Money in a Theory of Banking

Douglas W. Diamond and Raghuram G. Rajan

Proof of Corollary 1 to Proposition 4.

First, some definitions. Total date 2 output of produced goods is

$$X_2[\mu^B] = \frac{1}{1-t}(\theta^G C + (1-\theta^G)[\alpha^B C + (1-\alpha^B)\mu^B c]) \text{ and on date 4 it}$$

$$\text{is } X_4[\mu^B] = \frac{C}{1-t}(1-\theta^G)(1-\alpha^B)(1-\mu^B). \text{ Total date 2 liquidity is the sum of cash goods and}$$

produced goods, $L[\mu^B] = q_1 + X_2[\mu^B]$. Let M_0' and B_2' be the quantity of money and bonds

after the open market operations. We assume that the ratio of bond to money when refinancing

maturing debt at date 2, $\frac{B_4}{M_2 + B_4}$, is a constant, independent of changes in date 0 monetary

policy. The value of all nominal claims (money plus bonds) at date 2 is

$$N_2[r_{24}, \mu^B] \equiv tX_2[\mu^B] + \frac{1}{r_{24}} \text{Max}\{q_3 + \frac{B_4}{M_2 + B_4} tX_4[\mu^B], tX_4[\mu^B]\} \text{ and the date 2 present value of a}$$

type B bank's loans is $\Gamma[r_{24}, \mu^B] = \alpha^B \gamma C + \mu^B (1-\alpha^B)c + (1-\mu^B)(1-\alpha^B)\frac{\gamma C}{r_{24}}$. The value of all of

$$V_2^B[r_{24}, \mu^B, M_0', B_2'] = \frac{M_0'}{P_{02}} + \frac{B_2'}{P_{12}} + \Gamma[r_{24}, \mu^B] =$$

a type B bank's assets at date 2 is

$$\text{Max}\{q_1 + \frac{B_2'}{B_2 + M_0'} N_2[r_{24}, \mu^B], N_2[r_{24}, \mu^B]\} + \Gamma[r_{24}, \mu^B].$$

For a type B bank to be solvent with no restructuring and for the goods market to clear:

$$\frac{\delta_0}{P_{02}} = \delta_0 \max\left\{\frac{q_1}{M_0'}, \frac{N_2[r_{24}=1, \mu^B=0]}{M_0' + B_2'}\right\} \leq \min\{X_2[\mu^B=0] + q_1, V_2^B[r_{24}=1, \mu^B=0, M_0', B_2']\}.$$

For the goods market to clear with restructuring of $\mu^B \in (0,1]$ the condition is:

$$\frac{\delta_0}{P_{02}} = \delta_0 \max \left\{ \frac{q_1}{M_0}, \frac{N_2[r_{24} = \frac{\gamma C}{c}, \mu^B]}{M_0' + B_2'} \right\} = X_2[\mu^B] + q_1 \leq V_2^B[r_{24} = \frac{\gamma C}{c}, \mu^B, M_0', B_2'] \}. \text{ Depending on}$$

which min and max is binding, there are several cases. Rather than detail all the cases, we sketch

one. Let $q_1 \geq N_2[r_{24}, \mu^B]$, then $\frac{q_1}{M_0} > \frac{N_2[r_{24}, \mu^B]}{M_0' + B_2'}$ for all M_0 and B_2 , and the nominal interest rate

always exceeds one. Let $X_2[\mu^B] + q_1 < V_2^B[r_{24}, \mu^B, M_0', B_2']$. The money supply after open market

operations to keep the banking system solvent has to be

$$M_0' \geq \frac{\delta_0 q_1}{\min\{X_2[\mu^B] + q_1, V_2^B[r_{24}, \mu^B, M_0', B_2']\}} = \frac{\delta_0 q_1}{X_2[\mu^B] + q_1}. \text{ Let us determine if this can be}$$

achieved with an open market operation. Given an open market operation at the market clearing

bond price from Proposition 1, of $\frac{N_2[r_{24}, \mu^B]M_0'}{q_1(M_0' + B_2')}$, the quantities of money and bonds outstanding

adjust by $(M_0' - M_0)q_1(M_0' + B_2') = (B_2 - B_2')N_2[r_{24}, \mu^B]M_0'$. The feasibility of open market

operations then turns on whether there are enough bonds to buy back given the needed monetary

expansion. Solving for B_2' , substituting $M_0' = \frac{\delta_0 q_1}{X_2[\mu^B] + q_1}$, and then imposing the non-negativity

condition, we require $B_2 \geq \frac{q_1(\delta_0 q_1 - M_0(X_2[\mu^B] + q_1))}{N_2[\mu^B, r_{24}](q_1 + X_2[\mu^B])}$. If does not hold, open market operations

will repurchase bonds down to zero and additional helicopter drops of money will be necessary to

ensure the banking system is solvent.