Supplementary material for "A Theory of Demand Shocks"

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A. Log-linearization

First, let me derive the optimality conditions in their original form. The Euler equation is

$$Q_t \frac{1}{\overline{P}_{l,t} C_{l,t}} = \beta E_{l,t} \left[\frac{1}{\overline{P}_{l,t+1} C_{l,t+1}} \right], \tag{1}$$

the consumer budget constraint is, after some substitutions,

$$Q_t B_{l,t+1} + \overline{P}_{l,t} C_{l,t} = B_{l,t} + P_{l,t} Y_{l,t}, \tag{2}$$

the firm's optimality condition is

$$E_{l,t} \left[\sum_{\tau=t}^{\infty} \theta^{\tau} Q^{l}_{t+\tau|t} \left((\gamma - 1) P^*_{l,t} Y_{j,l,t+\tau} - \gamma \frac{W_{l,t+\tau}}{A_{l,t+\tau}} \frac{Y_{j,l,t+\tau}}{A_{l,t+\tau}} \right) \frac{Y_{j,l,t+\tau}}{P^*_{l,t}} \right] = 0. \tag{3}$$

As a reference point, I will consider the stochastic equilibrium of an economy where σ^2_ϵ is the same as in the economy I want to study, but all other variances are set to zero, implying that (i) there is no heterogeneity and (ii) there is complete information. Using stars to denote variables in the reference economy, I have $A^*_{l,t} = A_t \equiv e^{x_t}$ by definition. It is easy to show that in equilibrium

$$C^*_{l,t} = \bar{C} A_t, \quad Y^*_{j,l,t} = \bar{Y} A_t, \quad \frac{W^*_{l,t}}{P^*_{l,t}} = \bar{W} A_t,$$

the prices $P^*_{j,l,t}$ are all constant, the nominal bond price Q^*_t is constant and equal to $\beta e^{-\frac{1}{2}\sigma^2_\epsilon}$, and bond holdings are constant and equal to zero. To check that these values form an equilibrium and to derive the values of the scalars $\bar{C}$, $\bar{Y}$, and $\bar{W}$, it is sufficient to substitute these expressions in (1)-(3).

Now let me go back to the economy with heterogeneity and dispersed information. I will focus on equilibria that have the following property: conditional on a given sequence of realized shocks $\{\epsilon_t\}$, all equilibrium quantities, relative prices, and rates of inflation move in a neighborhood of the corresponding values in the reference economy. Denote with hats log deviations of quantities and relative prices from their values in the reference economy, e.g., $\hat{c}_{l,t} \equiv \ln C_{l,t} - \ln C^*_{l,t}$. For the rates of inflation, let $\pi_{l,t}, \overline{\pi}_{l,t}$ and π_t be defined, respectively, as $\pi_{l,t} \equiv \ln P_{l,t} - \ln P_{l,t-1}$, $\overline{\pi}_{l,t} \equiv \ln \overline{P}_{l,t} - \ln \overline{P}_{l,t-1}$, and $\pi_t = \ln P_t - \ln P_{t-1}$. Finally, recall that $h_{l,t} \equiv B_{l,t} / E_{l,t} [P_t Y_t]$. The linearized versions of (1)-(3) are obtained by taking Taylor expansions with respect to these variables, which are all zero in the reference economy.

To derive the log-linearized version of (1) multiply both sides by $\overline{P}_{l,t}\bar{C}A_{l,t}$ and take expectations, to obtain

$$Q_t E_{l,t}\left[\frac{\bar{C}A_{l,t}}{C_{l,t}}\right] = \beta E_{l,t}\left[\frac{\overline{P}_{l,t}}{\overline{P}_{l,t+1}}\frac{\bar{C}A_{l,t}}{C_{l,t+1}}\right],$$

which can be rewritten as

$$Q_t E_{l,t}\left[\frac{\bar{C}A_t}{C_{l,t}}\frac{A_{l,t}}{A_t}\right] = \beta E_{l,t}\left[\frac{\overline{P}_{l,t}}{\overline{P}_{l,t+1}}\frac{\bar{C}A_{t+1}}{C_{l,t+1}}\frac{A_{l,t}}{A_t}\frac{A_t}{A_{t+1}}\right],$$

and then as

$$Q_t E_{l,t}\left[e^{-\hat{c}_{l,t}+\eta_{l,t}}\right] = E_{l,t}\left[e^{-\overline{\pi}_{l,t+1}-\hat{c}_{l,t+1}+\eta_{l,t}}\right]\beta e^{-\frac{1}{2}\sigma_\epsilon^2}.$$

This gives the approximate relation

$$E_{l,t}\left[\hat{c}_{l,t}+\eta_{l,t}\right] = -i_t + E_{l,t}\left[\overline{\pi}_{l,t+1}+\hat{c}_{l,t+1}+\eta_{l,t}\right]. \tag{4}$$

The terms $E_{l,t}\left[\eta_{l,t}\right]$ on both sides cancel out. Moreover, the random walk assumption for x_t implies that $E_{l,t}\left[a_t\right] = E_{l,t}\left[a_{t+1}\right] = E_{l,t}\left[x_t\right]$. Adding $\ln\bar{C} + E_{l,t}\left[x_t\right]$ on both sides of (4) then gives

$$E_{l,t}\left[c_{l,t}\right] = E_{l,t}\left[c_{l,t+1}\right] - i_t + E_{l,t}\left[\overline{p}_{l,t+1}-\overline{p}_{l,t}\right].$$

Given that $c_{l,t}$ and $\overline{p}_{l,t}$ are in the information set of consumer l at time t, this gives (24) in the paper.

Proceeding in a similar way, (2) and (3) can be transformed to obtain (25) and (26) in the paper.

B. An extension with decreasing returns and variable capacity

To push a step further my quantitative exploration, I consider a variant of the model with decreasing returns and variable capacity utilization. The purpose of this extension is twofold. First, it allows me to explore a version of the model with stronger strategic complementarity in pricing decisions. Second, it allows me to show that the model can deliver procyclical labor productivity following noise shocks.

The model is identical to the model presented in Section II except for the firm's technology. The production function is

$$Y_{j,l,t} = A_{l,t}U_{j,l,t}^{\alpha}N_{j,l,t}^{1-\alpha},$$

where $U_{j,l,t}$ is a measure of capacity utilization which is chosen by firm j each period. To reach the level $U_{j,l,t}$ of capacity utilization, the firm needs $\chi_0 A_{l,t}U_{j,l,t}^{1+\chi}$ units of the local consumption good as an input, with $\chi > 0$.[1] This is a simple way of introducing a form of variable capacity without explicitly introducing capital in the model. It allows the firm to vary $U_{j,l,t}$ in response to increases in demand for their good.

The analysis of this case is presented in the supplementary material (Section 8). Here, I directly present the simulation results. The value of α is set to a standard value of 0.33 and

[1] This formulation is analogous to that in Christiano, Eichenbaum, and Evans (2005), except for the fact that the capital stock is fixed (i.e. adjustment costs are infinite) and for the presence of $A_{l,t}$ in the cost of capital utilization function. I assume that the cost of capacity utilization moves one-for-one with local productivity to ensure that $U_{j,l,t}$ is stationary despite the non-stationarity of $A_{l,t}$.

the value of χ to 0.1. The parameter χ determines how much the marginal cost of capacity utilization rises with the level of utilization. When χ is larger than a certain threshold $\hat{\chi}$, the response of $U_{j,l,t}$ is relatively inelastic. In this case, labor productivity tends to fall following a positive noise shock, because decreasing returns tend to dominate. When instead χ is below $\hat{\chi}$, the response of $U_{j,l,t}$ is sufficiently elastic, and labor productivity tends to increase following a positive noise shock. The value chosen implies that labor productivity is procyclical following a noise shock.[2]

Figure 1 reports the responses of the same variables reported in Figure 1 in the paper, except that in the second row I report the responses of labor productivity, $y_t - n_t$. The remaining model parameters are the same as in the baseline model except for σ_ω which is re-calibrated at 0.0004. The value of σ_ω has been reduced to keep the volatility of inflation noise in line with the inflation volatility generated by the other shocks, following my approach in the baseline parametrization.

The most immediate difference between the two figures is that the inflation responses to the shocks ϵ_t and e_t are smaller. This is due to the fact that, with decreasing returns in labor, the degree of strategic complementarity in pricing is stronger. This implies that, a given deviation of output from potential generates a milder response of inflation, in line with what happens in standard sticky price models.[3] Therefore, the real interest rate moves less and current consumption is closer to expected future consumption, narrowing the gap between the dotted and the solid lines in the top three panels. Comparing the figures shows that this implies larger real responses of output to noise shocks. However, there is an additional difference between the two cases. After re-calibrating the variance of ω_t, the total effect of the noise shocks ω_t is larger in the economy with variable capacity. The reason for this is that, with smaller responses of inflation to ϵ_t and e_t, it gets harder for consumers to infer the size of the average expectational error from observing aggregate inflation. Even after adjusting downward the volatility of ω_t, the joint combination of real and nominal signals observed by the agents remains less informative. This implies that the shock ω_t ends up having a bigger effect on output.

This discussion emphasizes once more that, in an economy with imperfect information, changes in parameter values have rich effects, since these parameters also affect the inference problems of the individual agents. This is true for variance parameters, as shown in the paper. But it also applies to other parameters, with relatively well-known effects in standard models (e.g., parameters affecting the degree of strategic complementarity in pricing), as they may have additional effects through informational channels.

Profits and marginal costs

It is useful to introduce the following change of variables

$$V_{j,l,t} \equiv U_{j,l,t}^{\frac{1}{1+\chi}}.$$

[2] I experimented with various values of χ and obtained that, all else equal, the choice of χ does not affect much the total response of output to a noise shock, but it affects how this response is decomposed into changes in labor supply and changes in labor productivity.

[3] See Woodford (2003, Chapter 3) for a discussion of the various determinants of strategic complementarity in pricing.

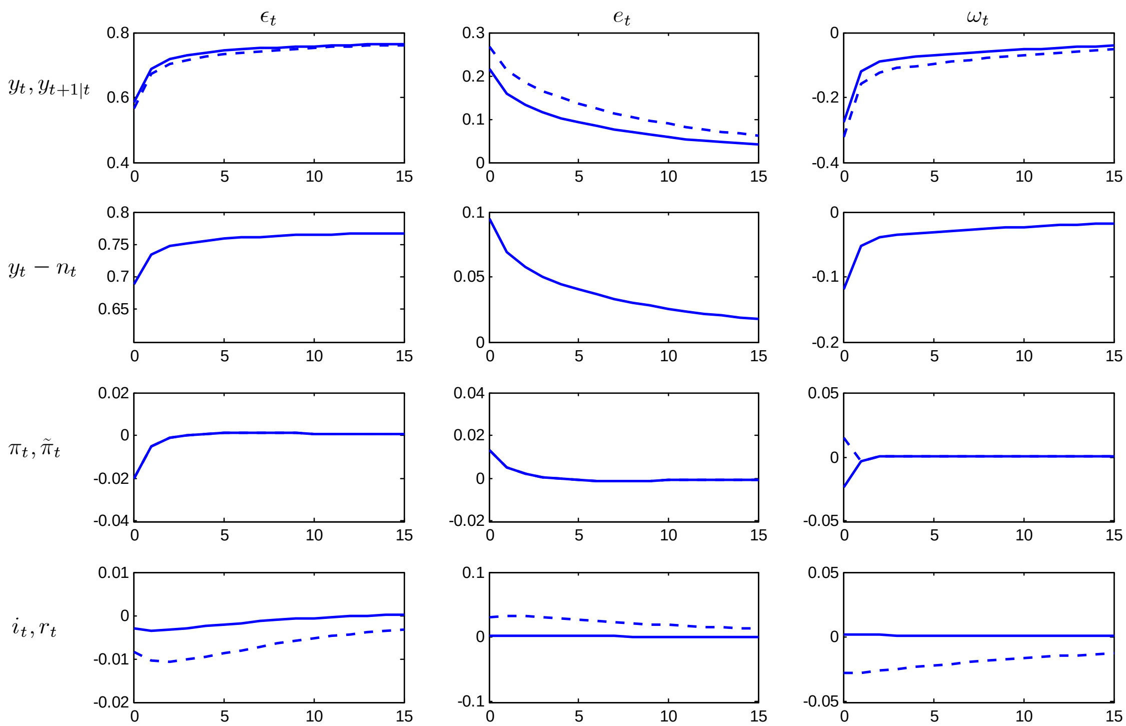

Figure 1: Impulse responses: model with decreasing returns and variable capacity.

Then, the technology takes the form

$$Y_{j,l,t} = A_{l,t} V_{j,l,t}^{\xi} N_{j,l,t}^{1-\alpha},$$

where $\xi \equiv \alpha / (1+\chi) \leq \alpha$. The profits of firm j, l are

$$\Pi_{j,l,t} = P_{j,l,t} A_{l,t} V_{j,l,t}^{\xi} N_{j,l,t}^{1-\alpha} - \chi_0 P_{l,t} A_{l,t} V_{j,l,t} - W_{l,t} N_{j,l,t}.$$

Total costs minimization shows that

$$\chi_0 P_{l,t} A_{l,t} V_{j,l,t} + W_{l,t} N_{j,l,t} = \chi_0^{\tilde{\alpha}} \tilde{\alpha}^{-\tilde{\alpha}} (1-\tilde{\alpha})^{-(1-\tilde{\alpha})} (P_{l,t} A_{l,t})^{\tilde{\alpha}} W_{l,t}^{1-\tilde{\alpha}} \left(\frac{Y_{j,l,t}}{A_{l,t}} \right)^{1+\mu}$$

where

$$\begin{aligned} \tilde{\alpha} &\equiv \frac{\xi}{\xi + 1 - \alpha}, \\ \mu &\equiv \frac{\alpha - \xi}{\xi + 1 - \alpha}. \end{aligned}$$

The marginal cost of firm j, l is then equal to

$$MC_{j,l,t} = \mu_0 \frac{1}{A_{l,t}} (P_{l,t} A_{l,t})^{\tilde{\alpha}} W_{l,t}^{1-\tilde{\alpha}} \left(\frac{Y_{j,l,t}}{A_{l,t}} \right)^{\mu}$$

where μ_0 is a constant term. Taking logs and using the demand function for good j, l, this gives

$$mc_{j,l,t} = mc_{l,t} + \gamma \mu (p_{l,t} - p_{j,l,t}),$$

where $mc_{l,t}$ is given by

$$mc_{l,t} = p_{l,t} + (1-\tilde{\alpha})(w_{l,t} - p_{l,t} - a_{l,t}) + \mu (y_{l,t} - a_{l,t}). \tag{5}$$

Labor market equilibrium

The cost minimization problem analyzed above, gives the relative input demand

$$\frac{V_{j,l,t}}{N_{j,l,t}} = \frac{\xi}{1-\alpha} \frac{W_{l,t}}{P_{l,t} A_{l,t}}.$$

Substituting in the production function gives

$$Y_j = A_{l,t} \left(\frac{\xi}{1-\alpha} \frac{W_{l,t}}{P_{l,t} A_{l,t}} \right)^{\xi} N_{j,l,t}^{1-\alpha+\xi},$$

which, in logs, gives

$$n_{j,l,t} = (1+\mu)(y_{j,l,t} - a_{l,t}) - \tilde{\alpha}(w_{l,t} - p_{l,t} - a_{l,t}),$$

and, aggregating across firms in island l,

$$n_{l,t} = (1+\mu)(y_{l,t} - a_{l,t}) - \tilde{\alpha}(w_{l,t} - p_{l,t} - a_{l,t}).$$

The optimal labor supply condition is

$$w_{l,t} = c_{l,t} + \overline{p}_{l,t} + \zeta n_{l,t}.$$

Substituting and rearranging gives the following expression for real wages in island l:

$$w_{l,t} - p_{l,t} = a_{l,t} + \frac{1}{1+\tilde{\alpha}\zeta}\left(\overline{p}_{l,t} + c_{l,t} - p_{l,t} - a_{l,t} + \zeta\left(1+\mu\right)\left(y_{l,t} - a_{l,t}\right)\right), \tag{6}$$

and the following expression for the capacity-input $v_{l,t}$:

$$\begin{aligned} v_{l,t} &= n_{l,t} + w_{l,t} - p_{l,t} - a_{l,t} = \\ &= \left(1 + \mu + \frac{1-\tilde{\alpha}}{1+\tilde{\alpha}\zeta}\zeta\left(1+\mu\right)\right)\left(y_{l,t} - a_{l,t}\right) + \frac{1-\tilde{\alpha}}{1+\tilde{\alpha}\zeta}\left(\overline{p}_{l,t} + c_{l,t} - p_{l,t} - a_{l,t}\right). \end{aligned} \tag{7}$$

I can also derive an expression for aggregate employment which is

$$n_t = \left(1+\mu\right)\left(y_t - a_t\right) - \frac{\tilde{\alpha}}{1+\tilde{\alpha}\zeta}\left(c_t - a_t + \zeta\left(1+\mu\right)\left(y_t - a_t\right)\right)$$

using (9), derived below, this gives

$$n_t = \left(\frac{1+\mu}{1+\tilde{\alpha}\zeta}\frac{\tau\left(1-\tilde{\alpha}\right) + \left(1-\tau\right)\left(1+\tilde{\alpha}\zeta\right)}{1+\tilde{\alpha}\zeta + \tau\left(1+\mu\right)\left(1+\zeta\right)} - \tilde{\alpha}\right)\left(c_t - a_t\right).$$

Firm-level demand

The demand for the output of firm j, l now comes from two sources: other producers located in island l and final consumers:

$$Y_{j,l,t} = \int_0^1 \left(\frac{P_{j,l,t}}{P_{l,t}}\right)^{-\gamma} \chi_0 A_{l,t} V_{j',l,t} dj' + \int_{\mathcal{C}_{l,t}} \left(\frac{P_{j,l,t}}{\overline{P}_{m,t}}\right)^{-\gamma} C_{m,t} dm.$$

In log approximation this gives

$$y_{j,l,t} = \tau\left(a_{l,t} + v_{l,t} - \gamma\left(p_{j,l,t} - p_{l,t}\right)\right) + \left(1-\tau\right)\left(c_t - \gamma\left(p_{j,l,t} - p_t\right) + \xi^2_{l,t}\right),$$

where the consumers' demand factor is

$$d_{l,t} \equiv c_t + \gamma p_t + \xi^2_{l,t},$$

and the ratio τ is equal to the value of $\chi_0 A^*_t V^*_t / Y^*_t$ in the reference equilibrium with no heterogeneity. Aggregating across producers in island l gives

$$y_{l,t} = \tau\left(a_{l,t} + v_{l,t}\right) + \left(1-\tau\right)\left(d_{l,t} - \gamma p_{l,t}\right).$$

Substituting (7) and rearranging gives

$$y_{l,t} - a_{l,t} = \nu_1\left(\overline{p}_{l,t} + c_{l,t} - p_{l,t} - a_{l,t}\right) + \nu_2\left(d_{l,t} - \gamma p_{l,t} - a_{l,t}\right), \tag{8}$$

where

$$\nu_1 \equiv \frac{1}{1+\tau(1+\mu)\frac{1+\zeta}{1+\tilde{\alpha}\zeta}}\tau\frac{1-\tilde{\alpha}}{1+\tilde{\alpha}\zeta},$$
$$\nu_2 \equiv \frac{1}{1+\tau(1+\mu)\frac{1+\zeta}{1+\tilde{\alpha}\zeta}}(1-\tau).$$

Aggregating gives a relation between $y_t - a_t$ and $c_t - a_t$:

$$y_t - a_t = \frac{\tau\frac{1-\tilde{\alpha}}{1+\tilde{\alpha}\zeta}+1-\tau}{1+\tau(1+\mu)\frac{1+\zeta}{1+\tilde{\alpha}\zeta}}(c_t - a_t). \tag{9}$$

Optimal prices

Optimality for firm j who can update at date t gives

$$\begin{aligned} p^*_{l,t} &= (1-\beta\theta)\sum(\beta\theta)^\tau E_{l,t}[mc_{j,l,t+\tau}] = \\ &= (1-\beta\theta)\sum(\beta\theta)^\tau E_{l,t}[mc_{l,t+\tau}+\gamma\mu p_{l,t}] - \gamma\mu p^*_{l,t}, \end{aligned}$$

which can be rewritten in recursive form as

$$p^*_{l,t} = \frac{1-\beta\theta}{1+\gamma\mu}(mc_{l,t}+\mu\gamma p_{l,t}) + \beta\theta E_{l,t}\left[p^*_{l,t+1}\right].$$

Combining this with

$$p_{l,t} = \theta p_{l,t-1} + (1-\theta)p^*_{l,t},$$

and rearranging, I obtain

$$p_{l,t} - p_{l,t-1} = \frac{1-\theta}{\theta}\frac{1-\beta\theta}{1+\mu\gamma}(mc_{l,t} - p_{l,t}) + \beta E_{l,t}[p_{l,t+1} - p_{l,t}].$$

Using (5) and (6), I get

$$mc_{l,t} - p_{l,t} = \frac{1-\tilde{\alpha}}{1+\tilde{\alpha}\zeta}\left(\overline{p}_{l,t} + c_{l,t} - p_{l,t} - a_{l,t}\right) + \left(\frac{1-\tilde{\alpha}}{1+\tilde{\alpha}\zeta}\zeta(1+\mu)+\mu\right)(y_{l,t} - a_{l,t}).$$

Finally, using (8) and rearranging, I obtain

$$p_{l,t} - p_{l,t-1} = \kappa_1\left(\overline{p}_{l,t} + c_{l,t} - p_{l,t} - a_{l,t}\right) + \kappa_2(d_{l,t} - \gamma p_{l,t} - a_{l,t}) + \beta E_{l,t}[p_{l,t+1} - p_{l,t}], \tag{10}$$

where

$$\kappa_1 \equiv \frac{1-\theta}{\theta}\frac{1-\beta\theta}{1+\mu\gamma}\left(\frac{1-\tilde{\alpha}}{1+\tilde{\alpha}\zeta} + \nu_1\left(\frac{1-\tilde{\alpha}}{1+\tilde{\alpha}\zeta}\zeta(1+\mu)+\mu\right)\right),$$
$$\kappa_2 \equiv \frac{1-\theta}{\theta}\frac{1-\beta\theta}{1+\mu\gamma}\left(\frac{1-\tilde{\alpha}}{1+\tilde{\alpha}\zeta}\zeta(1+\mu)+\mu\right)\nu_2.$$

Net nominal income

The budget constraint now takes the form

$$Q_t B_{l,t+1} + \overline{P}_{l,t} C_{l,t} = B_{l,t} + NI_{l,t},$$

or, in log-linearized form

$$\beta h_{l,t+1} = h_{l,t} + ni_{l,t} - \overline{p}_{l,t} - c_{l,t}. \tag{11}$$

$NI_{l,t}$ represents net nominal income in island l (the sum of nominal wages and profits), which can be written as

$$NI_{l,t} = W_{l,t} N_{l,t} + \int_0^1 \Pi_{j,l,t} dj,$$

or, equivalently, as

$$NI_{l,t} = \int_0^1 \left[P_{j,l,t} Y_{j,l,t} - \chi_0 P_{l,t} A_{l,t} V_{j,l,t} \right] dj.$$

After substituting the demand for $V_{j,l,t}$ in this expression and taking a log-linear approximation, I obtain

$$ni_{j,l,t} = \frac{1}{1-\tau} \left(p_{j,l,t} + y_{j,l,t} \right) - \frac{\tau}{1-\tau} \left(p_{l,t} + a_{l,t} + (1-\tilde{\alpha}) \left(w_{l,t} - p_{l,t} - a_{l,t} \right) + \mu \left(y_{j,l,t} - a_{l,t} \right) \right).$$

Aggregating across j and substituting (6)

$$\begin{aligned} ni_{l,t} &= p_{l,t} + a_{l,t} + \frac{1}{1-\tau} \left(y_{l,t} - a_{l,t} \right) + \\ &\quad - \frac{\tau}{1-\tau} \frac{1-\tilde{\alpha}}{1+\tilde{\alpha}\zeta} \left(\overline{p}_{l,t} + c_{l,t} - p_{l,t} - a_{l,t} + \zeta (1+\mu) \left(y_{l,t} - a_{l,t} \right) \right) - \frac{\tau}{1-\tau} \mu \left(y_{l,t} - a_{l,t} \right). \end{aligned}$$

Substituting (8) and rearranging gives

$$ni_{l,t} = p_{l,t} + a_{l,t} + \upsilon_1 \left(\overline{p}_{l,t} + c_{l,t} - p_{l,t} - a_{l,t} \right) + \upsilon_2 \left(d_{l,t} - \gamma p_{l,t} - a_{l,t} \right), \tag{12}$$

with

$$\begin{aligned} \upsilon_1 &\equiv \left(\frac{1}{1-\tau} - \frac{\tau}{1-\tau} \left(\mu + \zeta (1+\mu) \frac{1-\tilde{\alpha}}{1+\tilde{\alpha}\zeta} \right) \right) \nu_1 - \frac{\tau}{1-\tau} \frac{1-\tilde{\alpha}}{1+\tilde{\alpha}\zeta}, \\ \upsilon_2 &\equiv \left(\frac{1}{1-\tau} - \frac{\tau}{1-\tau} \left(\mu + \zeta (1+\mu) \frac{1-\tilde{\alpha}}{1+\tilde{\alpha}\zeta} \right) \right) \nu_2. \end{aligned}$$

Optimal decision rules

The individual decision rules take the form (30) and (31) in the paper. The consumer Euler equation takes the form (24) in the paper. The optimal pricing condition and the budget constraint, instead, take the form (10) and (11).

Prices Substitute (30) in the paper on the right-hand side of (10) and rearrange to get

$$\begin{aligned}\Lambda p_{l,t} &= p_{l,t-1} - (\kappa_1 + \kappa_2) a_{l,t} + \kappa_1 \left(\overline{p}_{l,t} + c_{l,t}\right) + \kappa_2 d_{l,t} + \\ &\quad + \beta \left(q_h h_{l,t+1} + q_p p_{l,t} + \left(q_a \mathbf{e}_x + q_d \left(\boldsymbol{\psi} + \gamma \mathbf{e}_p\right) + q_z\right) A E_{l,t} \left[\mathbf{z}_t\right]\right) \end{aligned} \tag{13}$$

where

$$\Lambda \equiv 1 + \beta + \kappa_1 + \gamma \kappa_2.$$

Using and (11) and (12), gives me

$$\beta h_{l,t+1} = h_{l,t} + p_{l,t} + a_{l,t} + \upsilon_1 \left(\overline{p}_{l,t} + c_{l,t} - p_{l,t} - a_{l,t}\right) + \upsilon_2 \left(d_{l,t} - \gamma p_{l,t} - a_{l,t}\right) - \overline{p}_{l,t} - c_{l,t}. \tag{14}$$

Substitute (14) in (13). Substitute (30) in the paper on the right-hand side of (13). Match coefficients to get

$$q_p = \frac{1}{\Lambda - q_h + q_h \left(\upsilon_1 + \gamma \upsilon_2\right) - \beta q_p}, \tag{15a}$$

$$q_h = q_p \left[q_h + \varpi b_h\right], \tag{15b}$$

$$q_a = q_p \left[q_h - (\kappa_1 + \kappa_2) - q_h \left(\upsilon_1 + \upsilon_2\right) + \varpi b_a\right], \tag{15c}$$

$$q_d = q_p \left[\kappa_2 + q_h \upsilon_2 + \varpi b_d\right], \tag{15d}$$

$$q_z = q_p \left[\beta \left(q_a \mathbf{e}_x + q_d \left(\boldsymbol{\psi} + \gamma \mathbf{e}_p\right) + q_z\right) A + \varpi b_z\right], \tag{15e}$$

where

$$\varpi \equiv \kappa_1 - q_h \left(1 - \upsilon_1\right).$$

Quantities Similar steps on the consumption side lead to

$$b_h = \frac{b_h}{\beta + b_h \left(1 - \upsilon_1\right)} + \varkappa q_h, \tag{16a}$$

$$b_p = \varkappa q_p, \tag{16b}$$

$$b_a = \frac{b_h}{\beta + b_h \left(1 - \upsilon_1\right)} \left(1 - \upsilon_1 - \upsilon_2\right) + \varkappa q_a, \tag{16c}$$

$$b_d = \frac{b_h}{\beta + b_h \left(1 - \upsilon_1\right)} \upsilon_2 + \varkappa q_d, \tag{16d}$$

$$b_z = \frac{\beta}{\beta + b_h \left(1 - \upsilon_1\right)} \left[-\mathbf{e}_i + \left(b_a \mathbf{e}_x + b_d \left(\boldsymbol{\psi} + \gamma \mathbf{e}_p\right) + b_z\right) A\right] + \varkappa q_z, \tag{16e}$$

where

$$\varkappa \equiv \frac{b_h \left(1 - \upsilon_1 - \gamma \upsilon_2\right) + \beta b_p}{\beta + b_h \left(1 - \upsilon_1\right)}.$$

The rest of the computation proceeds as in the main model (treated in the main Appendix), except that (12) and (35) in the paper are replaced, respectively, by (16) and (15) .