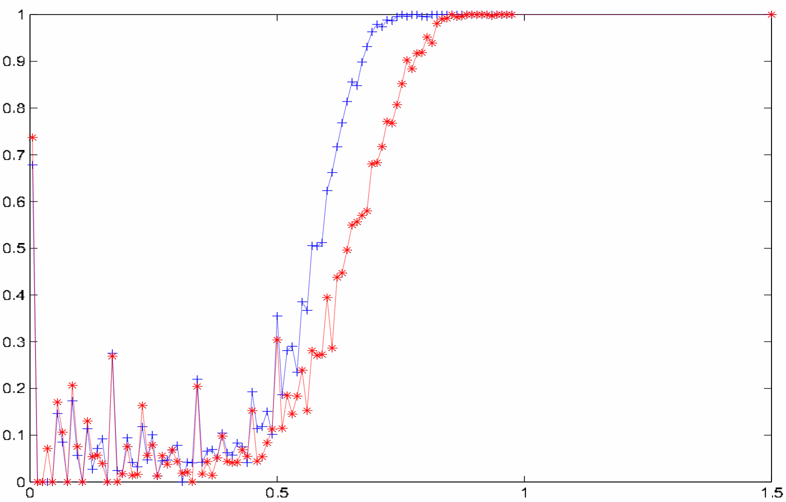

FIGURE 4. STABILITY & CLUSTERING

6. Threshold Discount Factors under Equal Sharing

To study the threshold discount factors at which some insurance network are stable, we need to precise the norm that we are studying. In the remaining of the paper we will study the stability of risk-sharing network that do follow an equal-sharing norm.

6.1. **Stability with Strong Punishment.** An important concept for the study of stability of network is the *bottleneck* agent.

Bottleneck Agents. In a component d of size m, for a state θ with income realization $\mathbf{y}$,

$$y_i + \sum_j x_{ij}(\theta) = \sum_i y_i/m$$

for all $i \in d$, such that $v_i^*(g)$ can easily be computed, by summing over all states. Since consumption is equalized across all agents in a component, $v_i(g)$ is the same for all agents i belonging to the same component d.

Under strong punishment, if i deviates on a subset of agents S his continuation value $v_i(g') = v_i(\emptyset)$ independently of the graph g and of the subset S. Hence, if she were to deviate an agent i would renege on all agents to whom she is making a positive net transfer.

It follows that for a given income realization, the only element differentiating incentives of different agents in a component is the maximal benefit that an agent can obtain by reneging on his current transfers,

$$u\left(y_i + \sum_{j|x_{ij}>0} x_{ij}(\theta)\right) - u\left(\sum_j y_j/m\right) \tag{4}$$

In order to check the stability of a graph, we thus need to check, component by component, and for every vector of income realization, the incentives of the agent(s) for which (4) is maximal. We call this agent or these agents the *bottleneck(s)* of the transfer scheme. If *all* agents face the same maximal short-term incentive to deviate (4) in at least one state, we say that there is no bottleneck agent.

In order to gain intuition about bottleneck agents, consider the simplest case in which the endowments are i.i.d. across agents and take on just two values: h with probability p and ℓ with probability $(1-p)$, with $h > \ell > 0$. Figure 5 and Figure 6 illustrate the concept of bottleneck agent for two different trees – a star and a line with four agents. k denotes the number of highs in the state realization. For values of $k \in \{1, ...3\}$, Figure 5 and 6 illustrate the state realization and underline the agent for which (4) is maximal.

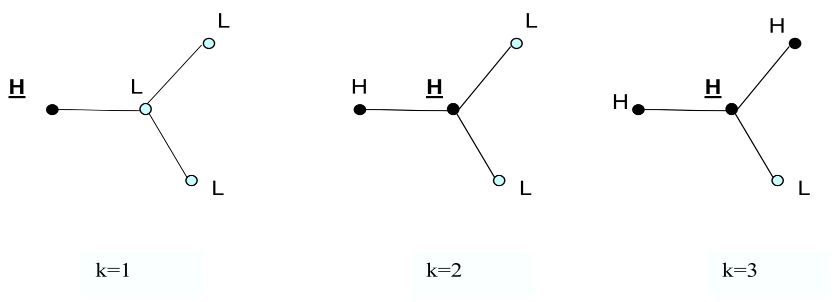

Figure 5. Bottleneck Agent in the Star.

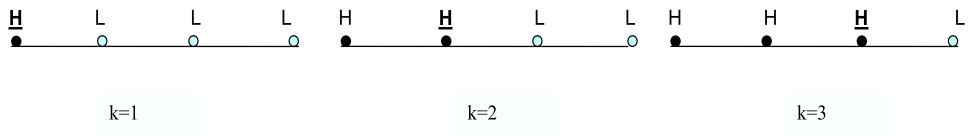

Figure 6. Bottleneck Agent in the Line.

In the star, not surprisingly, when the number of good states is greater than two, the bottleneck is the center of the star, which is called to make the maximal transfers. Similarly, in the line, the bottleneck agent is always one of the two central agents. What is more surprising is that the constraints faced by the bottleneck agents is the same in the two graphs. For any k good realizations, the bottleneck agent receives transfer from $k-1$ agents both in the star and the line, and her maximal deviation payoff is given by

$$h + \frac{(k-1)(n-k)(h-l)}{n}$$

in both graphs.

The next result shows that this is not an accident: for a large class of networks, the stability conditions are identical.

Decomposable Network. We define a *subnetwork* originating at i as a subgraph g' of a network g satisfying the following conditions:

(1) $i \in N(g')$
(2) For any $j, k \neq i$. If $jk \in g$ and $j \in N(g')$ then $k \in N(g')$

A node i is said to be *critical of degree* k if there exists a subnetwork of size k originating at i.

A connected graph g of size m is *decomposable* if for any $k = 1, ..., m-1$, there exists a critical node i of degree k.

To be sure, with strong punishment a higher discount rate helps stability. Hence, if a network is stable for a discount rate δ then it is stable for all $\delta' \geq \delta$. Let $\delta(g)$ (or $\delta(d)$) denote the minimal value of the discount factor for which graph g (or component d) is stable.

Proposition **3.** *For any two decomposable connected networks g and g' of size n, $\delta(g) = \delta(g')$.*

Proof. Consider two networks g and g' of same size n. Take a realization $\mathbf{y}$ for these networks. Denote as $\Pi(\mathbf{y})$ the set of permutations of $\mathbf{y}$. Call an individual a *giver* if $y_i \geq \sum_j y_j/n$ and a *recipient* otherwise. Let $\gamma(\mathbf{y})$ be the number of givers and $R(\mathbf{y})$ the set of recipients in $\mathbf{y}$. Let i be a critical node of degree γ. We claim that among the income realizations in $\Pi(\mathbf{y})$, the tightest constraints is obtained for node i when all agents in the subnetwork of size γ originating at i are givers. For all income realization $\mathbf{y}' \in \Pi(\mathbf{y})$ the average income is the same, such that (4) is maximal for i if it maximizes

$$y'_i + \sum_{j|x_{ij} \geq 0} x_{ij}(\mathbf{y}') \tag{5}$$

To equalize the incomes a total transfer of $\tau(\mathbf{y}) \equiv \sum_{j \in R(\mathbf{y})}[\sum_l y_l/n - y_j]$ needs to be made from givers to recipients. Since i is critical of degree γ, when all agents in the subnetwork of size γ originating in i are givers, all transfers to be made need to transit via her. In this case, (5) clearly takes its maximal value $\sum_l y_l/n + \tau(\mathbf{y})$ irrespective of i's particular income.

[The only other candidate would be a recipient j who receives exactly τ in transfers. But then (5) would be lower since $y_j < \sum_l y_l/n$.]

Hence, for any realization in $\Pi(\mathbf{y})$, the tightest constraint will be the same in any decomposable network. To compute δ, one needs to find the state $\mathbf{y}$ which results in the tightest constraint, i.e. that solves the problem:

$$\max_{\mathbf{y}} \; u\left(\sum_l y_l/n + \tau(\mathbf{y})\right) - u\left(\sum_l y_l/n\right). \tag{6}$$

The solution $\mathbf{y}^*$ of this problem depends on the shape of the utility function. Once $\mathbf{y}^*$ is known, the threshold value $\delta(g)$ can easily be computed. ∎

Moreover, the following Proposition shows that the threshold discount rate is higher for decomposable networks.

PROPOSITION **4.** *Suppose that g and g' are two connected networks of same size. If g is decomposable while g' is not, then $\delta(g') \leq \delta(g)$.*

Proof. Let $\mathbf{y}^*$ be the solution to the maximization problem (6). If the network g' has a critical node of degree $\gamma(\mathbf{y}^*)$ then clearly $\delta(g) = \delta(g')$. Otherwise, if g' does not have a critical node of degree $\gamma(\mathbf{y}^*)$, the consumption can be equalized in a way that the highest consumption a deviating agent could achieve when $\mathbf{y}^*$ is realized is strictly less than $\sum_l y_l/n + \tau(\mathbf{y})$. It follows that $\delta(g') < \delta(g)$. ∎

PROPOSITION **5.** *Consider two graphs g and g' with the same components and $g \subset g'$. Then $\delta(g) \geq \delta(g')$.*

Proof. Consider the binding constraint in graph g. Let θ be the corresponding realization of the shock, and i the agent whose constraint is binding. Compute a transfer scheme for graph g' as follows. For any $\theta' \neq \theta$, $\mathbf{x}(\theta')$ is the same for the two graphs. For θ, one may be able to relax the transfer constraint, by using additional links. In other words, if new links permit a rerouting of the transfers to bypass i, i's constraint will necessarily be relaxed. ∎

While very simple, Proposition 5 illustrates a very powerful fact. Under strong punishment, the addition of new links will always result in more stable graphs, as new links relax the constraint of the bottleneck agent. As a consequence, for any connected graph g, $\delta(g) \geq \delta(g^c)$ where g^c is the complete graph. Examples can easily be provided to show that this inequality may be strict.

Note that this remark does not show that the complete graph is the easiest graph to sustain. The inequality is only true among *connected* graphs and disconnected graphs, with smaller components, may be easier to sustain. In fact, there is no obvious reason why the enforcement constrained faced by community members should be monotonic in the number of agents in a component. To illustrate this point consider the simple case, already used earlier, in which the endowments are i.i.d. across agents and take on just two values: h with probability p and ℓ with probability $(1-p)$, with $h > \ell > 0$. The following example computes the threshold value $\delta(g)$ for complete graphs as a function of the size of the graph. It appears that the threshold value is *non monotonic* in the size of the graph.

EXAMPLE **1.** *Take utilities to be $u(c) = 2c^{1/2}$, $\ell = 0$, $h = 1$ and $p = 0.2$.*

In a complete graph, no transfer is ever mediated. This implies that the binding constraint for an agent is whether to keep her income when it receives a high shock or not. In other words, the maximal deviation is given by

$$u(h) - u\left(\frac{kh + (n-k)l}{n}\right)$$

This expression is clearly maximized when $k = 1$ *so the constraint becomes*

$$(1-\delta)u(h) + \delta v^*(1) \leq (1-\delta)u(\frac{h+(n-1)l}{n}) + \delta v^*(n)$$

where

$$v^*(n) = \sum_{k=0}^{n} \left(\begin{array}{c} n \\ m \end{array} \right) p^k (1-p)^{n-k} u(\frac{kh+(n-k)l}{n}).$$

We compute

$$\delta(n) \equiv \frac{1-(1/n)^{1/2}}{1-(1/n)^{1/2} - 0.2 + \sum_{k=0}^{n} \left(\begin{array}{c} n \\ k \end{array} \right) 0.2^k 0.8^{n-k} (k/n)^{1/2}}$$

Figure 7 pictures δ *as a function of* n *for* $n \in \{1, ..., 100\}$.

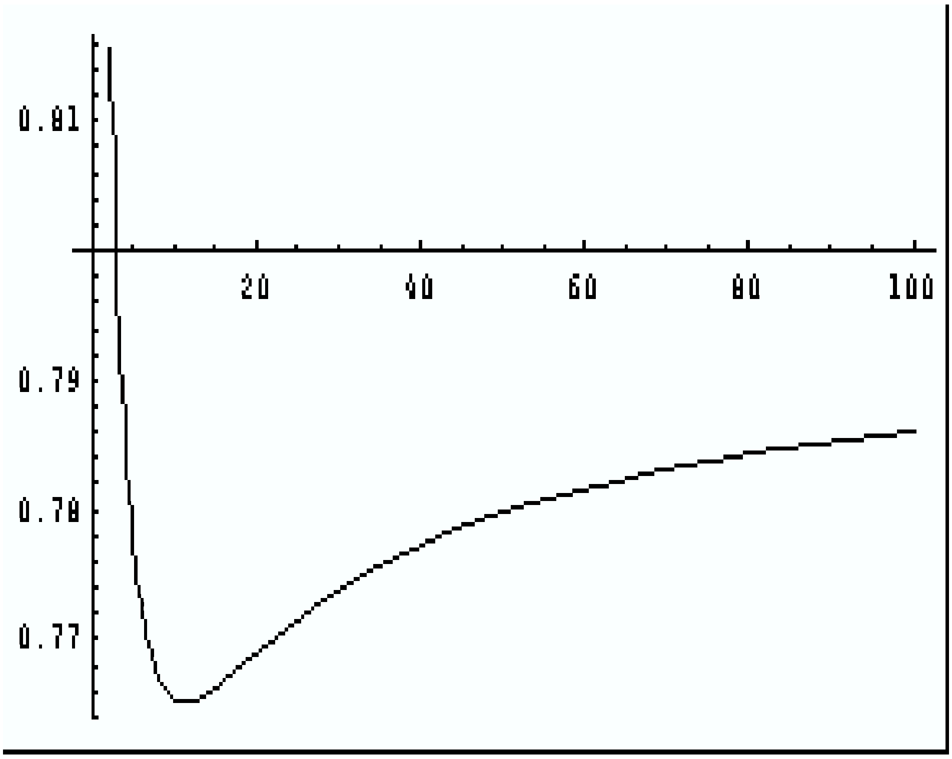

Figure 7. An Illustration of Example 1.

6.2. **Stability with Weak Punishment.** Under weak punishment, the analysis of stability of networks can no longer be done in terms of threshold discount factor. Indeed, a higher discount rate could, by helping the stability of subnetworks, increase the incentive of some individual to deviate in a network, thereby hurting stability. Hence, we now need to talk about *sets* of discount factor. Let $\Delta(g)$ (or $\Delta(d)$) denote the set of values of the discount factor for which graph g (or component d) is stable.

The temptations to deviate are clearly larger under weaker punishment, and insurance networks are harder to sustain. Our first result shows that again there is a family of networks (including stars and lines) for which the tightest incentive constraints are identical. As the argument relies on a recursive computation of stable networks, the property needed to show this equivalence is not decomposability but *full decomposability* – every subgraph of the graph must be decomposable.

PROPOSITION **6.** *Consider two fully decomposable networks g and g' connecting the same number of players, then $\Delta(g) = \Delta(g')$.*

Proof. The proof is by induction on the size of the network, denoted n. If $n = 2$, the statement is clearly true, as there is only one graph connecting two players. Suppose that the statement is true for $m \in \{2, ..., n-1\}$. By definition, any subnetwork of a fully decomposable network is also fully decomposable. Hence, by the induction hypothesis, the threshold value of the discount factor only depends on the size of the sub network, and we let $\delta(m)$ denote the value of this discount factor. Furthermore, because the norm is an equal sharing norm, the expected utility of player i in a stable graph g only depends on the size of the graph, and hence we write $v_i^*(g) = v^*(m)$ where m is the size of graph g. Clearly, v^* is increasing in m. We also define $s(\delta, m)\}$ as the largest stable subgraph of size smaller or equal to m given δ and $w(\delta, m) = v^*(s(\delta, m))$.

Now consider a connected and fully decomposable network g of size n. Assume that the bottleneck agent is an agent j who for a realization $\mathbf{y}$ has the tightest constraint for the following potential deviation: to satisfy his obligation towards the m agents in S and deviating on the $n - m$ others. Let i be an agent critical of degree m. By symmetry of the distribution, let all agents in the subnetwork of size m originating at i have the income of the agent in S under $\mathbf{y}$. To be sure i must be a bottleneck agent.

As for all decomposable network we can repeat the same operation, the tightest constraint will be the same in any fully decomposable network. It follows that $\Delta(g) = \Delta(g')$ for any two fully decomposable networks. ■

Proposition 6 shows that Proposition 3 can be extended to the case of optimistic beliefs, and that the threshold value of the discount factor is identical for a large family of networks including stars and lines.

As opposed to the case of strong punishment, the addition of new links typically does not result in a more stable insurance network. We saw in Section 4.3 that for high values of the discount factor network composed of trees are the only stable networks. In particular, the complete graph fails to be stable when under punishment. For lower values of the discount factor, when network composed of trees cease to be stable, denser networks can reemerge as stable outcomes.

This is illustrated in the following example with three agents with our simple high/low income distribution.

EXAMPLE **2.** *Endowments are i.i.d. across agents and take on just two values: h with probability p and ℓ with probability $(1 - p)$, with $h > \ell > 0$. The following parameters are set through the example: $\ell = 0$ and $h = 1$, and individuals are assumed to have utilities $u(c) = 2c^{1/2}$.*

In Figure 8, stable networks are computed for the entire range of discount factors δ for values of p equal to 0.2 in the upper part of the graph, then for $p = 0.5$ in the lower part.

Stability is clearly a complex object to check for. When for $\delta = .8$, the complete graph is the only stable graph. Thereafter, for $\delta = .9$, a line of two is stable and the gain enjoyed by the former is large enough to render the complete graph unstable. Then, for values of δ of 0.91, the complete

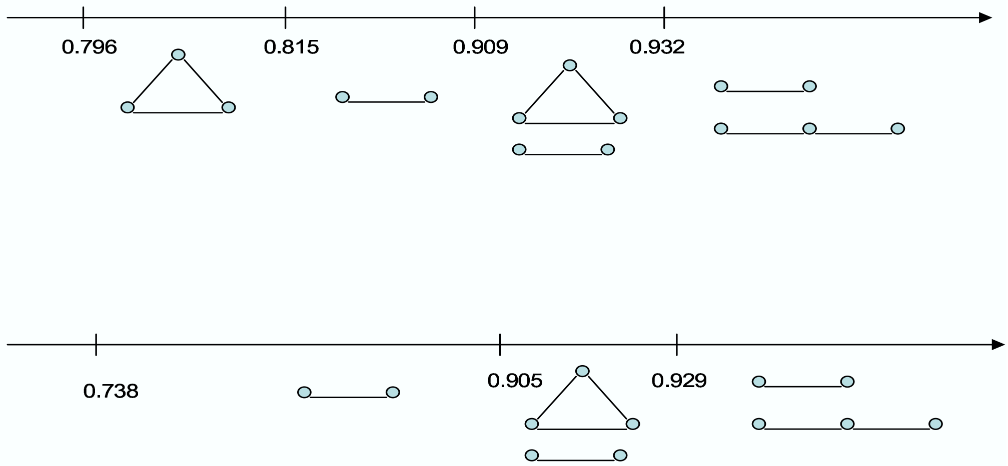

Figure 8. An Illustration of Example 2.

graph regains its stability. Yet, the fall is inevitable: for high values of δ only trees are stable, in line with our previous results.

7. Conclusion

In this paper, we develop a model of *risk-sharing in social networks.* Only individuals who are linked can make transfer to each other. There are two important features to our model. First, a risk-sharing arrangement at the level of network results from a collection of bilateral arrangement among linked individuals. Second, in choosing the transfer to make to each other, a pair of linked individuals follow a *transfer norm* that prescribe a transfer to be made as a function of the individuals' income, identities and their net obligations to others in the network. As these obligations are endogenous, we focus on transfer schemes that satisfy a society-wide consistency: a consistent transfer scheme is a *fixed point* of these bilateral state-contingent transfers.

After characterizing conditions for existence of consistent transfer schemes, we assess the stability of insurance networks stressing that to be stable a risk-sharing network must be self-enforcing. Two important factors affect the stability of insurance networks: a *transit* effect and an *information* effect determining the severity of punishment in the network.

Studying stability of networks for monotone social norms, we show that the severity of punishment in a network is closely related to a specific concept of sparsity. As a result, at high levels of discount rate, all networks are stable under strong punishment, while only networks of trees are stable under weak punishment. For intermediate level of punishment both very dense graphs and minimally connected graphs are stable while graphs of intermediary density are harder to sustain.

The structure of the network is also important for the transit effect. A high amount of transfers going through a particular agent provides her with a strong short term incentive to deviate from the insurance scheme. This effect can be clearly illustrated for the equal sharing norm in the case of strong punishment. We show that all decomposable networks share the same stability conditions, and have the most acute transit effect. Hence, they are the hardest network to sustain.

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