APPENDIX: Omitted Analysis and Proofs

Proof of Lemma 1. Differentiate $v(p, m)$ with respect to m and apply the envelope theorem ($G'(z)\, f'(x_i) - p_i = 0$ for all i),

$$\frac{\partial v(p,m)}{\partial m} = G'(z)\, f(x_m) - p_m x_m.$$

Evaluating inverse demand (1) for product $i = m$ and making the resulting substitution establishes the result.

Proofs of Propositions 1-4 follow the derivation of comparative statics results.

In the symmetric equilibrium, equations (3) and (4) can be simplified as follows. Differentiate $\pi^m(p, m)$ with respect to p_i, $i \in (0, m]$, and m, making use of the implicit function theorem on (1), and then impose the symmetry conditions $p_i = p = \overline{p}$ and $m = \overline{m}$ on the resulting expressions. Factoring terms gives

$$\begin{aligned} \frac{\partial \pi^m}{\partial p_i} &= \frac{[c + \tau - (1-\alpha)(1-\sigma(x))p(m,x)]\, x}{\sigma(x)p(m,x)}, \\ \frac{\partial \pi^m}{\partial m} &= \frac{(\sigma(x) - \varepsilon)\, [(1-\alpha)p(m,x) - c - \tau)]\, x}{\sigma(x)}, \end{aligned}$$

respectively. Substituting these effects and the symmetric market shares, $2\delta^* = 1/n$, into (3) and (4) and dropping arguments,

$$\text{(A1)} \qquad \Pi_p = -\frac{mx^2}{t}((1-\alpha)p - c - \tau) + \frac{[c + \tau - (1-\alpha)(1-\sigma)p]\, x}{\sigma p n} = 0,$$

$$\text{(A2)} \qquad \Pi_m = x((1-\alpha)p - c - \tau)\left[\frac{(1-\theta)pmx}{\theta t} + \frac{\sigma - \varepsilon}{\sigma n}\right] - F = 0.$$

To derive the comparative statics results, it is helpful to describe the market equilibrium in terms of output per product and product variety choices. Letting $\widetilde{\Pi}(x, m; n)$ denote the profit function of the representative retailer, and proceeding as above, the first-order conditions with respect to x and m are:

$$\text{(A3)} \qquad \widetilde{\Pi}_x = \frac{p\sigma mx}{t}((1-\alpha)p - c - \tau) + \frac{[(1-\alpha)(1-\sigma)p - c - \tau]}{n} = 0,$$

(A4) $$\widetilde{\Pi}_m = \frac{p\beta m x^2}{\theta t}\left((1-\alpha)p - c - \tau\right) + \frac{\left[(1-\alpha)(1-\varepsilon)p - c - \tau\right]x}{n} - f = 0,$$

where $\beta(x) = 1 - \theta(x) + \varepsilon\theta(x)$. The market equilibrium is characterized by these two conditions and the zero profit condition (5), which is denoted here by $\widetilde{\Pi}^s$. It is straightforward to verify that these conditions characterize the same equilibrium outcome as equations (A1) and (A2). Specifically, the left-hand side of condition (A3) can be written $\widetilde{\Pi}_x = \frac{-x\Pi_p}{p(m,x)\sigma(x)}$, so that $\widetilde{\Pi}_x = 0$ if and only if $\Pi_p = 0$, and the left-hand side of condition (A4) represents the transformation $\widetilde{\Pi}_m = \Pi_m + \frac{\varepsilon x}{\sigma(x)n}\widetilde{\Pi}_x$.

The goal is to examine and compare the equilibrium outcomes of the model in the short run and in the long run . To do so, it is helpful to evaluate all effects at an identical initial equilibrium position characterized by zero profit.

Consider, first, the long run equilibrium outcome. Letting $k(x) = \beta(x) - \sigma(x)(1-\theta(x))$, recursive substitution of (5) and (A4) into (A3) gives:

$$\frac{p(m,x)\sigma(x)mnx}{t} = \frac{\theta(x)\varepsilon}{1-\theta(x)},$$

and

$$p(x) = \frac{(c+\tau)\beta(x)}{(1-\alpha)k(x)}.$$

These results can be used to simplify terms that arise from the total derivation of (5), (A3) and (A4). Doing so (and dropping arguments) yields:

$$\begin{bmatrix} \widetilde{\Pi}_{xx} & \widetilde{\Pi}_{xm} & \widetilde{\Pi}_{xn} \\ \widetilde{\Pi}_{mx} & \widetilde{\Pi}_{mm} & \widetilde{\Pi}_{mn} \\ \widetilde{\Pi}^s_x & \widetilde{\Pi}^s_m & \widetilde{\Pi}^s_n \end{bmatrix} \begin{bmatrix} dx \\ dm \\ dn \end{bmatrix} = - \begin{bmatrix} \widetilde{\Pi}_{x\alpha} \\ \widetilde{\Pi}_{m\alpha} \\ \widetilde{\Pi}^s_\alpha \end{bmatrix} d\alpha - \begin{bmatrix} \widetilde{\Pi}_{x\tau} \\ \widetilde{\Pi}_{m\tau} \\ \widetilde{\Pi}^s_\tau \end{bmatrix} d\tau,$$

where

$$\begin{aligned}
\widetilde{\Pi}_{xx} &= \frac{-(c+\tau)\sigma}{(1-\theta)knx}\left[(1-\theta)k + (\varepsilon\theta)^2 + (1-\theta)^2 e^{\sigma}\right], \\
\widetilde{\Pi}_{xm} &= \frac{(c+\tau)\varepsilon}{(1-\theta)kmn}\left[(1-\theta)(\sigma-\beta) - \beta\varepsilon\theta\right], \\
\widetilde{\Pi}_{xn} &= \frac{(c+\tau)\varepsilon\theta\sigma}{kn^2}, \\
\widetilde{\Pi}_{x\tau} &= \frac{-\beta}{(1-\theta)n}, \\
\widetilde{\Pi}_{x\alpha} &= \frac{-pk}{(1-\theta)n},
\end{aligned}$$

$$
\begin{aligned}
\widetilde{\Pi}_{mx} &= \frac{-(c+\tau)}{(1-\theta)kn}\left[(\beta\varepsilon + (1-\theta)\sigma)\,\varepsilon\theta + (1-\theta)e^{\theta}\right], \\
\widetilde{\Pi}_{mm} &= \frac{-(c+\tau)\beta^2\varepsilon^2 x}{(1-\theta)\sigma kmn}, \\
\widetilde{\Pi}_{mn} &= \frac{-(c+\tau)x}{kn^2}\left[(1-\theta)\sigma - \beta\varepsilon\right], \\
\widetilde{\Pi}_{m\tau} &= \frac{-\left[(1-\theta)\sigma + \beta\varepsilon\right]x}{(1-\theta)\sigma n}, \\
\widetilde{\Pi}_{m\alpha} &= \frac{-\left[(1-\varepsilon)(1-\theta)\sigma + \beta\varepsilon\right]px}{(1-\theta)\sigma n}, \\
\widetilde{\Pi}^s_{x} &= \frac{-(c+\tau)\varepsilon\theta\sigma}{kn}, \\
\widetilde{\Pi}^s_{m} &= -\frac{(c+\tau)\beta\varepsilon x}{kmn}, \\
\widetilde{\Pi}^s_{n} &= -\frac{(c+\tau)(1-\theta)\sigma x}{kn^2}, \\
\widetilde{\Pi}^s_{\tau} &= -\frac{x}{n}, \\
\widetilde{\Pi}^s_{\alpha} &= \frac{px}{n}
\end{aligned}
$$

The comparative statics effects in the short run model do not qualitatively depend on particular values of n. To facilitate the comparison between short run and long run outcomes of the model, I evaluate the short run comparative statics effects at an initial zero profit position $(m^e(n^c), x^e(n^c))$. Accordingly, the market response to excise taxes in the short-run is characterized by the solution to:

$$
\begin{bmatrix} \widetilde{\Pi}_{xx} & \widetilde{\Pi}_{xm} \\ \widetilde{\Pi}_{mx} & \widetilde{\Pi}_{mm} \end{bmatrix} \begin{bmatrix} dx \\ dm \end{bmatrix} = -\begin{bmatrix} \widetilde{\Pi}_{x\alpha} \\ \widetilde{\Pi}_{m\alpha} \end{bmatrix} d\alpha - \begin{bmatrix} \widetilde{\Pi}_{x\tau} \\ \widetilde{\Pi}_{m\tau} \end{bmatrix} d\tau,
$$

The determinant of the coefficient matrix satisfies

$$
\frac{(c+\tau)^2\varepsilon}{(1-\theta)k^2mn^2}\Gamma > 0
$$

where

$$
\Gamma = (1-\theta)(\beta k + (\theta\sigma - k)\theta\sigma)\varepsilon + (1-\theta)\beta^2\varepsilon e^{\sigma} - (\beta\varepsilon\theta + (1-\theta)(k-\theta\sigma))e^{\theta} > 0.
$$

The effect of taxes on equilibrium output per product is

$$\begin{aligned}\frac{dx^e}{d\tau} &= \frac{-(1-\theta)(\beta\varepsilon+\theta\sigma-k)kx}{(c+\tau)\Gamma},\\ \frac{dx^e}{d\alpha} &= \frac{-(1-\varepsilon)(1-\theta)(\theta\sigma-k)kpx}{(c+\tau)\Gamma}.\end{aligned}$$

The effect of taxes on equilibrium product variety is

$$\begin{aligned}\frac{dm^e}{d\tau} &= \frac{-km}{(c+\tau)\varepsilon\Gamma}\left[(1-\theta)(\beta\varepsilon+(1-\theta)\sigma)(1-\sigma+e^{\sigma})-\beta e^{\theta}\right],\\ \frac{dm^e}{d\alpha} &= \frac{-kmp}{(c+\tau)\varepsilon\Gamma}\left[(1-\theta)(\beta\varepsilon+(1-\varepsilon)(1-\theta)\sigma)(1-\sigma+e^{\sigma})+\varepsilon\theta\sigma^2)-ke^{\theta}\right].\end{aligned}$$

Price effects per product in the short-run equilibrium are identified making use of consumer inverse demand, $p(m,x)$, as

$$\begin{aligned}\frac{dp^e}{d\tau} &= \frac{kp}{(c+\tau)\Gamma}[(1-\theta)(\beta\varepsilon+(\sigma-\varepsilon)\theta\sigma)+(1-\theta)(\beta\varepsilon+(1-\theta)\sigma)e^{\sigma}-\beta e^{\theta}],\\ \frac{dp^e}{d\alpha} &= \frac{kp^2}{(c+\tau)\Gamma}[(1-\theta)(\beta\varepsilon+(\theta\sigma-\varepsilon)\sigma)+(1-\theta)(\beta\varepsilon+(1-\varepsilon)(1-\theta)\sigma)e^{\sigma}-ke^{\theta}].\end{aligned}$$

Under free-entry oligopoly, the determinant of the coefficient matrix satisfies

$$\frac{-(c+\tau)^3\varepsilon\sigma x}{k^3mn^4}\Delta<0$$

where

$$\Delta=(1-\theta)(2\varepsilon-\sigma)\beta k+(1-\theta)(2\beta\varepsilon-(1-\theta)\sigma)\beta e^{\sigma}-(2\beta\varepsilon\theta+(1-\theta)(k-\theta\sigma))e^{\theta}>0.$$

The effect of taxes on equilibrium output per product is

$$\begin{aligned}\frac{dx^c}{d\tau} &= \frac{(1-2\varepsilon)(1-\theta)\beta kx}{(c+\tau)\Delta},\\ \frac{dx^c}{d\alpha} &= \frac{(1-\varepsilon)(1-\theta)k^2px}{(c+\tau)\Delta}>0.\end{aligned}$$

The effect of taxes on equilibrium product variety is

$$\begin{aligned}\frac{dm^c}{d\tau} &= \frac{-km}{(c+\tau)\varepsilon\Delta}\left[2(1-\theta)(1-\sigma+e^{\sigma})\beta\varepsilon-(\beta+\varepsilon\theta)e^{\theta}\right],\\ \frac{dm^c}{d\alpha} &= \frac{-kmp}{(c+\tau)\varepsilon\Delta}\left[(1-\theta)((2-\sigma)k+(k+\beta)e^{\sigma})\varepsilon-(k+\varepsilon\theta)e^{\theta}\right],\end{aligned}$$

and the effect of taxes on the equilibrium number of retailers is

$$\begin{aligned}\frac{dn^c}{d\tau} &= \frac{kn}{(c+\tau)\Delta}\left[(1-\theta)(1-\sigma+e^\sigma)\beta - e^\theta\right],\\ \frac{dn^c}{d\alpha} &= \frac{(1-\varepsilon)knp}{(c+\tau)\Delta}\left[(1-\theta)k+(1-\theta)\beta e^\sigma - \theta e^\theta\right].\end{aligned}$$

Price effects per product in the long-run equilibrium are identified making use of inverse demand, $p(m,x)$:

$$\begin{aligned}\frac{dp^c}{d\tau} &= \frac{kp}{(c+\tau)\Delta}[(1-\theta)(2\varepsilon-\sigma+2\varepsilon e^\sigma)\beta-(\beta+\varepsilon\theta)e^\theta],\\ \frac{dp^c}{d\alpha} &= \frac{kp^2}{(c+\tau)\Delta}[(1-\theta)((2\varepsilon-\sigma)k+(k+\beta)\varepsilon e^\sigma)-(k+\varepsilon\theta)e^\theta].\end{aligned}$$

Lemma A1. Assumptions 1 and 2 together with $e^\theta < 0$ *are sufficient conditions for strict concavity of* $\widetilde{\Pi}(x,m;n)$.

Proof. By inspection, $\widetilde{\Pi}_{mm} < 0$ and $\widetilde{\Pi}^s_n < 0$. Factoring terms in $\widetilde{\Pi}_{xx}$ and making the substitution $E = 1+\sigma-e^\sigma$ yields

$$\widetilde{\Pi}_{xx} \stackrel{s}{=} -\left[(1-\theta)^2(2-E)+\beta\varepsilon\theta\right] < 0.$$

Next, consider $\Gamma(x)$. Adding and subtracting the term $(1-\theta)(\sigma+\varepsilon\theta)\beta^2\varepsilon$ to Γ and factoring the resulting expression gives

$$\Gamma = (1-\theta)(2-E+\varepsilon\theta)\beta^2\varepsilon+(1-\theta)(\sigma-\beta)(\beta\varepsilon+\sigma)\varepsilon\theta-(\beta^2-(1-\theta)\sigma)e^\theta.$$

Making the substitution $e^\theta = \beta-\sigma$ and factoring terms yields

$$\begin{aligned}\Gamma &= (1-\theta)(2-E+\varepsilon\theta)\beta^2\varepsilon-(\beta^2+(1-\theta)\beta\varepsilon^2\theta-(1-\theta)(1-\varepsilon\theta)\sigma)e^\theta\\ &= (1-\theta)(2-E+\varepsilon\theta)\beta^2\varepsilon-((1-\varepsilon\theta)k+(2\varepsilon-1+(1-\theta)\varepsilon^2)\beta\theta)e^\theta,\end{aligned}$$

which is strictly positive for $e^\theta<0$, $\varepsilon\le 1$, $\theta<1$ by Assumption 2.

Proceeding similarly,

$$\begin{aligned}\widetilde{\Pi}_{xx}\widetilde{\Pi}^s_n-\widetilde{\Pi}^s_x\widetilde{\Pi}_{xn} &= \frac{(c+\tau)^2\sigma^2}{k^2n^3}\left[(1-\theta)^2(2-E)+(\beta+\varepsilon\theta)\varepsilon\theta\right] > 0,\\ \widetilde{\Pi}_{mm}\widetilde{\Pi}^s_n-\widetilde{\Pi}^s_m\widetilde{\Pi}_{mn} &= \frac{(c+\tau)^2\beta\varepsilon x^2}{k^2mn^3}[(2\varepsilon-1)\beta+k] > 0.\end{aligned}$$

It remains to show that $\Delta > 0$ for the coefficient matrix of equations (5), (A3), and (A4) to be negative definite. Factoring terms in Δ gives

$$\Delta = (1-\theta)\beta\left[(2\varepsilon-1)\varepsilon\theta\sigma + (k+(2\varepsilon-1)\beta)(2-E)\right] - (k+\beta\theta(2\varepsilon-1))e^{\theta} > 0.$$

Proof of Proposition 1. On substitution of $k(x)$, excise taxes qualitatively influence output per product under multi-product oligopoly as

$$\frac{dx^e}{d\tau} \stackrel{s}{=} (1-\varepsilon)\beta - \sigma,$$
$$\frac{dx^e}{d\alpha} \stackrel{s}{=} \beta - \sigma = -e^{\theta}.$$

When consumers have increasing preferences for variety, $\beta(x) < \sigma(x)$. Noting that $\varepsilon \leq 1$, it follows by inspection that excise taxes reduce output per product. The qualitative effect of excise taxes on product variety under multi-product oligopoly is

$$\frac{dm^e}{d\tau} \stackrel{s}{=} \beta e^{\theta} - (1-\theta)(\beta\varepsilon + (1-\theta)\sigma)(2-E),$$
$$\frac{dm^e}{d\alpha} \stackrel{s}{=} ke^{\theta} - \theta(1-\theta)\varepsilon\sigma^2 - (1-\theta)(\varepsilon k + (1-\theta)\sigma)(2-E).$$

It follows that $e^{\theta}(x) \leq 0$ and Assumption 1 are sufficient conditions for excise taxes to reduce product variety. Per product prices rise in response to excise taxes, since prices decrease monotonically in both output per product and in the product variety range.

In the free-entry oligopoly equilibrium, specific taxes qualitatively influence output per product as

$$\frac{dx^c}{d\tau} \stackrel{s}{=} 1 - 2\varepsilon \quad \leq \quad 0,$$
$$\frac{dx^c}{d\alpha} \stackrel{s}{=} 1 - \varepsilon \quad \geq \quad 0.$$

where the former inequality holds by Assumption 2.

The qualitative effects of excise taxes on product variety in the long-run oligopoly model are

$$\frac{dm^c}{d\tau} \stackrel{s}{=} (\beta + \varepsilon\theta)e^{\theta} - 2\beta\varepsilon(1-\theta)(2-E),$$
$$\frac{dm^c}{d\alpha} \stackrel{s}{=} (k + \varepsilon\theta)e^{\theta} - (1-\theta)\varepsilon^2\theta\sigma - (1-\theta)(k+\beta)(2-E)\varepsilon,$$

By inspection, Assumption 1 and $e^\theta(x) \leq 0$ are sufficient conditions for excise taxes to reduce product variety. The effect of excise taxes on prices is given by

$$\frac{dp^c}{d\tau} \overset{s}{=} (1-\theta)(2\varepsilon-1)\beta\sigma + 2(1-\theta)(2-E)\beta\varepsilon - (\beta+\varepsilon\theta)e^\theta,$$
$$\frac{dp^c}{d\alpha} \overset{s}{=} (1-\theta)((2\varepsilon-\sigma)k + (k+\beta)\varepsilon e^\sigma) - (k+\varepsilon\theta)e^\theta$$

The effect of ad valorem taxes on prices can be written

$$\frac{dp^c}{d\alpha} \overset{s}{=} (1-\theta)[(2\varepsilon-1)k - (1-\theta)(1-\sigma)\varepsilon]\sigma + (1-\theta)(k+\beta)(2-E)\varepsilon - (k+\varepsilon\theta)e^\theta.$$

To bound the conditions under which ad valorem taxes raise prices, set $E = 2$ and $2\varepsilon = 1$. A sufficient condition for prices to rise in response to an increase in ad valorem tax rates is

$$e^\theta \leq \frac{-(1-\theta)^2(1-\sigma)\varepsilon\sigma}{k+\theta\varepsilon}.$$

This inequality holds whenever $\sigma(x^c) > 1$.

Excise taxes alter the number of retailers according to

$$\frac{dn^c}{d\tau} \overset{s}{=} (1-\theta)(2-E)\beta - e^\theta,$$
$$\frac{dn^c}{d\alpha} \overset{s}{=} (1-\theta)\left[\varepsilon\theta\sigma + (2-E)\beta\right] - \theta e^\theta.$$

It follows by inspection of terms that Assumption 1 and $e^\theta(x) \leq 0$ are sufficient conditions for excise taxes to induce entry.

Proof of Proposition 2. Making use of the price effects calculated above and combining terms, the degree to which specific taxes shift into producer prices under multi-product oligopoly is

$$\frac{d(p^e-\tau)}{d\tau} \overset{s}{=} (1-\theta)(\sigma + (\varepsilon+1-E)\beta - e^\theta.$$

Given $e^\theta < 0$, a sufficient condition for the overshifting of specific taxes is $E < \varepsilon + 1 + \sigma/\beta$.

The degree to which specific taxes shift into producer prices in the long-run is

$$\frac{d(p^c-\tau)}{d\tau} \overset{s}{=} (1-\theta)(2\varepsilon+1-E)\beta - e^\theta.$$

Sufficient conditions for the overshifting of specific taxes are $e^{\theta} < 0$ and $E < 2\varepsilon + 1$.

The degree to which ad valorem taxes shift into producer prices in the short-run is

$$\frac{d(1-\alpha)p^e}{d\alpha} \overset{s}{=} (1-\theta)\left[(1+\varepsilon\theta)\sigma + (1-E)\beta\right] - \theta e^{\theta},$$

which implies the overshifting of ad valorem taxes when $e^{\theta} < 0$ and $E < 1 + (1 + \varepsilon\theta)\sigma/\beta$.

Ad valorem taxes shift into producer prices in the long-run as

$$\frac{d(1-\alpha)p^c}{d\alpha} \overset{s}{=} (1-\varepsilon)(1-\theta)(1+\sigma-E)\beta - \theta e^{\theta}.$$

Proof of Proposition 3. Under multi-product oligopoly, excise taxes reduce both output and product variety, and welfare unambiguously declines. Under free-entry oligopoly, the effect of an increase in specific taxes on aggregate welfare at the zero tax position is

$$\frac{\partial W(\phi)}{\partial \tau} = \frac{(1-\theta)cmx}{k}\left[\frac{\sigma}{x}\frac{\partial x}{\partial \tau} + \frac{\beta}{\theta m}\frac{\partial m}{\partial \tau} + \frac{(\beta - 4\varepsilon\theta)\sigma}{4\varepsilon\theta n}\frac{\partial n}{\partial \tau}\right].$$

Incorporating the values from above and factoring terms yields

$$\frac{\partial W(\phi)}{\partial \tau} \overset{s}{=} \left(4\beta^2 - 4(\sigma-\beta)\varepsilon\theta + \beta\sigma\right)e^{\theta} - (2-E)(8\beta\varepsilon + 4\varepsilon\theta\sigma - \beta\sigma) - 4(2\varepsilon-1)\varepsilon\theta\sigma.$$

Specific taxes reduce aggregate welfare when the sum of these three terms is negative. The third term is negative by Assumption 2. The second term reduces to

$$-(2-E)\left[(2\varepsilon-1)\beta + k + 3(2\beta+\theta\sigma)\varepsilon\right],$$

which is negative by Assumptions 1 and 2. The first term reduces to

$$(4(1-\theta)+\sigma)\beta + 4(2\beta-\sigma)\varepsilon\theta)\,e^{\theta}.$$

In the case of increasing preferences for variety, a sufficient condition for this term to be negative is $2\beta > \sigma$. If $\theta \leq 1/2$, $2\beta \geq \beta/(1-\theta) > \sigma$ by $0 < k(x)$. The remaining case to consider is $1/2 < \theta$. In this case, $3\varepsilon\theta > 3\theta/2 > 1-\theta$, where the first inequality holds by Assumption 2. Now suppose specific taxes increase aggregate welfare. Because taxes reduce both output and variety, a necessary condition for specific taxes to increase welfare is $\beta > 4\varepsilon\theta$. But this implies $1-\theta > 3\varepsilon\theta$, which contradicts.

Proceeding similarly, ad valorem taxes influence aggregate welfare as

$$\frac{\partial W(\phi)}{\partial \alpha} \overset{s}{=} \left[\frac{4k+\theta\omega}{1-\theta}\right] e^{\theta} - \left[4(2\varepsilon-1)+3(1-\varepsilon)\sigma\right]\varepsilon\theta\sigma - (2-E)\left[4k+\omega\right]\beta.$$

where $\omega = 4(2\varepsilon-1)\beta + 3(1-\varepsilon)\beta\sigma > 0$ by Assumption 2.

Proof of Proposition 4. In the short run model, the effect of a revenue-neutral tax reform towards ad valorem taxes on output is given by

$$\left.\frac{dx^e}{d\alpha}\right|_{T=0} = \frac{dx^e}{d\alpha} + \frac{dx^e}{d\tau}\left(\left.\frac{d\tau}{d\alpha}\right|_{T=0}\right).$$

Substitution of terms yields

$$\left.\frac{dx^e}{d\alpha}\right|_{T=0} = \frac{(1-\theta)\varepsilon\sigma pkx}{c\Gamma} > 0.$$

Proceeding similarly for the case of product variety provision,

$$\begin{aligned}\left.\frac{dm^e}{d\alpha}\right|_{T=0} &= \frac{kmp}{\varepsilon c\Gamma}\left[(k-\beta)e^{\theta} - (1-\theta)\varepsilon\theta\sigma^2 - (1-\theta)(k-\beta)(2-E)\varepsilon\right] \\ &= \frac{(1-\theta)\sigma kmp}{c\varepsilon\Gamma}\left[(1-\theta)(2-E)\varepsilon - \varepsilon\theta\sigma - e^{\theta}\right].\end{aligned}$$

A sufficient condition for a revenue-neutral tax reform towards the use of ad valorem taxes to increase product variety under short-run oligopoly is

$$e^{\theta} \leq -\varepsilon\theta\sigma$$

The change in aggregate welfare from a revenue-neutral tax reform towards the use of ad valorem taxes is

$$\left.\frac{dW}{d\alpha}\right|_{T=0} = \frac{(1-\theta)\sigma mpx}{\varepsilon\theta\Gamma}\left[(1-\theta)(2-E)\beta\varepsilon - (1-\varepsilon)(1-\theta)\varepsilon\theta\sigma - \beta e^{\theta}\right]$$

A sufficient condition for a revenue-neutral tax reform towards the use of ad valorem taxes to increase aggregate welfare in the short run is

$$e^{\theta} \leq \frac{-(1-\varepsilon)(1-\theta)\varepsilon\theta\sigma}{\beta}.$$

Under free-entry oligopoly, a revenue-neutral tax reform towards ad valorem taxes increases output by inspection of the terms above. The change in product variety from a revenue-neutral tax reform towards the use of ad valorem taxes is given by

$$\begin{aligned}\left.\frac{dm^c}{d\alpha}\right|_{T=0} &= \frac{kmp}{c\varepsilon\Delta}\left[(k-\beta)e^{\theta}-(1-\theta)\varepsilon^2\theta\sigma-(1-\theta)(k-\beta)(2-E)\varepsilon\right] \\ &= \frac{(1-\theta)\sigma kmp}{c\varepsilon\Delta}\left[(1-\theta)(2-E)\varepsilon-\varepsilon^2\theta-e^{\theta}\right].\end{aligned}$$

A sufficient condition for a revenue-neutral tax reform towards the use of ad valorem taxes to increase product variety in the long run is

$$e^{\theta}\le-\varepsilon^2\theta$$

A revenue-neutral tax reform towards the use of ad valorem taxes alters the equilibrium number of retailers as

$$\left.\frac{dn^c}{d\alpha}\right|_{T=0}=\frac{knp}{c\Delta}\left[(1-\varepsilon)(1-\theta)\varepsilon\theta\sigma-(2-E)\beta\varepsilon+\beta e^{\theta}\right].$$

A sufficient condition for a revenue-neutral tax reform towards the use of ad valorem taxes to decrease the equilibrium number of retailers is

$$e^{\theta}\le\frac{-(1-\varepsilon)(1-\theta)\varepsilon\theta\sigma}{\beta}.$$

The change in aggregate welfare from a revenue-neutral tax reform towards the use of ad valorem taxes is

$$\left.\frac{dW}{d\alpha}\right|_{T=0}=\frac{3(1-\theta)\beta\sigma mpx}{4\varepsilon\theta\Delta}\left[(1-\theta)(2-E)\beta\varepsilon-(1-\varepsilon)(1-\theta)\varepsilon\theta\sigma-\beta e^{\theta}\right]$$

A sufficient condition for a revenue-neutral tax reform towards the use of ad valorem taxes to increase aggregate welfare in the long run is

$$e^{\theta}\le\frac{-(1-\varepsilon)(1-\theta)\varepsilon\theta\sigma}{\beta}.$$