Online Appendix: "Performance and Turnover in a Stochastic Partnership" by David McAdams

Proof of Lemma 3.

Proof. Let $\Pi_{it}^{eqm}(x_t, \mathbf{e}_t; v) = \gamma E\left[\Pi_{\Sigma t+1}^{eqm}(X_{t+1}, \mathbf{e}_t; v)|x_t, \mathbf{e}_t\right]$ be shorthand for player i's expected time-t continuation payoff, after efforts e_t from history $(x_t, \mathbf{e}_{t-1})$, should both players subsequently choose to stay to period $t+1$ upon surviving period t. Figure 1 illustrates the key idea of Lemma 3. As long as $\Pi_{\Sigma t}^{eqm}(x_t, \mathbf{e}_t; v)$ exceeds the players' joint payoff after exit, γv_Σ, *plus* their joint incentive to shirk from efforts e_t, $c_{\Sigma t}(e_t; x_t)$, there exists a retention bonus promise given which both players have sufficient incentive to exert efforts e_t and then stay. Further, this promise is credible since each player promises less than his willingness to pay to avoid cooperation breakdown.

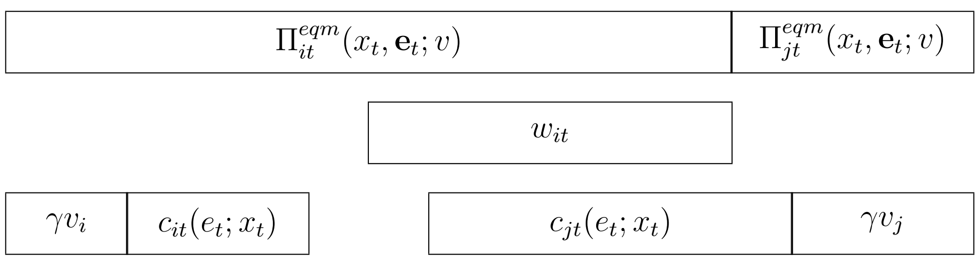

Figure 1: Efforts e_t are incentive-compatible when player i pays wage w_{it} (and $w_{jt} = 0$).

Let $\Delta_{it}(e_t) = \Pi_{it}^{eqm}(x_t, \mathbf{e}_t; v) - \gamma v_i - c_{it}(e_t; x_t)$ denote player i's "excess continuation payoff", the extra profit that he enjoys inside the partnership after efforts e_t, relative to deviating with zero effort and then quitting the relationship. $\Delta_{it}(e_t)$ is the most that player i can credibly promise to pay player j as a reward for not deviating from the prescribed efforts e_t and then not quitting.[1]

[1] Should efforts e_t be played, player i becomes willing to pay up to $\Delta_{it}(e_t) + c_{it}(e_t; x_t)$ to avoid exit. Then, should both players survive and stay to period $t+1$, player i becomes willing to pay more still to avoid a transition to an optimal punishment continuation SPE in which both players exert zero effort, pay zero wages, and exit for certain at time $t+1$. Thus, player i has sufficient incentive to exert his prescribed effort, then stay, then pay the specified bonus.

Without loss, suppose that $\Delta_{it}(e_t) \geq \Delta_{jt}(e_t)$. If $\Delta_{it}(e_t) + \Delta_{jt}(e_t) < 0$, then at least one player who exerts positive effort must strictly prefer to deviate by exerting zero effort and then quitting, given any credible wage. Otherwise, any retention bonus $w_{it} \in [\max\{0, -\Delta_{jt}(e_t)\}, \Delta_{it}(e_t)]$ from player i to player j can credibly support efforts e_t. Thus, effort-profile e_t can be supported in some SPE iff $\Delta_{it}(e_t) + \Delta_{jt}(e_t) \geq 0$, i.e. iff e_t satisfies (18) in the text. This completes the proof, since then the maximal SPE joint welfare given the specified continuation payoffs is the solution to (17) in the text. □

Proof of Theorems 1-2.

Proof. Let $\overline{\Pi}_t^{eqm}(x_t, \mathbf{e}_{t-1}; v) \in \mathbf{R}^2$ be the players' payoff profile in a SPE that maximizes *joint* welfare among all SPE from history $(x_t, \mathbf{e}_{t-1})$, and let $\overline{\Pi}_{\Sigma t}^{eqm}(x_t, \mathbf{e}_{t-1}; v) = \Sigma_i \overline{\Pi}_{it}^{eqm}(x_t, \mathbf{e}_{t-1}; v)$.

Outline of proof. I will construct a monotonically decreasing sequence of bounds on SPE joint welfare from each history, $(\overline{\Pi}_{\Sigma t}^{k}(x_t, \mathbf{e}_{t-1}; v) : k \geq 0)$, that converges pointwise to $\overline{\Pi}_{\Sigma t}^{eqm}(x_t, \mathbf{e}_{t-1}; v)$, and show that this maximal joint payoff is implemented by SPE strategies as specified in Theorem 1. Further, $\overline{\Pi}_{\Sigma t}^{k}(x_t, \mathbf{e}_{t-1}; v)$ is non-decreasing in x_t for each k, as well as in the limit $\overline{\Pi}_{\Sigma t}^{eqm}(x_t, \mathbf{e}_{t-1}; v)$, establishing Theorem 2.

Part I: Decreasing sequence of bounding payoff-profile sets. By Assumption 1, there exists a uniform upper bound M on players' joint payoff at any history. Define $\overline{\Pi}_{\Sigma t}^{0}(x_t, \mathbf{e}_{t-1}; v) = M$ at all histories. Clearly, $\overline{\Pi}_{\Sigma t}^{0} \geq \overline{\Pi}_{\Sigma t}^{eqm}$. Next, for all $k \geq 1$, define $\overline{\Pi}_{\Sigma t}^{k}(x_t, \mathbf{e}_{t-1}; v)$ recursively as follows (using shorthand $\mathbf{e}_t = (\mathbf{e}_{t-1}, e_t)$):

$$\overline{\Pi}_{\Sigma t}^{k}(x_t, \mathbf{e}_{t-1}; v) = \max_{e_t} \left(\pi_{\Sigma t}(e_t; x_t) + \gamma \max \left\{ v_\Sigma, E\left[\overline{\Pi}_{\Sigma t+1}^{k-1}(X_{t+1}, \mathbf{e}_t; v) | x_t, \mathbf{e}_t \right] \right\} \right) \tag{1}$$

$$\text{subject to } c_{\Sigma t}(e_t; x_t) \leq \gamma \max \left\{ 0, E\left[\overline{\Pi}_{\Sigma t+1}^{k-1}(X_{t+1}, \mathbf{e}_t; v) | x_t, \mathbf{e}_t \right] - v_\Sigma \right\}. \tag{2}$$

Assuming that $\overline{\Pi}_{\Sigma t+1}^{k-1}(x_{t+1}, \mathbf{e}_t; v)$ are upper bounds on joint payoff at time $t+1$, then (2) is a necessary condition for efforts e_t to be supported in any SPE from history $(x_{t+1}, \mathbf{e}_t)$. *Proof:* Players expect joint "inside continuation payoff" of at most $\gamma E\left[\overline{\Pi}_{\Sigma t+1}^{k-1}(X_{t+1}, \mathbf{e}_t; v) | x_t, \mathbf{e}_t \right]$ should they choose effort-profile e_t and then both stay should both survive. If players' joint outside option v_Σ exceeds this bound, then at least one player strictly prefers to quit and neither player can be incentivized to exert any costly effort. Otherwise, players' joint cost of effort $c_{\Sigma t}(e_t; x_t)$ must be less than or equal to the amount by which their joint inside continuation payoff exceeds their joint outside option. (If not, at least one player would strictly prefer to deviate by exerting zero effort and then quitting.) Indeed, by Lemma 3, (2) is also a sufficient condition to support time-t efforts e_t in SPE given continuation payoffs $\overline{\Pi}^{k-1}_{\Sigma t+1}(x_{t+1}, \mathbf{e}_t; v)$.

Since $\overline{\Pi}^0_{\Sigma t}(x_t, \mathbf{e}_{t-1}; v)$ is an upper bound on joint payoff, $\overline{\Pi}^0_{\Sigma t}(x_t, \mathbf{e}_{t-1}; v) \geq \overline{\Pi}^1_{\Sigma t}(x_t, \mathbf{e}_{t-1}; v)$. By induction, $\overline{\Pi}^k_{\Sigma t}(x_t, \mathbf{e}_{t-1}; v)$ is non-increasing in k. (The value of the maximization (1) is non-decreasing in continuation payoffs. Thus, $\overline{\Pi}^k_{\Sigma t+1}(x_{t+1}, \mathbf{e}_t; v) \leq \overline{\Pi}^{k-1}_{\Sigma t+1}(x_{t+1}, \mathbf{e}_t; v)$ for all $(x_{t+1}, \mathbf{e}_t)$ implies $\overline{\Pi}^{k+1}_{\Sigma t}(x_t, \mathbf{e}_{t-1}; v) \leq \overline{\Pi}^k_{\Sigma t}(x_t, \mathbf{e}_{t-1}; v)$.) Further, by induction, $\overline{\Pi}^k_{\Sigma t}(x_t, \mathbf{e}_{t-1}; v) \geq \overline{\Pi}^{eqm}_{\Sigma t}(x_t, \mathbf{e}_{t-1}; v)$ for all k. (Higher-than-equilibrium payoffs can be supported given higher-than-equilibrium continuation payoffs. Thus, the fact that $\overline{\Pi}^{k-1}_{\Sigma t}(x_t, \mathbf{e}_{t-1}; v) \geq \overline{\Pi}^{eqm}_{\Sigma t}(x_t, \mathbf{e}_{t-1}; v)$ implies $\overline{\Pi}^k_{\Sigma t}(x_t, \mathbf{e}_{t-1}; v) \geq \overline{\Pi}^{eqm}_{\Sigma t}(x_t, \mathbf{e}_{t-1}; v)$.)

Part II: These upper bounds on joint welfare are non-decreasing in x_t.

Base step: $k = 0$. $\overline{\Pi}^0_t(x_t, \mathbf{e}_{t-1}; v) = M$ is constant and hence trivially non-decreasing in x_t.

Induction step: $k \geq 1$. Suppose that $\overline{\Pi}^{k-1}_t(x_t, \mathbf{e}_{t-1}; v)$ is non-decreasing in x_t for all t. Observe that, for any $x_t^H \succeq x_t^L$,

$$\begin{aligned} E[\overline{\Pi}^{k-1}_{\Sigma t+1}(X_{t+1}; \mathbf{e}_t; v) | x_t^H, \mathbf{e}_t]) &= \int_0^\infty \Pr(\overline{\Pi}^{k-1}_{\Sigma t+1}(X_{t+1}; \mathbf{e}_t; v) > z | x_t^H, \mathbf{e}_t; v) \mathrm{d}z \\ &\geq \int_0^\infty \Pr(\overline{\Pi}^{k-1}_{\Sigma t+1}(X_{t+1}; \mathbf{e}_t; v) > z | x_t^L, \mathbf{e}_t; v) \mathrm{d}z \qquad (3) \\ &= E[\overline{\Pi}^{k-1}_{\Sigma t+1}(X_{t+1}; \mathbf{e}_t; v) | x_t^L, \mathbf{e}_t]) \end{aligned}$$

By the induction hypothesis, $\{x_{t+1} \in \mathcal{X}_{t+1} : \overline{\Pi}^{k-1}_{t+1}(X_{t+1}; \mathbf{e}_t; v) > z\}$ is an increasing subset of $\mathcal{X}_{t+1}$ for all z. Inequality (3) now follows from Assumption 3. Thus, for any given effort-history $\mathbf{e}_t$, $\max\left\{v_\Sigma, \delta E\left[\overline{\Pi}^{k-1}_{\Sigma t+1}(X_{t+1}, \mathbf{e}_t; v) | x_t, \mathbf{e}_t\right]\right\}$ is non-decreasing in x_t, so that higher x_t slackens the IC-constraint (2) while increasing the second term of (1). Finally, the first term of (1) is non-decreasing in x_t by Assumption 2. All together, we conclude that the value of the maximization (1) is non-decreasing in x_t. This completes the desired induction.

Let $\overline{\Pi}^\infty_{\Sigma t}(x_t, \mathbf{e}_{t-1}; v)$ denote the pointwise limit of $\overline{\Pi}^k_{\Sigma t}(x_t, \mathbf{e}_{t-1}; v)$ as $k \to \infty$. Since $\overline{\Pi}^k_{\Sigma t}(x_t, \mathbf{e}_{t-1}; v)$ is non-decreasing in x_t for all k, $\overline{\Pi}^\infty_{\Sigma t}(x_t, \mathbf{e}_{t-1}; v)$ inherits this monotonicity as well.

Part III: Limit of upper bounds can be achieved in SPE. It suffices now to show that $\overline{\Pi}_t^{\infty}(x_t, \mathbf{e}_{t-1}; v) = \overline{\Pi}_t^{eqm}(x_t, \mathbf{e}_{t-1}; v)$. As shown earlier, $\overline{\Pi}_t^{\infty}(x_t, \mathbf{e}_{t-1}; v) \geq \overline{\Pi}_t^{eqm}(x_t, \mathbf{e}_{t-1}; v)$. Let $e_t(x_t, \mathbf{e}_{t-1})$ denote a limit of any sequence of solutions to (1) subject to (2), as $k \to \infty$. By construction, efforts $e_t(x_t, \mathbf{e}_{t-1})$ are incentive-compatible if players expect continuation play in later periods that generates time-$(t+1)$ payoffs of $\overline{\Pi}_{t+1}^{\infty}(x_{t+1}, \mathbf{e}_t; v)$ for each player. Again by construction, these efforts generate continuation payoffs $\overline{\Pi}_{t+1}^{\infty}(x_{t+1}, \mathbf{e}_t; v)$; thus, these strategies constitute a joint welfare-maximizing SPE. Thus, $\overline{\Pi}_t^{\infty}(x_t, \mathbf{e}_{t-1}; v) \leq \overline{\Pi}_t^{eqm}(x_t, \mathbf{e}_{t-1}; v)$. This completes the proof. □

Proof of Theorem 3

By Theorem 2, joint payoff $\overline{\Pi}_{\Sigma t}^{eqm}(x_t, \mathbf{e}_{t-1}; v)$ in the joint welfare-maximizing SPE is weakly increasing in x_t for all $(\mathbf{e}_{t-1}, v)$. Given an exogenous stochastic process, further, such payoffs do not depend on the history of efforts. Since outside options $v = (v_i, v_j)$ are held fixed, I will henceforth use the simpler notation $\overline{\Pi}_{\Sigma t}^{eqm}(x_t)$ here.

Proof of (i). Recall that $\overline{\Pi}_{\Sigma t}^{eqm}(x_t) = \max_{e_t} \left(\pi_{\Sigma t}(e_t; x_t) + \gamma \max\left\{v_\Sigma, E\left[\overline{\Pi}_{\Sigma t+1}^{eqm}(X_{t+1})|x_t\right]\right)\right\}$ subject to the IC-constraint $c_{\Sigma t}(e_t; x_t) \leq \gamma \max\left\{0, E\left[\overline{\Pi}_{\Sigma t+1}^{eqm}(X_{t+1})|x_t\right] - v_\Sigma\right\}$. Joint continuation payoff should period $t+1$ be reached,

$$E\left[\overline{\Pi}_{\Sigma t+1}^{eqm}(X_{t+1})|x_t\right] = \int_0^{\infty} \Pr\left(\overline{\Pi}_{\Sigma t+1}^{eqm}(X_{t+1}) > z|x_t\right) \mathrm{d}z, \tag{4}$$

is weakly increasing in x_t: $\{x_{t+1} : \overline{\Pi}_{\Sigma t+1}^{eqm}(X_{t+1}) > z\}$ is an increasing subset of $\mathcal{X}_{t+1}$ so that, by Assumption 3, each of the probability terms in (4) is weakly increasing in x_t. Finally, since efforts do not control future payoffs, time-t efforts in the optimal SPE will be chosen to maximize joint stage-game payoff subject to the IC-constraint. Since joint continuation payoff is weakly increasing in x_t, so is the set of effort-profiles e_t satisfying the IC-constraint. Consequently, realized joint stage-game payoff is weakly increasing in x_t.

Proof of (ii). Let $QUIT_t = \{x_t \in \mathfrak{X}_t : \overline{\Pi}_{\Sigma t}^{eqm}(x_t) < v_\Sigma\}$ and $STAY_t = X_t \backslash QUIT_t$ denote the set of time-t states in which both players quit and stay, respectively, in the joint welfare-maximizing SPE of Theorem 1. Since joint continuation payoff is weakly increasing in x_t by Theorem 3(i), $STAY_t$ is an increasing set for all t.

Let $p_t^k(x_t)$ denote the probability that the partnership will survive until at least time $t+k$, conditional on $X_t = x_t$. I need to show that, for each $k \geq 1$, $p_t^k(x_t)$ is weakly increasing in x_t. The proof is by induction.

Base step. At any time t, the partnership is certain to end if $x_t \in QUIT_t$ and otherwise ends with probability $1-\gamma^2$ if $x_t \in STAY_t$. Thus, $p_t^1(x_t)$ being weakly decreasing in x_t follows directly from $STAY_t$ being an increasing subset of $\mathfrak{X}_t$.

Induction step. As the induction hypothesis, suppose that $p_t^m(x_t)$ is weakly increasing in x_t for all t and all $m = 1, ..., k-1$. I need to show that $p_t^k(x_t)$ is weakly increasing in x_t for all t. Note that

$$p_t^k(x_t) = p_t^1(x_t) E\left[p_{t+1}^{k-1}(X_{t+1}) | X_t = x_t\right] \tag{5}$$

(The partnership survives for k periods iff it survives for $k-1$ periods after first surviving for one period.) The base step showed that $p_t^1(x_t)$ is weakly increasing in x_t. It suffices now to show the same of the expectation term

$$E\left[p_{t+1}^{k-1}(X_{t+1}) \middle| X_t = x_t\right] = \int_0^1 \Pr\left(p_{t+1}^{k-1}(X_{t+1}) > p \middle| X_t = x_t\right) \mathrm{d}p \tag{6}$$

By the induction hypothesis, each set $\{x_{t+1} \in \mathcal{X}_{t+1} : p_{t+1}^{k-1}(x_{t+1}) > p\}$ is an increasing subset of $\mathcal{X}_{t+1}$. By Assumption 3, we conclude that each of the probability-terms in (6) is weakly increasing in x_t. This completes the proof. □

Proof of Theorem 4

As argued in the text, no joint outside option greater than $\overline{v}_\Sigma$ can possibly be supported in partnership-market equilibrium. To complete the proof, it suffices to verify that the strategies specified in Theorem 4 constitute a SPE and generate outside options $\overline{v}_\Sigma$. (The theorem specifies play on the equilibrium path; augment this with shirking and quitting to start a fresh relationship should either player deviate from this path of play.)

Let $\hat{p} = \frac{E\left[\overline{\Pi}_{\Sigma 0}^{eqm}(X_0;\overline{v}_\Sigma)\right] - \overline{v}_\Sigma}{E\left[\overline{\Pi}_{\Sigma 0}^{eqm}(X_0;\overline{v}_\Sigma)\right] - \gamma\overline{v}_\Sigma}$ be the probability with which players shirk and quit immediately based on the public randomization. Note that, by construction,

$$\hat{p}\gamma\overline{v}_\Sigma + (1-\hat{p}) E\left[\overline{\Pi}_{\Sigma 0}^{eqm}(X_0;\overline{v}_\Sigma)\right] = \overline{v}_\Sigma. \tag{7}$$

Thus, if players adopt the specified strategies, market-wide play generates ex ante expected joint payoff $\overline{v}_\Sigma$ at birth, supporting the maximal joint outside option $\overline{v}_\Sigma$. It suffices now to show that these strategies constitute a SPE. First, players are willing to shirk and quit when the public randomization is less than $\hat{p}$, since they expect uncooperative continuation play in the current relationship. Second, given joint outside option $\overline{v}_\Sigma$, Theorem 1 specifies SPE continuation play should the public randomization be more favorable. This completes the proof. $\square$

Proof of Theorem 5

Proof. To complete the proof, it suffices to show that $E[\overline{\Pi}^{eqm}_{\Sigma 0}(S_0, Y_0; v_\Sigma)]$ is continuous in v_Σ. (Recall that $X_t = (S_0, Y_t)$ where $S_0 \sim U[0,1]$ is the partners' "first impression"; see Assumption 6.) If so, the maximization (24) in the text requires that $\overline{v}_\Sigma = E[\overline{\Pi}^{eqm}_{\Sigma 0}(S_0, Y_0; \overline{v}_\Sigma)]$, which in turn is only possible if a probability-one measure of partnerships achieve the maximal joint equilibrium payoff $\overline{\Pi}^{eqm}_{\Sigma 0}(S_0, Y_0; \overline{v}_\Sigma)$ given their endogenous outside options.

An increase in joint outside option from v_Σ to $v_\Sigma + \varepsilon$ has two effects on the maximal SPE joint payoff. First, the direct effect is that players enjoy higher joint payoff when quitting and quit whenever they were previously almost indifferent to doing so. This direct effect increases joint payoff by at most ε. Second, since $E\left[\overline{\Pi}^{eqm}_{\Sigma t+1}(h_t, e_t(h_t; v_\Sigma), Y_{t+1}; v_\Sigma)\right] - \gamma v_\Sigma$ is non-increasing in v_Σ (see Part I of the proof of Theorem 6, which does not depend on Theorem 5), an indirect effect is that players can support (weakly) fewer effort-profiles at every history $h_t = (s_0, y_t, \mathbf{e}_{t-1})$. This decreases payoffs at those histories, inducing more exit and less effort at previous histories, and so on in a backward cascade that decreases joint payoff. This indirect effect of higher v_Σ may have a discontinuous effect on ex post payoffs but I will show that, when there are meaningful first impressions, it has a continuous effect on ex ante expected payoffs.

Recall that players' efforts $e_t(h_t; v_\Sigma)$ maximize joint payoff subject to the IC-constraint that joint continuation payoff is greater than or equal to joint outside option plus joint cost of effort:[2]

$$c_{\Sigma t}(e_t(h_t;v_\Sigma);h_t) \le E\left[\overline{\Pi}^{eqm}_{\Sigma t+1}(h_t,e_t(h_t;v_\Sigma),Y_{t+1};v_\Sigma)\right] - \gamma v_\Sigma. \tag{8}$$

I begin by showing that (8) binds with zero probability on the equilibrium path. Fix any joint outside option v_Σ, effort-profile e_t, history of effort profiles $\mathbf{e}_{t-1}$, and state $x_t = (s_0, y_t)$. By Assumption 6, $c_{it}(e_t; s_0, y_t)$ is strictly decreasing in s_0 for each player i while, by the proof of Theorem 2, $E\left[\overline{\Pi}^{eqm}_{\Sigma t+1}(s_0, y_t, \mathbf{e}_{t-1}, e_t, Y_{t+1}; v_\Sigma)\right]$ is weakly increasing in s_0. Thus, if the IC-constraint (8) binds for some efforts e_t at history $(s_0, y_t, \mathbf{e}_{t-1})$, then for all $s_0^l < s_0 < s_0^h$ it fails at history $(s_0^l, y_t, \mathbf{e}_{t-1})$ and is strictly satisfied at history $(s_0^h, y_t, \mathbf{e}_{t-1})$. Since by assumption there are finitely many effort-levels, (8) binds on $e_t(s_0, y_t, \mathbf{e}_{t-1})$ for finitely many partnership types $s_0 \in \mathbf{R}$. We conclude that, with probability one in the joint welfare-maximizing SPE, the IC-constraint will not be binding on *any* effort-profile prescribed on the equilibrium path.

Next, I prove right-continuity, that $\lim_{\varepsilon\to 0} \overline{\Pi}^{eqm}_{\Sigma t}(h_t; v_\Sigma + \varepsilon) = \overline{\Pi}^{eqm}_{\Sigma t}(h_t; v_\Sigma)$ for all v_Σ and all histories h_t reached with probability one on the equilibrium path. For this step, I employ a variation on the algorithm used in the proof of Theorem 2 (illustrated in Figure 5). Fix $\hat{v}_\Sigma$. For all histories h_t and $\varepsilon \ge 0$, define

$$\overline{\Pi}^{1}_{\Sigma t}(h_t; \hat{v}_\Sigma + \varepsilon) = \overline{\Pi}^{eqm}_{\Sigma t}(h_t; \hat{v}_\Sigma) + \varepsilon.$$

Since the positive "direct effect" of higher joint outside option discussed earlier is at most ε and the "indirect effect" is always negative, $\overline{\Pi}^{1}_{\Sigma t}(h_t; \hat{v}_\Sigma + \varepsilon) > \overline{\Pi}^{eqm}_{\Sigma t}(h_t; \hat{v}_\Sigma + \varepsilon)$. Also, clearly, $\overline{\Pi}^{1}_{\Sigma t}(h_t; v_\Sigma)$ is right-continuous at $v_\Sigma = \hat{v}_\Sigma$ for all histories h_t.

As in Steps A-C of the algorithm illustrated in Figure 5, define

$$\begin{aligned}
\overline{\Pi}^{1}_{\Sigma t}(h_t, e_t; v_\Sigma) &= \gamma \max\left\{v_\Sigma, E\left[\overline{\Pi}^{1}_{\Sigma t+1}(h_t, e_t, X_{t+1}; v_\Sigma)|h_t, e_t\right]\right\} \\
\mathcal{F}^2_t(h_t; v_\Sigma) &= \left\{e_t : \gamma v_\Sigma + c_{\Sigma t}(e_t; x_t) \le \overline{\Pi}^{1}_{\Sigma t}(h_t, e_t; v_\Sigma)\right\} \\
\overline{\Pi}^{2}_{\Sigma t}(h_t; v_\Sigma) &= \max_{e_t \in \mathcal{F}^2_t(h_t; v_\Sigma)} \left(\pi_{\Sigma t}(e_t; x_t) + \overline{\Pi}^{1}_{\Sigma t}(h_t, e_t; v_\Sigma)\right)
\end{aligned}$$

[2] To simplify the presentation, I focus on the case in which there is a unique such maximizer at almost all histories reached on the equilibrium path. More generally, the proof extends almost unchanged, when one recognizes that a discontinuity of $E\left[\overline{\Pi}^{eqm}_{\Sigma 0}(S_0, Y_0; v_\Sigma)\right]$ in v_Σ requires that the IC-constraint be binding on *all* such maximizers at a set of histories reached with positive probability.

As argued above, the IC-constraint (8) is not (exactly) binding for *any* effort-profile at a probability-one set of histories reached on the equilibrium path. At each such history, $\mathcal{F}_t^2(h_t; v_\Sigma)$ is unchanging in a neighborhood of $\hat{v}_\Sigma$. Thus, the right-continuity of $\overline{\Pi}^1_{\Sigma t}(h_t, e_t; v_\Sigma)$ in v_Σ implies right-continuity of $\overline{\Pi}^2_{\Sigma t}(h_t; v_\Sigma)$ in v_Σ, at a probability-one set of equilibrium histories. Repeating this argument for all $k \geq 1$, we conclude that $\overline{\Pi}^k_{\Sigma t}(h_t; v_\Sigma)$ is right-continuous in v_Σ at $\hat{v}_\Sigma$ at a probability-one set of equilibrium histories. Such continuity carries over to the limit as well, so that maximal equilibrium joint payoff $\overline{\Pi}^{eqm}_{\Sigma t}(h_t; v_\Sigma)$ is right-continuous in v_Σ at a probability-one set of histories. In particular, $E[\overline{\Pi}^{eqm}_{\Sigma t}(S_0; v_\Sigma)$ is right-continuous in v_Σ at $\hat{v}_\Sigma$. The proof of left-continuity is similar, and omitted to save space. □

Proof of Theorem 6

Proof. Part I: $v_\Sigma - E[\overline{\Pi}^{eqm}_{\Sigma 0}(X_0; v_\Sigma)]$ *is strictly increasing in* v_Σ. In a slight variation on the notation used in the text, let $\overline{\Pi}^{eqm}_{\Sigma t}(h_t; v^h_\Sigma)$ denote the maximal SPE joint payoff from history $h_t = (x_t, \mathbf{e}_{t-1})$ given joint outside option v^h_Σ. Consider now a lower joint outside option $v^l_\Sigma \in [0, v^h_\Sigma)$ and let $\widetilde{\Pi}_{\Sigma t}(h_t; v^l_\Sigma)$ denote the joint payoff that would result should players with joint outside option v^l_Σ mimic welfare-maximizing play as if it were v^h_Σ. Note that the stage-game payoff process and the partnership stopping time T are identically distributed when players follow the same strategies. Thus, the only difference in payoffs arises from the fact that players only get v^l_Σ when they survive but the partnership ends instead of v^h_Σ. In particular, for all histories h_t,

$$\overline{\Pi}^{eqm}_{\Sigma t}(h_t; v^h_\Sigma) - \widetilde{\Pi}_{\Sigma t}(h_t; v^l_\Sigma) = \gamma(v^h_\Sigma - v^l_\Sigma) \sum_{t' \geq t} \gamma^{t'-t} \Pr(T = t' | h_t) \leq \gamma \left(v^h_\Sigma - v^l_\Sigma\right) \tag{9}$$

Let $e_t(v^h_\Sigma)$ denote the efforts played in the optimal SPE given joint outside option v^h_Σ. Observe that these efforts remain incentive-compatible given lower joint outside option v^l_Σ:

$$\begin{aligned}
E\left[\widetilde{\Pi}_{\Sigma t+1}(H_{t+1}; v^l_\Sigma) | h_t, e_t(v^h_\Sigma)\right] &\geq E\left[\overline{\Pi}^{eqm}_{\Sigma t+1}(H_{t+1}; v^h_\Sigma) | h_t, e_t(v^h_\Sigma)\right] - \gamma\left(v^h_\Sigma - v^l_\Sigma\right) &(10)\\
&\geq \gamma v^h_\Sigma + c_{\Sigma t}(e_t(v^h_\Sigma); x_t) - \gamma\left(v^h_\Sigma - v^l_\Sigma\right) &(11)\\
&= c_{\Sigma t}(e_t(v^h_\Sigma); x_t) + \gamma v^l_\Sigma.
\end{aligned}$$

((10) follows from (9). (11) follows from the incentive-compatibility constraint (18) in the text, as applied to the optimal equilibrium given v_Σ^h.) By similar logic, staying is incentive-compatible given these mimicking strategies whenever players stay in the optimal equilibrium given joint outside option v_Σ^h. (Details omitted to save space.) Thus, these mimicking strategies constitute a SPE given v_Σ^l. In particular, $E[\overline{\Pi}_{\Sigma 0}^{eqm}(X_0; v_\Sigma^l)] \geq E[\widetilde{\Pi}_0(X_0; v_\Sigma^l)]$. Thus, $E[\overline{\Pi}_{\Sigma 0}^{eqm}(X_0; v_\Sigma^h)] - E[\overline{\Pi}_{\Sigma 0}^{eqm}(X_0; v_\Sigma^l)] \leq \gamma \left(v_\Sigma^h - v_\Sigma^l\right)$ and we conclude that $\gamma v_\Sigma - E[\overline{\Pi}_{\Sigma 0}^{eqm}(X_0; v_\Sigma)]$ is non-decreasing in v_Σ. Since $\gamma < 1$, this implies that $v_\Sigma - E[\overline{\Pi}_{\Sigma 0}^{eqm}(X_0; v_\Sigma)]$ is strictly increasing in v_Σ.

Part II: $v_\Sigma = E[\overline{\Pi}_{\Sigma 0}^{eqm}(X_0; v_\Sigma)]$ has a unique solution $\overline{v}_\Sigma$. (a) Given zero outside option, each player's expected payoff is non-negative in any SPE of the partnership game. In particular, $0 \leq E[\overline{\Pi}_{\Sigma 0}^{eqm}(X_0; 0)]$. (b) Since joint payoffs are bounded (Assumption 1), $v_\Sigma \geq E[\overline{\Pi}_{\Sigma 0}^{eqm}(X_0; v_\Sigma)]$ for all large enough v_Σ. (c) By Part I and the proof of Theorem 5, $v_\Sigma - E[\overline{\Pi}_{\Sigma 0}^{eqm}(X_0; v_\Sigma)]$ is strictly increasing and continuous in v_Σ. Thus, $v_\Sigma = E[\overline{\Pi}_{\Sigma 0}^{eqm}(X_0; v_\Sigma)]$ has a unique solution. $\square$