Crises and Capital Requirements in Banking: On Line Appendix

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Final Part of Proof of Proposition 3

Recall from the paper that the expected payment r_b from investment in a bank is given by

$$r_b = \alpha_b (R - Q) p_H + (1 - \alpha_b) \left\{ (1 - \lambda) (R - Q) p_L + \lambda R p_L \frac{k}{k-1} \right\},$$

where $\alpha_b = a + (1 - a) \frac{\mu}{b}$, and that the proportion of wealth which a depositor will invest in a bank given that the regulator is bad is given by

$$R_b = \frac{(\mu - 1) \mu}{N - 2\mu + \frac{\mu^2}{b}}.$$

Then straightforward manipulation yields:

$$\begin{aligned} \frac{\partial}{\partial b} [R_b (r_b - R p_L) + R p_L]_{a=0} &= \frac{\mu^2 (k-1)}{b^2 \left(N - 2\mu + \frac{\mu^2}{b} \right)} \left\{ \left[(R - Q) (\Delta p + p_L) - p_L R \lambda \frac{k}{k-1} \right] \left(\frac{\mu^2}{(N - 2\mu) b + \mu^2} - 1 \right) \right. \\ &\quad \left. + \frac{\mu}{N - 2\mu + \frac{\mu^2}{b}} \left[(1 - \lambda) (R - Q) p_L - \frac{R p_L}{k-1} (k(\lambda + 1) - 1) \right] \right\}. \end{aligned}$$

Since $N > 2\mu$, the first of these terms is clearly negative. The second term has the same sign as $(1 - \lambda) (R - Q) p_L - \frac{R p_L}{k-1} (k(\lambda + 1) - 1) < (R - Q) p_L - R p_L \frac{k}{k-1} < 0$: this concludes the proof.

Final Part of Proof of Proposition 4

Recall from the paper that

$$\begin{aligned} R^{IR}(a, k) &\equiv R \frac{(aN - 2a\mu + \mu) \Delta p + \frac{\lambda p_L}{k-1} (1 - a) (N - \mu)}{p_H (aN - 2a\mu + \mu) + p_L (1 - a) (N - \mu) (1 - \lambda)}, \\ B^P(a, k) &\equiv \frac{R ((k-1) \mu (a (N - 2\mu) + \mu) \Delta p + \lambda (N - \mu) (N - a\mu) p_L)}{(k-1) (\mu (a (N - 2\mu) + \mu) p_H + (1 - \lambda) (N - \mu) (N - a\mu) p_L)}. \end{aligned}$$

Differentiation of these expressions with respect to a and manipulation yields:

$$\begin{aligned}\frac{\partial}{\partial a} R^{IR}(a, k) &= \frac{R(N - \mu)^2 p_L \left((1 - \lambda) \Delta p - \frac{\lambda p_H}{k-1} \right)}{((a(N - 2\mu) + \mu) p_H + (1 - a)(1 - \lambda)(N - \mu) p_L)^2}; \\ \frac{\partial}{\partial a} B^P(a, k) &= \frac{R(N - \mu)^3 \mu p_L \left((1 - \lambda) \Delta p - \frac{\lambda p_H}{k-1} \right)}{(\mu(a(N - 2\mu) + \mu) p_H + (1 - \lambda)(N - \mu)(N - a\mu) p_L)^2}.\end{aligned}$$

As required, both of these expressions are positive multiples of $\left((1 - \lambda) \Delta p - \frac{\lambda p_H}{k-1} \right)$.

Recall further that

$$\begin{aligned}B^O(k) &\equiv \frac{R \left(\mu \Delta p + \lambda p_L \left(\frac{N - \mu}{k-1} \right) \right)}{N(1 - \lambda) p_L + \mu(p_H - (1 - \lambda) p_L)}; \\ B^{OP}(a, k) &\equiv \frac{R\lambda}{(k - 1)(1 - \lambda)}.\end{aligned}$$

Then manipulations yield the following:

$$\begin{aligned}B^P - B^{OP} &= \frac{R\mu(a(N - 2\mu) + \mu) \left((1 - \lambda) \Delta p - \frac{\lambda p_H}{k-1} \right)}{(1 - \lambda)(\mu(a(N - 2\mu) + \mu) p_H + (1 - \lambda)(N - \mu)(N - a\mu) p_L)}; \\ B^O - B^P &= \frac{(1 - a) R(N - \mu)^2 \mu p_L \left((1 - \lambda) \Delta p - \frac{\lambda p_H}{k-1} \right)}{(\mu p_H + (1 - \lambda)(N - \mu) p_L)(\mu(a(N - 2\mu) + \mu) p_H + (1 - \lambda)(N - \mu)(N - a\mu) p_L)}; \\ R^{IR} - B^O &= \frac{a R(N - \mu)^2 p_L \left((1 - \lambda) \Delta p - \frac{\lambda p_H}{k-1} \right)}{(\mu p_H + (1 - \lambda)(N - \mu) p_L)((a(N - 2\mu) + \mu) p_H + (1 - a)(1 - \lambda)(N - \mu) p_L)}.\end{aligned}$$

Once again, each of these expressions is a positive multiple of $\left((1 - \lambda) \Delta p - \frac{\lambda p_H}{k-1} \right)$. Note moreover that $(R^{IR} - B^O)|_{a=0} \equiv 0$.

Detailed Manipulations from the Proof of Proposition 5

Recall from the paper that the welfare difference between tightly and loosely regulated economies when pessimistic expectations obtain is:

$$G(a) = (k^P - 1) - (k^M - 1) \frac{\mu + a(N - \mu)}{N}.$$

Furthermore,

$$\begin{aligned}k^P &= 1 + \frac{R\lambda(N - \mu)(N - a\mu) p_H p_L}{C(1 - \lambda)(N - \mu)(N - a\mu) p_L - (\mu(a(N - 2\mu) + \mu) p_H (R\Delta p - C))}; \\ k^{BM} &= 1 + \frac{(1 - a) R\lambda(N - \mu) p_H p_L}{C(1 - \lambda)(N - \mu)(1 - a) p_L - p_H((a(N - 2\mu) + \mu) (R\Delta p - C))}; \\ k^{MM} &= \frac{R(N - a\mu) p_L (\Delta p + \lambda p_L)}{(1 - a) C(1 - \lambda)(N - \mu) p_L - (a(N - 2\mu) + \mu) (R\Delta p (\Delta p + \lambda p_L) - C p_H)}.\end{aligned}$$

We use these to determine the welfare gap. Firstly, when $k^{BM} < k^{MM}$, substitution and extensive manipulation yields

$$G(a)|_{k^{BM} < k^{MM}} = \frac{R\lambda(N-\mu)(N-a\mu)p_H p_L}{C(1-\lambda)(N-\mu)(N-a\mu)p_L - (\mu(a(N-2\mu) + \mu)p_H(R\Delta p - C))} - \left(\frac{(1-a)R\lambda(1-\frac{\mu}{N})(a(N-\mu) + \mu)p_H p_L}{C(1-a)(1-\lambda)(N-\mu)p_L - ((a(N-2\mu) + \mu)p_H(R\Delta p - C))} \right).$$

Whence further manipulation yields, when $k^{BM} < k^{MM}$,

$$(1) \quad G'(a)|_{k^{BM} < k^{MM}} = R\lambda(N-\mu)^2 p_H p_L \times \left\{ - \left(\frac{a(a(N-2\mu) + 2\mu)p_H(R\Delta p - C) + (1-a)^2 C(1-\lambda)(N-\mu)p_L}{N(C(1-a)(1-\lambda)(N-\mu)p_L - ((a(N-2\mu) + \mu)p_H(R\Delta p - C)))^2} \right) + \frac{(N-\mu)\mu p_H(R\Delta p - C)}{(C(1-\lambda)(N-\mu)(N-a\mu)p_L - (\mu(a(N-2\mu) + \mu)p_H(R\Delta p - C)))^2} \right\}.$$

When $k^{BM} > k^{MM}$, manipulation again yields the following:

$$G(a)|_{k^{BM} > k^{MM}} = \frac{R\lambda(N-\mu)(N-a\mu)p_H p_L}{C(1-\lambda)(N-\mu)(N-a\mu)p_L - (\mu(a(N-2\mu) + \mu)p_H(R\Delta p - C))} - \frac{a(N-\mu) + \mu}{N} \times \left\{ -1 + \frac{R(N-a\mu)p_L(\Delta p + \lambda p_L)}{C(1-a)(1-\lambda)(N-\mu)p_L - (a(N-2\mu) + \mu)(R\Delta p(\Delta p + \lambda p_L) - Cp_H)} \right\},$$

and

$$(2) \quad G'(a)|_{k^{BM} > k^{MM}} = \frac{N-\mu}{N} + \frac{R\lambda(N-\mu)^3 \mu p_H^2 (R\Delta p - C) p_L}{(C(1-\lambda)(N-\mu)(N-a\mu)p_L - (\mu(a(N-2\mu) + \mu)p_H(R\Delta p - C)))^2} - \frac{(a(N-\mu) + \mu)}{N} \frac{R(N-\mu)^2 (R\Delta p - C) p_L (\Delta p + \lambda p_L)^2}{(C(1-\lambda)(1-a)(N-\mu)p_L - (a(N-2\mu) + \mu)(R\Delta p(\Delta p + \lambda p_L) - Cp_H))^2} - \frac{(N-\mu)}{N} \frac{R(N-a\mu)p_L(\Delta p + \lambda p_L)}{C(1-\lambda)(1-a)(N-\mu)p_L - (a(N-2\mu) + \mu)(R\Delta p(\Delta p + \lambda p_L) - Cp_H)}.$$

Note that when $a = 0$, R^{IR} coincides with B^O , and hence that $k^{BM} < k^{MM}$. Hence:

$$G(0) = \frac{R\lambda\mu(1-\frac{\mu}{N})p_H p_L}{C(1-\lambda)(N-\mu)p_L - \mu p_H(R\Delta p - C)} + \frac{R\lambda(N-\mu)p_H p_L}{C(1-\lambda)(N-\mu)p_L - \mu\frac{\mu}{N}p_H(R\Delta p - C)},$$

as in the paper.

Substituting into equations (1) and (2) yields the following in both cases:

$$G'(a)|_{C=R\Delta p} = -\frac{\lambda(N-\mu)p_H}{N(1-\lambda)\Delta p} < 0,$$

as required.