

Online Appendix

Job Selection and Wages over the Business Cycle

Marcus Hagedorn*
Institute for Advanced Studies

Iourii Manovskii†
University of Pennsylvania

*Institute for Advanced Studies, Stumpergasse 56, A-1060 Vienna, Austria. Email: marcus.hagedorn07@gmail.com.

†Department of Economics, University of Pennsylvania, 160 McNeil Building, 3718 Locust Walk, Philadelphia, PA, 19104-6297 USA. E-mail: manovski@econ.upenn.edu.

I. Proofs and Derivations

A. Deriving $\tilde{F}^k(\epsilon|N_t^k)$

We consider a worker who not only received an offer in period $1 + T_{k-1}$ but also accepted this offer. Let G be the probability that this switcher accepts an offer less than $\hat{\epsilon}$. The information that the worker switches makes it necessary to modify the probability, $F(\hat{\epsilon})$, which describes the unconditional probability to accept an offer. For a switcher the probability is zero if $\hat{\epsilon} \leq \epsilon_{k-1}$, what is equivalent to $\epsilon_k \geq \epsilon_{k-1}$. As it still holds that $G(\bar{\epsilon}) = 1$, it follows that

$$(1) \quad G(\hat{\epsilon}) = \frac{F(\hat{\epsilon}) - F(\epsilon_{k-1})}{1 - F(\epsilon_{k-1})}$$

for $\hat{\epsilon} \geq \epsilon_{k-1}$. To derive the probability for later periods, consider a worker of type ϵ_{k-1} , who has received N_t^k offers. This worker declines all offers less than ϵ_{k-1} which happens with probability $F(\epsilon_{k-1})^{N_t^k}$. Thus the probability that the worker has a type less than $\hat{\epsilon}$ equals¹

$$(2) \quad \frac{(1 - F(\epsilon_{k-1}))(1 + N_t^k)}{1 - F(\epsilon_{k-1})^{1+N_t^k}} \int_{\epsilon_{k-1}}^{\hat{\epsilon}} F(\epsilon)^{N_t^k} dG(\epsilon) = \frac{F(\hat{\epsilon})^{1+N_t^k} - F(\epsilon_{k-1})^{1+N_t^k}}{1 - F(\epsilon_{k-1})^{1+N_t^k}}.$$

B. Determining Signs of c_1 and c_2

We first show that for $\tilde{N} > N$ and $\epsilon_{k-1} \leq \hat{\epsilon} \leq \bar{\epsilon}$

$$(3) \quad \Delta_{\tilde{N},N}(\hat{\epsilon}) = \Omega_{\tilde{N}}(\hat{\epsilon}) - \Omega_N(\hat{\epsilon}) \leq 0,$$

where

$$(4) \quad \Omega_N(\hat{\epsilon}) = \frac{F(\hat{\epsilon})^N - F(\epsilon_{k-1})^N}{1 - F(\epsilon_{k-1})^N}$$

$$(5) \quad \Omega_{\tilde{N}}(\hat{\epsilon}) = \frac{F(\hat{\epsilon})^{\tilde{N}} - F(\epsilon_{k-1})^{\tilde{N}}}{1 - F(\epsilon_{k-1})^{\tilde{N}}}.$$

$$(6) \quad \Delta_{\tilde{N},N}(\hat{\epsilon}) = \int_{\epsilon_{k-1}}^{\hat{\epsilon}} \left\{ \frac{\tilde{N}F(\epsilon)^{\tilde{N}-1}}{1 - F(\epsilon_{k-1})^{\tilde{N}}} - \frac{NF(\epsilon)^{N-1}}{1 - F(\epsilon_{k-1})^N} \right\} f(\epsilon) d\epsilon$$

$$(7) \quad = \int_{\epsilon_{k-1}}^{\hat{\epsilon}} \frac{NF(\epsilon)^{N-1}f(\epsilon)}{1 - F(\epsilon_{k-1})^N} (\omega F(\epsilon)^{\tilde{N}-N} - 1) d\epsilon,$$

¹Since $m := \int_{\epsilon_{k-1}}^{\bar{\epsilon}} F(\epsilon)^{N_t^k} dG(\epsilon) = \int_{\epsilon_{k-1}}^{\bar{\epsilon}} F(\epsilon)^{N_t^k} \frac{f(\epsilon)}{1 - F(\epsilon_{k-1})} d\epsilon = \frac{F(\bar{\epsilon})^{1+N_t^k} - F(\epsilon_{k-1})^{1+N_t^k}}{(1+N_t^k)(1 - F(\epsilon_{k-1}))} = \frac{1 - F(\epsilon_{k-1})^{1+N_t^k}}{(1+N_t^k)(1 - F(\epsilon_{k-1}))}$, we have to adjust by the factor $1/m$ to define a probability measure.

where

$$(8) \quad \omega = \frac{\tilde{N}(1 - F(\epsilon_{k-1})^N)}{N(1 - F(\epsilon_{k-1})^{\tilde{N}})}.$$

Since both Ω_N and $\Omega_{\tilde{N}}$ are probability measures on $[\epsilon_{k-1}, \bar{\epsilon}]$ it holds that

$$(9) \quad \Delta_{\tilde{N}, N}(\bar{\epsilon}) = 0.$$

Since $\Delta_{\tilde{N}, N}(\epsilon_{k-1}) = 0$ and $\omega F(\epsilon) - 1$ is increasing in ϵ it follows that an $\tilde{\epsilon}$ exists such that $\omega F(\tilde{\epsilon}) - 1 = 0$, $\omega F(\epsilon) - 1 < 0$ for $\epsilon < \tilde{\epsilon}$ and $\omega F(\epsilon) - 1 > 0$ for $\epsilon > \tilde{\epsilon}$. This implies that $\Delta_{\tilde{N}, N}(\hat{\epsilon}) \leq 0$ for all $\epsilon_{k-1} \leq \hat{\epsilon} \leq \bar{\epsilon}$.²

We can now turn to the linearization of

$$(10) \quad E_t(\epsilon_k | \epsilon_{k-1}, N_{T_k}^k) = \bar{\epsilon} - \int_{\epsilon_{k-1}}^{\bar{\epsilon}} \frac{F(\epsilon)^{1+N_{T_k}^k} - F(\epsilon_{k-1})^{1+N_{T_k}^k}}{1 - F(\epsilon_{k-1})^{1+N_{T_k}^k}} d\epsilon$$

w.r.t. $N_{T_k}^k$ and ϵ_{k-1} . We linearize around a steady state where all variables are evaluated at their expected values in a steady state.

Since we have established that $\Delta_{\tilde{N}, N}(\hat{\epsilon}) \leq 0$, the expected value of ϵ_k is increasing in $N_{T_k}^k$. The derivative of $E_t(\epsilon_k | \epsilon_{k-1}, N_{T_k}^k)$ w.r.t. ϵ_{k-1} equals

$$(11) \quad \int_{\hat{\epsilon}}^{\bar{\epsilon}} \frac{(1 - F(\epsilon)^{1+\bar{N}})^2}{(1 - F(\hat{\epsilon})^{1+\bar{N}})^2} (1 + \bar{N}) F(\hat{\epsilon})^{\bar{N}} f(\hat{\epsilon}) d\epsilon > 0,$$

where $\hat{\epsilon}$ is the steady state value of ϵ_{k-1} and \bar{N} is the steady state value of $N_{T_k}^k$.

C. Theory with Endogenous Separations

We now show how the results of the main text have to be modified if workers get separated endogenously. In particular we show that equation (18) in the main text approximates ϵ in this case.

The first modification is necessary for $E_t(\epsilon_k | \epsilon_{k-1}, N_t^k)$, which equals

$$(12) \quad E_t(\epsilon_k | \epsilon_{k-1}, N_t^k) = \int_{\tilde{\epsilon}_t^k}^{\bar{\epsilon}} \epsilon d\tilde{F}^k(\epsilon | N_t^k).$$

We now truncate at $\tilde{\epsilon}_t^k := \max\{\epsilon_{k-1}, \sigma_{1+T_{k-1}}, \dots, \sigma_t\}$. A worker separates if his type is lower than σ , so that a worker who has survived until period t must have a type larger or equal than $\sigma_t^k = \max\{\sigma_{1+T_{k-1}}, \dots, \sigma_t\}$.

²Since $\omega F(\epsilon) - 1 < 0$ for $\epsilon < \tilde{\epsilon}$ this is obvious for $\hat{\epsilon} \leq \tilde{\epsilon}$. Suppose now that $\Delta_{\tilde{N}, N}(\hat{\epsilon}) > 0$ for $\hat{\epsilon} > \tilde{\epsilon}$. Since $\omega F(\epsilon) - 1 > 0$ for $\epsilon \geq \hat{\epsilon} \geq \tilde{\epsilon}$ this would imply that $\Delta_{\tilde{N}, N}(\bar{\epsilon}) > 0$, contradicting $\Delta_{\tilde{N}, N}(\bar{\epsilon}) = 0$.

This truncation makes it also necessary to change the distribution $\tilde{F}^k(\epsilon|N_t^k)$. The probability G that a switcher accepts an offer less than $\hat{\epsilon}$ now equals

$$(13) \quad G(\hat{\epsilon}) = \frac{F(\hat{\epsilon}) - F(\tilde{\epsilon}_{1+T_{k-1}}^k)}{1 - F(\tilde{\epsilon}_{1+T_{k-1}}^k)}$$

for $\hat{\epsilon} \geq \tilde{\epsilon}_{1+T_{k-1}}^k$. The only difference, due to endogenous separations, is that we replace ϵ_{k-1} by $\tilde{\epsilon}_{1+T_{k-1}}^k$. To derive the probability for later periods, consider again a worker of type ϵ , who has received N_t^k offers. The probability that the worker has a type less than $\hat{\epsilon}$, taking into account endogenous separations, equals³

$$(14) \quad \frac{(1 - F(\tilde{\epsilon}_{1+T_{k-1}}^k))(1 + N_t^k)}{1 - F(\tilde{\epsilon}_t^k)^{1+N_t^k}} \int_{\tilde{\epsilon}_t^k}^{\hat{\epsilon}} F(\epsilon)^{N_t^k} dG(\epsilon) = \frac{F(\hat{\epsilon})^{1+N_t^k} - F(\tilde{\epsilon}_t^k)^{1+N_t^k}}{1 - F(\tilde{\epsilon}_t^k)^{1+N_t^k}},$$

where the only difference, due to endogenous separations, is that we replace ϵ_{k-1} by $\tilde{\epsilon}_t^k$.

We again use the predictor which contains the most information about this ϵ , the value at T_k . The expectation of ϵ_k at $1 + T_{k-1} \leq t \leq T_k$ then equals

$$(15) \quad E_t(\epsilon_k | \epsilon_{k-1}, N_{T_k}^k) = \int_{\tilde{\epsilon}_{T_k}^k}^{\bar{\epsilon}} \epsilon d\tilde{F}^k(\epsilon | N_{T_k}^k).$$

The expression for the expectation of ϵ_k conditional on ϵ_{k-1} stays the same (the modifications are of course incorporated in $E_t(\epsilon_k | \epsilon_{k-1}, N_{T_k}^k)$):

$$(16) \quad E_t(\epsilon_k | \epsilon_{k-1}) = \sum_{N_{T_k}^k} E_t(\epsilon_k | \epsilon_{k-1}, N_{T_k}^k) P_{T_k}^k(N_{T_k}^k).$$

Linearization.

Linearization of (16) w.r.t. $N_{T_k}^k$ and $\tilde{\epsilon}_{T_k}^k$ around a steady state where all variables are evaluated at their expected values in a steady state yields

$$(17) \quad E_t(\epsilon_k | \epsilon_{k-1}, N_{T_k}^k) \approx c_0 + c_1 N_{T_k}^k + c_2 \tilde{\epsilon}_{T_k}^k,$$

where the coefficients c_1 and c_2 are the first derivatives. The proof in appendix B., if ϵ_{k-1} is replaced by $\tilde{\epsilon}_{T_k}^k$, again shows that these coefficients are positive.

The same arguments as in the main text for the unconditional expectation establish

$$(18) \quad E_t(\epsilon_k) \approx c_0 + c_1 q_{T_k}^{HM} + c_2 E_{T_{k-1}}(\tilde{\epsilon}_{T_k}^k).$$

³Since $m := \int_{\tilde{\epsilon}_t^k}^{\bar{\epsilon}} F(\epsilon)^{N_t^k} dG(\epsilon) = \int_{\tilde{\epsilon}_t^k}^{\bar{\epsilon}} F(\epsilon)^{N_t^k} \frac{f(\epsilon)}{1 - F(\tilde{\epsilon}_{1+T_{k-1}}^k)} d\epsilon = \frac{F(\bar{\epsilon})^{1+N_t^k} - F(\tilde{\epsilon}_t^k)^{1+N_t^k}}{(1+N_t^k)(1 - F(\tilde{\epsilon}_{1+T_{k-1}}^k))} = \frac{1 - F(\tilde{\epsilon}_t^k)^{1+N_t^k}}{(1+N_t^k)(1 - F(\tilde{\epsilon}_{1+T_{k-1}}^k))}$, we have to adjust by the factor $1/m$ to define a probability measure.

The difference between this equation and the corresponding one without endogenous separations is that $\tilde{\epsilon}_{T_k}^k$ replaces ϵ_{k-1} (and of course the coefficients may be different).

To simplify $E_{T_{k-1}}(\tilde{\epsilon}_{T_k}^k)$ we use that

$$\begin{aligned} E_{T_{k-1}}(\tilde{\epsilon}_{T_k}^k) &= Prob_{T_{k-1}}(\epsilon_{k-1} \geq \sigma_{T_k}^k) E_{T_{k-1}}(\epsilon_{k-1} | \epsilon_{k-1} \geq \sigma_{T_k}^k) + Prob_{T_{k-1}}(\epsilon_{k-1} < \sigma_{T_k}^k) \sigma_{T_k}^k \\ (19) \quad &= E_{T_{k-1}}(\epsilon_{k-1}) + Prob_{T_{k-1}}(\epsilon_{k-1} < \sigma_{T_k}^k) (\sigma_{T_k}^k - E_{T_{k-1}}(\epsilon_{k-1} | \epsilon_{k-1} < \sigma_{T_k}^k)). \end{aligned}$$

We now use the fact that endogenous separations are a binding constraint in the current spell only if $\sigma_{T_k}^k > \Sigma_{k-1}^{max}$, where $\Sigma_{k-1}^{max} = \max\{\sigma_0, \dots, \sigma_{T_{k-1}}\}$ is the highest value of σ before the current job started. Workers with type $\epsilon < \sigma_k^{max} = \sigma_{T_k}^k$ would be separated but if $\Sigma_{k-1}^{max} > \sigma_k^{max}$ they were already separated before the current spell started. We thus know that $Prob_{T_{k-1}}(\epsilon_{k-1} < \sigma_k^{max}) = 0$ if $\sigma_k^{max} < \Sigma_{k-1}^{max}$ and is positive otherwise (if $\sigma_k^{max} < \bar{\epsilon}$). We therefore approximate the probability by an indicator \mathcal{I} which equals one if $\sigma_k^{max} > \Sigma_{k-1}^{max}$ and equals zero if $\sigma_k^{max} < \Sigma_{k-1}^{max}$. Finally we expect that $(\sigma_k^{max} - E_{T_{k-1}}(\epsilon_{k-1} | \epsilon_{k-1} < \sigma_k^{max}))$ is increasing in σ_k^{max} (Burdett (1996)), so that we get the following approximation:

$$(20) \quad c_2 E_{T_{k-1}}(\tilde{\epsilon}_{T_k}^k) \approx c_2 E_{T_{k-1}}(\epsilon_{k-1}) + c_3 \mathcal{I}_{\sigma_k^{max} > \Sigma_{k-1}^{max}} \sigma_k^{max},$$

where we expect c_3 to be positive (but do not impose this restriction). Using these derivations in (18) yields

$$(21) \quad E_t(\epsilon_k) \approx c_0 + c_1 q_{T_k}^{HM} + c_2 E_{T_{k-1}}(\epsilon_{k-1}) + c_3 \tilde{\sigma}_k^{max},$$

where $\tilde{\sigma}_k^{max} = \mathcal{I}_{\sigma_k^{max} > \Sigma_{k-1}^{max}} \sigma_k^{max}$.

Again we relate $E_t(\epsilon_k)$ to the worker's employment history before the current job started and apply the derivation for ϵ_k to ϵ_{k-1} . This yields the expected value of $E_t(\epsilon_{k-1})$, for $1 + T_{k-2} \leq t \leq T_{k-1}$:

$$(22) \quad E_t(\epsilon_{k-1}) \approx c_0 + c_1 q_{T_{k-1}}^{HM} + c_2 E_{T_{k-2}}(\epsilon_{k-2}) + c_3 \tilde{\sigma}_{k-1}^{max}$$

so that for $1 + T_{k-1} \leq t \leq T_k$

$$(23) \quad E_t(\epsilon_k) \approx c_0 + c_1 q_{T_k}^{HM} + c_3 \tilde{\sigma}_k^{max} + c_2 \{c_0 + c_1 q_{T_{k-1}}^{HM} + c_2 E_{T_{k-2}}(\epsilon_{k-2}) + c_3 \tilde{\sigma}_{k-1}^{max}\}.$$

Iterating these substitutions for $\epsilon_{k-2}, \epsilon_{k-3}, \dots$ shows that for any $0 \leq m \leq k-1$, $E_t(\epsilon_k)$ can be approximated as a function of $q_{T_k}^{HM}, \dots, q_{T_{k-m}}^{HM}$, $E_{T_{k-m-1}}(\epsilon_{k-m-1})$, and $\tilde{\sigma}_k^{max}, \tilde{\sigma}_{k-1}^{max}, \dots, \tilde{\sigma}_{k-m}^{max}$. In the extreme case, for $m = k-1$, $E_t(\epsilon_k)$ is a function of q^{HM} and σ^{max} only. Again, this inflates the number of regressors and we therefore truncate this iteration. It again holds that

$$(24) \quad E_{T_{k-1}}(\epsilon_{k-1}) = \sum_N E_{T_{k-1}}(\epsilon_{k-1} | N) P_{T_{k-1}}(N),$$

but where $\max\{\epsilon_{k-1}, \Sigma_{k-1}^{max}\}$ replaces ϵ_{k-1}

$$(25) \quad E_{T_{k-1}}(\epsilon_{k-1} | N_{T_{k-1}}) = \bar{\epsilon} - \int_{\max\{\epsilon_{k-1}, \Sigma_{k-1}^{max}\}}^{\bar{\epsilon}} F(\epsilon)^{1+N_{T_{k-1}}} d\epsilon.$$

The same linearization as before yields

$$(26) \quad E_{T_{k-1}}(\epsilon_{k-1}) \approx c_4 + c_5 q_{T_{k-1}}^{EH} + c_6 \Sigma_{k-1}^{max}.$$

Thus, as in the main text, we use the two regressors $q_{T_k}^{HM}$ and $q_{T_{k-1}}^{EH}$ to control for our selection effects (though on the job search) and add two further regressors $\tilde{\sigma}_k^{max}$ and Σ_{k-1}^{max} to control for endogenous separations.

We thus have that

$$(27) \quad E_t(\epsilon_k) \approx c_0 + c_1 q_{T_k}^{HM} + c_2 (c_4 + c_5 q_{T_{k-1}}^{EH} + c_6 \Sigma_{k-1}^{max}) + c_3 \tilde{\sigma}_k^{max}.$$

Finally, we approximate

$$(28) \quad \log(\epsilon) \approx \tilde{c}_0 + \tilde{c}_1 \log(q^{HM}) + \tilde{c}_2 \log(q^{EH}) + \tilde{c}_3 \log(\tilde{\sigma}^{max}) + \tilde{c}_4 \log(\Sigma^{max}),$$

for coefficients \tilde{c}_i .

D. Wage Volatility of Job Stayers and Switchers

Consider a worker who has already received N offers in his current employment spell. An unemployed worker is a special case for $N = 0$. The probability to switch from job-to-job for this worker who receives k offers in the current period equals

$$(29) \quad \int \frac{\partial F^N(\epsilon)}{\partial \epsilon} (1 - F^k(\epsilon)) d\epsilon = \frac{k}{N+k}.$$

Since the unconditional probability to receive k offers from M trials with a success probability q in each trial equals

$$(30) \quad \binom{M}{k} q^k (1-q)^{M-k},$$

the probability to switch equals

$$(31) \quad \sum_{k=1}^M \binom{M}{k} q^k (1-q)^{M-k} \frac{k}{N+k}.$$

Using Bayes' Law then shows that the probability for a switcher to have received k offers equals

$$(32) \quad \frac{k}{N+k} \frac{\binom{M}{k} q^k (1-q)^{M-k}}{\sum_l \frac{l \binom{M}{l} q^l (1-q)^{M-l}}{N+l}}.$$

The distribution of ϵ in the switching period then equals

$$(33) \quad \sum_{k=1}^M F(\epsilon)^{N+k} \frac{k}{N+k} \frac{\binom{M}{k} q^k (1-q)^{M-k}}{\sum_l \frac{l \binom{M}{l} q^l (1-q)^{M-l}}{N+l}} = F(\epsilon)^N \Psi(q, F(\epsilon)).$$

The difference between two distributions with different success probabilities, $\hat{q} > q$, is proportional to

$$(34) \quad \Delta(q, \hat{q}, x) = \Psi(q, x) - \Psi(\hat{q}, x),$$

where $x = F(\epsilon)$.

We now show that $\Delta \geq 0$ for all x which is equivalent to $\Psi(\hat{q}, F(\cdot))$ first-order stochastically dominating $\Psi(q, F(\cdot))$ and thus also $F(\epsilon)^N \Psi(\hat{q}, F(\cdot))$ first-order stochastically dominating $F(\epsilon)^N \Psi(q, F(\cdot))$.

The first derivative of Ψ w.r.t x equals

$$(35) \quad \Psi_x(q, x) = \frac{\sum_{k=1}^M k x^{k-1} \frac{k}{N+k} \binom{M}{k} q^k (1-q)^{M-k}}{\sum_{l=1}^M \frac{l}{N+l} \binom{M}{l} q^l (1-q)^{M-l}}.$$

Since $\Psi(q, x=0) = 0$ and $\Psi(q, x=1) = 1$

$$(36) \quad \Delta(q, \hat{q}, 0) = \Delta(q, \hat{q}, 1) = 0.$$

Thus

$$(37) \quad \Delta(q, \hat{q}, x) = \int_0^x (\Psi_x(q, z) - \Psi_x(\hat{q}, z)) dz = \int_0^x \Psi_x(q, z) \left(1 - \frac{\Psi_x(\hat{q}, z)}{\Psi_x(q, z)}\right) dz.$$

To determine the sign of this integral we now show that

$$(38) \quad 1 - \frac{\Psi_x(\hat{q}, z)}{\Psi_x(q, z)} = 1 - \frac{(\sum_{l=1}^M \frac{l}{N+l} \binom{M}{l} q^l (1-q)^{M-l}) (\sum_{k=1}^M k z^{k-1} \frac{k}{N+k} \binom{M}{k} \hat{q}^k (1-\hat{q})^{M-k})}{(\sum_{l=1}^M \frac{l}{N+l} \binom{M}{l} \hat{q}^l (1-\hat{q})^{M-l}) (\sum_{k=1}^M k z^{k-1} \frac{k}{N+k} \binom{M}{k} q^k (1-q)^{M-k})}$$

is decreasing in z . To establish this we show that

$$(39) \quad \frac{\sum_{k=1}^M k z^{k-1} \frac{k}{N+k} \binom{M}{k} \hat{q}^k (1-\hat{q})^{M-k}}{\sum_{k=1}^M k z^{k-1} \frac{k}{N+k} \binom{M}{k} q^k (1-q)^{M-k}}$$

is increasing in z . The derivative w.r.t z equals

$$(40) \quad - \frac{(\sum_{k=1}^M k(k-1) z^{k-2} \frac{k}{N+k} \binom{M}{k} \hat{q}^k (1-\hat{q})^{M-k}) (\sum_{k=1}^M k z^{k-1} \frac{k}{N+k} \binom{M}{k} q^k (1-q)^{M-k})}{(\sum_{k=1}^M k z^{k-1} \frac{k}{N+k} \binom{M}{k} q^k (1-q)^{M-k})^2} - \frac{(\sum_{k=1}^M k(k-1) z^{k-2} \frac{k}{N+k} \binom{M}{k} q^k (1-q)^{M-k}) (\sum_{k=1}^M k z^{k-1} \frac{k}{N+k} \binom{M}{k} \hat{q}^k (1-\hat{q})^{M-k})}{(\sum_{k=1}^M k z^{k-1} \frac{k}{N+k} \binom{M}{k} q^k (1-q)^{M-k})^2}.$$

For $\delta_{k,j} = k j \binom{M}{k} \binom{M}{j} \frac{k}{N+k} \frac{j}{N+j} z^{k+j-3} > 0$ the numerator equals

$$\begin{aligned}
& \sum_{k=1}^M \sum_{j=1}^M \hat{q}^k (1-\hat{q})^{M-k} q^j (1-q)^{M-j} (k-1) \delta_{k,j} \\
& - \sum_{k=1}^M \sum_{j=1}^M q^k (1-q)^{M-k} \hat{q}^j (1-\hat{q})^{M-j} (k-1) \delta_{k,j} \\
(41) \quad & = \sum_{k=1}^M \sum_{j=1}^M \{ \hat{q}^k (1-\hat{q})^{M-k} q^j (1-q)^{M-j} - \hat{q}^j (1-\hat{q})^{M-j} q^k (1-q)^{M-k} \} (k-j) \delta_{k,j}.
\end{aligned}$$

If $k > j$

$$\begin{aligned}
(42) \quad & \hat{q}^k (1-\hat{q})^{M-k} q^j (1-q)^{M-j} - \hat{q}^j (1-\hat{q})^{M-j} q^k (1-q)^{M-k} \\
(43) \quad & = \hat{q}^j (1-\hat{q})^{M-k} q^j (1-q)^{M-k} \{ \hat{q}^{k-j} (1-q)^{k-j} - q^{k-j} (1-\hat{q})^{k-j} \} > 0.
\end{aligned}$$

If $k < j$

$$\begin{aligned}
(44) \quad & \hat{q}^k (1-\hat{q})^{M-k} q^j (1-q)^{M-j} - \hat{q}^j (1-\hat{q})^{M-j} q^k (1-q)^{M-k} \\
(45) \quad & = \hat{q}^k (1-\hat{q})^{M-j} q^k (1-q)^{M-j} \{ q^{j-k} (1-\hat{q})^{j-k} - \hat{q}^{j-k} (1-q)^{j-k} \} < 0.
\end{aligned}$$

This establishes that the numerator in (41) is positive and thus that $1 - \frac{\Psi_x(\hat{q}, z)}{\Psi_x(q, z)}$ (equation 38) is decreasing in z . Since the derivative Ψ_x is positive (equation 35) and $\Delta(q, \hat{q}, 0) = \Delta(q, \hat{q}, 1) = 0$, it follows (by the same arguments as in footnote 2) that $\Delta(q, \hat{q}, x) \geq 0$.

II. Summary Statistics, NLSY Data

Table 1: Summary Statistics, NLSY Data.

Variable	Statistic		
	Min	Mean	Max
1. Age (years)	16	28.38	48
2. Years of Education	4	12.97	20
3. Marital status (1-married)	0	0.44	1
4. Race - white	0	0.82	1
5. Race - black	0	0.11	1
6. Race - other	0	0.07	1
7. Job tenure (weeks)	2	128.63	1322
8. Actual experience (weeks)	2	453.49	1398.5
9. Number of employment cycles	1	5.94	33
10. Jobs per employment cycle	1	1.62	13
Number of observations	42741		
Number of individuals	2785		

III. Wage Losses among Job-to-Job Switchers

Our NLSY data conforms with the findings on SIPP (Survey of Income and Program Participation) and other data sets that a sizable fraction of job-to-job transitions is associated with wage declines.

Wage losses in our model are due to loss of firm specific human capital, which completely depreciates when a worker switches firms. This is how papers that aim at estimating the returns to firm tenure (e.g. Altonji and Shakotko (1987), Topel (1991)) identify the returns to tenure. The change in wages between jobs is equal to the gain in match quality plus the loss in firm specific human capital. If we control for match quality (as we can through the inclusion of q^{HM} and q^{EH}), then the change in specific human capital is equal to the wage change minus the gain in match quality. If the observed wage change is negative, then since the gain in match quality is positive, the loss of specific human capital is positive.

How big are the wage losses generated by our model? Consider the following experiment. Using our NLSY sample we regress log wage on tenure, experience, q^{HM} , and q^{EH} and obtain fitted values from this regression. Using these fitted values we find that 28.4% of job-to-job moves are accompanied by a wage decline. The average wage decline among them is 9.8%. This accounts for all the systematic difference in wage cuts between job stayers and job-to-job switchers.⁴ Thus, our model has no difficulty generating wage declines upon job-to-job moves found in the data.

⁴Both job-stayers and job-to-job switchers occasionally experience substantial wage losses in the data and our model only accounts for the excess wage losses of job-to-job switchers. Adding idiosyncratic productivity shocks (or measurement error) would allow the model to account for the full amount of wage losses as these shocks are independent of a worker's mobility. Wage losses of job stayers are then due to negative productivity shocks only whereas wage losses of job-to-job switchers are a combination of changes in productivity and of losses of specific human capital.

IV. Alternative Specifications and Sensitivity Analysis using NLSY Data

A. Results based on Quadratic Specification.

Table 2: Controlling for Match Qualities in Beaudry-DiNardo Regressions. NLSY.

Variable		Specification					
		1	2	3	4	5	6
1.	u	-3.090 (0.519)	-1.527 (0.421)	-1.080 (0.734)	-1.369 (0.597)	-2.450 (0.697)	-1.524 (0.458)
2.	u^{min}	—	—	-3.023 (0.708)	-0.249 (0.592)	—	—
3.	u^{begin}	—	—	—	—	-1.183 (0.398)	-0.006 (0.334)
4.	q^{HM}	—	7.832 (0.444)	—	7.795 (0.443)	—	7.831 (0.447)
5.	q^{EH}	—	2.516 (0.521)	—	2.492 (0.536)	—	2.515 (0.540)
6.	$Tenure$	3.568 (0.347)	0.866 (0.284)	2.989 (0.346)	0.830 (0.278)	3.693 (0.351)	0.867 (0.296)
7.	$Tenure^2$	-0.131 (0.022)	-0.016 (0.014)	-0.107 (0.021)	-0.015 (0.014)	-0.129 (0.022)	-0.016 (0.014)
8.	$Experience$	7.206 (0.608)	7.495 (0.588)	7.217 (0.606)	7.495 (0.588)	7.156 (0.606)	7.495 (0.587)
9.	$Experience^2$	-0.132 (0.017)	-0.127 (0.015)	-0.130 (0.016)	-0.127 (0.015)	-0.130 (0.016)	-0.127 (0.015)
10.	$Grade$	-8.532 (5.179)	-6.981 (5.182)	-8.075 (5.207)	-6.951 (5.179)	-8.165 (5.204)	-6.980 (5.181)
11.	$Grade^2$	0.611 (0.193)	0.539 (0.193)	0.570 (0.194)	0.538 (0.193)	0.599 (0.194)	0.539 (0.193)

Note - Standard errors are in parentheses. All coefficients and standard errors are multiplied by 100.

Table 3: Wage Volatility of Job Stayers and Switchers. NLSY.

	Variable	Specification		
		Job Stayers	Job Switchers	
		1	2	3
1.	u	-2.234 (0.372)	-3.505 (0.487)	-1.872 (0.4497)
2.	q^{HM}	—	—	5.402 (0.402)
3.	q^{EH}	—	—	3.767 (0.701)

Note - Standard errors in parentheses. All coefficients and standard errors are multiplied by 100.

B. Results based on HP-Filtered Data.

Table 4: Controlling for Match Qualities in Beaudry-DiNardo Regressions. HP-Filtered NLSY Data.

Variable	Specification					
	1	2	3	4	5	6
1. u	-5.006 (0.910)	-2.516 (0.746)	-2.550 (1.179)	-2.455 (0.980)	-3.931 (0.964)	-2.324 (0.782)
2. u^{min}	—	—	-4.305 (1.107)	-0.112 (0.960)	—	—
3. u^{begin}	—	—	—	—	-2.269 (0.626)	-0.429 (0.549)
4. q^{HM}	—	7.621 (0.466)	—	7.611 (0.471)	—	7.589 (0.469)
5. q^{EH}	—	2.884 (0.507)	—	2.879 (0.517)	—	2.833 (0.515)

Note - Standard errors are in parentheses. All coefficients and standard errors are multiplied by 100.

Table 5: Wage Volatility of Job Stayers and Switchers. HP-Filtered NLSY Data.

	Variable	Specification		
		Job Stayers	Job Switchers	
		1	2	3
1.	u	-2.822 (0.545)	-5.238 (0.803)	-2.795 (0.815)
2.	q^{HM}	—	—	5.415 (0.400)
2.	q^{EH}	—	—	3.780 (0.703)

Note - Standard errors in parentheses. All coefficients and standard errors are multiplied by 100.

C. Additional Sensitivity Analysis.

Table 6: Controlling for Match Qualities in Beaudry-DiNardo Regressions. NLSY.
Downweighting Short Duration Jobs.

Variable	Specification					
	1	2	3	4	5	6
1. u	-3.309 (0.563)	-2.033 (0.494)	-2.116 (0.884)	-2.150 (0.840)	-3.060 (0.660)	-2.210 (0.604)
2. u^{min}	—	—	-2.030 (0.875)	0.207 (0.794)	—	—
3. u^{begin}	—	—	—	—	-0.600 (0.509)	0.452 (0.492)
4. q^{HM}	—	7.499 (0.738)	—	7.527 (0.720)	—	7.557 (0.736)
5. q^{EH}	—	3.833 (1.259)	—	3.844 (1.268)	—	3.929 (1.287)

Note - Standard errors are in parentheses. All coefficients and standard errors are multiplied by 100. This sensitivity analysis implements the procedure for taking into account the heterogeneity in the number of job spells across individuals. Two or three jobs are randomly selected from each individual in the sample and the regressions are estimated on this collection of jobs.

Table 7: Controlling for Match Qualities in Beaudry-DiNardo Regressions. NLSY.
No Unionized or Government Workers.

Variable	Specification					
	1	2	3	4	5	6
1. u	-3.336 (0.466)	-1.814 (0.407)	-1.851 (0.785)	-1.876 (0.717)	-2.763 (0.538)	-1.754 (0.462)
2. u^{min}	—	—	-2.175 (0.810)	0.095 (0.761)	—	—
3. u^{begin}	—	—	—	—	-1.034 (0.433)	-0.112 (0.414)
4. q^{HM}	—	7.061 (0.505)	—	7.073 (0.495)	—	7.048 (0.504)
5. q^{EH}	—	2.401 (0.572)	—	2.410 (0.593)	—	2.373 (0.599)

Note - Standard errors are in parentheses. All coefficients and standard errors are multiplied by 100.

Table 8: Controlling for Match Qualities in Beaudry-DiNardo Regressions. NLSY.
Workers Older than 30.

Variable	Specification					
	1	2	3	4	5	6
1. u	-5.235 (0.682)	-3.760 (0.635)	-3.006 (0.818)	-3.438 (0.843)	-4.869 (0.679)	-3.938 (0.652)
2. u^{min}	—	—	-4.293 (1.160)	-0.678 (1.186)	—	—
3. u^{begin}	—	—	—	—	-1.135 (0.640)	0.625 (0.658)
4. q^{HM}	—	7.111 (0.843)	—	6.967 (0.814)	—	7.226 (0.831)
5. q^{EH}	—	3.314 (0.877)	—	3.269 (0.887)	—	3.419 (0.895)

Note - Standard errors are in parentheses. All coefficients and standard errors are multiplied by 100.

Table 9: Controlling for Match Qualities in Beaudry-DiNardo Regressions. NLSY.
 q as Cyclical Indicator.

Variable	Specification					
	1	2	3	4	5	6
1. q	11.129 (1.504)	6.349 (1.312)	7.091 (2.198)	7.435 (2.034)	8.087 (1.686)	5.550 (1.538)
2. q^{max}	—	—	6.717 (2.230)	-1.945 (2.266)	—	—
3. q^{begin}	—	—	—	—	5.193 (1.292)	0.619 (1.289)
4. q^{HM}	—	7.336 (0.471)	—	7.462 (0.447)	—	7.214 (0.469)
5. q^{EH}	—	2.872 (0.502)	—	2.937 (0.526)	—	2.760 (0.553)

Note - Standard errors are in parentheses. All coefficients and standard errors are multiplied by 100.

Table 10: Controlling for Match Qualities in Beaudry-DiNardo Regressions. NLSY.
 q^{HM} and q^{EH} Constructed using the Job Finding Probability.

Variable	Specification					
	1	2	3	4	5	6
1. u	-3.455 (0.528)	-2.797 (0.444)	-1.804 (0.790)	-2.260 (0.698)	-2.884 (0.598)	-2.25 (0.500)
2. u^{min}	—	—	-2.439 (0.781)	-0.803 (0.696)	—	—
3. u^{begin}	—	—	—	—	-1.039 (0.399)	-0.502 (0.365)
4. q^{HM}	—	8.004 (0.486)	—	7.929 (0.476)	—	7.972 (0.485)
5. q^{EH}	—	3.082 (0.554)	—	3.036 (0.566)	—	3.006 (0.569)

Note - Standard errors are in parentheses. All coefficients and standard errors are multiplied by 100.

D. Allowing for Returns to Specific Human Capital

Our empirical analysis follows much of the empirical literature (e.g., Altonji and Shakotko (1987), Topel (1991)) and allows for three reasons for wage growth: (1) accumulation of general human capital – transferable across employers – with experience, (2) accumulation of firm specific human capital with firm tenure, and (3) selection into matches of higher quality over time. In the main text these three reasons for wage growth are independent. Theoretically however this is not the case as the accumulation of firm-specific human capital and the selection into better matches are not independent.⁵ The reason is quite simple. The probability to switch employers depends negatively on the amount of firm specific human capital (measured as firm tenure) since a switch leads to a complete loss of this specific capital. The probability to switch employers also depends negatively on the match quality since a higher current match quality makes a better offer less likely. Thus observing someone with high completed tenure is not necessarily an indication of a high match quality but may just reflect a large amount of human capital. In other words, rejecting offers is only partially informative about match quality since high amount of specific human capital also leads workers to reject more offers.

Intuitively, if workers accumulate firm specific human capital, the timing of receiving offers matters. At the beginning of the firm spell, workers have little firm-specific human capital and the decision to switch is largely determined by match quality. The switching decision is different for a worker with high tenure since this worker would lose his specific human capital by switching. Receiving offers for such a worker is thus less likely to result in a switch. More generally, it matters when the worker receives the offer. Early offers are more informative than later offers about match quality (under some distributional assumptions). However, the magnitude of these differences depends on the quantitative significance of specific human capital. If workers accumulate little firm-specific human capital then we would not expect large differences between early and late offers.

In a companion paper “Estimating the Returns to Tenure and Experience” we formalize this intuition and show how the regressors q^{HM} and q^{EH} need to be adjusted. Assume that the worker switched employers in periods $1 + S_1, 1 + S_2, \dots, 1 + S_k$, so that this worker stayed with his first employer between periods 0 and S_1 , with the second employer between period $1 + S_1$ and S_2 and with employer j between period $1 + S_{j-1}$ and S_j . (We set $S_0 = -1$.) For such an employment cycle, a sequence q_0, \dots, q_{S_j} of job-offer probabilities and coefficients $\{\rho_t\}_{t=1, \dots}$, define

$$(46) \quad \tilde{q}_t^{HM} = \rho_1 q_{1+S_{j-1}} + \dots + \rho_{S_j - S_{j-1}} q_{S_j} \quad \text{for} \quad 1 + S_{j-1} \leq t \leq S_j$$

⁵General human capital is by definition perfectly transferable across employers and has no effect on the mobility decisions.

and

$$(47) \quad \tilde{q}_t^{EH} = \tilde{q}_{S_1}^{HM} + \dots + \tilde{q}_{S_{j-1}}^{HM} \quad \text{for } 1 + S_{j-2} \leq t \leq S_{j-1}.$$

The variable \tilde{q}_t^{HM} is constant within every job spell and equals the weighted (by ρ_t) sum of q 's from the start of the current job spell until the the last period of this job spell. The variable \tilde{q}_t^{EH} summarizes the employment history in the current employment cycle until the start of the current job spell. Theory requires that receiving an offer in different period has different predictive power for match quality. The coefficients ρ_k describe the importance of receiving an offer in period k of a spell.⁶ For example the ‘‘marginal effect’’ of receiving an offer in period $S_{j-1} + k$ is given by the coefficient ρ_k . Whereas our variable q^{HM} weighs all periods equally, taking into account firm specific human capital requires an adjustment for periods $2, \dots$ in every spell. As a result we modify our regression which includes only q_t^{HM} and q_t^{EH} by adding the variables $q_{S_{j-1}+2}, \dots, q_{S_{j-1}+k}, \dots$ to capture the different marginal effects in periods $2, \dots, k, \dots$. Theory requires also to modify how we capture the employment history. For every $k \geq 2$ and $0 \leq m \leq j - 2$ define $\tilde{q}_{k+S_m} = q_{k+S_m}$ if $k + S_m \leq S_{m+1}$ and $\tilde{q}_{k+S_m} = 0$ if $k + S_m > S_{m+1}$. We can then define

$$(48) \quad \tilde{q}_{t,k}^{EH} = \tilde{q}_{k+S_0} + \dots + \tilde{q}_{k+S_m} \dots + \tilde{q}_{k+S_{j-2}} \quad \text{for } 1 + S_{j-1} \leq t \leq S_j$$

as the sum of the q 's in period k of every spell (\tilde{q}_{k+S_m} is the probability to receive an offer in period $k + S_m \leq S_{m+1}$, the k^{th} period in the spell that lasts from period $1 + S_m$ until S_{m+1}). The variable $\tilde{q}_{t,k}^{EH}$ thus summarizes the information \tilde{q} from the past from receiving an offer when tenure equals k .

Thus we should include the following regressors in period $1 + S_{j-1} \leq t \leq S_j$

$$(49) \quad q_{S_j}^{HM}, q_{S_{j-1}+2}, \dots, q_{S_{j-1}+k}, \dots, q_{S_j}$$

as well as

$$(50) \quad q_{S_{j-1}}^{EH}, \tilde{q}_{t,2}^{EH}, \tilde{q}_{t,3}^{EH}, \dots, \tilde{q}_{t,k}^{EH}, \dots$$

Including so many variables is not productive given the size of our data because many of these variables would be statistically insignificant. We therefore reduce the number of regressors substantially. We consider only four intervals - less than 3 years, between 3 and 6 years, between 6

⁶We can also show that ρ_k is smaller for higher tenure levels if $\frac{f_x}{F}$ is decreasing in x (F is the offer distribution). This condition is stronger than F being *log-concave* ($\log(F)$ being concave), so that we do not necessarily expect this property to hold.

and 9 years, and 9 years or more - and we consider those intervals as one period when constructing our regressors. Thus, for the empirical implementation we define (for the current job)

$$(51) \quad \tilde{q}_{t,2}^{HM} = \sum_{S_{j-1}+3years \leq s < S_{j-1}+6years} q_s \quad \text{for } 1 + S_{j-1} \leq t \leq S_j,$$

$$(52) \quad \tilde{q}_{t,3}^{HM} = \sum_{S_{j-1}+6years \leq s < S_{j-1}+9years} q_s \quad \text{for } 1 + S_{j-1} \leq t \leq S_j,$$

$$(53) \quad \tilde{q}_{t,4}^{HM} = \sum_{S_{j-1}+9years \leq s} q_s \quad \text{for } 1 + S_{j-1} \leq t \leq S_j,$$

and for the employment history

$$(54) \quad \tilde{q}_{t,2}^{EH} = \tilde{q}_{S_{1,2}}^{HM} + \dots + \tilde{q}_{S_{j-1,2}}^{HM} \quad \text{for } 1 + S_{j-1} \leq t \leq S_j,$$

$$(55) \quad \tilde{q}_{t,3}^{EH} = \tilde{q}_{S_{1,3}}^{HM} + \dots + \tilde{q}_{S_{j-1,3}}^{HM} \quad \text{for } 1 + S_{j-1} \leq t \leq S_j,$$

$$(56) \quad \tilde{q}_{t,4}^{EH} = \tilde{q}_{S_{1,4}}^{HM} + \dots + \tilde{q}_{S_{j-1,4}}^{HM} \quad \text{for } 1 + S_{j-1} \leq t \leq S_j.$$

Thus we include the following regressors in period $1 + S_{j-1} \leq t \leq S_j$

$$(57) \quad q_{S_j}^{HM}, \tilde{q}_{S_j,2}^{HM}, \tilde{q}_{S_j,3}^{HM}, \tilde{q}_{S_j,4}^{HM}$$

as well as

$$(58) \quad q_{S_{j-1}}^{EH}, \tilde{q}_{S_{j-1,2}}^{EH}, \tilde{q}_{S_{j-1,3}}^{EH}, \tilde{q}_{S_{j-1,4}}^{EH}.$$

In the companion paper we estimate the returns to tenure and experience adding these additional variables. We find that the wage growth is relatively fast over the first three years with a firm (three years of tenure are associated with an increase in wages of about 5%) and is close to zero afterwards. As the returns to tenure are fairly small and concentrated at the very beginning of the job spell, they do not substantially affect the firm switching behavior of the workers and the switching behavior does not change much over the job spell. As a result, the timing of offers matters very little rendering the additional regressors implied by the extended theory insignificant as is illustrated in Table 11. Adding Beaudry-DiNardo regressors to these regressions in columns (3) - (6) leaves all the conclusion from the paper unchanged. Both regressors u^{min} and u^{begin} are strongly insignificant while q^{HM} and q^{EH} are strongly significant.

Table 11: Controlling for Match Qualities in Beaudry-DiNardo Regressions. Regressors Adjusted for the Presence of Returns to Specific Human Capital.

Variable	Specification					
	1	2	3	4	5	6
1. u	-3.455 (0.528)	-1.876 (0.445)	-1.804 (0.790)	-1.882 (0.714)	-2.884 (0.598)	-1.849 (0.506)
2. u^{min}	—	—	-2.439 (0.781)	0.010 (0.709)	—	—
3. u^{begin}	—	—	—	—	-1.039 (0.399)	-0.051 (0.386)
4. q^{HM}	—	7.426 (0.480)	—	7.427 (0.471)	—	7.418 (0.479)
5. q^{EH}	—	2.747 (0.488)	—	2.748 (0.509)	—	2.736 (0.514)
6. \tilde{q}_2^{HM}	—	-0.320 (0.764)	—	-0.322 (0.761)	—	-0.296 (0.816)
7. \tilde{q}_3^{HM}	—	2.275 (1.327)	—	2.274 (1.333)	—	2.288 (1.322)
8. \tilde{q}_4^{HM}	—	2.597 (2.640)	—	2.595 (2.664)	—	2.619 (2.686)

Note - Standard errors are in parentheses. All coefficients and standard errors are multiplied by 100.

E. Completed Tenure and the Number of Offers per Period

In this section we investigate the effects of breaking $\log(q^{HM})$ into its two components, the realized duration $\log(\bar{T})$ and the offer rate per unit of time, $\log(OR) = \log(q^{HM}) - \log(\bar{T})$. This experiment is purely empirical since the theory developed in this paper clearly implies to include $\log(q^{HM})$ and not to break it up. The total number of offers received determines the selection into better matches. Whether a worker received many offers because of high duration or because of favorable business cycle conditions does not matter. Theory therefore suggests that breaking our regressor will lead to a misspecified model. Such misspecifications become particularly troublesome in multivariate regressions as the ones we are implementing. A main concern with the misspecified model is the strong correlation of completed tenure $\log(\bar{T})$ and tenure, which becomes an issue when we break our regressor. This biases the estimated return on tenure and as a result the estimates of all other coefficients.

To deal with this concern we proceed in two steps. In a first step we implement a wage regression, that includes all the regressors implied by the theory to control for match quality, q^{EH} , q^{HM} , $\tilde{\sigma}, \dots$, (as well as our standard additional regressors, such as u , marital status, etc). Since we add q^{EH} and q^{HM} to control for match quality, we obtain unbiased estimates of the returns to tenure and experience. We then subtract the estimates (for tenure, experience, marital status, ...) from wages, to obtain a wage residual. In the second step we then implement two regressions, one with u^{min} and one without u^{min} , where in both regressions we replace $\log(q^{HM})$ with $\log(\bar{T})$ and $\log(OR)$. Table 12 reports the results. Both $\log(\bar{T})$ and $\log(OR)$ are significant and u^{min} is strongly insignificant even in this regression. The estimated coefficients on $\log(\bar{T})$ and $\log(OR)$ are statistically not distinguishable.⁷

Table 12: Controlling for Match Qualities in Beaudry-DiNardo Regressions. NLSY. Specification with Completed Tenure and the Offer Rate per Unit of Time.

	u	$\log(\bar{T})$	$\log(OR)$	q^{EH}	$\tilde{\sigma}^{max}$	Σ^{max}	u^{min}
1.	-2.840 (0.420)	7.087 (0.436)	5.265 (1.322)	1.186 (0.439)	0.428 (0.048)	0.208 (0.074)	- -
2.	-2.697 (0.675)	7.030 (0.407)	5.214 (1.312)	1.149 (0.470)	0.431 (0.048)	0.217 (0.074)	-0.208 (0.683)

Note - All coefficients and standard errors (except those on $\tilde{\sigma}^{max}$ and Σ^{max}) are multiplied by 100.

⁷The latter finding might be an artifact of a relatively small sample in the NLSY. When we add the two components $\log(OR)$ and $\log(\bar{T})$ instead of $\log(q^{HM})$ to the wage regression in the data generated by our model, we find that the coefficient on $\log(\bar{T})$ is also larger than the coefficient on $\log(OR)$.

V. Results based on the Panel Study of Income Dynamics Data.

A. PSID Data

We use the PSID data over the 1976-1997 period. The PSID has the advantage of being a panel representative of the population in every year. Moreover, it is the dataset originally used by Beaudry and DiNardo (1991). Unfortunately, it does not permit the construction of q^{EH} because unemployment data is not available in some of the years making it impossible to construct histories of job spells uninterrupted by unemployment. Thus, we are only able to include q^{HM} into the regression.

Identifying jobs is notoriously difficult in the PSID. Results below are based on the same procedure for constructing job spells and making tenure consistent within spells as in Beaudry and DiNardo (1991). The results are not sensitive to this.

B. PSID Results

The results of estimating the regressions that evaluate the influence of implicit contracts on wages are presented in Table 14. Despite our limited ability to control for selection in the PSID data, the inclusion of q^{HM} into the regression renders minimum unemployment highly insignificant. Unemployment at the start of the job flips sign.⁸

Table 13 shows that our results and those of Beaudry and DiNardo (1991) are not driven by the restrictive curvature specification on the returns to tenure and experience. Instead of the quadratic specification in the benchmark specification, the estimates reported in this table are based on a regression that includes a full set of annual tenure and experience dummies.

In Table 15 we compare the wage volatility of job stayers and job switchers. As in the NLSY, wages of job switchers are more cyclical. However, once we control for selection, we find little difference in the cyclical behavior of wages for job stayers and job switchers.

⁸A similar flipping of a sign of unemployment at start of a job was noted by McDonald and Worswick (1999). We also find it in simulations of the model. This is not unexpected in multiple regressions where one or more regressors are imperfect proxies for match quality (Greene (2002)). Coefficients can not only be attenuated but can also flip signs.

Table 13: Controlling for Match Qualities in Beaudry-DiNardo Regressions. PSID.
Specification with Tenure and Experience Dummies.

Variable	Specification					
	1	2	3	4	5	6
1. u	-1.216 (0.144)	-0.848 (0.146)	-0.905 (0.169)	-0.988 (0.169)	-1.240 (0.151)	-1.012 (0.151)
2. u^{min}	—	—	-0.789 (0.224)	0.382 (0.236)	—	—
3. u^{begin}	—	—	—	—	0.099 (0.194)	0.835 (0.198)
4. q^{HM}	—	5.584 (0.325)	—	5.746 (0.3414)	—	5.879 (0.333)

Note - Standard errors in parentheses. All coefficients and standard errors are multiplied by 100. The model includes a full set of tenure and experience dummies.

Table 14: Controlling for Match Qualities in Beaudry-DiNardo Regressions. PSID.
Specification with Quadratic in Tenure and Experience.

Variable	Specification					
	1	2	3	4	5	6
1. u	-1.160 (0.145)	-0.715 (0.145)	-0.545 (0.169)	-0.758 (0.168)	-1.163 (0.151)	-0.902 (0.151)
2. u^{min}	—	—	-1.567 (0.220)	0.120 (0.234)	—	—
3. u^{begin}	—	—	—	—	-0.023 (0.195)	0.940 (0.198)
4. q^{HM}	—	7.066 (0.305)	—	7.122 (0.324)	—	7.370 (0.312)

Note - Standard errors in parentheses. All coefficients and standard errors are multiplied by 100.

Table 15: Wage Volatility of Job Stayers and Switchers. PSID.

Variable		Specification		
		Job Stayers	Job Switchers	
		1	2	3
1.	u	-1.200 (0.199)	-1.527 (0.435)	-1.256 (0.443)
2.	q^{HM}	— —	— —	1.863 (0.592)

Note - Standard errors in parentheses. All coefficients and standard errors are multiplied by 100.

VI. Simulations of Contracting Models.

In this section we simulate data from three calibrated contracting models and estimate various wage regressions on it. The models are the canonical insurance against aggregate risk model analyzed in Beaudry and DiNardo (1991) and the business cycle versions of the models in Postel-Vinay and Robin (2002) and Cahuc, Postel-Vinay and Robin (2006). In each of these models wages depend on the best aggregate conditions since the start of the job. Thus, the lowest unemployment rate since the start of the job should be predicting wages in these models. We verify that unemployment since the start of the job remains a significant predictor of wages even when q^{EH} and q^{HM} are controlled for in the calibrated versions of the models. In addition, these models feature commitment of employers to matching outside offers. This implies that wages at date t on the job depend on the sum of offers received from the start of the job until date t , i.e. on the variable $q_t^{Contract}$. We verify that this remains the case in a regression that includes q^{EH} and q^{HM} .

A. Insurance Model of Beaudry and DiNardo (1991)

We now describe and simulate a model in which wages are set by contracts that insure risk-averse workers against aggregate fluctuations, and firms can commit to a contract but workers cannot. This is the version of the model analyzed by Beaudry and DiNardo (1991). The original model was cast in a perfectly competitive environment. To have a well-defined notion of jobs (over which u^{min} , u^{begin} , q^{EH} , q^{HM} , and $q_t^{Contract}$ are measured) we introduce search frictions and on-the-job search. The goal is, however, to remain as close as possible to the spirit and implications of the original model. Thus, all jobs have the same productivity. When an unemployed individual meets a firm, they bargain over wages that depend on aggregate labor market conditions at the time of hiring. From then on the wage is fixed unless aggregate conditions improve sufficiently so that the firm has to raise the wage to prevent the worker from separating into unemployment and looking for another job. Similarly, if an employed worker meets another firm his wages with the current firm are rebargained to equal the wage of a newly hired worker at that date.⁹ Since productivity of all matches is the same, the worker never accepts an outside offer. Separations into unemployment occur exogenously.

Denote the quality of all matches by ϵ . The productivity of a match when business cycle conditions are θ equals

$$(59) \quad \pi(\theta, \epsilon) = \alpha\theta + \epsilon.$$

⁹At a technical level this assumption means that firms do not engage in bargaining over the worker. In a richer contracting model analyzed below the worker will extract full surplus from one of the competing firms in this situation.

Workers have no access to financial markets, so consumption equals the wage w if they are employed and they consume b if unemployed. Workers' bargaining power is denoted β and the instantaneous utility function is u . Workers' and firms' common discount factor is denoted δ . The remaining notation is as before.

The lifetime utility of an unemployed worker who does not receive an offer in period t when business cycle conditions are θ_t is given by

$$(60) \quad U(\theta_t) = u(b) + \delta E_t[f(\theta_{t+1})W^{nh}(\theta_{t+1}) + (1 - f(\theta_{t+1}))U(\theta_{t+1})],$$

where $W^{nh}(\theta_t)$ denotes the value of a newly hired worker when aggregate conditions are θ_t . This value (and corresponding wages) is determined through bargaining according to

$$(61) \quad W^{nh}(\theta_t, w^{nh}) = U(\theta_t) + \beta[\hat{W}(\hat{\theta}, \theta_t) - U(\theta_t)].$$

$\hat{W}(\hat{\theta}, \theta_t)$ is the lifetime utility of a worker who extracts the full surplus from its current employer. The value function $\hat{W}(\hat{\theta}, \theta_t)$ depends on the aggregate business conditions at the time when the contract is negotiated, denoted $\hat{\theta}$, and the current aggregate conditions θ_t . When the newly hired worker bargains with the firm $\hat{\theta} = \theta_t$. This value is straightforward to compute. Recall that we assumed that the stochastic process for log market tightness follows an AR(1) process with persistence parameter ρ , so that, in levels, $\theta_{t+1} = \theta_t^\rho e^{\nu_{t+1}}$, where $\nu \sim N(0, \sigma_\nu^2)$. Since separations are exogenous at rate s , the present value of output of a match started when aggregate conditions are θ_t is given by

$$(62) \quad \bar{\pi}(\theta_t, \epsilon) := E_t \sum_t^\infty \pi(\theta_t, \epsilon) = \\ (\alpha\theta_t + \epsilon) + \delta(1-s)(\alpha E_t(\theta_{t+1}) + \epsilon) + \dots + (\delta(1-s))^k(\alpha E_t(\theta_{t+k}) + \epsilon) + \dots = \\ \frac{\epsilon}{1 - \delta(1-s)} + \alpha(\theta_t + \delta(1-s)\theta_t^\rho e^{\frac{1}{2}\sigma_\nu^2} + \dots + (\delta(1-s))^{k+1}\theta_t^{\rho^{k+1}} e^{\frac{1}{2}\sigma_\nu^2(1+\sum_{m=1}^k \rho^{2m})} + \dots).$$

Since workers prefer a constant consumption stream, the utility maximizing wage $\hat{w}(\hat{\theta})$ negotiated when aggregate conditions are $\hat{\theta}$ is constant and its present value equals to the present value of output:

$$(63) \quad \hat{w}(\hat{\theta}) = (1 - \delta(1-s))\bar{\pi}(\hat{\theta}, \epsilon).$$

This implies that

$$(64) \quad \hat{W}(\hat{\theta}, \theta_t) = \max\{U(\theta_t), u(\hat{w}(\hat{\theta})) + \delta E_t((1-s)\hat{W}(\hat{\theta}, \theta_{t+1}) + sU(\theta_{t+1}))\}.$$

The lifetime utility of an employed worker at wage w who does not receive an offer in period t and is not separated from his current employer by an exogenous separation shock is given by

$$(65) \quad \tilde{W}(\theta_t, w) = \max\{u(w) + \delta E_t W(\theta_{t+1}, w), u(\tilde{w}) + \delta E_t W(\theta_{t+1}, \tilde{w})\},$$

where $W(\theta, w)$ is the beginning of period value of having a job (before the worker receives offers or gets separated), when aggregate conditions are θ and the worker is employed at wage w . The max operator appears because if the outside option of the worker is binding wages are renegotiated to prevent the worker from leaving the job. That is, if at the current wage w the worker prefers unemployment, the wage is adjusted to \tilde{w} to make the worker indifferent between quitting and staying, i.e.

$$(66) \quad \tilde{W}(\theta_t, \tilde{w}) = U(\theta_t).$$

Otherwise, the wage w remains unchanged.

The lifetime utility of an employed worker at wage w who does not receive an offer in period t is given by

$$(67) \quad W^{no\ offer}(\theta_t, w) = (1 - s)\tilde{W}(\theta_t, w) + sU(\theta_t).$$

An employed worker at wage w who receives an offer in period t obtains the value

$$(68) \quad W^{offer}(\theta_t, w) = (1 - s) \max\{U(\theta_t), \tilde{W}(\theta_t, w), W^{nh}(\theta_t, w^{nh})\} + sU(\theta_t).$$

Finally,

$$(69) \quad W(\theta_{t+1}, w) = q(\theta_{t+1})W^{offer}(\theta_{t+1}, w) + (1 - q(\theta_{t+1}))W^{no\ offer}(\theta_{t+1}, w).$$

Calibration and Results

We calibrate the model according to the same procedure we used to calibrate the job ladder model in the main text. In particular, the driving process remains market tightness which determines the job finding rates for employed and unemployed workers. The workers' outside option b is normalized to 1. The utility function is assumed to be logarithmic. The parameter α in this model relates fluctuations in market tightness to fluctuations in aggregate productivity. We choose its value to match the standard deviation of HP-filtered (1600) log quarterly output per worker in the data equal to 0.013.

The values of the following five parameters remain to be determined: the average probabilities of receiving an offer for unemployed and employed workers $\bar{\lambda}$ and \bar{q} , the elasticity of the offer probabilities κ , the mean match quality ϵ , and the workers bargaining weight β . We determine their values by targeting the same targets as in the job ladder model in the main text except for the targets defined for job-to-job switchers because such switches do not take place in this model.

The full list of targets and the performance of the model in matching them are described in Table 16. The calibrated parameter values can be found in Table 17. The model can match the targets quite well.

When the wage regression contains only the current unemployment rate u , its coefficient is estimated to be significantly negative because wages in the model are higher in booms because (1) participation constraints bind more often resulting in an upward wage adjustments, and (2) more offers arrive in a boom resulting in bidding up of wages. However, u^{min} is a better measure of these selection effects and its inclusion into the model renders the coefficient on u small and statistically insignificant. The estimated coefficient on u^{min} is large and statistically significant.

We then add our regressor q^{HM} to these regressions in the same way we did in the data (q^{EH} is not defined in this model because there are no job-to-job transitions). The results from these regressions are presented in Table 18. We find that q^{HM} is not a significant predictor of wages and that the coefficients on u , u^{min} , and u^{begin} are little affected when it is added to the regression. This is in contrast to what we find in the data.

Since in this model wages are bid up when a worker receives an outside offer, the wage at date t on the job depends on the sum of offers received from the start of the job until date t , i.e. on the variable $q_t^{Contract}$. In Table 19 we report the results of a regression that includes $q_t^{Contract}$ and q^{HM} . The coefficient on $q_t^{Contract}$ is estimated to be positive and significant, while the coefficient on q^{HM} is not. This is also in contrast to what we find in the data.

These results are expected. In this model u^{min} and $q_t^{Contract}$ are indeed important predictors of wages. Once their effects are accounted for q^{HM} should be irrelevant. The results indicate that despite the facts that these variables are correlated, the regression has no difficulty in disentangling their effects. The fact that the estimated coefficient on q^{HM} is small and insignificant in the data generated from the insurance model but is large and significant in the data suggest that it is unlikely that the canonical insurance model of Beaudry and DiNardo (1991) can be thought of as the true data generating process.

Table 16: Insurance Model of Beaudry and DiNardo (1991): Matching the Calibration Targets.

Target	Value	
	Data	Model
1. Semi-Elasticity of wages wrt agg. unemployment u	-3.090	-2.489
2. Semi-Elasticity of wages wrt minimum unemployment u_{min}	-4.039	-4.025
3. Semi-Elasticity of wages wrt agg. unemployment u (joint reg. with u^{min})	-1.080	-0.458
4. Semi-Elasticity of wages wrt minimum unemployment u^{min} (joint reg. with u)	-3.023	-3.712
5. Semi-Elasticity of wages wrt starting unemployment u^{begin}	-2.563	-2.547
6. Semi-Elasticity of wages wrt agg. unemployment u (joint reg. with u^{begin})	-2.450	-1.742
7. Semi-Elasticity of wages wrt starting unemployment u^{begin} (joint reg. with u)	-1.183	-1.847
8. Monthly job-finding rate for unemployed	0.430	0.437
9. Std. of aggregate unemployment	0.090	0.071
10. Std. of aggregate productivity	0.013	0.014

Note - The table describes the performance of the insurance model in matching the calibration targets.

Table 17: Insurance Model of Beaudry and DiNardo (1991): Calibrated Parameter Values.

Parameter	Definition	Value
α	parameter of the output function	0.118
β	workers' bargaining weight	0.201
$\bar{\lambda}$	avg. prob to receive an offer for unemployed	0.431
\bar{q}	avg. prob to receive an offer for employed	0.207
κ	elasticity of the offer probability	0.261
ϵ	match productivity	1.818
ρ	persistence of aggregate process	0.990
σ_ν	std. of aggregate process	0.095

Note - The table contains the calibrated parameter values of the insurance model.

Table 18: Controlling for Match Qualities in Beaudry-DiNardo Regressions. Insurance Model of Beaudry and DiNardo (1991).

Variable	Specification					
	1	2	3	4	5	6
u	-2.489 [-4.12,-0.93]	-2.365 [-3.97,-0.91]	-0.458 [-1.24,0.12]	-0.467 [-1.27,0.11]	-1.742 [-2.98,-0.76]	-1.660 [-2.82,-0.75]
u^{min}	—	—	-3.712 [-5.41,-2.05]	-3.677 [-5.43,-2.03]	—	—
u^{begin}	—	—	—	—	-1.847 [-3.13,-0.85]	-1.783 [-2.84,-0.90]
q^{HM}	—	1.379 [0.59,2.32]	—	0.045 [-0.31,0.42]	—	1.287 [0.66,1.89]

Note - All coefficients are multiplied by 100. Bold font denotes significance at a 95% level based on bootstrapped confidence intervals in parentheses.

Table 19: Offers up to date t . Insurance Model of Beaudry and DiNardo (1991).

u	$q^{Contract}$	q^{HM}
-2.350 [-3.91, -0.89]	1.229 [0.73, 1.77]	0.237 [-0.24, 0.77]

Note - All coefficients are multiplied by 100. Bold font denotes significance at a 95% level based on bootstrapped confidence intervals in parentheses.

B. Offer Matching Models

We now describe and simulate benchmark models that combine on-the-job search as in our model, wages set by contracts that insulate workers from aggregate fluctuations, and with contracts renegotiated when a worker obtains an outside offer. These are effectively business cycle versions of Postel-Vinay and Robin (2002) and Cahuc, Postel-Vinay and Robin (2006). For simplicity we maintain the assumption that workers are risk-neutral but this is inessential for our qualitative findings.

B.1 Contracting Model of Cahuc, Postel-Vinay and Robin (2006)

We start with the business cycle version of the model of Cahuc, Postel-Vinay and Robin (2006). This model extends Postel-Vinay and Robin (2002) by explicitly modeling the bargaining process between the workers and the employer(s). Because this model nests Postel-Vinay and Robin (2002) as a special case, we present it first.

The productivity of a match of quality ϵ when business cycle conditions are θ equals

$$(70) \quad \pi(\theta, \epsilon) = \alpha\theta + \epsilon.$$

Workers have no access to financial markets, so consumption equals the wage w if they are employed and they consume b if unemployed. Workers' bargaining power is denoted β . Workers' and firms' common discount factor is denoted δ . The remaining notation is as before.

The lifetime utility of an unemployed worker who does not receive an offer in period t when business cycle conditions are θ_t is given by

$$(71) \quad U(\theta_t) = b + \delta E_t[f(\theta_{t+1}) \max\{W^{nh}(\theta_{t+1}, w^{nh}, \epsilon), U(\theta_{t+1})\} + (1 - f(\theta_{t+1}))U(\theta_{t+1})].$$

where $W^{nh}(\theta_{t+1}, w^{nh}, \epsilon)$ denotes the value of a newly hired worker in a firm with idiosyncratic productivity ϵ when aggregate conditions are θ_{t+1} . This value (and the corresponding wage $w^{nh}(\theta_{t+1}, \epsilon)$) is determined through bargaining according to

$$(72) \quad W^{nh}(\theta_t, w^{nh}, \epsilon) = U(\theta_t) + \beta[\hat{W}(\theta_t, \epsilon) - U(\theta_t)].$$

$\hat{W}(\theta_t, \epsilon)$ is the lifetime utility of a worker who extracts the full surplus from its current employer if business cycle conditions are θ_t . It is described below.

The lifetime utility of an employed worker at wage w and match quality ϵ who does not receive an offer in period t and is not separated from his current employer by an exogenous separation shock is given by

$$(73) \quad \tilde{W}(\theta_t, w, \epsilon) = \max\{w + \delta E_t W(\theta_{t+1}, w, \epsilon), \tilde{w} + \delta E_t W(\theta_{t+1}, \tilde{w}, \epsilon), U(\theta_t)\},$$

where $W(\theta, w, \epsilon)$ is the beginning of period value of having a job (before the worker receives offers or gets separated), when aggregate conditions are θ , the worker is employed at wage w and the match quality is ϵ . The max operator appears because if the outside option of the worker is binding wages are renegotiated to prevent the worker from leaving the job. That is, if at the current wage w the worker prefers unemployment, the wage is adjusted (if feasible) to \tilde{w} to make the worker indifferent between quitting and staying, i.e.

$$(74) \quad \tilde{W}(\theta_t, \tilde{w}, \epsilon) = U(\theta_t).$$

If the outside option is not binding, the wage w remains unchanged. If the outside option is binding and the wage adjustment is not feasible (extracting full surplus from the firm is less attractive than becoming unemployed) the worker separates into unemployment.

An employed worker who is not exogenously separated and receives an offer $\hat{\epsilon}$ that is higher than his current match quality ϵ will switch to the new job (if the highest value he can obtain in the new job dominates the option of becoming unemployed). Utility equals

$$(75) \quad \tilde{W}^s(\theta_t, \epsilon, \hat{\epsilon}) = \max\{\hat{W}(\theta_t, \epsilon) + \beta[\hat{W}(\theta_t, \hat{\epsilon}) - \hat{W}(\theta_t, \epsilon)], \tilde{W}(\theta_t, \tilde{w}, \hat{\epsilon})\}.$$

Thus, the workers' outside option in bargaining with the new potential employer is to remain with the incumbent employer and to extract full surplus from that relationship.

If the new job encountered by the employed has quality $\hat{\epsilon}$ that is lower than the current quality ϵ , the worker does not switch. Utility equals

$$(76) \quad \tilde{W}^n(\theta_t, \epsilon, \hat{\epsilon}) = \max\{\hat{W}(\theta_t, \hat{\epsilon}) + \beta[\hat{W}(\theta_t, \epsilon) - \hat{W}(\theta_t, \hat{\epsilon})], \tilde{W}(\theta_t, w, \epsilon)\}.$$

That is, if $\hat{\epsilon}$ is sufficiently high relative to ϵ , rebargaining the wage with the current employer results in a wage increase. In this case the first argument of the max dominates. If, on the other hand, $\hat{\epsilon}$ is sufficiently low or the current wage w is sufficiently high, receiving the outside offer does not affect the current wage and the second argument dominates.

Thus, the expected value of starting next period employed at wage w with match quality ϵ (where the expectation is over the aggregate conditions next period and the outside offer $\hat{\epsilon}$ that might be received) is

$$(77) E_t W(\theta_{t+1}, w, \epsilon) = E_t \{ sU(\theta_{t+1}) \\ + (1-s)(1-q(\theta_{t+1}))\tilde{W}(\theta_{t+1}, w, \epsilon) \\ + (1-s)q(\theta_{t+1})[prob_{t+1}(\hat{\epsilon} > \epsilon)E_{\hat{\epsilon}}(\tilde{W}^s(\theta_{t+1}, \epsilon, \hat{\epsilon}) \mid \hat{\epsilon} > \epsilon) \\ + prob_{t+1}(\hat{\epsilon} \leq \epsilon)E_{\hat{\epsilon}}(\tilde{W}^n(\theta_{t+1}, \epsilon, \hat{\epsilon}) \mid \hat{\epsilon} \leq \epsilon)] \}.$$

The value of the worker who extracts full surplus from the current employer is defined similarly, with the only exception that receiving an offer $\hat{\epsilon} \leq \epsilon$ does not affect neither the value nor the wage.

Patterns of Wages

The behavior of wages in this model can be summarized as follows. The unemployed worker hired when aggregate conditions are θ at the job of quality ϵ receives the wage $w^{nh}(\theta, \epsilon)$. After the worker is hired the wage remains constant unless due to the change in aggregate conditions the worker prefers unemployment to remaining employed at the current wage. In this case the wage is rebargained, if possible. When the workers encounters a job that is more productive than the current one, he switches and his threat point in bargaining with the new firm is remaining with the incumbent and extracting full surplus from that relationship. If the quality of the outside job is lower than the current one, but the worker can obtain a higher wage by moving to a new job and extracting its full surplus, he stays with the current job but his wages are raised. If extracting full surplus from the job that the worker encountered results in lower wages than in the current job, the wage remains unchanged.

Calibration

We calibrate the model according to the same procedure we used to calibrate the job ladder model in the main text. In particular, the driving process remains market tightness which determines the job finding rates for employed and unemployed workers. Workers can receive up to M offers per period and we continue to assume that match qualities ϵ are drawn from $F = \mathcal{N}(\mu_\epsilon, \sigma_\epsilon^2)$, truncated at two standard deviations. Workers' flow utility of unemployment b is normalized to 1. The parameter α in this model relates fluctuations in market tightness to fluctuations in aggregate productivity. We choose its value to match the standard deviation of HP-filtered (1600) log quarterly output per worker in the data equal to 0.013.

Values of the following seven parameters remain to be determined: the average levels of receiving an offer for unemployed and employed workers $\bar{\lambda}$ and \bar{q} , the elasticity of the offer probabilities κ , the mean and the volatility of idiosyncratic productivity μ_ϵ and σ_ϵ^2 , the maximum number of offers, M , and the workers' bargaining power β . We determine their values by targeting the same targets as in the job ladder model.

Results

The full list of targets and the performance of the model in matching them are described in Table 20. The calibrated parameter values can be found in Table 21. We find that the model is not able to match the calibration targets very well. In particular, it cannot generate sufficient elasticity of wages with respect to the unemployment variables. The root of the problem appears to be the option value effect embedded in this model. Recall that the model can generate wage declines of some job-to-job switchers because they may be willing to accept a wage cut upon a move to a more productive firm in expectation of higher wage growth when the firm will be

matching outside offers. The same effect operates at cyclical frequencies. In a boom newly hired workers are willing to accept a relatively low wage because offers arrive often and their wages are likely to be bid up soon. In recessions, when offers arrive less frequently, starting wages are higher. Thus, it is hard to generate strongly pro-cyclical wages of newly hired workers in this model.

We then add our regressors q^{HM} and q^{EH} to wage regressions in the same way we did in the data. The results from these regressions are presented in Table 27. Contrary to what we find in the data, the estimated coefficient on u^{min} remains large and statistically significant while the coefficient on u is small and insignificant.

Note that the model in Postel-Vinay and Robin (2002) and Cahuc, Postel-Vinay and Robin (2006) is much closer to our model than is the insurance model. In both Postel-Vinay and Robin (2002) and in our model, workers receive offers with a certain probability and switch to another job if the match quality in the offered job is higher than the current one. The key difference between the two models are the different implications in response to receiving an offer that is worse than the current one. In our model nothing changes, in particular the wage remains unchanged. In Postel-Vinay and Robin (2002), however, the wage can increase. This is because the current firm commits to matching outside offers. If the worker receives an offer that is worse the current one (and thus stays with the current firm), the current firm increases the worker's wage as a result of bargaining between the worker and the firm, where the offer serves as the outside option. Thus workers who have received more offers on the job can have higher wages just because they have had more opportunities to bargain their wages up. This implies that the offers received since the start of the job is an important determinant of wages in this model. In terms of regressors the key difference between our model and Postel-Vinay and Robin (2002) is that in our model knowing the match quality is sufficient to know the wage and thus it is sufficient to add the two regressors q^{EH} and q^{HM} which measure match quality. In Postel-Vinay and Robin (2002) knowing match quality is not sufficient as bargaining leads to wage increases during a job spell (where match quality is constant). Thus an additional regressor, the expected number of offers since the beginning of the job, $q^{Contract}$, has explanatory power. The results reported in Table 24 confirm this prediction. All three regressors, q^{EH} and q^{HM} and $q^{Contract}$, are significant. In our model and in the data, however, only q^{HM} and q^{EH} are significant and $q^{Contract}$ is insignificant in such a regression.

Table 20: Offer Matching Model of Cahuc, Postel-Vinay and Robin (2006): Matching the Calibration Targets.

Target	Value	
	Data	Model
1. Semi-Elasticity of wages wrt agg. unemployment u	-3.090	-1.006
2. Semi-Elasticity of wages wrt minimum unemployment u_{min}	-4.039	-1.815
3. Semi-Elasticity of wages wrt agg. unemployment u (joint reg. with u^{min})	-1.080	0.438
4. Semi-Elasticity of wages wrt minimum unemployment u^{min} (joint reg. with u)	-3.023	-2.179
5. Semi-Elasticity of wages wrt starting unemployment u^{begin}	-2.563	-1.165
6. Semi-Elasticity of wages wrt agg. unemployment u (joint reg. with u^{begin})	-2.450	-0.483
7. Semi-Elasticity of wages wrt starting unemployment u^{begin} (joint reg. with u)	-1.183	-0.880
8. Semi-Elasticity of wages wrt unemployment for stayers, β^{Stay}	-2.233	-0.031
9. Semi-Elasticity of wages wrt unemployment for switchers, β^{Switch}	-3.505	-0.982
10. Monthly job-finding rate for unemployed	0.430	0.443
11. Monthly job-to-job probability for employed	0.029	0.027
12. Std. of aggregate unemployment	0.090	0.129
13. Std. of aggregate productivity	0.013	0.012

Note - The table describes the performance of the offer matching model in matching the calibration targets.

Table 21: Offer Matching Model of Cahuc, Postel-Vinay and Robin (2006): Calibrated Parameter Values.

Parameter	Definition	Value
α	parameter of the output function	0.051
β	workers' bargaining weight	0.557
$\bar{\lambda}$	avg. prob to receive an offer for unemployed	0.104
\bar{q}	avg. prob to receive an offer for employed	0.030
κ	elasticity of the offer probability	0.783
M	max number of offers per period	5
μ_ϵ	mean of idiosyncratic productivity	1.155
σ_ϵ	std. of idiosyncratic productivity	0.043
ρ	persistence of aggregate process	0.990
σ_ν	std. of aggregate process	0.095

Note - The table contains the calibrated parameter values of the offer matching model.

Table 22: Controlling for Match Qualities in Beaudry-DiNardo Regressions. Offer Matching Model of Cahuc, Postel-Vinay and Robin (2006).

	Variable		Specification			
	1	2	3	4	5	6
u	-1.006 [-2.02,-0.36]	-0.668 [-1.52,-0.22]	0.438 [0.10,0.81]	0.075 [-0.19,0.27]	-0.483 [-1.30,-0.05]	-0.367 [-1.00,-0.07]
u^{min}	—	—	-2.179 [-4.35,-1.08]	-1.253 [-2.79,-0.58]	—	—
u^{begin}	—	—	—	—	-0.880 [-1.82,-0.29]	-0.544 [-1.21,-0.21]
q^{HM}	—	2.324 [1.95,2.74]	—	1.595 [1.41,1.93]	—	2.223 [1.95,2.50]
q^{EH}	—	1.735 [1.50,1.97]	—	1.580 [1.39,1.72]	—	1.589 [1.40,1.72]

Note - All coefficients are multiplied by 100. Bold font denotes significance at a 95% level based on bootstrapped confidence intervals in parentheses.

Table 23: Wage Volatility of Job Stayers and Switchers. Offer Matching Model of Cahuc, Postel-Vinay and Robin (2006).

	Variable	Specification		
		Job Stayers		Job Switchers
		1	2	3
1.	u	-0.031 [-0.05, -0.02]	-0.982 [-1.90, -0.41]	-0.653 [-1.26, -0.29]
2.	q^{HM}	—	—	0.990 [0.73, 1.16]
3.	q^{EH}	—	—	3.638 [2.42, 4.44]

Note - All coefficients are multiplied by 100. Bold font denotes significance at a 95% level based on bootstrapped confidence intervals in parentheses.

Table 24: Offers up to date t . Offer Matching Model of Cahuc, Postel-Vinay and Robin (2006).

u	$q^{Contract}$	q^{HM}	q^{EH}
-0.658	0.900	1.525	1.751
[-1.49, -0.21]	[0.58, 1.23]	[1.37, 1.74]	[1.52, 1.97]

Note - All coefficients are multiplied by 100. Bold font denotes significance at a 95% level based on bootstrapped confidence intervals in parentheses.

B.2 Contracting Model of Postel-Vinay and Robin (2002)

The model in Postel-Vinay and Robin (2002) is obtained by setting the workers' bargaining weight β to zero in the model of Cahuc, Postel-Vinay and Robin (2006) discussed above. Although it is a special case of that model, it is somewhat closer to our job ladder model and might be harder to tell apart. We would like to verify whether this is the case.

Calibration and Results

We impose $\beta = 0$ and calibrate the remaining parameters model to match the same targets as in our calibration of the model in Cahuc, Postel-Vinay and Robin (2006). The full list of targets and the performance of the model in matching them are described in Table 25. The calibrated parameter values can be found in Table 26.

The results parallel those for the Cahuc, Postel-Vinay and Robin (2006) discussed above. The model continues to have difficulties in matching the calibration targets. Due to the strong option value effect embedded in this model, workers hired in a boom are willing to accept lower wages which makes it difficult to generate strongly pro-cyclical wages of newly hired workers in this model.

When we add our regressors q^{HM} and q^{EH} to wage regressions as we did in the data, the estimated coefficient on u^{min} remains large and statistically significant while the coefficient on u is small and insignificant (Table 27).

As discussed above, in this model the offers received since the start of the job are an important determinant of wages. In terms of regressors the key difference between our model and Postel-Vinay and Robin (2002) is that in our model knowing the match quality is sufficient to know the wage and thus it is sufficient to add the two regressors q^{EH} and q^{HM} which measure match quality. In Postel-Vinay and Robin (2002) knowing match quality is not sufficient as matching of outside offers by firms leads to wage increases during a job spell (where match quality is constant). Thus an additional regressor, the expected number of offers since the beginning of the job, $q^{Contract}$, has explanatory power. The results reported in Table 29 confirm this prediction. All three regressors, q^{EH} and q^{HM} and $q^{Contract}$, are significant. In our model and in the data, however, only q^{HM} and q^{EH} are significant and $q^{Contract}$ is insignificant in such a regression.

Table 25: Offer Matching Model of Postel-Vinay and Robin (2002): Matching the Calibration Targets.

Target	Value	
	Data	Model
1. Semi-Elasticity of wages wrt agg. unemployment u	-3.090	-1.026
2. Semi-Elasticity of wages wrt minimum unemployment u_{min}	-4.039	-2.057
3. Semi-Elasticity of wages wrt agg. unemployment u (joint reg. with u^{min})	-1.080	0.847
4. Semi-Elasticity of wages wrt minimum unemployment u^{min} (joint reg. with u)	-3.023	-2.829
5. Semi-Elasticity of wages wrt starting unemployment u^{begin}	-2.563	-1.206
6. Semi-Elasticity of wages wrt agg. unemployment u (joint reg. with u^{begin})	-2.450	-0.443
7. Semi-Elasticity of wages wrt starting unemployment u^{begin} (joint reg. with u)	-1.183	-0.937
8. Semi-Elasticity of wages wrt unemployment for stayers, β^{Stay}	-2.233	-0.211
9. Semi-Elasticity of wages wrt unemployment for switchers, β^{Switch}	-3.505	-0.863
10. Monthly job-finding rate for unemployed	0.430	0.443
11. Monthly job-to-job probability for employed	0.029	0.027
12. Std. of aggregate unemployment	0.090	0.137
13. Std. of aggregate productivity	0.013	0.013

Note - The table describes the performance of the offer matching model in matching the calibration targets.

Table 26: Offer Matching Model of Postel-Vinay and Robin (2002): Calibrated Parameter Values.

Parameter	Definition	Value
α	parameter of the output function	0.053
β	workers' bargaining weight	0.000
$\bar{\lambda}$	avg. prob to receive an offer for unemployed	0.123
\bar{q}	avg. prob to receive an offer for employed	0.037
κ	elasticity of the offer probability	0.995
M	max number of offers per period	4
μ_ϵ	mean of idiosyncratic productivity	1.047
σ_ϵ	std. of idiosyncratic productivity	0.009
ρ	persistence of aggregate process	0.990
σ_ν	std. of aggregate process	0.095

Note - The table contains the calibrated parameter values of the offer matching model.

Table 27: Controlling for Match Qualities in Beaudry-DiNardo Regressions. Offer Matching Model of Postel-Vinay and Robin (2002).

	Variable		Specification			
	1	2	3	4	5	6
u	-1.026 [-2.35,-0.30]	-0.534 [-1.55,-0.09]	0.847 [0.30,1.91]	0.401 [0.07,0.90]	-0.443 [-1.49, 0.08]	-0.286 [-1.02, 0.05]
u^{min}	—	—	-2.829 [-6.66,-1.23]	-1.594 [-4.22,-0.65]	—	—
u^{begin}	—	—	—	—	-0.937 [-2.23,-0.24]	-0.441 [-1.19,-0.11]
q^{HM}	—	3.147 [2.79,3.52]	—	2.251 [1.81,2.69]	—	3.036 [2.76,3.30]
q^{EH}	—	2.264 [1.676,2.56]	—	2.037 [1.45,2.27]	—	2.118 [1.57,2.38]

Note - All coefficients are multiplied by 100. Bold font denotes significance at a 95% level based on bootstrapped confidence intervals in parentheses.

Table 28: Wage Volatility of Job Stayers and Switchers. Offer Matching Model of Postel-Vinay and Robin (2002).

	Variable	Specification		
		Job Stayers	Job Switchers	
		1	2	3
1.	u	-0.211 [-0.38,-0.11]	-1.195 [-3.66,-0.17]	-0.466 [-1.76, 0.08]
2.	q^{HM}	—	—	0.983 [0.36, 1.20]
3.	q^{EH}	—	—	7.239 [2.83, 8.87]

Note - All coefficients are multiplied by 100. Bold font denotes significance at a 95% level based on bootstrapped confidence intervals in parentheses.

Table 29: Offers up to date t . Offer Matching Model of Postel-Vinay and Robin (2002).

u	$q^{Contract}$	q^{HM}	q^{EH}
-0.510	1.905	1.457	2.278
[-1.48, -0.09]	[1.593, 2.184]	[1.09, 1.72]	[1.64, 2.57]

Note - All coefficients are multiplied by 100. Bold font denotes significance at a 95% level based on bootstrapped confidence intervals in parentheses.

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