A Web Appendix for "Matching with Contracts: Corrigendum"

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A.1 Error in the Proof

We investigate which part of the proof of Claim 1 contains an error. Hatfield and Milgrom (2005) first note that we can assume that there are only two hospitals h and h' without loss of generality, as we can assume that doctors find no contract with other hospitals acceptable. Let there exist $x, y \in X$ and $X' \subset X$ such that $z_H = h$ for all $z \in X'$ and $x \in R_h(X') \setminus R_h(X' \cup \{y\})$, noting that $d_1 \equiv x_D \neq y_D \equiv d_2$ since both x and y are elements of $C_h(X' \cup \{y\})$. Then they specify preferences of hospital h' and doctors as follows; $\{x'\} \succ_{h'} \{y'\} \succ_{h'} \emptyset$ where x' and y' are some fixed contracts with $x'_D = d_1, y'_D = d_2$, and all other contracts are unacceptable to h'; $x \succ_{d_1} x' \succ_{d_1} \emptyset$, $y' \succ_{d_2} y \succ_{d_2} \emptyset$ and other contracts are unacceptable; for every doctor in $x_D(C_H(X') \cup C_H(X' \cup \{y\})) \setminus \{d_1, d_2\}$, her elements in $C_H(X') \cup C_H(X' \cup \{y\})$ are the most preferred; all other doctors find all contracts unacceptable.

Below is a quote (p.931) from one of the cases Hatfield and Milgrom (2005) consider.

> Consider a feasible, acceptable allocation X'' such that $y' \in X''$. Since h' and d_2 can have only one contract in X'', $x', y \notin X''$. Then, h's contracts in X'' form a subset of X', <u>so x is not included</u> ... (the underline is added by the current authors)

The underlined part does not hold in general. To see this point, in Example 1 let $x = (d_1, h_c), y = (d_2, h_r), y' = (d_2, h'), X' = \{(d_1, h_r), (d_1, h_c)\}$, and $X'' = \{(d_1, h_c), (d_2, h')\}$ (note that the required assumption $x \in R_h(X') \setminus$

$R_h(X' \cup \{y\})$ is satisfied under this definition). Clearly h's contract in X'' forms a subset of X' and yet $x \in X''$, hence this is a counterexample to the underlined conclusion.

A.2 Proof of Observation 1

Assume that X' is a stable allocation in the associated problem. X' is an allocation in the original problem, since each doctor signs at most one contract by the assumption that X' is an allocation in the associated problem.

In order to show that X' is stable in the original problem, we first show that condition (1) of stability holds. Since the set of contracts and preferences of all the agents except for h are identical in the original and associated problems, $C_a(X')$ is identical in both problems for every agent $a \neq h$. To show $C_h(X') = C_{h_r}(X') \cup C_{h_c}(X')$, first note that $C_{h_r}(X') \cup C_{h_c}(X')$ is equal to $\emptyset, \{(d_1, h_c)\}, \{(d_1, h_r)\}, \{(d_2, h_r)\}$ or $\{(d_2, h_r), (d_1, h_c)\}$ since X' is an allocation and hence d_1 can sign at most one contract. By definition of $\succ_h$, h is willing to keep all the contracts in any of the above sets of contracts. Therefore $C_h(X') = C_{h_r}(X') \cup C_{h_c}(X')$. Hence condition (1) holds in the original problem.

In order to show that condition (2) of stability holds in the original problem, note first that no hospital other than h can form a blocking coalition since preferences of agents except for h are unchanged between the original and associated problems. So suppose on the contrary that there is a blocking coalition including h in the original problem, that is, there exists $X'' \neq C_h(X')$ such that $X'' = C_h(X' \cup X'') \subset C_D(X' \cup X'')$. By enumerating all possible cases of $C_h(X')$ and its block X'' (see Table 1), we see that there exists $x \in X''$ such that $x \succ_{x_D} C_{x_D}(X')$ and either $\{x\} \succ_{h_r} C_{h_r}(X')$ or $\{x\} \succ_{h_c} C_{h_c}(X')$.[10] The existence of such x implies that X' violates condi-

[10]For example, the first row of Table 1 shows that if $C_h(X') = \{(d_1, h_r)\}$ and X' is

tion (2) of stability in the associated problem (x blocks X' in the associated problem), which by contradiction shows that X' is stable in the original problem.

$C_h(X')$	X''	x
$\{(d_1, h_r)\}$	$\{(d_2, h_r), (d_1, h_c)\}$	(d_1, h_c)
$\{(d_2, h_r)\}$	$\{(d_2, h_r), (d_1, h_c)\}$	(d_1, h_c)
$\{(d_2, h_r)\}$	$\{(d_1, h_r)\}$	(d_1, h_r)
$\{(d_1, h_c)\}$	$\{(d_2, h_r), (d_1, h_c)\}$	(d_2, h_r)
$\{(d_1, h_c)\}$	$\{(d_1, h_r)\}$	(d_1, h_r)
$\{(d_1, h_c)\}$	$\{(d_2, h_r)\}$	(d_2, h_r)
$\emptyset$	$\{(d_2, h_r), (d_1, h_c)\}$	(d_2, h_r) or (d_1, h_c)
$\emptyset$	$\{(d_1, h_r)\}$	(d_1, h_r)
$\emptyset$	$\{(d_2, h_r)\}$	(d_2, h_r)
$\emptyset$	$\{(d_1, h_c)\}$	(d_1, h_c)

Table 1: Possible blockings of X' including h.

A.3 Proof of Proposition 1

Following Hatfield and Milgrom (2005) we limit our attention to cases in which there are only two hospitals h and h' without loss of generality. Assume that there exist $x, y \in X$ and $X' \subset X$ such that $z_H = h$ for every $z \in X' \cup \{y\}$, $z, w \in X' \cup \{y\}$ and $z_D = w_D$ imply $z = w$, and

$$x \in R_h(X') \setminus R_h(X' \cup \{y\}). \tag{1}$$

blocked by $X'' = \{(d_2, h_r), (d_1, h_c)\}$, then d_1 should prefer $x = (d_1, h_c)$ to $C_{d_1}(X') = (d_1, h_r)$ and also h_c prefers $\{(d_1, h_c)\}$ to $C_{h_c}(X') = \emptyset$.

Note that $d_1 \equiv x_D$ and $d_2 \equiv y_D$ are different doctors by assumption. By a maintained assumption, there are contracts x' and y' with $x'_H = y'_H = h'$, $x'_D = d_1$ and $y'_D = d_2$.

We specify preferences of h' and doctors as follows: $\{x'\} \succ_{h'} \{y'\} \succ_{h'} \emptyset$ and all other contracts are unacceptable to h'; $x \succ_{d_1} x' \succ_{d_1} \emptyset$ and all other contracts are unacceptable to d_1; $y' \succ_{d_2} y \succ_{d_2} \emptyset$ and all other contracts are unacceptable to d_2; for every doctor d in $x_D(X') \setminus \{d_1\}$, the element $z \in X'$ with $z_D = d$ is acceptable and all other contracts are unacceptable.[11] Finally, all other doctors prefer the null contract most.

Suppose, in order to derive a contradiction, that there is a stable allocation X'' such that $y' \in X''$. Since h' and d_2 can have only one contract in X'', $x', y \notin X''$. Since every doctor $d \in x_D(X')$ prefers the contract in X' most, stability of X'' implies $C_h(X'') = C_h(X')$, so $x \notin X''$ by assumption. Thus d_1 signs a null contract, which is less preferred to x'. Therefore d_1 and h' can block X'' by contract x', which is a contradiction.

Suppose that there exists a stable allocation X'' with $y' \notin X''$.[12] Then, either $x, y \in X''$ or X'' is blocked by a coalition including h, d_1 and d_2 using the contracts x and y. However, if $x, y \in X''$, then a deviation by (d_2, h') to contract y' blocks X'', a contradiction.

[11] Note that $d_2 \notin x_D(X')$ since $z, w \in X' \cup \{y\}$ and $z_D = w_D$ imply $z = w$ and $d_2 = y_D$ by assumption. Also note that z is well-defined since for each doctor $d \in x_D(X') \setminus \{d_1\}$, there is exactly one contract $z \in X'$ with $z_D = d$ by assumption on X'.

[12] The argument of this paragraph is a minor modification of the original proof of Hatfield and Milgrom (2005). The proof is included for completeness.

A.4 Nonexistence of stable allocations with weak substitutes

Example 2. Let $D = \{d_1, d_2, d_3\}$, $H = \{h, h'\}$ and

$$X = \{(d_1, h), (d_2, h), (d_2, h)^*, (d_3, h), (d_3, h)^*, (d_1, h'), (d_3, h')\}$$

with $(d_1, h)_D = (d_1, h')_D = d_1, (d_2, h)_D = (d_2, h)^*_D = d_2, (d_3, h)_D = (d_3, h)^*_D = (d_3, h')_D = d_3$ and $(d_1, h)_H = (d_2, h)_H = (d_2, h)^*_H = (d_3, h)_H = (d_3, h)^*_H = h, (d_1, h')_H = (d_3, h')_H = h'$.

Preferences are given as follows:

$P_{d_1} : (d_1, h') \succ_{d_1} (d_1, h),$

$P_{d_2} : (d_2, h) \succ_{d_2} (d_2, h)^*,$

$P_{d_3} : (d_3, h)^* \succ_{d_3} (d_3, h) \succ_{d_3} (d_3, h'),$

$P_h : \{(d_1, h), (d_2, h), (d_3, h)\} \succ_h \{(d_2, h)^*\} \succ_h \{(d_3, h)^*\} \succ_h \{(d_1, h), (d_3, h)\}$

$\succ_h \{(d_2, h), (d_3, h)\} \succ_h \{(d_1, h), (d_2, h)\} \succ_h \{(d_1, h)\} \succ_h \{(d_3, h)\} \succ_h \{(d_2, h)\},$

$P_{h'} : \{(d_3, h')\} \succ_{h'} \{(d_1, h')\}.$

Preference relation $\succ_h$ satisfies weak substitutes but violates substitutes, while all other preferences satisfy substitutes. A tedious but simple calculation shows that there exists no stable allocation in this problem.

The next example shows that even a pairwise-stable matching (see Roth and Sotomayor (1990)) may fail to exist with the weak substitutes condition.

Example 3. Let $D = \{d_1, d_2, d_3\}$, $H = \{h, h'\}$ and

$$X = \{(d_1, h), (d_2, h), (d_2, h)^*, (d_3, h), (d_3, h)^*, (d_1, h'), (d_3, h')\}$$

with $(d_1, h)_D = (d_1, h')_D = d_1, (d_2, h)_D = (d_2, h)^*_D = d_2, (d_3, h)_D = (d_3, h)^*_D = (d_3, h')_D = d_3$ and $(d_1, h)_H = (d_2, h)_H = (d_2, h)^*_H = (d_3, h)_H = (d_3, h)^*_H = h, (d_1, h')_H = (d_3, h')_H = h'$.

Preferences are given as follows:

P_{d_1} : $(d_1, h) \succ_{d_1} (d_1, h')$,

P_{d_2} : $(d_2, h) \succ_{d_2} (d_2, h)^*$,

P_{d_3} : $(d_3, h)^* \succ_{d_3} (d_3, h') \succ_{d_3} (d_3, h)$,

P_h : $\{(d_1, h), (d_2, h), (d_3, h)\} \succ_h \{(d_2, h), (d_3, h)\} \succ_h \{(d_2, h)^*, (d_3, h)\} \succ_h \{(d_2, h)^*\} \succ_h \{(d_3, h)^*\}$
$\succ_h \{(d_1, h), (d_2, h)\} \succ_h \{(d_1, h), (d_3, h)\} \succ_h \{(d_2, h)\} \succ_h \{(d_1, h)\} \succ_h \{(d_3, h)\}$,

$P_{h'}$: $\{(d_1, h')\} \succ_{h'} \{(d_3, h')\}$.

Preference relation $\succ_h$ satisfies weak substitutes but violates substitutes, while all other preferences satisfy substitutes. By calculation there exists no pairwise-stable matching.

A.5 Nonexistence of stable allocations with weak substitutes and the law of aggregate demand

Let $D = \{d_1, d_2, d_3, d_4\}$, $H = \{h, h'\}$ and

$$X = \{(d_1, h), (d_2, h), (d_2, h)^*, (d_3, h), (d_3, h)^*, (d_4, h), (d_1, h'), (d_3, h')\}$$

with $(d_1, h)_D = (d_1, h')_D = d_1, (d_2, h)_D = (d_2, h)^*_D = d_2, (d_3, h)_D = (d_3, h)^*_D = (d_3, h')_D = d_3, (d_4, h)_D = d_4$ and $(d_1, h)_H = (d_2, h)_H = (d_2, h)^*_H = (d_3, h)_H =$

$(d_3,h)^*_H = (d_4,h)_H = h, (d_1,h')_H = (d_3,h')_H = h'$. Preferences are given by

$P_{d_1} : (d_1,h) \succ_{d_1} (d_1,h'),$

$P_{d_2} : (d_2,h) \succ_{d_2} (d_2,h)^*,$

$P_{d_3} : (d_3,h)^* \succ_{d_3} (d_3,h') \succ_{d_3} (d_3,h),$

$P_{d_4} : (d_4,h),$

$P_h : \{(d_1,h),(d_2,h),(d_3,h)\} \succ_h \{(d_2,h)^*,(d_4,h)\} \succ_h \{(d_3,h)^*,(d_4,h)\} \succ_h \{(d_1,h),(d_2,h)\}$

$\succ_h \{(d_2,h),(d_3,h)\} \succ_h \{(d_1,h),(d_3,h)\} \succ_h \{(d_2,h),(d_4,h)\} \succ_h \{(d_1,h),(d_4,h)\}$

$\succ_h \{(d_3,h),(d_4,h)\} \succ_h \{(d_2,h)^*\} \succ_h \{(d_3,h)^*\} \succ_h \{(d_2,h)\} \succ_h \{(d_1,h)\}$

$\succ_h \{(d_3,h)\} \succ_h \{(d_4,h)\},$

$P_{h'} : \{(d_1,h')\} \succ_{h'} \{(d_3,h')\}.$

Preference relation $\succ_h$ satisfies weak substitutes and the law of aggregate demand but violates substitutes, while all other preferences satisfy substitutes. By calculation there exists no stable allocation in this problem.

A.6 Kelso and Crawford (1982) Labor Matching Model

Although Claim 1 does not hold in the general matching model with contracts, this section shows that the Claim holds in a simplified version of Kelso and Crawford (1982) with finite wages.

In the simplified Kelso and Crawford (1982) environment we assume that $X = D \times H \times W$, where $W = \{\underline{w}, \dots, \overline{w}\}$ is a finite set of possible wages with $\underline{w} = \min W$ and $\overline{w} = \max W$. Given contract $x \in X$, let x_W be the wage component of x, that is, $x_W = w$ if $x = (d,h,w)$. We assume that, keeping the matched hospitals fixed, doctors prefer higher wages, that is, for any $h \in H$ and $d \in D$, $w_d \geq w'_d$ implies $(d,h,w_d) \succeq_d (d,h,w'_d)$. Moreover we assume that preference of each hospital h is quasilinear in wages, that is, the utility of signing contracts $X' \subset D \times \{h\} \times W$ is of the form

$v_h(x_D(X')) - \sum_{x \in X'} x_W$, where v_h is a utility function of h depending on the set of doctors matched with h.[13] In particular, hospitals prefer lower wages, keeping the matched doctors fixed. While the environment is quite general, it excludes more complex terms of contracts such as those of Example 1.

Proposition 2. *Consider the simplified Kelso-Crawford environment, and suppose H contains at least two hospitals, which we denote by h and h'. Further suppose that contracts are not substitutes for h. Then there exist preference orderings for the doctors in set D, a preference ordering for a hospital h' with a single job opening such that, regardless of the preferences of the other hospitals, no stable set of contracts exists.*

Proof. Following Hatfield and Milgrom (2005) we limit our attention to cases in which there are only two hospitals h and h' without loss of generality. Assume that there exist $x, y \in X$ and $X' \subset X$ such that $z_H = h$ for every $z \in X'$ and

$$x \in R_h(X') \setminus R_h(X' \cup \{y\}). \tag{2}$$

Note that $d_1 \equiv x_D$ and $d_2 \equiv y_D$ are different doctors, since $x, y \in C_h(X' \cup \{y\})$ and one doctor can sign at most one contract. Take y to be the contract with the highest wage paid to d_2 that is consistent with (2), that is, y satisfies (2) and $(d_2, h, w) \notin C_h(X' \cup \{(d_2, h, w)\})$ if $w > y_W$.

We can assume without loss of generality that $z \in C_h(X') \cup C_h(X' \cup \{y\})$ for any $z \in X'$ (if $X' \setminus (C_h(X') \cup C_h(X' \cup \{y\}))$ is nonempty, then we can

[13]Following Hatfield and Milgrom (2005), we assume that preferences of hospitals are strict. More specifically, we assume that, for each hospital $h \in H$, the utility function v_h is such that if different sets of doctors are hired in two sets of contracts, then these sets of contracts are not indifferent irrespective of wages, and if the same set of doctors are hired with the same total wages in two sets of contracts, then ties are broken in some deterministic way.

redefine X' by removing such a subset). Moreover, since the preference of h is quasilinear in wages, without loss of generality we can further assume $d_2 \notin x_D(X')$ and, for each doctor d, there is at most one contract $z \in X'$ with $z_D = d$.

We specify preferences of h' and doctors as follows: $\{(d_1, h', \overline{w})\} \succ_{h'} \{(d_2, h', \overline{w})\} \succ_{h'} \emptyset$ and contracts with other doctors are unacceptable to h'[14]; $x \succ_{d_1} (d_1, h', \overline{w}) \succ_{d_1} \emptyset$, $(d_1, h, w) \succ_{d_1} x$ if $w > x_W$, and all other contracts are unacceptable to d_1; $(d_2, h', \overline{w}) \succ_{d_2} y \succ_{d_2} \emptyset$, $(d_2, h, w) \succ_{d_2} (d_2, h', \overline{w})$ if $w > y_W$, and all other contracts are unacceptable to d_2; for every doctor d in $x_D(X') \setminus \{d_1\}$, the element $z \in X'$ with $z_D = d$ is acceptable and all contracts (d, h, w) with $w < z_W$ and contracts with h' are unacceptable.[15] Finally, all other doctors prefer the null contract most.

Suppose, in order to derive a contradiction, that there is a stable allocation X'' such that $(d_2, h', \overline{w}) \in X''$. The following lemma shows that d_1 signs a null contract under such an allocation, which is later used to reach a contradiction. Note that the special assumptions such as quasilinear preferences of hospitals are used in the proof: the corresponding claim (the underlined part of the quote in Section A.1) fails in the general problem with contracts.

Lemma 1. *Suppose that X'' is stable and $(d_2, h', \overline{w}) \in X''$. Then $d_1 \notin x_D(X'')$, that is, d_1 signs a null contract under X''.*

Proof. Since no doctor $d \notin x_D(X') \cup \{d_2\}$ finds a contract with h acceptable, and d_2 is matched with h' and d_2 cannot sign more than one contract, h signs contracts with a subset of $x_D(X')$ under X''. Moreover, every doctor

[14] It is part of our assumption that h' prefers lower wages, for example $\{(d_1, h', w)\} \succ_h \{(d_1, h', w')\}$ if $w < w'$. We suppress such preference relations which are implied by assumption of the model whenever they do not cause confusion.

[15] z is well-defined since for each doctor $d \in x_D(X') \setminus \{d_1\}$, there is exactly one contract $z \in X'$ with $z_D = d$ by assumption on X'.

in $x_D(X')$ who does not sign a contract with h signs a null contract since h' has only one position, h' has a contract $(d_2, h', \overline{w})$ with d_2, and there are only two hospitals h and h'.

Now assume on the contrary that $d_1 \in x_D(X'')$. We shall show that X'' is blocked by an allocation X''' constructed as follows: (i) $x_D(X''') = x_D(C_h(X'))$, (ii) $x' \in X'''$, $x'' \in X''$ and $x'_D = x''_D$ imply $x' = x''$, and (iii) $x' \in X'''$, $x'' \in C_h(X')$ and $x'_D = x''_D \notin x_D(X'')$ imply $x' = x''$. In words, X''' is a set of contracts such that (i) an identical set of doctors sign contracts with h under X''' and under $C_h(X')$, (ii) if a doctor has an existing contract with h under X'', then she signs the same contract under X'' and X''', and (iii) if a doctor who does not sign a contract with h under X'' signs a contract under X''', then she signs a contract that is in X'.

We will show that $X''' \succ_h X''_h$, where $X''_h = \{z \in X'' | z_H = h\}$. To show this, suppose $X''_h \succeq_h X'''$ for contradiction. The assumption $d_1 \in x_D(X'')$ implies $d_1 \in x_D(X''_h)$ since h' can sign at most one contract and $(d_2, h', \overline{w}) \in X''$ by assumption. Since $d_1 \in x_D(X''_h)$ and $d_1 \notin x_D(C_h(X')) = x_D(X''')$ by assumption we have $X''_h \neq X'''$, and hence $X''_h \succ_h X'''$ since preferences of h are strict. Now, consider the following allocation,

$$X^{(4)} = \{z \in X''_h | z_D \in x_D(X''_h) \cap x_D(X''')\} \cup \left(\bigcup_{d \in x_D(X''_h) \setminus x_D(X''')} \{z | z \in X', z_D = d\} \right).$$

In words, $X^{(4)}$ is an allocation in which doctors in $x_D(X''_h)$ sign contracts, wages for doctors common in $x_D(X''_h)$ and $x_D(X''')$ are the same as in X''_h, and wages for doctors in $x_D(X''_h)$ but not in $x_D(X''')$ are changed to the level in X'. By construction, $X^{(4)} \succ_h X''_h$. Therefore $X^{(4)} \succ_h X'''$. Now note that $X^{(5)} = \left(\bigcup_{d \in x_D(X''_h)} \{z | z \in X', z_D = d\} \right)$ and $C_h(X')$ are allocations that reduce the same amount of wages from $X^{(4)}$ and X''', respectively (wages for doctors in $x_D(X''_h) \cap x_D(X''')$ were changed from the levels in X''_h to the levels in X'.). Since h has quasi-linear utility, this fact and $X^{(4)} \succ_h X'''$

imply $X^{(5)} \succ_h C_h(X')$. This is a contradiction to the definition of $C_h(X')$ since $X^{(5)} \subseteq X'$.

From the last paragraph, we have $X''' \succ_h X''_h$. Moreover, it is easily seen that all doctors in $x_D(X''')$ weakly prefer their contracts under X''' to those in X'' (doctors who sign new contracts under X''', i.e. doctors in $x_D(X''') \setminus x_D(X'')$, are strictly better off under X''', since they are assigned null contracts under X''). Hence X''' blocks X'', which contradicts the assumption that X'' is stable. This completes the proof, showing that $d_1 \notin x_D(X'')$. $\square$

Doctor d_1 is assigned the null contract under X'' by Lemma 1, which d_1 prefers less to $(d_1, h', \overline{w})$. By construction of preferences, h' prefers $\{(d_1, h', \overline{w})\}$ to the contract $\{(d_2, h', \overline{w})\}$ assigned under X''. Therefore d_1 and h' can block X'' by $\{(d_1, h', \overline{w})\}$, which contradicts the assumption that X'' is stable. This shows that there exists no stable allocation X'' with $(d_2, h', \overline{w}) \in X''$.

Suppose that there exists a stable allocation X'' with $(d_2, h', \overline{w}) \notin X''$. Since y is a contract under which the highest wage is paid to d_2 consistent with (2) by assumption, X'' is blocked by a coalition including h, d_1 and d_2 unless $x, y \in X''$. However, if $x, y \in X''$, then $\{(d_2, h', \overline{w})\}$ blocks X'', a contradiction. $\square$

Proposition 2 is similar to Proposition 1, as the weak substitutes condition coincides with the substitutes condition in the simplified Kelso-Crawford environment. Note that Proposition 2 is not a special case of Proposition 1, however. This is because, in the simplified Kelso-Crawford model, doctor (hospital) preferences are assumed to be quasilinear and increasing (decreasing) in wages and hence it is not possible to construct preferences of hospital h' and doctors freely, as in a general model of matching with contracts.

Close connections have been established between the substitutes condition and other concepts in the literature. Milgrom (2000) and Ausubel and Milgrom (2002) study the relationship in the auction context. The current result is most closely related to Gul and Stacchetti (1999), who study Walrasian equilibria in a market with discrete goods and continuous prices. They show that if goods are not substitutes for one consumer and if there are a sufficiently large number of consumers, then there are unit demand preferences of consumers such that Walrasian equilibria do not exist. Their Walrasian equilibrium concept is closely related to stability. Our result is the corresponding result in a labor matching model in which wages are discrete and finite.[16] Another study closely related to ours is Sönmez and Ünver (2003). They establish a result analogous to Claim 1 and Gul and Stacchetti (1999) in a many-to-many matching environment in which, in our terminology, a contract is identified by the doctor-hospital pair who sign it.

[16]The connection becomes clear if one associates doctors in our environment with goods in Gul and Stacchetti (1999) and hospitals with consumers. There are some differences, however. For example, the current result holds if there are at least two hospitals, whereas a large number of consumers are needed for the result of Gul and Stacchetti (1999).