Appendix to "Disentangling Insurance and Information in Intertemporal Consumption Choices"

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March 5, 2009

1 Accounting for the non-annual frequency of the data: SHIW 1995, 1998, and 2000

1.1 Income data

Set $t = 1998$. We observe the following income realizations:

$$\begin{aligned} y_{it+2} - y_{it} &= (X_{it+2} - X_{it})'\beta + (\varepsilon_{it+2}^U - \varepsilon_{it}^U) + (\varepsilon_{it+2}^A - \varepsilon_{it}^A) + (\zeta_{it+2}^A + \zeta_{it+1}^A) + (\zeta_{it+2}^U + \zeta_{it+1}^U) \\ y_{it} - y_{it-3} &= (X_{it} - X_{it-3})'\beta + (\varepsilon_{it}^U - \varepsilon_{it-3}^U) + (\varepsilon_{it}^A - \varepsilon_{it-3}^A) + (\zeta_{it}^A + \zeta_{it-1}^A + \zeta_{it-2}^A) + (\zeta_{it}^U + \zeta_{it-1}^U + \zeta_{it-2}^U) \end{aligned}$$

and the following expectations:

$$\begin{aligned} E(\Delta y_{it+1} | \Omega_t^i) &= \Delta X'_{it+1}\beta - \varepsilon_{it}^U + \Delta \varepsilon_{it+1}^A + \zeta_{it+1}^A \\ E(\Delta y_{it-2} | \Omega_{t-3}^i) &= \Delta X'_{it-2}\beta - \varepsilon_{it-3}^U + \Delta \varepsilon_{it-2}^A + \zeta_{it-2}^A \end{aligned}$$

It is useful to net out the effect of the observables and define the terms:

$$\begin{aligned} \omega_{it+1} &= E(\Delta y_{it+1} | \Omega_t^i) - \Delta X'_{it+1}\beta = -\varepsilon_{it}^U + \Delta \varepsilon_{it+1}^A + \zeta_{it+1}^A \\ \omega_{it-2} &= E(\Delta y_{it-2} | \Omega_{t-3}^i) - \Delta X'_{it-2}\beta = -\varepsilon_{it-3}^U + \Delta \varepsilon_{it-2}^A + \zeta_{it-2}^A \\ v_{it+2} &= y_{it+2} - y_{it} - (X_{it+2} - X_{it})'\beta = (\varepsilon_{it+2}^U - \varepsilon_{it}^U) + (\varepsilon_{it+2}^A - \varepsilon_{it}^A) + (\zeta_{it+2}^A + \zeta_{it+1}^A) + (\zeta_{it+2}^U + \zeta_{it+1}^U) \\ v_{it} &= y_{it} - y_{it-3} - (X_{it} - X_{it-3})'\beta = (\varepsilon_{it}^U - \varepsilon_{it-3}^U) + (\varepsilon_{it}^A - \varepsilon_{it-3}^A) + (\zeta_{it}^A + \zeta_{it-1}^A + \zeta_{it-2}^A) + (\zeta_{it}^U + \zeta_{it-1}^U + \zeta_{it-2}^U) \end{aligned}$$

Define:

$$\boldsymbol{\omega} = \begin{pmatrix} \omega_{it+1} \\ \omega_{it-2} \end{pmatrix}, \mathbf{v} = \begin{pmatrix} v_{it+2} \\ v_{it} \end{pmatrix}, \mathbf{e} = \begin{pmatrix} \boldsymbol{\omega} \\ \mathbf{v} \end{pmatrix}$$

Then we can derive the following moments

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$$\begin{aligned}
E(\omega\omega') &= \begin{pmatrix} \sigma_{\varepsilon^U}^2 + 2\sigma_{\varepsilon^A}^2 + \sigma_{\zeta^A}^2 & 0 \\ \sigma_{\varepsilon^U}^2 + 2\sigma_{\varepsilon^A}^2 + \sigma_{\zeta^A}^2 & \end{pmatrix} \\
E(\mathbf{v}\mathbf{v}') &= \begin{pmatrix} 2(\sigma_{\varepsilon^U}^2 + \sigma_{\varepsilon^A}^2 + \sigma_{\zeta^U}^2 + \sigma_{\zeta^A}^2) & -(\sigma_{\varepsilon^U}^2 + \sigma_{\varepsilon^A}^2) \\ 2(\sigma_{\varepsilon^U}^2 + \sigma_{\varepsilon^A}^2) + 3(\sigma_{\zeta^U}^2 + \sigma_{\zeta^A}^2) & \end{pmatrix} \\
E(\omega\mathbf{v}') &= \begin{pmatrix} \sigma_{\varepsilon^U}^2 + \sigma_{\varepsilon^A}^2 + \sigma_{\zeta^A}^2 & -(\sigma_{\varepsilon^U}^2 + \sigma_{\varepsilon^A}^2) \\ 0 & \sigma_{\varepsilon^U}^2 + \sigma_{\varepsilon^A}^2 + \sigma_{\zeta^A}^2 \end{pmatrix}
\end{aligned}$$

where

$$E(\mathbf{e}\mathbf{e}') = \begin{pmatrix} E(\omega\omega') & E(\omega\mathbf{v}') \\ E(\mathbf{v}\mathbf{v}') & \end{pmatrix}$$

One can easily see that the model is identified with four overidentifying restrictions.

1.2 Consumption Data

The consumption observations are:

$$\begin{aligned}
c_{it+2} - c_{it} &= (X_{it+2} - X_{it})'\beta + \phi(\zeta_{it+2}^U + \zeta_{it+1}^U) + \psi(\varepsilon_{it+2}^U + \varepsilon_{it+1}^U) \\
c_{it} - c_{it-3} &= (X_{it} - X_{it-3})'\beta + \phi(\zeta_{it}^U + \zeta_{it-1}^U + \zeta_{it-2}^U) + \psi(\varepsilon_{it}^U + \varepsilon_{it-1}^U + \varepsilon_{it-2}^U)
\end{aligned}$$

Netting out the effect of the observables yields

$$\begin{aligned}
u_{it+2} &= c_{it+2} - c_{it} - (X_{it+2} - X_{it})'\beta = \phi(\zeta_{it+2}^U + \zeta_{it+1}^U) + \psi(\varepsilon_{it+2}^U + \varepsilon_{it+1}^U) \\
u_{it} &= c_{it} - c_{it-3} - (X_{it} - X_{it-3})'\beta = \phi(\zeta_{it}^U + \zeta_{it-1}^U + \zeta_{it-2}^U) + \psi(\varepsilon_{it}^U + \varepsilon_{it-1}^U + \varepsilon_{it-2}^U)
\end{aligned}$$

Define:

$$\omega = \begin{pmatrix} \omega_{it+1} \\ \omega_{it-2} \end{pmatrix}, \mathbf{v} = \begin{pmatrix} v_{it+2} \\ v_{it} \end{pmatrix}, \mathbf{u} = \begin{pmatrix} u_{it+2} \\ u_{it} \end{pmatrix}, \mathbf{z} = \begin{pmatrix} \omega \\ \mathbf{v} \\ \mathbf{u} \end{pmatrix}$$

We have

$$\begin{aligned}
E(\omega\omega') &= \begin{pmatrix} \sigma_{\varepsilon^U}^2 + 2\sigma_{\varepsilon^A}^2 + \sigma_{\zeta^A}^2 & 0 \\ \sigma_{\varepsilon^U}^2 + 2\sigma_{\varepsilon^A}^2 + \sigma_{\zeta^A}^2 & \end{pmatrix} \\
E(\mathbf{v}\mathbf{v}') &= \begin{pmatrix} 2(\sigma_{\varepsilon^U}^2 + \sigma_{\varepsilon^A}^2 + \sigma_{\zeta^U}^2 + \sigma_{\zeta^A}^2) & -(\sigma_{\varepsilon^U}^2 + \sigma_{\varepsilon^A}^2) \\ 2(\sigma_{\varepsilon^U}^2 + \sigma_{\varepsilon^A}^2) + 3(\sigma_{\zeta^U}^2 + \sigma_{\zeta^A}^2) & \end{pmatrix} \\
E(\mathbf{u}\mathbf{u}') &= \begin{pmatrix} 2\psi^2\sigma_{\varepsilon^U}^2 + 2\phi^2\sigma_{\zeta^U}^2 & 0 \\ 3\psi^2\sigma_{\varepsilon^U}^2 + 3\phi^2\sigma_{\zeta^U}^2 & \end{pmatrix} \\
E(\omega\mathbf{v}') &= \begin{pmatrix} \sigma_{\varepsilon^U}^2 + \sigma_{\varepsilon^A}^2 + \sigma_{\zeta^A}^2 & -(\sigma_{\varepsilon^U}^2 + \sigma_{\varepsilon^A}^2) \\ 0 & \sigma_{\varepsilon^U}^2 + \sigma_{\varepsilon^A}^2 + \sigma_{\zeta^A}^2 \end{pmatrix} \\
E(\omega\mathbf{u}') &= \begin{pmatrix} 0 & -\psi\sigma_{\varepsilon^U}^2 \\ 0 & 0 \end{pmatrix} \\
E(\mathbf{u}\mathbf{v}') &= \begin{pmatrix} \psi\sigma_{\varepsilon^U}^2 + 2\phi\sigma_{\zeta^U}^2 & 0 \\ -\psi\sigma_{\varepsilon^U}^2 & \psi\sigma_{\varepsilon^U}^2 + 3\phi\sigma_{\zeta^U}^2 \end{pmatrix}
\end{aligned}$$

where

$$E(\mathbf{z}\mathbf{z}') = \begin{pmatrix} E(\omega\omega') & E(\omega\mathbf{v}') & E(\omega\mathbf{u}') \\ E(\mathbf{v}\mathbf{v}') & E(\mathbf{u}\mathbf{v}') \\ E(\mathbf{u}\mathbf{u}') \end{pmatrix}$$

With consumption data we have six more moment conditions, but only two additional parameters to identify. Thus we have eight overidentifying restrictions in this case. This enables us to allow for measurement error in income and consumption and for a fixed effect in the subjective expectations reports, while still being overidentified.