Cummins, Hassett, and Oliner, “Investment Behavior, Observable Expectations, and Internal Funds”

Appendix A: Derivation of Equation (1)

This appendix provides the details for the derivation of equation (1), our investment equation under the null of perfect capital markets. As noted in the paper, this equation is a standard one in the investment literature. Firm *i* is assumed to maximize the expected discounted value of future payouts to shareholders:

$$V_{i,t} = E_t\left[\sum_{s=0}^{\infty} \beta_{t+s} \Pi_{i,t+s}\right],$$

where $\Pi_{i,t+s}$ denotes the payout in period *t+s*; β_{t+s} is the discount factor used in period *t* to discount the expected payout in period *t+s,* with $\beta_t = 1$; and $E_t[\cdot]$ denotes an expectation conditioned on information available in period *t*. For simplicity, we abstract from taxes and debt finance, although we account for both in our empirical work.

Assuming that capital is the only quasi-fixed factor and that variable factors have been maximized out of Π, the payout function is

(A1) $$\Pi_{i,t}\left(K_{i,t}, I_{i,t}\right) = p_t\left[F\left(K_{i,t}\right) - G\left(I_{i,t}, K_{i,t}\right)\right] - p_t^k I_{i,t},$$

where $K_{i,t}$ is the firm’s stock of capital, $I_{i,t}$ is gross investment, p_t is the price of output, p_t^k is the price of capital goods, $F\left(K_{i,t}\right)$ is the production function, and $G\left(I_{i,t}, K_{i,t}\right)$ is the adjustment cost function. We assume that $F\left(K_{i,t}\right)$ and $G\left(I_{i,t}, K_{i,t}\right)$ exhibit constant returns to scale and that the firm is a price taker in all markets. These assumptions imply that Π is linear homogeneous in *K* and *I*.

The firm maximizes $V_{i,t}$ by choosing investment, subject to the capital stock accounting identity:

$$K_{i,t+s} = (1-\delta)K_{i,t+s-1} + I_{i,t+s},$$

where δ is the rate of depreciation. Our timing assumption is that investment becomes productive without a lag and that current prices and realizations of current technology shocks are known to the firm when choosing current investment.

Let $\lambda_{i,t}$ denote the shadow value of an additional unit of installed capital in period *t*. Then, the first-order conditions for the constrained maximization are:

(A2) $$-\left(\frac{\partial \Pi_{i,t}}{\partial I_{i,t}}\right) = \lambda_{i,t}$$

and

(A3) $$\lambda_{i,t} = \left(\frac{\partial \Pi_{i,t}}{\partial K_{i,t}}\right) + (1-\delta)\beta_{t+1}E_t\left[\lambda_{i,t+1}\right],$$

where (A3) can be expressed as

(A4) $$\lambda_{i,t} = E_t\left[\sum_{s=0}^{\infty}\beta_{t+s}(1-\delta)^s \frac{\partial \Pi_{i,t+s}}{\partial K_{i,t+s}}\right].$$

Given equation (A1) and price-taking behavior, equation (A2) can be rearranged as

(A5) $$\left(\frac{\partial G_{i,t}}{\partial I_{i,t}}\right) = (q_{i,t}-1)\frac{p_t^k}{p_t},$$

where $q_{i,t} = \lambda_{i,t} / p_t^k$ is marginal *q*, or the ratio of the shadow value of an additional unit of installed capital to its purchase cost.

Combining equations (A2) and (A3) with the capital stock identity, and using the linear homogeneity of Π, we have

$$\lambda_{i,t}(1-\delta)K_{i,t-1} = \Pi_{i,t} + \beta_{t+1}E_t\left[\lambda_{i,t+1}(1-\delta)K_{i,t}\right],$$

which implies that

(A6) $$\lambda_{i,t}(1-\delta)K_{i,t-1} = E_t\left[\sum_{s=0}^{\infty}\beta_{t+s}\Pi_{i,t+s}\right] = V_{i,t}.$$

Recalling that $q_{i,t} = \lambda_{i,t} / p_t^k$, equation (A6) implies that

(A7) $$q_{i,t} = \frac{V_{i,t}}{p_t^k(1-\delta)K_{i,t-1}},$$

where the right-hand side of the equation is the ratio of the firm's value entering period *t* to the replacement value of its capital stock.

The final step in the derivation is to specify an explicit form for the adjustment cost function. We use the standard quadratic function:

$$G(I_{i,t}, K_{i,t}) = \frac{b}{2}\left(\frac{I_{i,t}}{K_{i,t}} - a - \varepsilon_{i,t}\right)^2 K_{i,t},$$

in which adjustment costs depend on the technical coefficients of adjustment, *a* and *b*, and a technology shock $\varepsilon_{i,t}$. If we substitute $\partial G_{i,t} / \partial I_{i,t}$ into equation (A5) and rearrange terms, we obtain equation (1) in the text:

$$\left(\frac{I}{K}\right)_{i,t} = a + \frac{1}{b}(q_{i,t} - 1)\frac{p_t^k}{p_t} + \varepsilon_{i,t} = a + \frac{1}{b}\left[\frac{V_{i,t}}{p_t^k\left(1-\delta\right)K_{i,t-1}} - 1\right]\frac{p_t^k}{p_t} + \varepsilon_{i,t} \equiv a + \frac{1}{b}Q_{i,t} + \varepsilon_{i,t},$$

where the second equality relies on equation (A7), and Q denotes average q.