# Appendix

**Derivation of the expression for $I_j^*(w)$ in equation (25)**: Setting $h(L) = L^{1+\frac{1}{\varepsilon}}$ simplifies the expression for $\Psi(F_j)$ in (4), $j = l, h, hl$, to

$$\Psi(F_j) = \left\{ I_j(w) - \left[1 + \left(1 + \frac{1}{\varepsilon}\right)\Omega_j(w)\right]\left(\frac{I_j(w)}{w}\right)^{1+\frac{1}{\varepsilon}} \right\} f_j(w)dw. \tag{A1}$$

Rearranging the first-order condition for the maximization of $\Psi(F_j)$ with respect to $I_j(w)$ then yields

$$I_j^*(w) = wL_j^*(w) = w^{1+\varepsilon}\left(\frac{\varepsilon}{1+\varepsilon}\right)^{\varepsilon}\left[1 + \left(1 + \frac{1}{\varepsilon}\right)\Omega_j(w)\right]^{-\varepsilon}, \quad j = l, h, hl. \tag{A2}$$

**Derivation of the expressions for $\underline{u}_{hl}^*$ in (27) and $\underline{u}^*$ in (29)**: First, substitute the expression for $I_j^*(w)$ from (A2) into (A1) and simplify to get, for $j = l, h, hl$,

$$\Psi^*(F_j) = \varepsilon^{\varepsilon}(1+\varepsilon)^{-1-\varepsilon}\int_{\underline{w}}^{\overline{w}} w^{1+\varepsilon}\left[1 + \left(1 + \frac{1}{\varepsilon}\right)\Omega_j(w)\right]^{-\varepsilon} f_j(w)dw. \tag{A3}$$

Second, given the expression for $\Psi^*(F_{hl})$ and using (15), we have

$$\underline{u}_{hl}^* = \Psi^*(F_{hl}) = \varepsilon^{\varepsilon}(1+\varepsilon)^{-1-\varepsilon}\int_{\underline{w}}^{\overline{w}} w^{1+\varepsilon}\left[1 + \left(1 + \frac{1}{\varepsilon}\right)\Omega_{hl}(w)\right]^{-\varepsilon} f_{hl}(w)dw.$$

Finally, given the expression for $\Psi^*(F_j)$, $j = l, h$, and using (20), we have

$$\begin{aligned}\underline{u}^* &= \frac{1}{2}\varepsilon^{\varepsilon}(1+\varepsilon)^{-1-\varepsilon}\left\{\int_{\underline{w}}^{\overline{w}} w^{1+\varepsilon}\left[1 + \left(1 + \frac{1}{\varepsilon}\right)\Omega_h(w)\right]^{-\varepsilon} f_h(w)dw\right. \\ &\left.+\int_{\underline{w}}^{\overline{w}} w^{1+\varepsilon}\left[1 + \left(1 + \frac{1}{\varepsilon}\right)\Omega_l(w)\right]^{-\varepsilon} f_l(w)dw\right\}.\end{aligned}$$

**Derivation of equations (27)–(30):** With $h(L) = L^{1+\frac{1}{\varepsilon}}$, we have, for $j = l, h, hl$,

$$\int_{\underline{w}}^{w} \frac{I_j^*(s)}{s^2} h'\left(\frac{I_j^*(s)}{s}\right) ds = \left(1 + \frac{1}{\varepsilon}\right)\int_{\underline{w}}^{w} s^{-1}\, L_j^*(s)^{1+\frac{1}{\varepsilon}} ds.$$

Substitute for $L_j^*(w)$ from (A2) into above. This gives

$$\int_{\underline{w}}^{w} \frac{I_j^*(s)}{s^2} h'\left(\frac{I_j^*(s)}{s}\right) ds = \left(\frac{\varepsilon}{1+\varepsilon}\right)^{\varepsilon} \int_{\underline{w}}^{w} s^{\varepsilon} \left[1 + \left(1 + \frac{1}{\varepsilon}\right) \Omega_j(s)\right]^{-1-\varepsilon} ds.$$

Given this expression, one can simplify the expressions for $u_{hl}^*(w)$ and $u_j^*(w)$, $j = l, h$, derived in (16) and (21), to (27) and (29). Similarly, one simplifies the expressions for $c_{hl}^*(w)$ and $c_j^*(w)$, $j = l, h$, derived in (17) and (22), to (28) and (30).

**Proof of inequality (26):** Let

$$\begin{aligned} \Delta &\equiv \int_{\underline{w}}^{\overline{w}} I_h^*(w)\ f_h(w)dw + \int_{\underline{w}}^{\overline{w}} I_l^*(w)\ f_l(w)dw - 2\int_{\underline{w}}^{\overline{w}} I_{hl}^*(w)\ f_{hl}(w)dw \\ &= \int_{\underline{w}}^{\overline{w}} \left[I_h^*(w) - I_{hl}^*(w)\right] f_h(w)dw + \int_{\underline{w}}^{\overline{w}} \left[I_l^*(w) - I_{hl}^*(w)\right] f_l(w)dw. \end{aligned}$$

Substitute for $I_i^*(w)$, $j = l, h, hl$, from (A2) in above and simplify. We have

$$\begin{aligned} &\Delta \left(\frac{1+\varepsilon}{\varepsilon}\right)^{\varepsilon} = \\ &\int_{\underline{w}}^{\overline{w}} w^{1+\varepsilon} \left\{ \left[1 + \left(1 + \frac{1}{\varepsilon}\right) \Omega_h(w)\right]^{-\varepsilon} - \left[1 + \left(1 + \frac{1}{\varepsilon}\right) \Omega_{hl}(w)\right]^{-\varepsilon} \right\} f_h(w)dw + \\ &\int_{\underline{w}}^{\overline{w}} w^{1+\varepsilon} \left\{ \left[1 + \left(1 + \frac{1}{\varepsilon}\right) \Omega_l(w)\right]^{-\varepsilon} - \left[1 + \left(1 + \frac{1}{\varepsilon}\right) \Omega_{hl}(w)\right]^{-\varepsilon} \right\} f_l(w)dw. \quad \text{(A4)} \end{aligned}$$

Now subtract the expression for $\underline{u}_{hl}^*$ from the expression for $\underline{u}^*$:

$$\begin{aligned} \underline{u}^* - \underline{u}_{hl}^* &= \frac{1}{2}\varepsilon^{\varepsilon} (1+\varepsilon)^{-1-\varepsilon} \left\{ \int_{\underline{w}}^{\overline{w}} w^{1+\varepsilon} \left[1 + \left(1 + \frac{1}{\varepsilon}\right) \Omega_h(w)\right]^{-\varepsilon} f_h(w)dw + \right. \\ &\left. \int_{\underline{w}}^{\overline{w}} w^{1+\varepsilon} \left[1 + \left(1 + \frac{1}{\varepsilon}\right) \Omega_l(w)\right]^{-\varepsilon} f_l(w)dw \right\} - \\ &\varepsilon^{\varepsilon} (1+\varepsilon)^{-1-\varepsilon} \int_{\underline{w}}^{\overline{w}} w^{1+\varepsilon} \left[1 + \left(1 + \frac{1}{\varepsilon}\right) \Omega_{hl}(w)\right]^{-\varepsilon} f_{hl}(w)dw \end{aligned}$$

$$
\begin{aligned}
= \quad & \frac{1}{2}\varepsilon^{\varepsilon}\left(1+\varepsilon\right)^{-1-\varepsilon}\left\{\int_{\underline{w}}^{\overline{w}} w^{1+\varepsilon}\left[1+\left(1+\frac{1}{\varepsilon}\right)\Omega_h(w)\right]^{-\varepsilon} f_h(w)dw - \right.\\
& \left.\frac{1}{2}\int_{\underline{w}}^{\overline{w}} w^{1+\varepsilon}\left[1+\left(1+\frac{1}{\varepsilon}\right)\Omega_{hl}(w)\right]^{-\varepsilon}\left[f_h(w)+f_l(w)\right]dw\right\} + \\
& \frac{1}{2}\varepsilon^{\varepsilon}\left(1+\varepsilon\right)^{-1-\varepsilon}\left\{\int_{\underline{w}}^{\overline{w}} w^{1+\varepsilon}\left[1+\left(1+\frac{1}{\varepsilon}\right)\Omega_l(w)\right]^{-\varepsilon} f_l(w)dw - \right.\\
& \left.\frac{1}{2}\int_{\underline{w}}^{\overline{w}} w^{1+\varepsilon}\left[1+\left(1+\frac{1}{\varepsilon}\right)\Omega_{hl}(w)\right]^{-\varepsilon}\left[f_h(w)+f_l(w)\right]dw\right\}\\
= \quad & \frac{1}{2}\varepsilon^{\varepsilon}\left(1+\varepsilon\right)^{-1-\varepsilon}\int_{\underline{w}}^{\overline{w}} w^{1+\varepsilon}\left\{\left[1+\left(1+\frac{1}{\varepsilon}\right)\Omega_h(w)\right]^{-\varepsilon} - \right.\\
& \left.\left[1+\left(1+\frac{1}{\varepsilon}\right)\Omega_{hl}(w)\right]^{-\varepsilon}\right\} f_h(w)dw + \frac{1}{2}\varepsilon^{\varepsilon}\left(1+\varepsilon\right)^{-1-\varepsilon}\int_{\underline{w}}^{\overline{w}} w^{1+\varepsilon}\times\\
& \left\{\left[1+\left(1+\frac{1}{\varepsilon}\right)\Omega_l(w)\right]^{-\varepsilon} - \left[1+\left(1+\frac{1}{\varepsilon}\right)\Omega_{hl}(w)\right]^{-\varepsilon}\right\} f_l(w)dw. \qquad \text{(A5)}
\end{aligned}
$$

Substituting from (A5) into (A4), we get

$$
\Delta = 2(1+\varepsilon)\left(\underline{u}^{*} - \underline{u}_{hl}^{*}\right) \geqq 0.
$$