Appendix to "A Biological Model of Unions"

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A Appendix

This Appendix provides the proofs for all propositions stated in "A Biological Model of Unions." Additional transitional dynamics can be found in the working paper version of the paper (Kremer and Olken 2001, NBER Working Paper #8257).

Proof of Proposition 1. The steady state pattern of unionization must satisfy two criteria. First, the number of unionized firms, denoted U, must remain constant. Second, the distribution of organizing difficulties, c, of union and non-union firms must also remain constant, given the (differing) death rates of unionized and non-unionized firms and the distribution of c among newly-created firms.

Suppose that at a given moment all firms with organizing difficulty below some cutoff point p are unionized and all firms with difficulty above p are non-unionized. This will be the case in steady state. There will be two types of non-unionized firms, firms that have just been created with difficulty distributed according to the initial distribution and pre-existing firms with difficulties greater than p. Unions will optimally spend their organizing budget first to organize newly emerged firms with organizing difficulty below p. Once the union has organized those firms, it will spend what remains of its budget on the remaining previously existing firms with marginal difficulty of organizing p.

Normalize the number of firms, F, to 1, so that U becomes the fraction of firms that are unionized. During an instant of time of length dt, $[\delta(\alpha+B)U+\delta(0)(1-U)]\,dt$ firms will have just exited due to a negative productivity shock. As those firms die, new firms will be born with difficulties of being unionized distributed according to the initial distribution. For a union to organize all newborn firms with difficulty level below p, the union will have to spend

$$[\delta(\alpha+B)U+\delta(0)(1-U)]\,dt\int_0^p c\,dH(c), \tag{A.1}$$

which, since $H(c)$ is Uniform[0,1], is just $[\delta(\alpha+B)U+\delta(0)(1-U)]\,dt\frac{p^2}{2}$. In order for p, the threshold below which all firms are organized, to remain constant, the union's effective organizing budget must exactly correspond to the total cost of organizing all newly created firms with cost less than or equal to p, i.e.:

$$A(\alpha,B)BU=[\delta(\alpha+B)U+\delta(0)(1-U)]\frac{p^2}{2}. \tag{A.2}$$

This condition, that p must not change, is one of the two conditions that must be satisfied in the steady state. If the union had a surplus, i.e. if $A(\alpha,B)BU>[\delta(\alpha+B)U+\delta(0)(1-U)]\frac{p^2}{2}$, then it would spend that surplus organizing non-union firms in the "thick" segment with difficulty greater than p, and p would increase. Conversely, if the union's budget was not sufficient to organize all of the newly born firms with difficulty below p, then p would decrease.

The second condition for the steady state is that the number of unionized firms, U, must not change, which implies that the number of newly born firms the union organizes must exactly equal the number of firms it loses to attrition. This yields the condition

$$[\delta(\alpha+B)U+\delta(0)(1-U)]p=\delta(\alpha+B)U. \tag{A.3}$$

Equations (4) and (3) in the text can be obtained by combining equation (A.2) and equation (A.3). The derivation for the condition that guarantees an interior solution, $2A(\alpha,B)B<\delta(\alpha+B)$, can be seen by setting the algebraic expressions for U^* and p^* equal to 1, the maximum value they can take, given that the maximum proportion of firms that can be unionized is 1 and that the difficulties of unionization are distributed on the interval $[0,1]$. ■

Proof of Proposition 2. The comparative statics with respect to $A(\alpha,B)$, and a level shift in δ are immediate from equation (4). To see the effect of a proportional shift, suppose that the ratio $\frac{\delta(\alpha+B)}{\delta(0)}$ is fixed at $\lambda(\alpha+B)$. Then equation 4 can be rewritten as

$$U^*=\frac{2A(\alpha+B)B}{\lambda(\alpha+B)^2\delta(0)-2A(\alpha+B)B[\lambda(\alpha+B)-1]}. \tag{A.4}$$

Taking the derivative with respect to $\delta(0)$ yields

$$\frac{dU^*}{d\delta(0)}=-\frac{U^*\lambda(\alpha+B)^2}{\lambda(\alpha+B)^2\delta(0)-2A(\alpha+B)B[\lambda(\alpha+B)-1]}. \tag{A.5}$$

Condition (5) guarantees that $2A(\alpha+B)B\leq\lambda(\alpha+B)\delta(0)$, which in turn guarantees that $\frac{dU^*}{d\delta(0)}$ will be less than zero. ■

Proof of Proposition 3. Suppose the incumbent union follows the policy (α_1,B_1) and the entrant follows the policy (α_2,B_2), with $p_2^*=\frac{2A(\alpha_2,B_2)B_2}{\delta(\alpha_2+B_2)}>\frac{2A(\alpha_1,B_1)B_1}{\delta(\alpha_1+B_1)}=p_1^*$. Denote the sizes of the two unions by U_1 and U_2 respectively. It suffices to show that $U_1\to 0$ as $t\to\infty$.

To see this, we first need to introduce the dynamics of the system with a single union. At any instant, assuming that there is no discontinuous increase in the number of firms, there are two different sets of firms that the union may chose to organize: the "thick" set of firms that are non-unionized and the "thin" set of firms that were created that instant to replace firms that exited due to a negative shock. The measure of non-unionized firms in the thick set is $1-U$ and the measure of firms in the thin segment is

$$[\delta(\alpha+B)U+\delta(0)(1-U)]dt \tag{A.6}$$

Facing this profile of non-unionized firms, the union will organize the easiest firms it can. These will be all of the firms in the thin segment with cost less than p and then as many firms in the thick segment as it can with whatever remains of its organizing budget at that moment. Note that p represents the lower bound of the "thick" set of non-unionized firms – it will be possible in certain transitions that there are unionized firms whose difficulties are greater than p. Since the distribution of firms in the thin segment is uniform, the cost of organizing all firms in the thin segment with cost less than p will be

$$[\delta(\alpha+B)U+\delta(0)(1-U)]dt\frac{p^2}{2} \tag{A.7}$$

so that the flow effective organizing budget surplus or deficit becomes

$$A(\alpha,B)BU-[\delta(\alpha+B)U+\delta(0)(1-U)]\frac{p^2}{2} \tag{A.8}$$

If the effective organizing budget has a surplus, then the growth of the union will be the number of firms in the thin segment with difficulty levels less than or equal to p plus however many older firms the union can afford to organize at marginal cost p with whatever remains of its budget, minus the number of its member firms it lost due to negative shocks:

$$\dot{U} = [\delta(\alpha + B) U + \delta(0)(1-U)] p + \frac{A(\alpha, B) BU - [\delta(\alpha + B) U + \delta(0)(1-U)] \frac{p^2}{2}}{p} - \delta(\alpha + B) U \tag{A.9}$$

$$= \frac{A(\alpha, B) BU + [\delta(\alpha+B)U + \delta(0)(1-U)] \frac{p^2}{2}}{p} - \delta(\alpha+B)U \tag{A.10}$$

On the other hand, if the union's effective organizing budget is not sufficient to organize all firms in the thin segment with costs less than or equal to p, the union will organize as many of those firms as it can. This will be all newly created firms with difficulty levels less than or equal to some cutoff level l such that the total budget exactly equals the cost of organizing the firms, i.e.

$$[A(\alpha, B) BU] = [\delta(\alpha + B) U + \delta(0)(1-U)] \frac{l^2}{2} \tag{A.11}$$

However, since the newly created firms of difficulty levels between l and p will not be unionized, they will become part of the thick segment, and p will immediately decrease to be equal to l. This implies that when the union organizing budget is insufficient,

$$p = l = \sqrt{\frac{2A(\alpha, B) BU}{[\delta(\alpha + B) U + \delta(0)(1-U)]}} \tag{A.12}$$

The change in the number of unionized firms in this case will therefore be the fraction p of thin firms unionized, multiplied by the total number of thin firms, less the number of unionized firms that exit:

$$\dot{U} = [\delta(\alpha + B) U + \delta(0)(1-U)] p - \delta(\alpha + B) U \tag{A.13}$$

In the case of two coexisting unions, as in the case of a single union, there are two different equations for the change in the number of unionized firms according to whether there is a budget surplus or a budget deficit.[1] First, note that the number of firms that die in an instant of time dt becomes

$$\delta(\alpha_1 + B_1) U_1 + \delta(\alpha_2 + B_2)U_2 + \delta(0)(1 - U_1 - U_2). \tag{A.14}$$

As discussed in the text, when there are multiple unions, they do not compete over the same firms in the "thin" segment, but rather divide them according to their effective organizing expenditures.

[1] Note that at any instant p is the same for both unions, since new firms are allocated in proportion to the unions' effective organizing budgets, and either both unions have a budget surplus and can organize firms in the thick segment (of difficulty level p) or neither does. This can be seen by substituting the formulas for dividing newly created firms ($\frac{A(\alpha_1,B_1)B_1U_1}{A(\alpha_1,B_1)B_1U_1+A(\alpha_2,B_2)B_2U_2}$ and $\frac{A(\alpha_2,B_2)B_2U_2}{A(\alpha_1,B_1)B_1U_1+A(\alpha_2,B_2)B_2U_2}$) into equation (A.8).

Given this, in the case of a budget surplus, equation (A.10) for the spread of unions becomes:

$$\dot{U}_1 = \frac{A(\alpha_1,B_1)B_1U_1 + \frac{A(\alpha_1,B_1)B_1U_1}{A(\alpha_1,B_1)B_1U_1+A(\alpha_2,B_2)B_2U_2}\left[\begin{array}{c}\delta\left(\alpha_1+B_1\right)U_1+\delta(\alpha_2+B_2)U_2\\ +\delta\left(0\right)\left(1-U_1-U_2\right)\end{array}\right]\frac{p^2}{2}}{p} - \delta\left(\alpha_1+B_1\right)U_1 \tag{A.15}$$

$$\dot{U}_2 = \frac{A(\alpha_2,B_2)B_2U_2 + \frac{A(\alpha_2,B_2)B_2U_2}{A(\alpha_1,B_1)B_1U_1+A(\alpha_2,B_2)B_2U_2}\left[\begin{array}{c}\delta\left(\alpha_1+B_1\right)U_1+\delta(\alpha_2+B_2)U_2\\ +\delta\left(0\right)\left(1-U_1-U_2\right)\end{array}\right]\frac{p^2}{2}}{p} - \delta\left(\alpha_2+B_2\right)U_2. \tag{A.16}$$

In the case of a budget deficit with two unions, equation (A.13) for the spread of unions becomes

$$\dot{U}_1 = \frac{A(\alpha_1,B_1)B_1U_1}{A(\alpha_1,B_1)B_1U_1+A(\alpha_2,B_2)B_2U_2}\left[\begin{array}{c}\delta\left(\alpha_1+B_1\right)U_1+\delta(\alpha_2+B_2)U_2\\ +\delta\left(0\right)\left(1-U_1-U_2\right)\end{array}\right]p - \delta\left(\alpha_1+B_1\right)U_1 \tag{A.17}$$

$$\dot{U}_2 = \frac{A(\alpha_2,B_2)B_2U_2}{A(\alpha_1,B_1)B_1U_1+A(\alpha_2,B_2)B_2U_2}\left[\begin{array}{c}\delta\left(\alpha_1+B_1\right)U_1+\delta(\alpha_2+B_2)U_2\\ +\delta\left(0\right)\left(1-U_1-U_2\right)\end{array}\right]p - \delta\left(\alpha_1+B_2\right)U_2. \tag{A.18}$$

In either case, examination of the formulae above shows that

$$\left(\frac{dU_1}{dt}+\delta(\alpha_1+B_1)U_1\right)\frac{1}{A(\alpha_1,B_1)B_1U_1} = \left(\frac{dU_2}{dt}+\delta(\alpha_2+B_2)U_2\right)\frac{1}{A(\alpha_2,B_2)B_2U_2} \tag{A.19}$$

which implies that

$$\frac{1}{A(\alpha_1,B_1)B_1}\frac{d\left(\log U_1\right)}{dt}+\frac{2}{p_1^*} = \frac{1}{A(\alpha_2,B_2)B_2}\frac{d\left(\log U_2\right)}{dt}+\frac{2}{p_2^*}. \tag{A.20}$$

Hence

$$\frac{d\log\left(\frac{U_2^{\frac{1}{A(\alpha_2,B_2)B_2}}}{U_1^{\frac{1}{A(\alpha_1,B_1)B_1}}}\right)}{dt} = \frac{2}{p_1^*}-\frac{2}{p_2^*} \tag{A.21}$$

The right hand side of this equation is a constant which is strictly positive since $p_2^* > p_1^*$ by hypothesis. It follows that $\frac{U_2^{\frac{1}{A(\alpha_2,B_2)B_2}}}{U_1^{\frac{1}{A(\alpha_1,B_1)B_1}}} \to \infty$. However, since $0 \le U_i \le 1$ and $A(\alpha_i,B_i)B_i > 0$, the only way this can happen is if $U_1 \to 0$. Hence the incumbent union is wiped out by the entrant union. It follows that we must converge to the steady state containing only the (α_2,B_2) union. ■

Proof of Proposition 4. The fact that the (a_S,B_S) union will be evolutionarily stable follows directly from Proposition 3, since by definition (a_S,B_S) maximizes p^*. To see that $(\alpha_S,B_S) \neq (\alpha_W,B_W)$, suppose not, i.e., that $\alpha_S = \alpha_W$ and $B_S = B_W$. Consider $\widetilde{\alpha}$, which is slightly less than α_W. Since (α_W,B_W) maximizes $A\left(\alpha_W,B_W\right)$, moving from α_W to $\widetilde{\alpha}$ results in a first order decrease in $\delta\left(\alpha+B\right)$ but only a second-order decrease in $A\left(\alpha,B\right)$. Therefore, $\frac{2A(\widetilde{\alpha},B_W)B_W}{\delta(\widetilde{\alpha}+B_W)} > \frac{2A(\alpha_W,B_W)B_W}{\delta(\alpha_W+B_W)}$, which is a contradiction of the assumption that $\alpha_S = \alpha_W$ and $B_S = B_W$, since by definition (a_S,B_S) maximizes $p^* = \frac{2A(\alpha,B)B}{\delta(\alpha+B)}$. Therefore $(\alpha_S,B_S) \neq (\alpha_W,B_W)$. ■

Proof of Proposition 5. To prove the proposition, suppose not, i.e. that $\alpha_S + B_S \ge \alpha_W + B_W$ and $B_S \le B_W$. Then in that case, $\frac{2A(\alpha_S,B_S)B_S}{\delta(\alpha_S+B_S)} \le \frac{2A(\alpha_W,B_W)B_W}{\delta(\alpha_W+B_W)}$, since $\delta\left(\alpha_S+B_S\right) \ge \delta\left(\alpha_W+B_W\right)$, $B_S \le B_W$, and $A\left(\alpha_S,B_S\right) \le A\left(\alpha_W,B_W\right)$ since (α_W,B_W) maximizes $A\left(\alpha_W,B_W\right)$. Since Proposition 4 implies that $(\alpha_S,B_S) \neq (\alpha_W,B_W)$, at least one of these inequalities must be strict, in which case $\frac{2A(\alpha_S,B_S)B_S}{\delta(\alpha_S+B_S)} < \frac{2A(\alpha_W,B_W)B_W}{\delta(\alpha_W+B_W)}$. This violates the definition of (a_S, B_S), which is that it maximizes $p^* = \frac{2A(\alpha,B)B}{\delta(\alpha+B)}$. ■