Online Appendix for “Inferring Strategic Voting”

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1 Supplementary Material A

In our estimation, we run regressions in Step 1 and Step 3 in order to obtain $\beta_{k,d}^{data}$ and $\beta_{k,d}(T^d, \alpha_d, \xi_d; \theta)$, which are

$$\begin{aligned}\beta_{k,d}^{data}(T^d, \alpha_d, \xi_d; \theta) &= \arg \min_{\beta} \left[\sum_{m=1}^{M^d} 1_{\{z_{km}=z\}} (v_{k,m}^{data} - \beta \cdot f_m)^2 \right], \text{ and} \\ \beta_{k,d}(T^d, \alpha_d, \xi_d; \theta) &= \arg \min_{\beta} \left[\sum_{m=1}^{M^d} 1_{\{z_{km}=z\}} (v_{k,m}^{PRED}(T^d, \alpha_{M^d}, \xi_{M^d}; \theta) - \beta \cdot f_m)^2 \right].\end{aligned}$$

We run 9 different types of regressions (fourty eight regressions in total) for each district as follows.

1. $f_m = (1, \text{“fraction of population above 65”})$, i.e. we regress the vote shares onto a constant and the fraction of population above 65 years old. If we let P denote $\{LDP, DPJ, JCP, YUS\}$, we run this regression for each combination of $(z_1^{POS}, \dots, z_K^{POS}) \in P^K$.
2. f_m is a constant and the fraction of population with years of schooling between 12 to 14 years. Regression is run for each combination of $(z_1^{POS}, \dots, z_K^{POS}) \in P^K$.
3. f_m is a constant and the fraction of population with years of schooling between 15 to 16 years. Regression is run for each combination of $(z_1^{POS}, \dots, z_K^{POS}) \in P^K$.
4. f_m is a constant and the fraction of population with years of schooling over 16 years. Regression is run for each combination of $(z_1^{POS}, \dots, z_K^{POS}) \in P^K$.
5. f_m is a constant and the fraction of population with income in the first quartile (lower than 1.870 million yen). Regression is run for each combination of $(z_1^{POS}, \dots, z_K^{POS}) \in P^K$.
6. f_m is a constant and the fraction of population with income in the second quartile (between 1.870 million yen and 2.704 million yen). Regression is run for each combination

of $(z_1^{POS}, \dots, z_K^{POS}) \in P^K$.

7. f_m is a constant and the fraction of population with income in the third quartile (between 2.704 million yen and 3.911 million yen). Regression is run for each combination of $(z_1^{POS}, \dots, z_K^{POS}) \in P^K$.

8. f_m is a constant. Regression is run for each combination of $(z_1^{POS}, \dots, z_K^{POS}) \in P^K$.

9. f_m is a constant. Regression is run for each combination of $(z_1^{POS}, \dots, z_K^{POS}) \in P^K$, and $(z_{km}^{EXPR}, z_{km}^{HOME})$ where $z_{km}^{EXPR} \in \{\text{incumbent, previous political experience, no previous political experience}\}$, and $z_{km}^{HOME} \in \{\text{hometown of the candidate is outside the prefecture, hometown of the candidate is inside the prefecture (but outside the district), hometown of the candidate is in the district (but outside municipality } m), \text{ hometown of the candidate is in municipality } m\}$.

In order to improve the sharpness of the identified set, we include another type of moment inequalities that harnesses the comovements in β that results from changes in T as discussed in Step 6 of Appendix B. We augment the moment conditions by using restrictions on the comovement of the coefficients for the 9th type of regressions. This allows us to add restrictions on the pairwise difference in the β s that relate to the effect of candidates' experience and hometowns, e.g., the difference in the vote share for a *LDP* candidate whose hometown is outside of the prefecture compared to a *LDP* candidate whose hometown is within the prefecture. In practice, the matrix A used in Step 6 in our estimation is $A^T = \begin{pmatrix} I_{60} & \mathbf{0} \\ & B \end{pmatrix}$

$$\text{where } B = \begin{pmatrix} 1 & \dots & \dots & 1 & 0 & \dots & 0 & \dots & \dots \\ -1 & 0 & \dots & 0 & 1 & \dots & 1 & 0 & \dots \\ 0 & -1 & \ddots & \vdots & -1 & 0 & 0 & 1 & \dots \\ \vdots & \ddots & \ddots & 0 & 0 & \ddots & 0 & -1 & 0 \\ 0 & \dots & 0 & -1 & \vdots & \ddots & -1 & 0 & \ddots \end{pmatrix} \text{ and } I_{60} \text{ is the identity matrix}$$

of size 60×60 .

2 Supplementary Material B

In this Supplementary Material, we prove that the bounds $ub(\{\Delta^k\})$ and $lb(\{\Delta^k\})$ we have used in Appendix C in fact constitute bounds and that they are sharp. Because the bounds are different for $K = 3$ and $K = 4$, we prove each case in turn. We drop subscripts d and m for the rest of the section.

2.0.1 Case of $K = 3$

First, we prove that, for the case of $K = 3$, the extent of strategic voting is bound by $lb(\{\Delta^k\})$ and $ub(\{\Delta^k\})$, where

$$\begin{aligned} lb(\{\Delta^k\}) &= \max_k \{|\Delta^k|\}, \text{ and} \\ ub(\{\Delta^k\}) &= 1\{\#\{\Delta^k > 0\} = 2\}(\max_k \{\Delta^k | \Delta^k > 0\} - \min_k \{\Delta^k\}) \\ &\quad + 1\{\#\{\Delta^k > 0\} = 1\}(\max_k \{\Delta^k\} - \min_k \{\Delta^k | \Delta^k < 0\}) \\ &= \max_k \{\Delta^k\} - \min_k \{\Delta^k\}, \end{aligned}$$

and $\#\{\Delta^k > 0\}$ indicates the number of Δ^k s that are positive, and $1\{\cdot\}$ is an indicator function. Let D_{kl} denote the votes cast for candidate k by strategic voters who prefers candidate l most. Then the amount of misaligned voting is $\sum_{kl} D_{kl}$ (Note that **C1** implies that if $D_{kl} > 0$, then $D_{lk} = 0$).

First, we prove that the extent of strategic voting is bound by $lb(\{\Delta^k\})$ and $ub(\{\Delta^k\})$. Without loss of generality, index the candidates as 1, 2, and 3 such that the beliefs regarding the tie probabilities satisfy $T_{12} \geq T_{13} \geq T_{23}$. Then the amount of misaligned voting is $D = D_{12} + D_{13} + D_{23}$ (Note that $D_{21} = D_{31} = D_{32} = 0$). Now, we can write

$$\Delta^1 = D_{12} + D_{13}, \tag{A1}$$

$$\Delta^2 = D_{23} - D_{12}, \tag{A2}$$

$$\Delta^3 = -D_{13} - D_{23}. \tag{A3}$$

Note that $|\Delta^1| + |\Delta^3| = D_{12} + 2D_{13} + D_{23} \geq D$, thus $|\Delta^1| + |\Delta^3|$ is an upper bound. We consider two cases; (i) $\{\#\{\Delta^k > 0\} = 1\}$, and (ii) $\{\#\{\Delta^k > 0\} = 2\}$. In case (i), we know that the positive number we observe is Δ^1 , but cannot identify which of the two negative numbers correspond to Δ^2 or Δ^3 . In case (ii), we know that the negative number we observe is Δ^3 , but we cannot identify which of the two positive numbers correspond to Δ^1 or Δ^2 . These two cases are exhaustive as $\Delta^1 + \Delta^2 + \Delta^3 = 0$. In case (i),

$$\begin{aligned} ub(\{\Delta^k\}) &= \max_k \{\Delta^k\} - \min_k \{\Delta^k | \Delta^k < 0\} = \Delta^1 - \min\{\Delta^2, \Delta^3\} \\ &= |\Delta^1| + \max\{|\Delta^2|, |\Delta^3|\} \\ &\geq |\Delta^1| + |\Delta^3|. \end{aligned}$$

In case (ii),

$$\begin{aligned}
ub(\{\Delta^k\}) &= \max_k \{\Delta^k | \Delta^k > 0\} - \min_k \{\Delta^k\} = \max\{\Delta^1, \Delta^2\} - \Delta^3 \\
&= \max\{|\Delta^1|, |\Delta^2|\} + |\Delta^3| \\
&\geq |\Delta^1| + |\Delta^3|.
\end{aligned}$$

We can also see that $\max_k \{|\Delta^k|\}$ is the lower bound because $|\Delta^1| = D_{12} + D_{13} \leq D$, $|\Delta^2| \leq D_{23} + D_{12} \leq D$, and $|\Delta^3| = D_{13} + D_{23} \leq D$.

Second, we prove by contradiction that the upper bound $ub(\{\Delta^k\})$ is sharp. Let $h(\Delta^1, \Delta^2, \Delta^3) \leq ub(\{\Delta^k\})$ for all $\{\Delta_{d,m}^k\}$, and moreover $h(\Delta^{1*}, \Delta^{2*}, \Delta^{3*}) < ub(\{\Delta^k\})$. Without loss of generality, consider the following two cases (i) $\Delta^{1*} > 0 > \max\{\Delta^{2*}, \Delta^{3*}\}$ and (ii) $\min\{\Delta^{1*}, \Delta^{2*}\} > 0 > \Delta^{3*}$. Note that we cannot identify whether the two negative numbers in case (i) correspond to Δ^{2*} or Δ^{3*} , and similarly, in case (ii), we cannot identify whether the two positive numbers correspond to Δ^{1*} or Δ^{2*} . This is the reason why we use the min and the max operators. In case (i), if we let $D_{12} = \Delta^{1*}$, $D_{23} = -\min\{\Delta^{2*}, \Delta^{3*}\}$ and $D_{13} = 0$, then the three equations (A1)-(A3) can be satisfied. In this instance, $D_{12} + D_{13} + D_{23} = \Delta^{1*} - \min\{\Delta^{2*}, \Delta^{3*}\} = ub(\{\Delta^{k*}\})$, achieving our bound. Hence, h cannot be an upper bound. In case (ii), let $D_{12} = \max\{\Delta^{1*}, \Delta^{2*}\}$, $D_{13} = 0$, $D_{23} = -\Delta^{3*}$. Then (A1)-(A3) are satisfied, and moreover, $D_{12} + D_{13} + D_{23} = \max\{\Delta^{1*}, \Delta^{2*}\} - \Delta^{3*} = ub(\{\Delta^{k*}\})$.

Third, we prove by contradiction that the lower bound $lb(\{\Delta^k\})$ is sharp. Let $h(\Delta^1, \Delta^2, \Delta^3) \geq lb(\{\Delta^k\})$ for all $\{\Delta_{d,m}^k\}$, and moreover $h(\Delta^{1*}, \Delta^{2*}, \Delta^{3*}) > lb(\{\Delta^k\})$. Without loss of generality, consider the following two cases (i) $\Delta^{1*} > 0 > \max\{\Delta^{2*}, \Delta^{3*}\}$ and (ii) $\min\{\Delta^{1*}, \Delta^{2*}\} > 0 > \Delta^{3*}$. In case (i), let $D_{12} = -\Delta^{2*}$, $D_{13} = -\Delta^{3*}$, and $D_{23} = 0$. This satisfies the three equations (A1)-(A3) and moreover, $D_{12} + D_{13} + D_{23} = -\Delta^{2*} - \Delta^{3*} = \Delta^{1*} = lb(\{\Delta^{k*}\})$. In case (ii) let $D_{12} = 0$ and $D_{23} = \Delta^{2*}$ and $D_{13} = -\Delta^{3*} - \Delta^{2*}$. This also satisfies equations (A1)-(A3), and implies $D_{12} + D_{13} + D_{23} = -\Delta^{3*} = lb(\{\Delta^{k*}\})$. Thus, h cannot be a lower bound.

2.0.2 Case of $K = 4$

For the case of $K = 4$, the lower and upper bounds $lb(\{\Delta^k\})$ and $ub(\{\Delta^k\})$ are written as

$$\begin{aligned}
lb(\{\Delta^k\}) &= 1\{\#\{\Delta^k > 0\} = 3\} \max \left\{ \min_{k,l \neq k} \{\Delta^k + \Delta^l | \Delta^k, \Delta^l > 0\}, -\min_k \{\Delta^k | \Delta^k < 0\} \right\} \\
&\quad + 1\{\#\{\Delta^k > 0\} = 2\} \max \left\{ \min_k \{\Delta^k | \Delta^k > 0\}, -\min_k \{\Delta^k | \Delta^k < 0\} \right\} \\
&\quad + 1\{\#\{\Delta^k > 0\} = 1\} \max \left\{ \max_k \{\Delta^k\}, -\max_k \{\Delta^k | \Delta^k < 0\} \right\}, \text{ and} \\
ub(\{\Delta^k\}) &= \max_{k,l \neq k} \{2\Delta^k + \Delta^l\} - \max_k \{\Delta^k | \Delta^k < 0\}
\end{aligned}$$

The proof is similar to the case of $K = 3$.