Online Appendix:
“A Theory of Charitable Fund-raising with Costly Solicitations,”
by Alvaro J. Name-Correa and Huseyin Yildirim

In this appendix, we provide the proofs of Propositions 2 and 4. We also formalize the arguments that were intuitively explained in the text due to space constraints.

FORMAL DERIVATION OF $\hat{C}_i$:

Consider person i 's *solo* decision to cover the entire fund-raising cost, C . Note that person i would receive utility $u_i(w_i, 0)$, if he contributed nothing. Otherwise, he would have to choose $g_i \geq C$ to maximize $u_i(w_i - g_i, g_i - C)$. The solution to this problem is $g_i = C + f_i(w_i - C)$, resulting in indirect utility: $V_i(w_i - C) = u_i(w_i - C - f_i(w_i - C), f_i(w_i - C))$. Clearly, $V_i(w_i - C)$ is increasing in $(w_i - C)$. For $C = 0$, we have $V_i(w_i) > u_i(w_i, 0)$ because $f_i(w_i) > 0$, whereas for $C = w_i$, we have $V_i(0) \leq u_i(w_i, 0)$. Hence, there is a unique cutoff cost, $\hat{C}_i \in (0, w_i]$ such that when alone, person i would consume some public good if and only if $C < \hat{C}_i$.

In the text, we assume that $c_i < \hat{C}_i$ for all $i \in N$. That is, in a one-person economy, person i would always prefer to cover his solicitation cost and consume some public good. While this assumption is easier to interpret, the following lemma offers a weaker condition.

LEMMA A3: *Let $k \in N$ be the largest index such that $\Delta_k > 0$ as in Proposition 1. Then, if, for all $i \in N$,*

$$(1) \quad f_i(w_i - \hat{C}_i) < \Phi_k^{-1}(\sum_{j=1}^k (w_j - c_j)),$$

then $\sum_{j=1}^k c_j - G_{-i}^ < \hat{C}_i$ for $i = 1, \dots, k$. Moreover, (1) holds if $c_i < \hat{C}_i$ for all $i \in N$.*

PROOF: Note that $\bar{G}^* = \Phi_k^{-1}(\sum_{j=1}^k (w_j - c_j)) > 0$ is the equilibrium among $i = 1, \dots, k$ if $\sum_{j=1}^k c_j - G_{-i}^* = g_i^* - \bar{G}^* < \hat{C}_i$. Note also that in any equilibrium, person i gives less than his stand-alone contribution at the cutoff cost: $g_i^* < \hat{g}_i = \hat{C}_i + f_i(w_i - \hat{C}_i)$. This means that if $\hat{g}_i - \bar{G}^* \leq \hat{C}_i$ is satisfied, so is $g_i^* - \bar{G}^* < \hat{C}_i$. Inserting and simplifying terms into the former condition result in (1).

Next, we show that $f_i(w_i - \hat{C}_i) \leq \Phi_k^{-1}(\sum_{j=1}^k (w_j - \hat{C}_j))$ is automatically satisfied for $i = 1, \dots, k$. Let $\hat{w}_i \equiv w_i - \hat{C}_i$, which is nonnegative. If $\hat{w}_i = 0$ for all i , then the inequality trivially follows because $f_i(0) = \Phi_k(0) = 0$. If $\hat{w}_i > 0$ for some i , then, by Bergstrom,

Blume, and Varian (1986), there is a unique equilibrium $G^* > 0$ and $\Phi_k(G^*) = \sum_{j=1}^k \hat{w}_j$. But since $G^* \geq f_i(\hat{w}_i + G_{-i}^*) \geq f_i(\hat{w}_i)$ for $i = 1, \dots, k$, the inequality again follows. Finally, if, as assumed in the text, $c_i < \hat{C}_i$ for $i \in N$, then $\Phi_k^{-1}(\sum_{j=1}^k (w_j - \hat{C}_j)) < \Phi_k^{-1}(\sum_{j=1}^k (w_j - c_j))$, which implies (1).

PROOFS OF PROPOSITIONS 2 AND 4

PROOF OF PROPOSITION 2: Let $\mathcal{F}_i$ be donor i 's belief about the fund-raiser set when he is contacted. Then, as stated in the text, $\mathcal{F}_i = F^o$ if $i \in F^o$, and $\mathcal{F}_i = F^o \cup \{i\}$ if $i \notin F^o$. We will show that given the beliefs $\{\mathcal{F}_i\}_{i=1}^n$, contacting $j \notin F^o$ is not a profitable deviation for the fund-raiser. In particular, letting g_j^o be j 's contribution in this case, we will show that $g_j^o \leq c_j$. To the contrary, suppose $g_j^o > c_j$. Then, upon being contacted, person j would expect others' gross contributions to be $G^*(F^o)$, resulting in

$$(2) \quad \phi_j(\bar{G}^*(F^o) + g_j^o - c_j) - (\bar{G}^*(F^o) + g_j^o - c_j) = w_j - g_j^o.$$

On the other hand, if the individuals in F^o knew about the presence of j before contributing, then, by Lemma 1, we would have: (1) $g_j^*(F_{+j}^o) \leq c_j$, and (2) $\bar{G}^*(F^o) \geq \bar{G}^*(F_{+j}^o)$, where $F_{+j}^o \equiv F^o \cup \{j\}$. This is because F^o is optimal and $j \notin F^o$. Furthermore,

$$(3) \quad \phi_j(\bar{G}^*(F_{+j}^o)) - \bar{G}^*(F_{+j}^o) \geq w_j - g_j^*(F_{+j}^o).$$

Note that $g_j^*(F_{+j}^o) \leq c_j$ and $c_j < g_j^o$ imply that $g_j^*(F_{+j}^o) < g_j^o$, which, in turn, implies that $w_j - g_j^*(F_{+j}^o) > w_j - g_j^o$. Then, since $\phi_j' > 1$, eq.(2) and (3) reveal that $\bar{G}^*(F_{+j}^o) > \bar{G}^*(F^o) + g_j^o - c_j$. But, since $g_j^o > c_j$ by hypothesis, we have $\bar{G}^*(F_{+j}^o) > \bar{G}^*(F^o)$, contradicting the optimality of the set F^o . Hence, $g_j^o \leq c_j$.

PROOF OF PROPOSITION 4: Note that since $\bar{G}^*(F_R^o) = \max_F \{G(F) - C(F) + R, 0\}$, in equilibrium $G^*(F_R^o) - C(F_R^o) \geq 0$; otherwise $F_R^o = \emptyset$, which guarantees the public good level R . That is, even with an outside grant R , the fund-raiser tries to maximize $G(F) - C(F)$ in equilibrium. In particular, Lemma 1 still holds. Since $\bar{G}^*(F_R^o) > 0$, equilibrium condition for individual $i \in F_R^o$ implies that $\bar{G}^*(F_R^o) = f_i(w_i + G_{-i}^*(F_R^o) - (C(F_R^o) - R))$. This has two implications: First, as in Lemma A1, summing over all $i \in F_R^o$, we obtain $\Phi_{F_R^o}(\bar{G}^*(F_R^o)) = \sum_{i \in F_R^o} (w_i - c_i) + R$. Second, re-arranging terms, we have $g_i^*(F_R^o) - c_i = f_i(w_i - c_i + \sum_{j \neq i} (g_j^*(F_R^o) - c_j) + R) - [\sum_{j \neq i} (g_j^*(F_R^o) - c_j) + R]$. But, this means that $g_i^*(F_R^o) - c_i > 0$ if and only if $\sum_{j \neq i} (g_j^*(F_R^o) - c_j) + R > G_i^0$, where G_i^0 is exactly as defined in the text for $R = 0$. Thus, i is a net contributor in equilibrium if and only if

$\Phi_{F_R^o}(G_i^0) > \sum_{i \in F_R^o} (w_i - c_i) + R$, or equivalently $\Delta_i > R$, where Δ_i is stated in Proposition 1.

To prove part (a), suppose that $\bar{G}^*(F_0^o) > 0$. Then, $F_0^o \neq \emptyset$. If $F_R^o = \emptyset$, clearly $F_R^o \subset F_0^o$. Suppose $F_R^o \neq \emptyset$. If $i \in F_R^o$, then $\Delta_i > R$, which implies that $\Delta_i > 0$ and thus $i \in F_0^o$. Hence, $F_R^o \subseteq F_0^o$. Next, we prove part (b), i.e., $\bar{G}^*(F_0^o) < \bar{G}^*(F_R^o)$. Suppose not. That is, suppose $\bar{G}^*(F_0^o) \geq \bar{G}^*(F_R^o)$ even though $F_R^o \subseteq F_0^o$. Then,

$$\sum_{i \in F_0^o} (w_i - c_i) = \Phi_{F_0^o}(\bar{G}^*(F_0^o)) \geq \Phi_{F_0^o}(\bar{G}^*(F_R^o)) \geq \sum_{i \in F_0^o} (w_i - c_i) + R,$$

implying that $R \leq 0$, a contradiction. Hence, $\bar{G}^*(F_0^o) < \bar{G}^*(F_R^o)$. Finally, note that in equilibrium $g_i^*(F_R^o) = w_i + \bar{G}^*(F_R^o) - \phi_i(\bar{G}^*(F_R^o))$. Since $\bar{G}^*(F_0^o) < \bar{G}^*(F_R^o)$ and $\phi_i' > 1$, it follows that $g_i^*(F_R^o) < g_i^*(F_0^o)$. As a result, by the strict normality, $\bar{G}^*(F_R^o) < R + \bar{G}_0^*$.

FORMAL DETAILS OF THE EXTENSIONS

SECTION V.A: Warm-Glow Giving

With warm-glow giving, we assume that $u_i = u_i(x_i, \bar{G}, \max\{g_i - c_i, 0\})$; in particular, we assume that the warm-glow benefit is received from the net contribution. Our objective here is to prove our claim that Lemma 1 holds under the warm-glow extension, and explicitly solve the CES example. Note first that the existence of a unique $\hat{C}_i \in (0, w_i]$ can be established exactly as in the pure altruism case above: simply compare payoffs $u_i(w_i, 0, 0)$ and $V_i(w_i - C) = \max u_i(w_i - g_i, g_i - C, g_i - C)$ subject to $g_i \geq C$. Next, as in the text, define $\hat{\phi}_i(G, w_i)$ to be the unique solution to: $\hat{f}_i(w_i + \hat{\phi}_i, \hat{\phi}_i) = G$. Using the properties of $\hat{f}_i$, it follows that $\hat{\phi}_{i1} > 1$ and $-1 \leq \hat{\phi}_{i2} < 0$, where the second index represents respective partial derivative. In addition, recall from the text that $\hat{\Phi}_F(G, \mathbf{w}) \equiv \sum_{i \in F} (\hat{\phi}_i(G, w_i) - G) + G$, and $F_C \equiv \{i \in F | g_i^*(F) > 0\}$.

LEMMA A1': If $\bar{G}^*(F) > 0$, then $\hat{\Phi}_{F_C}(\bar{G}^*(F), \mathbf{w} - \mathbf{c}) = -\sum_{i \in F \setminus F_C} c_i$, and $\hat{\Phi}_F(\bar{G}^*(F), \mathbf{w} - \mathbf{c}) \geq 0$.

PROOF: Suppose $\bar{G}^*(F) > 0$. If $i \in F_C$, then $\bar{G}^* = \hat{f}_i(w_i - c_i + G_{-i}^* - C + c_i, G_{-i}^* - C + c_i)$, which implies $\hat{\phi}_i(\bar{G}^*(F), w_i - c_i) - c_i = G_{-i}^* - C(F)$. Summing over all $i \in F_C$, and arranging terms yield $\hat{\Phi}_{F_C}(\bar{G}^*(F), \mathbf{w} - \mathbf{c}) = -\sum_{i \in F \setminus F_C} c_i$. Moreover, since $\hat{\phi}_i(\bar{G}^*(F), w_i - c_i) - c_i \geq \bar{G}^*(F)$ for $i \in F \setminus F_C$, summing over all $i \in F$, we obtain $\hat{\Phi}_F(\bar{G}^*(F), \mathbf{w} - \mathbf{c}) \geq 0$, as desired.

PROOF OF LEMMA 1 (with Warm-Glow): ($\implies$) Let F^o be the unique optimal fundraiser set. Suppose that $i \in F^o$ but, contrary to C1, $g_i^*(F^o) \leq c_i$. Since $F^o \neq \emptyset$, clearly

$\bar{G}^*(F^o) > 0$. Next, we show that $F^o = F_C^o$. Since $F_C^o \subseteq F^o$ by definition, we only show that $F^o \subseteq F_C^o$. Suppose not. Then, $j \in F^o$ but $j \notin F_C^o$ for some j . That is, person j is contacted even though $g_j^*(F^o) = 0$. Then, Lemma A1' reveals that $\hat{\Phi}_{F_C^o}(\bar{G}^*(F^o), \mathbf{w} - \mathbf{c}) < -\sum_{i \in F^o \setminus F_C^o} c_i + c_j \leq \hat{\Phi}_{F_C^o}(G^*(F_C^o) - C(F_{-j}^o), \mathbf{w} - \mathbf{c})$. Since $\partial \hat{\Phi}_{F_C^o} / \partial G > 0$, we have $G^*(F_C^o) - C(F_{-j}^o) > \bar{G}^*(F^o)$. Given this, note that if $i \notin F_C^o$ under cost $C(F^o)$, then $i \notin F_C^o$ under cost $C(F_{-j}^o)$. Thus, $F_{-j,C}^o \subseteq F_C^o$, which implies $\bar{G}^*(F_{-j}^o) = G^*(F_C^o) - C(F_{-j}^o)$, and in turn, $\bar{G}^*(F_{-j}^o) > \bar{G}^*(F^o)$, contradicting the optimality of F^o . Hence, $F^o = F_C^o$.

Now, recall our hypothesis that $i \in F^o$ and $g_i^*(F^o) \leq c_i$. We also know that $g_i^*(F^o) > 0$, and thus $\hat{\phi}_i(\bar{G}^*(F^o), w_i - c_i) - \bar{G}^*(F^o) = -(g_i^*(F^o) - c_i)$. Inserting this into the equilibrium condition in Lemma A1': $\hat{\Phi}_{F^o}(\bar{G}^*(F^o), \mathbf{w} - \mathbf{c}) = 0$, we obtain

$$\hat{\Phi}_{F_{-i}^o}(\bar{G}^*(F^o), \mathbf{w} - \mathbf{c}) = g_i^*(F^o) - c_i \leq 0 \leq \hat{\Phi}_{F_{-i}^o}(\bar{G}^*(F_{-i}^o), \mathbf{w} - \mathbf{c}),$$

where the last inequality is due to Lemma A1'. Then, given that $\partial \hat{\Phi}_{F_{-i}^o} / \partial G > 0$, we have $\bar{G}^*(F^o) \leq \bar{G}^*(F_{-i}^o)$. But, this contradicts the optimality of F^o either because $\bar{G}^*(F^o) < \bar{G}^*(F_{-i}^o)$, or because $\bar{G}^*(F^o) = \bar{G}^*(F_{-i}^o)$ and $C(F^o) > C(F_{-i}^o)$. As a result, $g_i^*(F^o) > c_i$.

To prove that F^o must also satisfy C2, suppose, by way of contradiction, that individual i is not in F^o , but that if added to F^o , i 's contribution would satisfy $g_i^*(F^o \cup \{i\}) - c_i > 0$. To economize on notation, let $F^o \cup \{i\} \equiv F^+$ and $F_{C,-i}^+ \equiv F_C^+ \setminus \{i\}$. By definition, $F_{C,-i}^+ \subseteq F^o$. Moreover, since $c_i > 0$, we have $g_i^*(F^+) > 0$, which means that $\hat{\phi}_i(\bar{G}^*(F^+), w_i - c_i) - \bar{G}^*(F^+) = -(g_i^*(F^+) - c_i)$. Inserting this into the equilibrium condition, $\hat{\Phi}_{F_C^+}(\bar{G}^*(F^+), \mathbf{w} - \mathbf{c}) = -\sum_{j \in F^+ \setminus F_C^+} c_j$, we obtain $\hat{\Phi}_{F_{C,-i}^+}(\bar{G}^*(F^+), \mathbf{w} - \mathbf{c}) = -\sum_{j \in F^+ \setminus F_C^+} c_j + (g_i^*(F^+) - c_i)$. If $F_{C,-i}^+ = F^o$, then

$$\hat{\Phi}_{F^o}(\bar{G}^*(F^+), \mathbf{w} - \mathbf{c}) = (g_i^*(F^+) - c_i) > 0 = \hat{\Phi}_{F^o}(\bar{G}^*(F^o), \mathbf{w} - \mathbf{c}),$$

where the last equality follows because $F^o = F_C^o$. But, given that $\partial \hat{\Phi}_{F^o} / \partial G > 0$, we then have $\bar{G}^*(F^+) > \bar{G}^*(F^o)$, which contradicts the optimality of F^o .

If $F_{C,-i}^+ \neq F^o$, or equivalently $F_{C,-i}^+ \subset F^o$, then, by definition of $\hat{\Phi}_{F^o}$,

$$\hat{\Phi}_{F^o}(\bar{G}^*(F^+), \mathbf{w} - \mathbf{c}) = \hat{\Phi}_{F_{C,-i}^+}(\bar{G}^*(F^+)) + \sum_{j \in F^o \setminus F_{C,-i}^+} (\hat{\phi}_j(\bar{G}^*(F^+), w_j - c_j) - \bar{G}^*(F^+)).$$

Since $\hat{\Phi}_{F_{C,-i}^+}(\bar{G}^*(F^+)) = -\sum_{j \in F^+ \setminus F_C^+} c_j + (g_i^*(F^+) - c_i)$ and $\hat{\phi}_j(\bar{G}^*(F^+), w_j - c_j) - \bar{G}^*(F^+) \geq c_j$ (because $j \in F^o \setminus F_{C,-i}^+$ and thus a free-rider in the set F^+), it follows that

$$\hat{\Phi}_{F^o}(\bar{G}^*(F^+), \mathbf{w} - \mathbf{c}) \geq (g_i^*(F^+) - c_i) > 0.$$

Note again that $\widehat{\Phi}_{F^o}(\overline{G}^*(F^o), \mathbf{w} - \mathbf{c}) = 0$ because $F^o = F_C^o$. This implies that $\widehat{\Phi}_{F^o}(\overline{G}^*(F^+), \mathbf{w} - \mathbf{c}) > \widehat{\Phi}_{F^o}(\overline{G}^*(F^o), \mathbf{w} - \mathbf{c})$, which, in turn, implies that $\overline{G}^*(F^+) > \overline{G}^*(F^o)$, contradicting the optimality of F^o . As a result, i is in F^o , which means F^o also satisfies C2.

($\Leftarrow$): We prove the uniqueness of the equilibrium fund-raiser set. Suppose, on the contrary, that there are two distinct sets F and F' each satisfying C1 and C2. Note that $F \subset F'$ or $F' \subset F$ cannot be the case: otherwise, either C1 or C2 would be violated for at least one set. Next, take any i such that $i \in F'$ but $i \notin F$. By C2, i would be a net free-rider in $F \cup \{i\} = F^+$, i.e., $g_i^*(F^+) - c_i \leq 0$, which implies that $G_i^0 \leq G_{-i}^*(F^+) - C(F)$. Therefore,

$$G_i^0 = \widehat{f}_i(w_i - c_i + G_i^0, G_i^0) \leq \widehat{f}_i(w_i - c_i + G_{-i}^*(F^+) - C(F), G_{-i}^*(F^+) - C(F)) \leq \overline{G}^*(F^+).$$

Note also that $\overline{G}^*(F^+) \leq \overline{G}^*(F)$ because, by the first part, removing a net free-rider increases the equilibrium public good. Together, $G_i^0 \leq \overline{G}^*(F)$. In addition, since i is a net contributor in F' by C1, i.e., $g_i^*(F') - c_i > 0$, we have $G_i^0 > G_{-i}^*(F') - C(F'_{-i})$ and thus,

$$G_i^0 = \widehat{f}_i(w_i - c_i + G_i^0, G_i^0) > \widehat{f}_i(w_i - c_i + G_{-i}^*(F') - C(F'_{-i}), G_{-i}^*(F') - C(F'_{-i})) = \overline{G}^*(F'),$$

implying that $G_i^0 > \overline{G}^*(F')$. Together, the two inequalities reveal that $\overline{G}^*(F) \geq G_i^0 > \overline{G}^*(F')$, which, in turn, reveals $\overline{G}^*(F) > \overline{G}^*(F')$. But, a symmetric argument shows that $\overline{G}^*(F) < \overline{G}^*(F')$, yielding a contradiction. Hence, $F = F'$.

SOLUTION TO THE CES EXAMPLE IN SECTION V.A

As in the text, let

$$u_i = [(1 - \alpha)x_i^\rho + \alpha((1 - \omega)G + \omega g_i)^\rho]^{1/\rho},$$

where $\rho \in (-\infty, 1)$, $\alpha \in (0, 1)$, and $\omega \in [0, 1]$.

Substituting for $x_i = w_i + G_{-i} - G$ and $g_i = G - G_{-i}$, and maximizing u_i with respect to G reveal that

$$G = \widehat{f}_i(w_i + G_{-i}, G_{-i}) = \frac{1}{1 + \eta} [w_i + (1 + \eta\omega)G_{-i}],$$

where $\eta = \left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{1-\rho}}$. From here, the inverse Nash supply is found to be:

$$\widehat{\phi}(G_{-i}; w_i) = \frac{1 + \eta}{1 + \eta\omega} G_{-i} - \frac{1}{1 + \eta\omega} w_i.$$

Moreover, from the equation: $\widehat{f}_i(w_i - c + G_i^0, G_i^0) = G_i^0$, we obtain $G_i^0 = \frac{w_i - c}{\eta(1 - \omega)}$. Then,

$$\begin{aligned}\widehat{\Delta}_i &= \sum_{j=1}^i (\widehat{\phi}(G_i^0; w_j - c) - G_i^0) + G_i^0 \\ &= \sum_{j=1}^i \left[\frac{1 + \eta}{1 + \eta\omega} G_i^0 - \frac{1}{1 + \eta\omega} (w_j - c) - G_i^0 \right] + G_i^0 \\ &= \sum_{j=1}^i \left[\frac{1}{1 + \eta\omega} (w_i - c) - \frac{1}{1 + \eta\omega} (w_j - c) \right] + G_i^0\end{aligned}$$

Thus,

$$\widehat{\Delta}_i = \frac{w_i - c}{\eta(1 - \omega)} - \frac{1}{1 + \eta\omega} \sum_{j=1}^i (w_j - w_i).$$

Since, with the government grant R , person i is solicited if and only if $\widehat{\Delta}_i > R$, it follows that i is solicited if and only if c is less than this cutoff:

$$\bar{c}_i = w_i - \eta(1 - \omega)R - \frac{\eta(1 - \omega)}{1 + \eta\omega} \sum_{j=1}^i (w_j - w_i),$$

as recorded in the text.

SECTION V.B: Fund-raising with Income Uncertainty

As described in the text, suppose that there are $m \geq 1$ groups of donors in the population, with n_i being the size of group i , and $n = \sum_{i=1}^m n_i$. The fund-raiser commonly believes that each member of group i independently draws his income from a discrete distribution, $\tilde{w}_i$, with support $w^1 > w^2 \dots > w^T$ and mean $E[\tilde{w}_i]$. Let $p_{it} = \Pr\{\tilde{w}_i = w^t\}$. The fund-raiser's objective is to solicit $k_i \in \{0, 1, \dots, n_i\}$ donors from group i , where $k = \sum_{i=1}^m k_i$ denotes total solicitations.

For technical ease, we assume that all individuals are contributors (not necessarily net contributors) in equilibrium for any fund-raiser set and any realization of incomes. This assumption holds if $w^T > \frac{1 - \alpha}{n(1 - \alpha) + (2\alpha - 1)} [(n - 1)w^1 - nc]$, i.e., if the highest and lowest incomes are not too different. If preferences are homothetic and identical, then $f(w) = \alpha w$ for some $\alpha \in (0, 1)$. In this case, simple algebra shows that the *expected* level of equilibrium public good is:

$$(4) \quad \bar{G}^*(k) = \frac{\sum_{i=1}^m k_i (E[\tilde{w}_i] - c)}{1 + \frac{1 - \alpha}{\alpha} k}.$$

PROPOSITION A1: Without loss of generality, let $E[\tilde{w}_1] \geq \dots \geq E[\tilde{w}_m]$, and define

$$\bar{c}_i = E[\tilde{w}_i] - \frac{1-\alpha}{\alpha} \sum_{j=1}^i n_j (E[\tilde{w}_j] - E[\tilde{w}_i]).$$

Then, it is optimal to solicit all members of group i if $c < \bar{c}_i$; and no members, otherwise.

PROOF: Simple differentiation of (4) yields

$$(5) \quad \frac{\partial \bar{G}^*(k)}{\partial k_i} = \frac{E[\tilde{w}_i] - c - \frac{1-\alpha}{\alpha} \left[\sum_{j=1}^m k_j (E[\tilde{w}_j] - E[\tilde{w}_i]) \right]}{\left[1 + \frac{1-\alpha}{\alpha} k \right]^2}.$$

Note that the sign of $\frac{\partial \bar{G}^*(k^*)}{\partial k_i}$ does not depend on k_i because, in the numerator, the coefficient of k_i is 0. Therefore, $k_i^* = n_i$ if $\frac{\partial \bar{G}^*(k^*)}{\partial k_i} > 0$, and $k_i^* = 0$ otherwise. Next, note that $\frac{\partial \bar{G}^*(n)}{\partial k_i}$ is decreasing in i , i.e., $\frac{\partial \bar{G}^*(n)}{\partial k_i} > 0$ implies $\frac{\partial \bar{G}^*(n)}{\partial k_{i+1}} > 0$. Thus, $k_i^* = n_i$ if and only if $E[\tilde{w}_i] - c - \frac{1-\alpha}{\alpha} \left[\sum_{j=1}^i n_j (E[\tilde{w}_j] - E[\tilde{w}_i]) \right] > 0$, or equivalently, if and only if $c < \bar{c}_i$.

DEFINITION A1 (Indistinguishable Groups). We say that members of two groups i and j are indistinguishable by the fund-raiser if she knows n_i and n_j , but cannot distinguish between group i and group j members.

Without loss of generality, suppose $i < j$, i.e., $E[\tilde{w}_i] \geq E[\tilde{w}_j]$, and let $\tilde{w}_{ij}$ denote the income distribution of the merged group of size $n_i + n_j$. Refer to $\bar{c}_l$ and $\bar{c}'_l$ as group l 's cutoff costs when all m groups are distinguishable, and when groups i and j are indistinguishable, respectively. Note that by definition, $\Pr\{\tilde{w}_{ij} = w^t\} = \frac{n_i}{n_i+n_j} p_{it} + \frac{n_j}{n_i+n_j} p_{jt}$, and $E[\tilde{w}_{ij}] = \frac{n_i}{n_i+n_j} E[\tilde{w}_i] + \frac{n_j}{n_i+n_j} E[\tilde{w}_j]$. We, furthermore, have

FACT A1: $\bar{c}_i \geq \bar{c}'_l \geq \bar{c}_l$ for any $i < l < j$.

PROOF: Note that since $i < j$, we must have $E[\tilde{w}_i] \geq E[\tilde{w}_{ij}] \geq E[\tilde{w}_j]$. Then, either $\bar{c}'_l = \bar{c}_l + \frac{1-\alpha}{\alpha} n_i (E[\tilde{w}_i] - E[\tilde{w}_l]) \geq \bar{c}_l$ or $\bar{c}'_l = \bar{c}_l - \frac{1-\alpha}{\alpha} n_j (E[\tilde{w}_j] - E[\tilde{w}_l])$. In any case $\bar{c}'_l \geq \bar{c}_l$ because $E[\tilde{w}_i] \geq E[\tilde{w}_l] \geq E[\tilde{w}_j]$ by hypothesis. Finally,

$$\bar{c}'_l \leq E[\tilde{w}_l] - \frac{1-\alpha}{\alpha} \sum_{q=1}^{i-1} n_q (E[\tilde{w}_q] - E[\tilde{w}_l]) \leq \bar{c}_i.$$

Refer to $\bar{G}^*$ and $\bar{G}^{*'} as the public good supply when all m groups are distinguishable and when groups i and j are indistinguishable, respectively.$

PROPOSITION A2: *If any two groups i and j become indistinguishable by the fund-raiser, then the equilibrium supply of the public good decreases, i.e., $\bar{G}^* \geq \bar{G}^{*'}.$*

PROOF: We first prove that if either both groups i and j are originally fundraised, or neither is originally fundraised, then $\bar{G}^* = \bar{G}^{*'}$. Since $E[\tilde{w}_i] \geq E[\tilde{w}_{ij}] \geq E[\tilde{w}_j]$, it easily follows that $\bar{c}_i \geq \bar{c}_{ij} \geq \bar{c}_j$, which implies that the joint group ij is solicited if i and j are originally solicited. Conversely, the joint group is not solicited if neither i nor j is originally fundraised. It also easily follows that $\bar{c}_l = \bar{c}'_l$ for any $l < i$ and for any $l > j$. Thus, the fund-raiser's strategy for group l does not depend on whether or not groups i and j are distinguishable. Finally, since, by Fact A1 above, $\bar{c}_i \geq \bar{c}'_l \geq \bar{c}_l$ for any $i < l < j$, group l is solicited if i and j are solicited. Conversely, group l is not solicited if neither i nor j is originally fundraised. As a result, the set of solicited individuals remains the same when groups i and j become indistinguishable by the fund-raiser, implying that $\bar{G}^* = \bar{G}^{*'}$.

Next, we prove that if, originally, only group i is solicited, then $\bar{G}^* > \bar{G}^{*'}$. Note that we have $\bar{c}_i > c \geq \bar{c}_j$ by hypothesis, and $\bar{c}_i > \bar{c}_{ij} > \bar{c}_j$ by definition. If $c > \bar{c}_{ij}$, then, when indistinguishable, group i is not solicited, either. If, on the other hand, $c < \bar{c}_{ij}$, then both groups i and j are solicited (even when soliciting group j would not be optimal originally). In either case, the equilibrium public good level strictly decreases, i.e., $\bar{G}^* > \bar{G}^{*'}$.

COROLLARY A1: *The equilibrium public good level is the lowest if all of the m groups are indistinguishable by the fund-raiser.*

PROOF: Since this economy can be obtained successively by making two groups indistinguishable in each step, the equilibrium level of the public good is monotonically decreasing by Proposition A2.

SECTION V.C: Learning-by-Fundraising

As defined in the text, let $c(i)$ be the marginal cost of soliciting i th individual in sequence such that $c(1) > c(2) \geq \dots \geq c(n)$. Furthermore, let $a_n \equiv \frac{\sum_{i=1}^n c(i)}{n}$ be the average cost of solicitation. Clearly, a_n is monotone decreasing and thus converges to some $a_\ell < c(1)$ as $n \rightarrow \infty$. Assuming that $a_n < \hat{C}$, the following proposition formally states our findings in Section V.C.

PROPOSITION A3: *Suppose that donors are identical. Then, it is optimal to contact every donor. Moreover, in the unique equilibrium,*

- (a) *the supply of the public good, $\bar{G}_n^*$ is increasing in n and converges to $\bar{\phi}^{-1}(w - a_\ell)$;*
- (b) *each gift, g_n^* , is decreasing in n and converges to a_ℓ .*

PROOF: Since a_n is decreasing in n and less than $\hat{C}$, it is optimal to contact all n donors. From Proposition 1, $\bar{G}_n^*$ uniquely solves:

$$(6) \quad \phi(\bar{G}_n^*) - \bar{G}_n^* + \frac{\bar{G}_n^*}{n} = w - a_n.$$

Clearly, $\bar{G}_n^*$ is increasing in n and bounded. Thus, $\bar{G}_n^*$ converges to some $\bar{G}_\infty^*$ where $\phi(\bar{G}_\infty^*) - \bar{G}_\infty^* = w - a_\ell$, or by re-writing, $\bar{G}_\infty^* = \bar{\phi}^{-1}(w - a_\ell)$, proving part (a). Next, since the equilibrium is unique, each gift must be equal, which we denote by g_n^* . From (6), note that $g_n^* - a_n = w - a_n - (\phi(\bar{G}_n^*) - \bar{G}_n^*)$, or equivalently

$$g_n^* = w - (\phi(\bar{G}_n^*) - \bar{G}_n^*).$$

Given that $\bar{G}_n^*$ is increasing in n , g_n^* is decreasing. In addition, since $\phi(\bar{G}_\infty^*) - \bar{G}_\infty^* = w - a_\ell$, we have $g_n^* \rightarrow a_\ell$ as $n \rightarrow \infty$, completing the proof.