Resolving Conflicting Preferences in School Choice: The "Boston" Mechanism Reconsidered

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Web Appendix

Proposition 1. *With complete information, common ordinal preferences and strict school priorities under the Boston mechanism, if a naive student i^* becomes strategically sophisticated, that student becomes weakly better off but every other student, strategic as well as naive, becomes weakly worse off. If i^* becomes strictly better off, then some student becomes strictly worse off.*

Proof of Proposition 1: Assume that students have the same ordinal preferences $s_1 \succ s_2 \succ ... \succ s_m$ and schools have strict priorities $\pi = (\pi_{s_1}, ..., \pi_{s_m})$, where π_{s_a} is school a's priorities, represented by an ordered list of all students. Since the students have the same ordinal rankings, every such economy $(\succ, \pi)$ has a unique stable matching, which can be obtained by the following procedure: *Assign the top q_1 students in π_{s_1} to s_1; given the assignments at $s_1, ..., s_{k-1}$, assign the top q_k unassigned students in π_{s_k} to s_k. That matching is also Pareto efficient.* Given a set of sophisticated students M and naive students N and $(\succ, \pi)$, let $(\succ, \tilde{\pi})$ be the associated augmented economy à la Pathak and Sönmez (2008): $\tilde{\pi}_{s_1} = \pi_{s_1}$ and for every $s \neq s_1$, $\tilde{\pi}_s$ ranks all sophisticated students at the top according to π_s then all naive students below according to π_s. The augmented economy $(\succ, \tilde{\pi})$ has a unique stable matching μ. Therefore, μ is the unique complete information Nash equilibrium of the Boston mechanism in the economy $(\succ, \pi)$ with sophisticated students M and naive students N (Proposition 1, Pathak and Sönmez, 2008). Suppose that some $i^* \in N$ becomes sophisticated. Let $(\succ, \tilde{\pi}^*)$ be the associated augmented economy and μ^* be the unique stable matching of $(\succ, \tilde{\pi}^*)$, which is also the unique complete information Nash equilibrium outcome of the Boston mechanism in the economy $(\succ, \pi)$ with sophisticated students $M \cup \{i^*\}$ and naive students $N \backslash \{i^*\}$. Then by construction, i^* improves his standing at every school $s \neq s_1$ in $\tilde{\pi}^*$ in comparison to $\tilde{\pi}$. If $\mu^*(i^*) = \mu(i^*)$, then $\mu^*(i) = \mu(i)$ for every $i \neq i^*$, which follows immediately from the construction of μ and μ^*. If $\mu^*(i^*) \neq \mu(i^*)$, then $\mu^*(i^*) \succ \mu(i^*)$, since i^* improves his standing at every school but s_1 in $\tilde{\pi}^*$. Since μ and μ^* are Pareto efficient, $\mu^*(i^*) \succ \mu(i^*)$ implies that there exists $i_1 \in M \cup N \backslash \{i^*\}$ such that $\mu(i_1) = \mu^*(i^*) \neq \mu^*(i_1)$ and $\mu(i_1) \succ \mu^*(i_1)$. Then either $\mu^*(i_1) = \mu(i^*)$ or there exists $i_2 \in M \cup N \backslash \{i^*, i_1\}$ such that $\mu(i_2) = \mu^*(i_1) \neq \mu^*(i_2)$ and $\mu(i_2) \succ \mu^*(i_2)$. In general, given $\{i^*, i_1, ..., i_k\}$, $k \geq 1$, such that $\mu(i_{l+1}) = \mu^*(i_l) \neq \mu^*(i_{l+1})$, $\mu(i_{l+1}) \succ \mu^*(i_{l+1})$ for all $l = 1, ..., k-1$ and $\mu^*(i_{l+1}) \neq \mu(i^*)$, Pareto efficiency of μ and μ^* implies that there exists $i_{k+1} \in M \cup N \backslash \{i^*, i_1, ..., i_k\}$ such that $\mu(i_{k+1}) = \mu^*(i_k) \neq \mu^*(i_{k+1})$ and $\mu(i_{k+1}) \succ \mu^*(i_{k+1})$. Continuing this iteration, by finiteness we obtain some K such that $\mu^*(i_K) = \mu(i^*)$. Then for every $i \in \{i_1, ..., i_K\}$, $\mu(i) \succ \mu^*(i)$, i.e. i becomes strictly worse off at the unique complete Nash equilibrium of the Boston mechanism when i^* becomes sophisticated. For every $i \in M \cup N \backslash \{i^*, i_1, ..., i_K\}$, $\mu(i) = \mu^*(i)$. This completes the proof. ▌