Web Appendix: Credible Sales Mechanisms and Intermediaries, by D. McAdams and M. Schwarz

We consider perfect Bayesian equilibria of a repeated version of our Bargaining Game in which "seller reputation R" emerges endogenously. Furthermore, we allow for the possibility that the seller may "commit to reserve price r" by never accepting any offer less than r on the equilibrium path of play.

To simplify the analysis, we will focus on the special case in which buyer values are iid uniform on $[0, 1]$ and in which the seller's per-round delay cost $c(T) = c > 0$ for all T. In this case, the first-price auction with reserve price $r^* = 1/2$ is an optimal auction.

Model. Each "period" the seller plays a multi-round bargaining game with N new buyers who have observed the full past history of play. This game is the same as the Bargaining Game described in the text, except that it allows for a pre-round to determine initial bidder activity. That is, before the beginning of round $T = 0$, each buyer has a round of cheap talk – announcing "continue" or "end" – and the seller indicates which buyers will be "active". Let M_0 denote the number of initially active buyers.

The seller's goal is to maximize the expected present value of her stream of revenues, with respect to discount factor $\delta \in (0, 1)$.

A class of strategies. We consider perfect Bayesian equilibria having strategies indexed by (r, R) (called "(r, R)-strategies"), that proceed as follows: Prior to round $T = 0$, each buyer says "continue" iff his value exceeds r and the seller designates every such buyer as active. (When $r = 0$, as in the text, this pre-round of communication is redundant as it always leads every bidder to be active.) If only one buyer is active at $T = 0$, that buyer offers r and the seller accepts. If $M_0 > 1$ buyer is active at $T = 0$, all players believe that active buyers" values exceed r while inactive buyers" values are less than r. Play then proceeds as in the subgame of the equilibrium presented in the text when active buyers all have values in $[r, \infty)$ and the seller has reputation R. (Since all offers in future rounds are strictly higher than r, the reserve price is never binding in equilibrium in rounds $T > 0$.)

Buyer best response. Given others' (r, R)-strategies, each buyer finds his (r, R)-strategy to be a best response. Any buyer with value less than r believes that he will never win the object and so is willing to announce "end". When $M_0 > 1$, continuation strategies constitute an equilibrium by the argument in the text. When $M_0 = 1$, finally, the active buyer offers r since he believes that the seller will never accept less.

Seller best response. To check whether there is a perfect Bayesian equilibrium corresponding to reserve price r and reputation R, it therefore suffices to check that the seller finds her (r, R)-strategy to be a best response.

Let $\Pi(r, R, c, N)$ be the seller's expected per-period profit under (r, R)-strategies given per-round delay cost c and N bidders. The present value of the seller's future expected profit given (r, R)-strategies is $\frac{\delta}{1-\delta}\Pi(r, R, c, N)$, and any seller deviation can be "punished" by $(0, 0)$-strategies in continuation play for future expected profit $\frac{\delta}{1-\delta}\Pi(0, 0, c, N)$. ($(0, 0)$-strategies always constitute an equilibrium; see Theorem 1 in the text.) Thus, (r, R)-strategies constitute a perfect Bayesian equilibrium if

$$r \leq \frac{\delta}{1-\delta} \left(\Pi(r, R, c, N) - \Pi(0, 0, c, N)\right) \tag{1}$$

$$R \leq \frac{\delta}{1-\delta} \left(\Pi(r, R, c, N) - \Pi(0, 0, c, N)\right) \tag{2}$$

If (1) fails, the seller has an incentive to accept an offer slightly less than r when there is just one active buyer. If (2) fails, the seller has an incentive to solicit surprise rounds of offers.

Let $r(c, N), R(c, N)$ denote the reserve price and reputation that maximize the seller's expected profit, subject to the equilibrium incentive constraints (1,2). By the analysis in the text, note that $\Pi(r, R, c, N)$ is non-decreasing in R for all fixed (r, c, N). Thus,

$$\begin{aligned} r(c,N) &\leq R(c,N) \\ R(c,N) &= \frac{\delta}{1-\delta}\left(\Pi(r(c,N), R(c,N), c, N) - \Pi(0,0,c,N)\right) \end{aligned} \tag{3}$$

Define R^* implicitly by $R^* = \frac{\delta}{1-\delta}\left(\Pi(r^*, R^*, c, N) - \Pi(0,0,c,N)\right)$, where $r^* = 1/2$ is the optimal reserve price. When $R^* \geq r^*$, the seller can credibly commit to a first-price auction with optimal reserve price. In this case, $r(c,N) = r^*$ and $R(c,N)$ is implicitly defined by (3). What if $R^* < r^*$, so that the optimal reserve price is not credible? Over the range of reserve prices less than r^*, the seller's equilibrium profit $\Pi(r, R, c, N)$ is increasing in r, for all fixed (R, c, N). (A higher reserve price increases expected gross revenue, as usual, but also decreases delay by starting the bidding at a relatively high level.) Thus, in this case, $r(c,N), R(c,N)$ are implicitly defined by $r(c,N) = R(c,N)$ and (3).

For the rest of the analysis, we shall focus on the case with two buyers.

Committing to leave money on the table given small per-round cost of delay. Consider first the case in which per-round delay costs are very small ($c \approx 0$). Can the seller sustain a reputation of leaving R on the table during negotiations? If the seller solicits a surprise round of offers, buyers will believe that the seller will never leave any money on the table in future periods. By Theorem 2 in the text, $\Pi(0,0,c,2) \approx \frac{1}{6}$ given two bidders, half of the revenue from a first-price auction with zero reserve price. (We shall refer to this equilibrium outcome as "grim punishment".) By maintaining any reputation $R >> c$ and any reserve price $r \in [0, 1/2]$, on the other hand, the seller will bring in $\Pi(r, R, c, 2) \geq \Pi(0, R, c, 2) \approx \frac{1}{3}$ each period. Thus, the seller can establish a credible reputation to leave $\frac{\delta}{1-\delta}\left(\frac{1}{3} - \frac{1}{6}\right)$ on the table. When $\delta > 0$ and $c \approx 0$, this is enough so that the seller experiences negligible delay in equilibrium.

Committing to a reserve price given small per-round cost of delay. As shown above, there exist equilibria with negligible delay when $c \approx 0$ as long as the seller has some chance of being a repeat player. The seller's expected revenue each period when committing to reserve price $r \in [0, 1/2]$ is $1/3 + r^2 - 4r^3/3$, which exceeds the present value of the expected revenue in grim punishment by $\frac{\delta}{1-\delta}(1/6 + r^2 - 4r^3/3)$. To credibly commit to reserve r, it must be that

$$r \leq \frac{\delta}{1-\delta}\left(\frac{1}{6} + r^2 - 4r^3/3\right) \tag{4}$$

Observe that $r^2 > 4r^3/3$ for all relevant reserve prices $r \in [0, 1/2]$. So, reserve prices greater than or equal to $\min\{1/2, \frac{\delta}{6(1-\delta)}\}$ are credible. Careful examination of (4) shows that, indeed, the optimal reserve $r = 1/2$ is credible iff $\frac{\delta}{1-\delta} > 2$. Theorem A summarizes these arguments.

Theorem A. *Suppose that $\delta > 0$ and $N = 2$. There exists $c^* > 0$ such that, whenever $c < c^*$, the seller can credibly commit to a sales mechanism with (i) negligible delay and (ii) reserve price* $\min\left\{\frac{1}{2}, \frac{\delta}{1-\delta}\frac{1}{6}\right\}$. *Further, the seller's expected revenue can be (approximately) as high as in an optimal auction whenever $\frac{\delta}{1-\delta} > 2$.*

Committing to a reserve price with high per-round cost of delay. Consider next the case in which per-round delay costs are so large ($c > 1/2$) that the seller can credibly commit to a first-price auction with zero reserve price. Now, the seller's "grim punishment" is not so grim, since she can still get 1/3 expected revenue from a first-price auction with zero reserve. The long-run benefit from committing to reserve r is therefore only $\frac{\delta}{1-\delta}(r^2 - 4r^3/3)$. To credibly commit to reserve r, it must be that

$$r \leq \frac{\delta}{1-\delta}(r^2 - 4r^3/3) \tag{5}$$

When $\frac{\delta}{1-\delta} < 6$, routine calculations show that the seller can not credibly commit to *any* reserve price. When $\frac{\delta}{1-\delta} \geq 6$, however, the seller can commit to the optimal reserve $r^* = 1/2$. (When $\frac{\delta}{1-\delta} = 6$, the seller can *only* credibly commit to the optimal reserve!) Theorem B summarizes these arguments.

Theorem B. *Suppose that $\delta > 0$, $N = 2$, and $c > 1/2$. If $\delta \geq \frac{6}{7}$, the seller can credibly commit to a first-price auction with optimal reserve price. If $\delta < \frac{6}{7}$, however, the seller can not credibly commit to any reserve price.*