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"A Framework for Exploring the Macroeconomic Determinants of Systematic Risk"

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A Framework for Exploring the Macroeconomic Determinants of Systematic Risk

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The increasing availability of high-frequency asset return data has had a fundamental impact on empirical financial economics, focusing attention on asset return volatility and correlation dynamics, with key applications in portfolio and risk management. So-called "realized" volatilities and correlations have featured prominently in the recent literature, and numerous studies have provided direct characterizations of the unconditional and conditional distributions of realized volatilities and correlations across different assets, asset classes, countries, and sample periods. For overviews see Torben G. Andersen, Tim Bollerslev, Peter F. Christoffersen and Francis X. Diebold (2005a, b).

In this paper we selectively survey, unify and extend that literature. Rather than focusing exclusively on characterization of the properties of realized volatility, we progress by examining economically interesting *functions* of realized volatility, namely realized betas for equity portfolios, relating them both to their underlying realized variance and covariance parts and to underlying macroeconomic fundamentals.

We proceed as follows. In part I we introduce realized volatility and basic theoretical results concerning its convergence to integrated volatility. In part II we move to realized beta and characterize its dynamics relative to those of its variance and covariance components. In part III we introduce a state space representation that facilitates extraction and prediction of true (latent) betas based on their realized values, and which also allows for simple incorporation and joint modeling of macroeconomic fundamentals. In part IV we provide an illustrative empirical example, and we conclude in part V.

I. Realized Volatility

Let the $N \times 1$ logarithmic vector price process, p_t, follow a multivariate continuous-time stochastic volatility diffusion,

$$dp_t \quad = \quad \mu_t \, dt \quad + \quad \Omega_t \, dW_t \, , \tag{1}$$

where W_t denotes a standard N-dimensional Brownian motion, both the $N \times N$ positive definite diffusion matrix, Ω_t, and the N-dimensional instantaneous drift, μ_t, are strictly stationary and jointly independent of W_t (extensions to allow for leverage effects, or non-zero correlations between W_t and Ω_t, and/or jumps in the price process could in principle be incorporated as well). Also, suppose that the N'th element of p_t contains the log price of the market, and the i'th element of p_t contains the log price of the i'th individual stock, so that the corresponding covariance matrix contains both the market variance, say $\sigma^2_{M,t} = \Omega_{(NN),t}$, and the individual equity covariance with the market, say $\sigma_{iM,t} = \Omega_{(iN),t}$.

Conditional on the realized sample paths of μ_t and Ω_t, the distribution of the continuously compounded h-period return, $r_{t+h,h} \equiv p_{t+h} - p_t$, is then

$$r_{t+h,h} \;\; | \;\; \sigma\{ \mu_{t+\tau} \, , \Omega_{t+\tau} \}_{\tau=0}^{h} \quad \sim \quad N(\int_0^h \mu_{t+\tau} \, d\tau \, , \;\; \int_0^h \Omega_{t+\tau} \, d\tau) \, , \tag{2}$$

where $\sigma\{ \mu_{t+\tau} \, , \Omega_{t+\tau} \}_{\tau=0}^{h}$ denotes the σ-field generated by the sample paths of $\mu_{t+\tau}$ and $\Omega_{t+\tau}$ for $0 \leq \tau \leq h$. The integrated diffusion matrix $\int_0^h \Omega_{t+\tau} \, d\tau$ therefore provides a natural measure of the true latent h-period volatility. Under weak regularity conditions, it follows from the theory of quadratic variation that

$$\Sigma_{j=1,\dots,[h/\Delta]} \, r_{t+j\cdot\Delta,\Delta} \cdot r'_{t+j\cdot\Delta,\Delta} \;\; - \;\; \int_0^h \Omega_{t+\tau} \, d\tau \quad \rightarrow \quad 0, \tag{3}$$

almost surely (a.s.) for all t as the return sampling frequency increases ($\Delta \rightarrow 0$). Thus, by using sufficiently finely-sampled high-frequency returns, it is possible in theory to construct a *realized* diffusion matrix that is arbitrarily close to the integrated diffusion matrix (for a survey of the relevant theory, see Andersen, Bollerslev and Diebold, 2005). In practice, market microstructure frictions limits the highest feasible sampling frequency ($\Delta \geq \delta > 0$), and the best way to deal with this, whether using the simple estimator in (3) or some variant thereof, is currently a very active area of research.

Meanwhile, key empirical findings for realized volatility include lognormality and long memory of volatilities and correlations (Andersen, Bollerslev, Diebold and Paul Labys, 2001; Andersen, Bollerslev, Diebold and Heiko Ebens, 2001), as well as normality of returns standardized by realized volatility (Andersen, Bollerslev, Diebold and Labys, 2000). Those properties, as distilled in the lognormal / normal mixture model of Andersen, Bollerslev, Diebold and Labys (2003), have important implications for risk management and asset allocation.

II. Realized Beta and its Components

Although characterizations of the properties of realized variances and covariances are of interest, alternative objects are often of greater economic significance with a leading example being the market beta of a portfolio. If either the market volatility or its covariance with portfolio returns is time-varying, then the portfolio beta will generally be time-varying. Hence it is clearly of interest to explore the links between time-varying volatilities, time-varying correlations, and time-varying betas. One may construct realized betas from underlying realized covariance and variance components, or conversely, decompose realized betas into realized variance and covariance components.

Armed with the relevant realized market variance and realized covariance measures, we can readily define and empirically construct "realized betas." Using an initial subscript to indicate the corresponding element of a vector, we denote the realized market volatility by

$$\hat{v}^2_{M,t,t+h} = \Sigma_{j=1,\ldots,[h/\Delta]}\ r^2_{(N),t+j\cdot\Delta,\Delta}, \tag{4}$$

and the realized covariance between the market and the ith portfolio return by

$$\hat{v}_{iM,t,t+h} = \Sigma_{j=1,\ldots,[h/\Delta]}\ r_{(i),t+j\cdot\Delta,\Delta} \cdot r_{(N),t+j\cdot\Delta,\Delta}. \tag{5}$$

Now defining the realized beta as the ratio between the two, it follows under the assumptions above that

$$\hat{\beta}_{i,t,t+h} \equiv \frac{\hat{v}_{iM,t,t+h}}{\hat{v}^2_{M,t,t+h}} \rightarrow \beta_{i,t,t+h} \equiv \int_0^h \Omega_{(iN),t+\tau}\, d\tau \;/\; \int_0^h \Omega_{(NN),t+\tau}\, d\tau, \tag{6}$$

a.s. for all t as $\Delta \rightarrow 0$, so that realized beta is consistent for the corresponding true integrated beta.

By comparing the properties of directly-measured betas to those of directly-measured variances and covariances, we can decompose movements in betas in informative ways. In particular, because the long memory in underlying variances and covariances may be common, it is possible that betas may be only weakly persistent (short-memory, $I(d)$, with $d \approx 0$), despite the widespread finding that realized variances and covariances are long-memory (fractionally- integrated, $I(d)$, with $d \approx 0.4$). Recent work by Andersen, Bollerslev, Diebold and Ginger Wu (2005a) indicates that the relevant realized variances and covariances are indeed reasonably well-characterized as nonlinearly fractionally cointegrated in this fashion (as beta is an a priori known ratio of the two measures).

III. A State Space Framework Facilitating the Inclusion of Macroeconomic Fundamentals

Although the decomposition of realized betas into contributions from underlying variances and covariances is intriguing, a more thorough economic analysis would seek to identify the fundamental determinants of realized variances and covariances that impact realized betas. Here we take some steps in that direction, directly allowing for dependence of betas on underlying macroeconomic fundamentals.

First, in parallel to the volatility model in Ole Barndorff-Nielsen and Neil Shephard (2002), the time-varying integrated/realized beta may be conveniently cast in state space form. The realized beta equals the true latent integrated beta, plus a weak white noise measurement error, asymptotically Gaussian in the sampling frequency ($\Delta \rightarrow 0$). Normalizing $h \equiv 1$ and suppressing the subscripts:

$$\hat{\beta}_t = \beta_t + u_t. \tag{7a}$$

We can easily allow for dynamics in β_t, as exemplified by the first-order autoregressive representation

$$\beta_t = \gamma_0 + \gamma_1 \beta_{t-1} + v_t, \tag{7b}$$

where v_t is weak white noise. We therefore have a state space system, with measurement equation (7a) and transition equation (7b), so that the Kalman filter may be used for extraction and prediction of the latent integrated β_t based on the observed $\hat{\beta}_t$ (a more refined approach in which the nonconstant variance of u_t is equated to the asymptotic, for $\Delta \rightarrow 0$, expression in Barndorff-Nielsen and Shephard, 2004, could also be applied). Note, that the system in (7a,b) is distinctly different from the one in which the measurement equation is replaced by a conditional CAPM model, $r_t = \alpha + \beta_t r_{M,t} + \varepsilon_t$ (see, e.g., Andrew Ang and Josephn Chen, 2004, and Gergana Jostova and Alexander Philipov, 2005, and the references therein). The *smoothed* version of β_t extracted by the Kalman filter from (7a,b), in particular, should compare favorably to the standard practice of assuming that the sampling frequency is so high that $\hat{\beta}_t$ is effectively indistinguishable from β_t, or $u_t \approx 0$ (see also Dean Foster and Dan Nelson, 1996, who argue for smoothing of realized betas, from a very different and complementary perspective).

Second, note that we may readily include macroeconomic fundamentals in the state space dynamics, by augmenting the state vector as in the system:

$$\hat{\beta}_t = z' B_t + u_t \tag{8a}$$

$$B_t = \Gamma_0 + \Gamma_1 B_{t-1} + v_t, \tag{8b}$$

where $z' \equiv (1, 0, ..., 0)$, $u_t \sim (0, \sigma_u^2)$, Γ_0 is a vector of intercepts, Γ_1 is a matrix of coefficients, $B_t' \equiv (\beta_t, x_t')$, x_t is a column vector of macroeconomic variables, and $v_t \sim (0, \Sigma)$ is a vector of transition disturbances. The vector autoregressive transition equation (8b) permits interaction between beta and macroeconomic fundamentals, both dynamically (via Γ_1) and contemporaneously (via the covariances in Σ). For illustration, in this paper, we only explore macroeconomic indicators one at a time, under an assumption of recursive transition dynamics. That is, letting $\Sigma = diag(\sigma_{v1}^2,\ \sigma_{v2}^2)$, we estimate the system

$$\hat{\beta}_t = \beta_t + u_t \tag{9a}$$

$$\begin{pmatrix} \beta_t \\ x_t \end{pmatrix} = \begin{pmatrix} \gamma_{01} \\ \gamma_{02} \end{pmatrix} + \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ 0 & \gamma_{22} \end{pmatrix} \begin{pmatrix} \beta_{t-1} \\ x_{t-1} \end{pmatrix} + \begin{pmatrix} \nu_{1t} \\ \nu_{2t} \end{pmatrix}. \tag{9b}$$

For simplicity, we further assume homoskedastic measurement errors for monthly realized betas. This is clearly not true for daily data, but a more palatable approximation at the monthly level that is relevant for the analysis below. It follows that inference based on the standard Kalman Filter is valid.

IV. An Illustrative Application

We use underlying fifteen-minute returns for individual NYSE-listed stocks and the value-weighted market portfolio. We construct all returns from the TAQ dataset, February 1, 1993 through May 31, 2003, excluding real estate investment trusts, stocks of companies incorporated outside the United States, and closed-end mutual funds. Next, we sort the firms into twenty-five portfolios, corresponding to various combinations of the five market capitalization ("size") and five book-to-market ("value") quintiles, month-by-month, re-balancing each month. We denote the twenty-five portfolios by ij, for $i, j = 1, \ldots, 5$, where i refers to size quintile and j refers to value quintile (from low to high). Finally, for each of the twenty-five portfolios, we use the fifteen-minute portfolio and market returns to construct monthly realized covariances of each portfolio return with the market return, the realized variance of the market return, and the ratio, or "realized beta." To adjust for asynchronous trading, we use an equally-weighted average of contemporaneous realized beta and four leads and lags.

In Figure 1, we show extractions of the latent integrated betas obtained using the Kalman smoother. Substantial and highly-persistent time variation is evident for all the realized betas, but they do not appear to be trending or otherwise nonstationary; instead reverting to fixed means. We have also shaded the March-November 2001 recession for visual reference. Looking across the columns from low- to high-value portfolios, the betas for many portfolios appear to increase substantially during and around the recession, and the high-value portfolio betas seem to be more responsive over the cycle.

We now assess these graphically-motivated conjectures more rigorously by estimating the time-varying beta model in (9a,b), explicitly allowing for macroeconomic influences. In Andersen, Bollerslev, Diebold and Wu (2005b), we study all twenty-five portfolios and several macroeconomic indicators, alone and in combination, including industrial production, the term premium, the default premium, the consumption/wealth ratio, the consumer price index, and the consumer confidence index. Here we merely sketch some illustrative results, focusing on representative large-capitalization portfolios 51, 53 and 55, and a central macroeconomic indicator, industrial production growth (*IP*).

We display the estimation results in Table 1. The β own-lag coefficients γ_{11} indicate substantial own persistence, while the *IP* own-lag coefficients γ_{22} are obviously much smaller. This is natural as the *IP* variable is a growth rate (change in logarithm). The key macro-finance interaction coefficient, γ_{12}, summarizes the response of β_t to movements in IP_{t-1}. Interestingly, and in keeping with our earlier conjecture, both the statistical and economic significance of the estimates of γ_{12} increase with value, as measured by book-to-market. For portfolio 51, the point estimate of γ_{12} is near zero and statistically *in*significant at any conventional level, while for portfolio 53, the point estimate is substantially larger in magnitude (-3.4) and significant at the ten percent level. For portfolio 55, the point estimate is statistically significant at the one percent level, and quite large at -6.1, implying that an additional percentage point of IP_{t-1} growth produces a -.061 decrease in $\beta_{55,t}$. Hence as IP_{t-1} varies over the cycle from, say, -0.05 to +0.05, $\beta_{55,t}$ will move substantially.

Impulse response functions provide a more complete distillation of the dynamic response patterns. Although the recursive structure automatically identifies the vector autoregression (10b), we still normalize by the Cholesky factor of Σ to express all shocks in standard deviation units. We report results in Figure 2. In parallel with the impact estimates in Table 1, the beta for the growth portfolio 51 shows no dynamic response but, as we move upward through the value spectrum, we find progressively larger effects, with positive IP_{t-1} shocks producing sharp decreases in β_t, followed by very slow reversion to the mean. These are, of course, only partial effects, and a more complete analysis would have to jointly consider the influence of other business cycle variables as in (8a,b).

V. Concluding Remarks

There is an emerging empirical consensus that expected excess returns are counter-cyclical – not only for stocks, as in Martin Lettau and Sydney Ludvigson (2001a), but also for bonds, as in John H. Cochrane and Monika Piazzesi (2005) – whether because risk is higher in recessions, as in George M. Constantinides and Darrell Duffie (1996), or because risk aversion is higher in recessions, as in John Campbell and Cochrane (1999). The preliminary results reported here indicate that equity market betas do indeed vary with macroeconomic indicators such as industrial production growth, and that the macroeconomic effects on expected returns are large enough to be economically important. Moreover, the preliminary results strongly indicate that the counter-cyclicality of beta is primarily a value stock phenomenon, suggesting that the well-documented and much-debated value premium (see also the related studies by Andrew Ang and Jun Liu, 2004; Ravi Jagannathan and Zhenyu Wang, 1996; Lettau and Ludvigson, 2001b; Jonathan Lewellen and Stefan Nagel, 2004; Ralitsa Petkova and Lu Zhang, 2004, and the many references therein) may at least in part be explained by an increase in expected returns for value stocks during bad economic times.

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Footnotes

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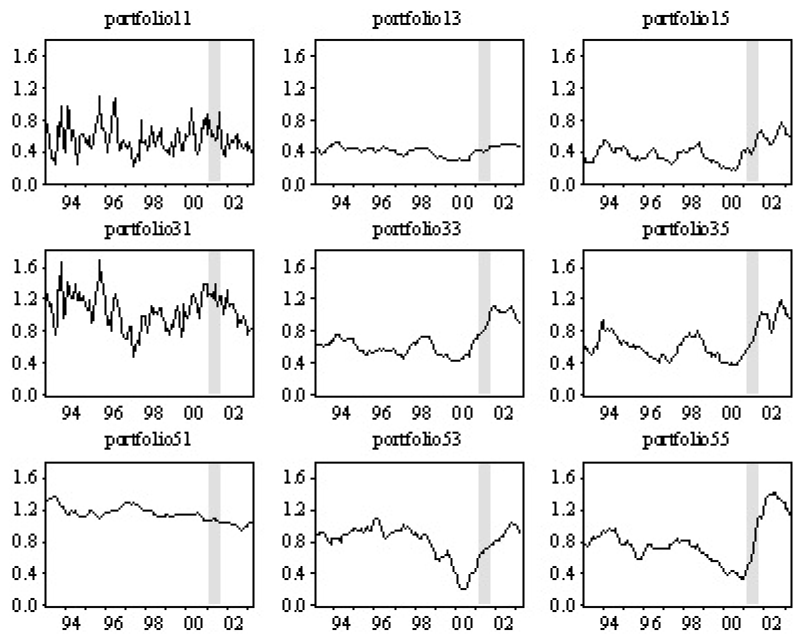
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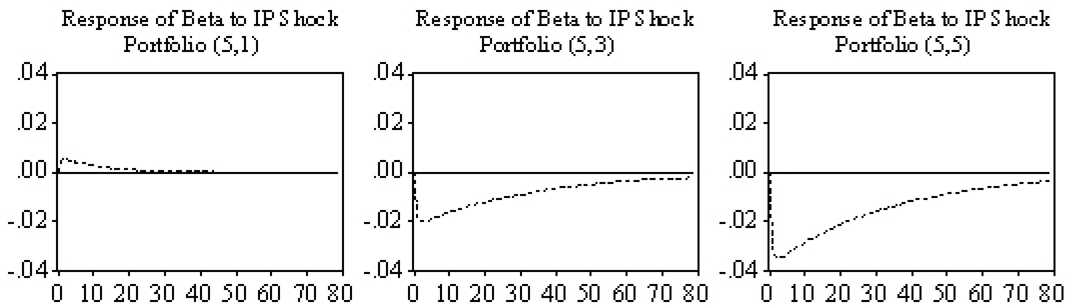

Figure 2. Impulse Response Functions

Table 1 – Parameter Estimates for Model (10a, b)

	Portfolio 51		Portfolio 53		Portfolio 55	
	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.
γ_{01}	0.092**	0.036	0.032	0.028	0.042**	0.020
γ_{02}	0.002***	0.0005	0.002***	0.0005	0.002***	0.0005
γ_{11}	0.915***	0.031	0.971***	0.034	0.971***	0.024
γ_{12}	0.920	1.114	-3.486*	2.156	-6.101***	1.706
γ_{22}	0.191**	0.088	0.191**	0.088	0.191**	0.088

Notes: *, ** and *** denote statistical significance at the ten percent, five percent and one percent levels, respectively.