

# Top Incomes, Rising Inequality, and Welfare\*

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## Abstract

This paper develops a general-equilibrium model of skill-biased technological change that approximates the dramatic upward shift in the share of total income going to the top decile of U.S. households since 1980. Under realistic assumptions, we show that all agents in the economy can benefit from the technology change, provided that the observed rise in U.S. redistributive transfers over this period is taken into account. We show that the increase in capital's share of total income and the presence of capital-entrepreneurial skill complementarity are two key features that help support the wages of ordinary workers as the new technology diffuses. Depending on assumptions, the model can produce simulations in which rising income inequality is associated with either faster or slower output growth relative to the no-change trend.

Keywords: *Income Inequality, Skill-biased Technological Change, Capital-skill Complementarity, Redistribution, Welfare.*

JEL Classification: D31, E32, E44, H23, O33.

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# 1 Introduction

Income inequality in many industrial countries increased markedly over the past three decades. Most of the increase can be traced to gains made by those near the top of the income distribution. According to a recent study by the OECD (2011), “the highest 10% of earners have been leaving the middle earners behind more rapidly than the lowest earners have been drifting away from the middle.” The study asserts that technological progress and a more integrated global economy have brought profound changes in the ways that firms produce and distribute goods and services, and that these changes have shifted production technologies in favor of highly-skilled individuals.

Rising inequality from top incomes is particularly evident in the U.S. economy. Autor, et al. (2006) show that since the mid-1980s, upper tail U.S. wage dispersion has increased significantly while lower tail wage dispersion has actually declined. The share of total pre-tax income including capital gains going to the top decile of U.S. households rose from 35% in 1980 to about 50% in 2012 (Piketty and Saez 2003, 2013). One striking statistic noted by Piketty and Saez (2013, p. 3) is that “more than 15% of U.S. national income was shifted from the bottom 90% to the top 10% [of households] in the U.S. over the past 30 years.” The aim of this paper is to investigate the welfare consequences of this income shift for agents within and outside the top decile. Along the lines of the OECD (2011) study, we postulate that the increase in U.S. pre-tax income inequality over the past three decades was driven by a slow moving technological change that made production processes more capital intensive and raised the wages of highly-skilled entrepreneurs in the top decile. Our quantitative modeling approach allows us to take into account the effects of rising income inequality on the trajectory of real output during the transition period.

The increase in the U.S. top decile income share was driven by shifts in both labor and capital incomes. Changes in capital gains and dividend income were the two largest contributors to the increase in the Gini coefficient from 1996 to 2006 according to a study by the Congressional Research Service (Hungerford 2011). Capital’s share of total income in the U.S. economy increased from about 35% in 1980 to around 41% in 2012. Given that the distribution of wealth in the U.S. economy is highly skewed, the observed increase in capital’s share of income would be expected to disproportionately benefit households near the top of the income distribution. As a mitigating factor, transfer payments from the government and businesses to individuals increased from 10% of GDP in 1980 to around 15% in 2012. These transfers would be expected to disproportionately benefit households outside the top decile of the income distribution.

Unlike income inequality, the degree of wealth inequality in the U.S. economy has remained

relatively steady going back to the 1970s (Kopczuk and Saez, 2004). The top decile of U.S. households owns approximately 80 percent of financial wealth and about 70 percent of total wealth including real estate.<sup>1</sup> Shares of corporate stock are an important component of financial wealth, representing claims to the physical capital of firms. The framework of our analysis is a general equilibrium model in which the top decile of households owns 100 percent of the productive capital stock—a setup that roughly approximates the U.S. wealth distribution.

The consumption of the capital owners in the model is funded from wages and dividends while the consumption of the remaining agents, called workers, is funded from wages and redistributive government transfers. All agents supply labor endogenously to firms. Capital owners are interpreted as entrepreneurs whose labor input exhibits complementarity with the stock of physical capital. This effect, which we label as “capital-entrepreneurial skill complementarity” works in much the same way as the mechanism proposed by Krusell, et al. (2000), except that here the complementarity effect applies more narrowly to the labor supply of the top decile, as opposed to the broader population of college-educated workers. An empirical study by Lemieux (2006) provides support for our specification. Specifically, he finds that wage inequality among college-educated workers has increased significantly in recent decades. The study concludes (p. 199) that “changes in wage inequality are increasingly concentrated in the very top end of the wage distribution.”

We show that the welfare effects of the technology change in the model depend crucially on several features. These include: (1) the nature of capital owners’ expectations (which affects perceptions of permanent income and the resulting investment/saving response), (2) the assumed paths for redistributive government transfers and capital’s share of total income, and (3) the degree of complementarity between physical capital and entrepreneurial labor. Under realistic assumptions, we show that all agents can benefit from the technology change, provided that the observed rise in U.S. redistributive transfers over this period is taken into account. The increase in capital’s share of total income and the presence of capital-entrepreneurial skill complementarity are two key features that help support the wages of ordinary workers as the new technology diffuses. Depending on assumptions, the model can produce simulations in which rising income inequality is associated with either faster or slower output growth relative to the no-change trend. This theoretical ambiguity may provide some basis for the conflicting findings in the empirical literature on the links between inequality and growth (Forbes 2000, Berg and Ostry 2011).

We model skill-biased technological change as a diffusion process that shifts the parameters of the representative firm’s constant elasticity of substitution (CES) production function in

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<sup>1</sup>See Wolff (2006), Table 4.2, p. 113.

a way that approximates the significant upward shift in the income share of the top decile of U.S. households since 1980. Specifically, the share parameters for the three productive inputs (physical capital, entrepreneurial labor, and ordinary worker labor) are allowed to evolve according to an S-shaped trajectory, consistent with empirical studies on the manner in which new innovations are adopted over time (Comin, et al. 2008). We calibrate the law of motion for the diffusion process to approximately match the average U.S. adoption rate for three important technology innovations, namely, personal computers, mobile telephones, and internet use. Coincident with the technology diffusion process, we allow redistributive government transfers from the top decile to the remainder of households to increase in a manner consistent with U.S. data. We show that the model’s simulated paths for the top decile income share and the ratio of redistributive transfers to GDP are similar to the corresponding paths in the data.

Our approach to modeling skill-biased technological change follows Goldin and Katz (2007) who allow CES production function share parameters to shift over time as a way of capturing technology-induced changes in the demand for skilled versus unskilled labor. According to Acemoglu and Autor (2012), shifts in these parameters can also be interpreted as capturing “skill-replacing technical changes” that increase firms’ demand for one type of skill at the expense of another.<sup>2</sup>

The introduction of any new technology naturally involves considerable uncertainty about its potential widespread use in the future. We therefore examine the role of expectations in shaping the transition paths of the endogenous variables and the resulting welfare and growth effects. We first consider the case where capital owners have rational expectations about the transition path.<sup>3</sup> In our setting, rational expectations corresponds to perfect foresight because the laws of motion that govern trend growth and the diffusion of new technology abstract from stochastic variation. While the perfect foresight assumption may be viewed as extreme, it serves as a useful benchmark. Next, we examine the case where capital owners employ myopic (or random walk) expectations. Specifically, their forecasts for variables dated  $t + 1$  or later are given by the most recently observed value of the same variable. Such a forecast rule can be viewed as boundedly-rational because it economizes on the costs of collecting and processing information. Finally, we consider a formulation labeled “learning” in which the share of capital owners with knowledge about the laws of motion governing the transition increases gradually over time as the new technology is adopted.

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<sup>2</sup>Along somewhat similar lines, Young (2004) and Ríos-Rull and Santaeulàlia-Llopis (2010) allow for stochastic variation in the share parameters of a Cobb-Douglas production function to account for various business cycle facts.

<sup>3</sup>Workers consume their wage income plus transfers each period, so they make no intertemporal decision.

The welfare outcomes for both types of agents are sensitive to the way that expectations are formed. Capital owners always benefit from the technology change but the size of their welfare gains depend on their degree of foresight. Their optimal investment response and the resulting path for their consumption depend crucially on whether they foresee the permanent shift in their income. Workers' welfare may either rise or fall, depending on the magnitude of the capital owners' investment response which in turn influences the equilibrium path of workers' wages. Under perfect foresight, welfare gains are highest for capital owners but workers suffer a welfare loss. In this case, capital owners immediately increase their consumption at the expense of investment because they foresee the large increase in their permanent income. The initial jump in their consumption yields a large welfare gain—in excess of 30% of per-period consumption for the baseline calibration. However, the resulting slowdown in capital accumulation lowers the paths of workers' wages and consumption relative to the model's no-change trend. As a result, workers suffer a welfare loss of 1.3% of per-period consumption in the baseline model under perfect foresight.

In the case of myopic expectations, capital owners do not foresee the large increase in their permanent income. Consequently, their consumption does not jump at the beginning of the transition, but rather increases gradually along with their current income. We view this scenario as more realistic than the perfect foresight regime. Similarly, investment increases gradually relative to the no-change trend which boosts capital accumulation and raises the paths of workers' wages and consumption. At the same time, redistributive government transfers are growing faster than GDP, as observed in the data. For the baseline model, the welfare gain for capital owners is about 9% of per-period consumption whereas workers now achieve a welfare *gain* of about 1.5%. The welfare results for the learning regime fall in between those for perfect foresight and myopic expectations. Similar to myopic expectations, the learning mechanism precludes an immediate jump in capital owners consumption at the beginning of the transition path. However, as more capital owners learn about the process governing their future income, their consumption starts increasing faster, eventually catching up to the perfect foresight trajectory. Under learning, capital owners' achieve a welfare gain of about 15% of per-period consumption whereas workers achieve a welfare gain of about 0.6%.

As part of our analysis, we consider how different categories of income contribute to the welfare effects of the transition. When the ratio of U.S. redistributive government transfers to GDP is held constant at the 1980 level of 10% (rather than increasing to 15% as in the data), capital owners enjoy a welfare gain of 16% of per-period consumption under myopic expectations versus a gain of 9% in the baseline scenario. Workers now suffer a small welfare loss of 0.15% versus a baseline gain of 1.5%. This experiment highlights the importance of the

rising trend of redistributive transfers in allowing workers to achieve a positive welfare gain in the baseline scenario. We also consider an experiment where capital's share of total income is held constant at its initial level while the share of labor income going to the top decile continues to rise in a manner consistent with the U.S. data. Both types of agents are made worse-off relative to the baseline scenario. Interestingly, both types of agents benefit from an increase in capital's share of total income even though capital ownership is concentrated in the hands of the top decile. As discussed further below, this result is due to the positive wage impacts of a technology-induced increase in the productivity of physical capital. The positive wage impacts are stronger in the presence of capital-entrepreneurial skill complementarity.

To gauge the influence of capital-entrepreneurial skill complementarity, we compare the baseline model to one with a standard Cobb-Douglas production function. In the Cobb-Douglas model, both types of labor exhibit the same (unitary) elasticity of substitution with physical capital. The share parameters of the Cobb-Douglas production function are assumed to shift over time in manner that matches the U.S. income distribution data. We find that both types of agents are considerably worse-off in the Cobb-Douglas world. The absence of capital-entrepreneurial skill complementarity means that a technology change which raises the productivity of physical capital now bestows less benefits on entrepreneurial labor, thus lowering the capital owner's wage path relative to the baseline model. The wage path of workers is also lowered, as dictated by the equilibrium conditions of the competitive labor market. This experiment shows that capital-entrepreneurial skill complementarity is an important feature that not only benefits the suppliers of entrepreneurial labor; it can also deliver benefits to ordinary workers via higher wages than otherwise. In the Cobb-Douglas model, lower wage paths for both types of agents bring about lower labor supplies, which in turn slows the growth rate of aggregate output during the transition period. In contrast to the baseline model in which rising income inequality is associated with faster output growth during the transition, the Cobb-Douglas model delivers much slower output growth during the transition.

We also investigate the sensitivity of the welfare results to changes in the values of other key parameters, including the speed of technology diffusion, the elasticities of intertemporal substitution for consumption and for labor supply, and the subjective time discount factor. We show that each of these parameters can have a significant impact on welfare outcomes. Overall, we find that the range of possible welfare outcomes from skill-biased technological change is enormous, even in the relatively simple framework considered here with only two types of agents. These findings suggest that conclusions regarding the appropriate policy response to rising income equality can be strongly influenced by the details of any particular model.

## 1.1 Related Literature

Our analysis focuses on the consequences of inequality that is driven by gains in top incomes, i.e., the highest 10% of earners. In contrast, the vast majority of previous research has focused on inequality that is driven by the rising wage premium of college-educated workers.<sup>4</sup> Moreover, our welfare analysis takes into account the simultaneous shifts in the distribution of both labor and capital incomes in U.S. data. According to Alvaredo, et al. (2013), the increased correlation between top labor incomes and top capital incomes since 1980 is an important but often-overlooked factor contributing to the rise in U.S. income inequality.

As an alternative to skill-biased technological change, Piketty, et al. (2011) argue that the dramatic rise in top incomes has been driven mainly by institutional changes which strengthened the bargaining power of top earners at the expense of lower earners. According to this “grabbing hand” theory, the shift in bargaining power has enabled rent-seeking top earners to successfully push their pay above their marginal product. Along these lines, Kumhof and Ranciere (2011) consider a model where rising income inequality (as measured by the income share of the top 5% of households) is driven by a decline in the bargaining power of workers. However, in reduced form, the worker’s loss of bargaining power can be interpreted as an exogenous shift in the firm’s production technology in favor of the firm’s owners. In either case, the wages of ordinary workers will rise less than otherwise. Kumhof and Ranciere (2011) focus on the link between rising inequality and a shock-induced financial crisis. In contrast, our aim is to gauge the welfare consequences of the observed three-decade rise in the U.S. top decile income share. While the grabbing-hand theory may have different implications for social welfare, the welfare consequences of the transition for each class of agents would still be linked to the resulting paths for their income, consumption, and leisure, which our analysis takes into account.

Our finding that all agents can achieve welfare gains in a economy with rising income inequality complements the results of Heathcote, et al. (2010, 2011) who focus on the college versus no-college wage premium. As in our analysis, they obtain smaller welfare gains for agents who are myopic. This is because myopic agents in their model fail to anticipate the future rise in the college wage premium and thus do not invest in a college education. In our model, welfare gains are smaller for myopic capital owners because they fail to anticipate the future rise in their permanent income, and thus postpone consumption relative to the perfect foresight trajectory. However, the capital owners’ myopia is actually beneficial for

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<sup>4</sup>A partial list of research in this area includes: Katz and Autor (1999), Krusell, et al. (2000), Acemoglu (2002), Aghion (2002), Card and DiNardo (2002), Hornstein, et al. (2005), Goldin and Katz (2007, 2008), Heathcote, et al. (2010, 2011), and Acemoglu and Autor (2012). For a recent overview of the literature, see Violante (2012).

workers because it leads to faster capital accumulation which in turn boosts workers' wages and consumption.

In contrast to the structural model approach adopted here and by Heathcote, et al. (2010, 2011), empirical studies have mostly found large welfare losses from rising income inequality (Attanasio and Davis 1996, Krueger and Perri 2004). As a caveat, it must be noted that empirical data on shifts in relative wages may not give an accurate picture of the quantities that matter for household welfare, namely consumption and leisure. Krueger and Perri (2006) argue that the impact of rising income inequality on consumption inequality was partially mitigated by an increase in household borrowing to finance consumption at the lower end of the income distribution. A study by Meyer and Sullivan (2010) also finds that the rise in consumption inequality is less pronounced than the rise in income inequality, owing to the redistributive effects of taxes and transfers—a feature that our structural model explicitly takes into account. More recently, Aguiar and Bils (2011) and Attanasio, et al. (2012) argue that consumption inequality, when measured in a different fashion, appears to closely mirror income inequality.

## 2 Stylized Facts

### 2.1 Income Shares

Figure 1 plots various statistics that document the dramatic upward shift in distribution of income that has taken place in the U.S. economy since 1980. The top left panel plots the share of pre-tax income (including capital gains) going to the top decile of U.S. households, as compiled by Piketty and Saez (2003, 2013). The top decile income share rose from 35% in 1980 to about 50% in 2012.<sup>5</sup> The top right panel plots data from the U.S. Census Bureau which shows that the income share of the top 5 percent of households increased from 17% to around 22% over the same period. In the bottom left panel, capital's share of total income, as defined by the U.S. Bureau of Labor Statistics, increased from about 35% in 1980 to almost 42% in 2012.<sup>6</sup> Finally, again using Census Bureau data, the bottom right panel shows that the growth in mean real household income significantly outpaced the growth in the median income over the past three decades, indicating a shift in the mass of income towards the upper tail of the distribution.<sup>7</sup>

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<sup>5</sup>Updated annual data through 2012 are available from Emmanuel Saez's website: <http://elsa.berkeley.edu/~saez/>.

<sup>6</sup>Capital's share is defined here as one minus labor's share where labor's share is obtained from [www.bls.gov/data](http://www.bls.gov/data) using series ID PRS85006173. The tabulated series is indexed to 100 in 2009 which corresponds to a labor share of 59.8%. For additional details, see Gomme and Rupert (2004).

<sup>7</sup>The U.S. Census Bureau data plotted in Figure 1 are from Tables H.3 and H.9, available at [www.census.gov/hhes/www/income/data/historical/household/](http://www.census.gov/hhes/www/income/data/historical/household/)

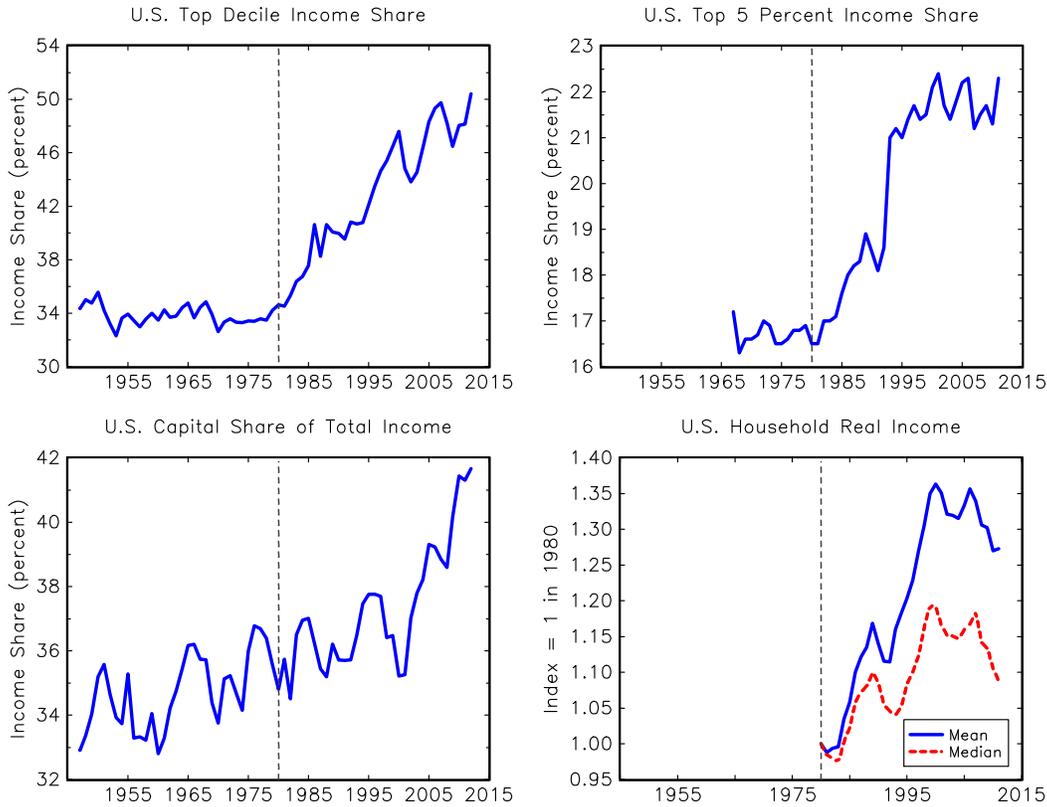


Figure 1: The top decile income share increased from 35% in 1980 to about 50% in 2012. The top 5 percent income share increased from around 17% to 22% over the same period. Capital’s share of total income increased from about 35% in 1980 to around 41% in 2012. The growth in mean real household income has significantly outpaced the growth in median income over the past three decades.

## 2.2 Decomposition of Top Decile Income Share

Figure 2 shows the decomposition of the top decile income share into its component parts. Income from wage and non-wage sources both contributed to the rise. Non-wage sources of income for the top decile (roughly in order of importance) include: entrepreneurial income, capital gains, dividends, interest income, and rents. It is worth noting, however, that the category of wages includes income derived from the exercise of employee stock options—a component that blurs the distinction between labor and capital incomes.

The income threshold for the top decile of household earners in 2011 is \$110,651 (Piketty and Saez, updated). Households within the top decile are very heterogeneous, ranging from upper-middle class families deriving most of their income from wages to the super-rich living off large fortunes. Gordon and Dew-Becker (2007) identify the top decile of earners with

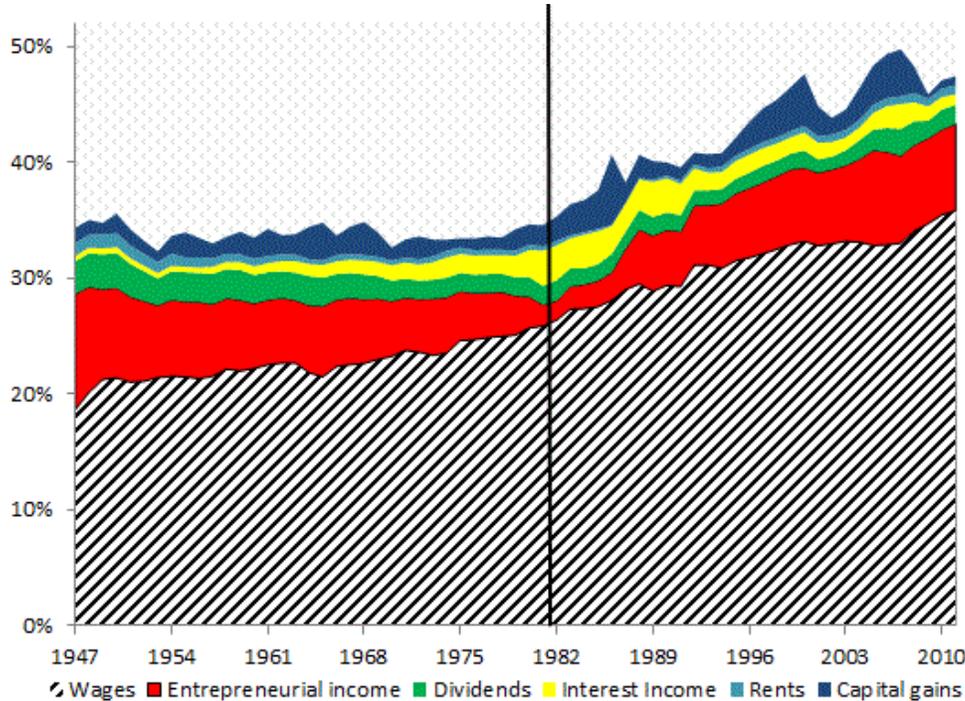


Figure 2: Decomposition of top decile income share into wage and non-wage sources. Non-wage sources of income for the top decile (roughly in order of importance) include: entrepreneurial income, capital gains, dividends, interest income, and rents.

three different groups of professional occupations: superstars in the sports and entertainment industries, highly-paid and highly-skilled workers such as lawyers and investment bankers, and CEOs and other top corporate managers. Abstracting from this heterogeneity, we model the top decile of earners as a single group that owns the entire stock of productive capital. We view this as a reasonable approximation to the U.S. wealth distribution in which the top decile of U.S. households owns approximately 80 percent of financial wealth. Hence, we associate the top decile of households in the model with U.S. stockholders.<sup>8</sup>

### 2.3 Redistributive Transfers

Figure 3 plots transfer payments to individuals as a percentage of GDP from 1959 to 2010. These are payments from governments and businesses to individuals or nonprofit institutions serving individuals.<sup>9</sup> Examples include benefits from Old Age, Survivors, and Disability In-

<sup>8</sup>Similar concentrated capital ownership models have been applied recently to asset pricing. See, for example, Danthine and Donaldson (2008), Guvenen (2009), and Lansing (2011). Mankiw (2000) examines the implications of such a model for fiscal policy.

<sup>9</sup>Data on transfer payments and GDP are from the Federal Reserve Bank of St. Louis' FRED data base. Payments from businesses accounted for only about 1% of total transfers in 2005. For a detailed description of the various transfer programs, see

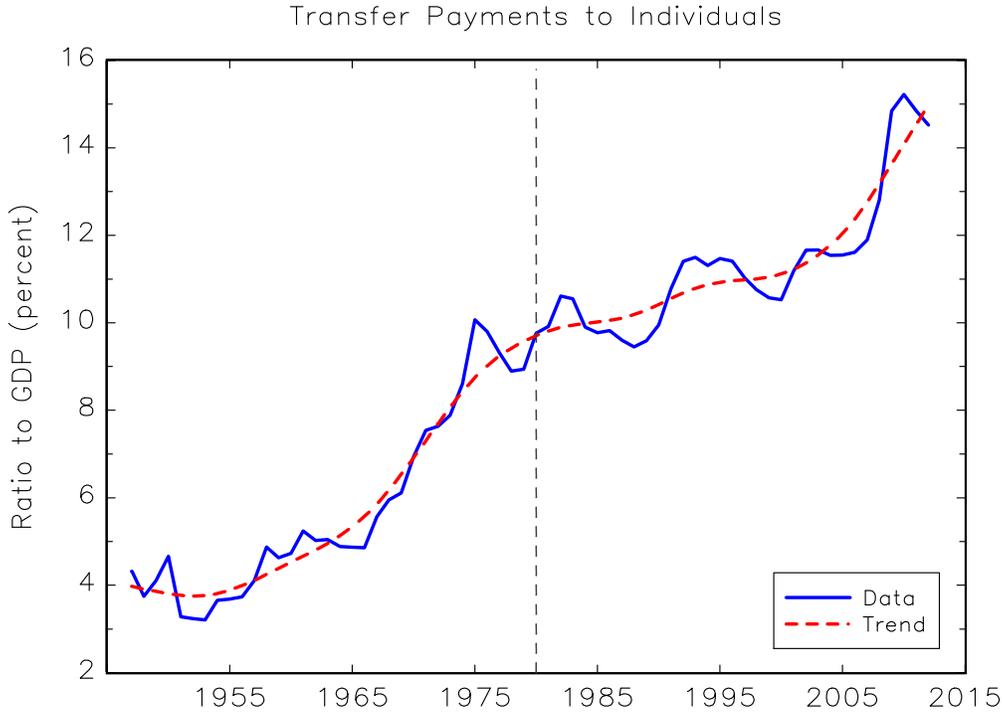


Figure 3: Transfer payments from the government and businesses to individuals increased from 10% of GDP in 1980 to around 15% of GDP in 2012.

insurance (OASDI), Medicare and Medicaid benefits, Supplemental Security Income, Family Assistance, Food Stamps, and Unemployment Insurance Compensation. The figure shows that the ratio of transfer payments to GDP increased from 10% of GDP in 1980 to around 15% in 2010.

While some of the run-up in transfer payments in recent years appears to have been triggered by the government’s response to the financial crisis of 2007-2009, it is also true that pre-tax income inequality, as measured by the top decile income share, continued to trend upward over this period. More generally, it seems reasonable to view the upward trend in transfer payments from 1980 to 2012 as a deliberate effort by the government to address the trend of rising pre-tax income inequality. In the model, we make the simplifying assumption that transfer payments represent a pure redistribution from the top decile to the remainder of households, accomplished via a lump-sum tax on capital owners administered by the government. We investigate the sensitivity of our results the assumed path for these transfers.

<http://www.bea.gov/regional/pdf/spi2005/06%20Personal%20Current%20Transfer%20Receipts.pdf>. The trend in the figure is constructed using the Hodrick-Prescott filter with a smoothing parameter of 100.

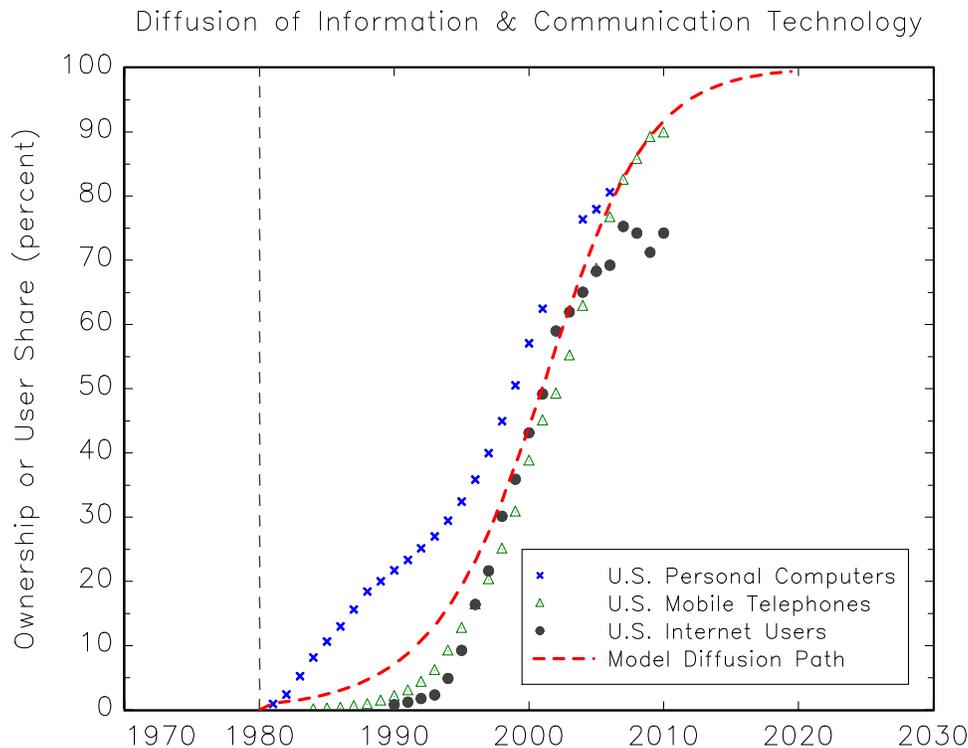


Figure 4: The diffusion path for information and communication technology in the U.S. economy can be approximated by the law of motion  $\pi_t = \pi_{t-1} + \kappa\pi_{t-1}(1 - \pi_{t-1})$ , where  $\pi_t$  is the ownership/user share and  $\kappa = 0.25$  is the speed of diffusion parameter.

## 2.4 ICT Diffusion

A basic assumption of our analysis is that the increase in U.S. pre-tax income inequality over the past three decades was driven by a slow moving technological change that made production processes more capital intensive and raised the wages of highly-skilled entrepreneurs in the top decile. As evidence of technological change, Figure 4 plots the U.S. adoption trajectories for three important technology innovations, namely, personal computers, mobile cellular telephones, and internet use—three series which measure the spread of information and communication technology (ICT).<sup>10</sup> All three series exhibit an S-shaped trajectory—a typical pattern for the manner in which new innovations are adopted over time (Comin, et al. 2008). Comparing Figure 4 to Figure 1 shows that the spread of ICT in the U.S. economy follows roughly the same trajectory as the rise in the top decile income share. While suggestive,

<sup>10</sup>Personal computer ownership data are from the NBER’s Cross-country Historical Adoption of Technology (CHAT) data set available at <http://www.nber.org/data/chat/>. Data for years 2002 and 2003 are missing. Data on mobile cellular telephones and internet use are from the World Bank’s infrastructure data set available at <http://data.worldbank.org/indicator>.

this comovement does not prove causation running from ICT diffusion to income inequality. However, it is consistent with the mechanism of skill-biased technological change emphasized by many authors. There are other examples in history when major technological change was accompanied by a rise in income inequality. These include the Industrial Revolution in Great Britain from 1760 to 1860 (Greenwood, 1999) and the U.S. economy during the 1920s (Atkinson, et al. 2011). Regarding the latter period, Nicholas (2008) presents evidence that the 1920s was “a period of unprecedented technological advance.”

To formalize the process of technology diffusion in the model, we employ the following nonlinear law of motion

$$\pi_t = \pi_{t-1} + \kappa\pi_{t-1}(1 - \pi_{t-1}), \quad (1)$$

where  $\pi_t \in [0, 1]$  represents the share of firms employing the new technology and  $\kappa > 0$  governs the speed of diffusion. Starting from a small positive value, the law of motion implies  $\pi_t \rightarrow 1$  as  $t \rightarrow \infty$ . Figure 4 plots the theoretical diffusion path with  $\kappa = 0.25$  which is the baseline value employed in our quantitative analysis. Starting at  $\pi_0 = 0$  in 1980, we assume that 1% of firms unilaterally adopt the new technology at  $t = 1$ , corresponding to the year 1981. For  $t > 1$ , the theoretical diffusion path tracks roughly in between the observed diffusion paths for personal computers, mobile telephones, and internet use, reaching an adoption share of about 92% in 2010. The theoretical diffusion path takes about 18 years to move from a 10% adoption share to 90%. This result is close to the corresponding average period of 15 years estimated by Jovanovich and Lach (1997) for a wide variety of new product innovations. In our sensitivity analysis, we also consider the value  $\kappa = 0.35$ , which yields a 13-year period to go from from a 10% adoption share to 90%.

### 3 Model

The model economy consists of workers, capital-owner/entrepreneurs, competitive firms, and the government. Throughout the paper, we use the terms “capital-owners” and “entrepreneurs” interchangeably to denote the top decile of earners in the model. There are  $n = 9$  times more workers than capital owners, with the total number of capital owners normalized to one. Naturally, firms are owned by the capital owners. Both types of agents supply labor endogenously to firms. The government’s only role is to redistribute income from capital owners to workers via a lump-sum tax and transfer scheme.

### 3.1 Workers

The individual worker's decision problem is to maximize

$$\widehat{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[ c_t^w - \frac{D^w}{\gamma} H_t (\ell_t^w)^\gamma \right]^{1-\alpha} - 1}{1-\alpha}, \quad (2)$$

subject to the budget constraint

$$c_t^w = w_t^w \ell_t^w + T_t/n, \quad (3)$$

where the symbol  $\widehat{E}_t$  represents the agent's subjective expectation conditional on information available time  $t$ . Under rational expectations,  $\widehat{E}_t$  corresponds to the mathematical expectation operator  $E_t$  evaluated using the laws of motion that govern the technology diffusion process. The parameter  $\beta$  is the subjective time discount factor,  $c_t^w$  is the individual worker's consumption, and  $\ell_t^w$  is labor supply. Along the lines of Greenwood, et al. (1988), the disutility of non-leisure time is governed by the functional form  $(D^w/\gamma^w) H_t (\ell_t^w)^\gamma$ , where  $D^w > 0$ , and  $\gamma > 1$ . This specification implies that foregone leisure is adjusted to reflect trend growth according to  $H_t = \exp(z_t)$ , where  $z_t$  represents labor-augmenting technological progress, to be described more fully below. The labor disutility function may be interpreted as the reduced form of a more-elaborate specification that incorporates home production.<sup>11</sup> The elasticity of intertemporal substitution in labor supply is given by  $1/(\gamma - 1)$ . As  $\gamma \rightarrow \infty$ , the model reduces to one with fixed labor supply. The parameter  $\alpha$  represents the inverse of the elasticity of intertemporal substitution (EIS) for the worker's composite consumption basket.

Workers are assumed to incur a transaction cost for saving or borrowing small amounts which prohibits their participation in financial markets. As a result, they simply consume their income each period, consisting of labor income  $w_t^w \ell_t^w$  and a per-worker transfer payment  $T_t/n$  received from the government.

The worker's first-order conditions with respect to  $c_t^w$  and  $\ell_t^w$  are given by

$$\left[ c_t^w - \frac{D^w}{\gamma} H_t (\ell_t^w)^\gamma \right]^{-\alpha} = \lambda_t^w, \quad (4)$$

$$D^w H_t (\ell_t^w)^{\gamma-1} \left[ c_t^w - \frac{D^w}{\gamma} H_t (\ell_t^w)^\gamma \right]^{-\alpha} = \lambda_t^w w_t^w, \quad (5)$$

where  $\lambda_t^w$  is the Lagrange multiplier associated with the budget constraint (3). Since the worker makes no intertemporal decision, the subjective expectation operator  $\widehat{E}_t$  does not appear in

<sup>11</sup>The linearity in  $H_t$  ensures that agents' time allocations are stationary along the model's balanced growth path. See Greenwood, Rogerson, and Wright (1995, p. 161).

the first-order conditions. The first-order conditions imply the following labor supply equation

$$\ell_t^w = \left( \frac{w_t^w}{D^w H_t} \right)^{\frac{1}{\gamma-1}}. \quad (6)$$

### 3.2 Capital Owner/Entrepreneurs

Capital owner/entrepreneurs represent the top decile of earners. Their decision problem is to maximize

$$\widehat{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[ c_t^c - \frac{D^c}{\gamma} H_t (\ell_t^c)^\gamma \right]^{1-\alpha} - 1}{1-\alpha}, \quad (7)$$

subject to the budget constraint

$$c_t^c + p_t s_{t+1} = w_t^c \ell_t^c + (p_t + d_t) s_t - T_t, \quad (8)$$

where  $c_t^c$  is the individual capital owner's consumption and  $\ell_t^c$  is labor supply. For simplicity, we assume that the functional form of the utility function and the preference parameters  $\beta$ ,  $\gamma$ , and  $\alpha$  are the same for both capital owners and workers. Capital owners earn labor income in the amount  $w_t^c \ell_t^c$  and may invest in shares of the firm's equity in the amount  $s_{t+1}$  at the ex-dividend price  $p_t$ . Shares owned in the previous period yield a dividend  $d_t$ .<sup>12</sup>

Equity shares are assumed to exist in unit net supply. Market clearing therefore implies  $s_t = 1$  for all  $t$ . In equilibrium, the capital owner's budget constraint becomes  $c_t^c = w_t^c \ell_t^c + d_t - T_t$ , which shows that the capital owner's consumption is funded from wage income and dividends, after subtracting a lump-sum tax levied by the government.

The capital owner's first-order conditions with respect to  $c_t^c$ ,  $\ell_t^c$ , and  $s_{t+1}$  are given by

$$\left[ c_t^c - \frac{D^c}{\gamma} H_t (\ell_t^c)^\gamma \right]^{-\alpha} = \lambda_t^c, \quad (9)$$

$$D^c H_t (\ell_t^c)^{\gamma-1} \left[ c_t^c - \frac{D^c}{\gamma} H_t (\ell_t^c)^\gamma \right]^{-\alpha} = \lambda_t^c w_t^c, \quad (10)$$

$$p_t = \widehat{E}_t \beta \frac{\lambda_{t+1}^c}{\lambda_t^c} (p_{t+1} + d_{t+1}), \quad (11)$$

where  $\lambda_t^c$  is the Lagrange multiplier associated with the budget constraint (8). The capital owner's labor supply equation is given by

$$\ell_t^c = \left( \frac{w_t^c}{D^c H_t} \right)^{\frac{1}{\gamma-1}}. \quad (12)$$

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<sup>12</sup>The capital owner's decision problem can be represented in different ways. We employ this particular decentralization because it identifies the top decile in the model as stockholders and shows the link between the firm's equity price and investment.

As  $D^c \rightarrow \infty$  we have  $\ell_t^c \rightarrow 0$  such that only the workers supply labor. This case corresponds to a standard framework that is often used to study optimal redistributive capital taxation (Judd 1985, Lansing 1999, and Krusell 2002).

### 3.3 Firms

Competitive firms are owned by the capital owner/entrepreneurs and produce output according to the technology

$$y_t = A \left\{ \theta_t \left[ (1 - \rho_t) k_t^{\psi_k} + \rho_t [\exp(z_t) \ell_t^c]^{\psi_k} \right]^{\frac{\psi_\ell}{\psi_k}} + (1 - \theta_t) [\exp(z_t) n \ell_t^w]^{\psi_\ell} \right\}^{\frac{1}{\psi_\ell}} \quad (13)$$

$$\text{where } \psi_k \equiv \frac{\sigma_k - 1}{\sigma_k}, \quad \psi_\ell \equiv \frac{\sigma_\ell - 1}{\sigma_\ell},$$

$$z_t = z_{t-1} + \mu, \quad (14)$$

$$\theta_t = \theta_0 \exp[\delta_\theta (\pi_t - \pi_0)], \quad (15)$$

$$\rho_t = \rho_0 \exp[\delta_\rho (\pi_t - \pi_0)], \quad (16)$$

$$\pi_t = \pi_{t-1} + \kappa \pi_{t-1} (1 - \pi_{t-1}), \quad (17)$$

with  $z_0$ ,  $\theta_0$ ,  $\rho_0$ , and  $\pi_0$  given. The symbol  $k_t$  is the firm's stock of physical capital and  $z_t$  is a labor-augmenting technology process that evolves as a random walk with drift. The drift parameter  $\mu$  determines the trend growth rate of output. We abstract from stochastic variation in trend growth because we wish to focus on the dynamics that arise from shifts in the income shares, as opposed to ordinary business cycle fluctuations. The parameter  $\psi_k$  depends on the elasticity of substitution between physical capital and entrepreneurial labor, denoted by  $\sigma_k$ . The parameter  $\psi_\ell$  depends on the elasticity of substitution between entrepreneurial inputs and workers' labor, denoted by  $\sigma_\ell$ . When  $\sigma_\ell > \sigma_k$ , the production function exhibits what we call "capital-entrepreneurial skill complementarity." This means that the capital owners' entrepreneurial labor is more complementary to physical capital than ordinary workers' labor. In other words, the capital owners' entrepreneurial skills are more closely coupled to the physical assets of the firm than are workers' skills.

Motivated by the technology diffusion process shown in Figure 4, our production specification (13) is intended to capture the emergence of unique business skills tied to the spread of ICT. Examples might be the skills associated with setting up and operating a technology company such as Microsoft, Apple, Amazon, Ebay, Oracle, Google, etc. These type of skills yielded significant monetary rewards (mainly in the form of valuable stock options) to the

founders and early employees who conceived and executed the firms’ original business strategies. Another example would be the skills needed to set up and operate a successful web-based business—a platform that did not exist prior to the mid-1990s. In the finance industry, ICT advances have allowed fund managers to quickly process market-moving information that has led to more efficient trade execution and larger pools of assets under management, thus raising the associated fees paid to the fund managers (Kaplan and Rauh 2009).<sup>13</sup>

The entrepreneurial-type skills we have in mind are much more concentrated than the broader college education-based skills emphasized by Krusell, et al. (2000), Goldin and Katz (2007, 2008), Heathcote, et al. (2010, 2011) and others. In our view, a more-concentrated skill premium better reflects the wide variety of empirical evidence which shows that the observed trends in U.S. income inequality over the past three decades were driven mainly by gains in top incomes, as documented in various ways by Autor, et al. (2006), Lemieux (2006), Atkinson, et al. (2011), and OECD (2011).

When  $\sigma_k = \sigma_\ell = 1$  (or  $\psi_k = \psi_\ell = 0$ ), we recover the standard Cobb-Douglas production technology which does not exhibit capital-entrepreneurial skill complementarity. When  $\sigma_k \rightarrow 0$  and  $\sigma_\ell \rightarrow 0$  (or  $\psi_k \rightarrow -\infty$  and  $\psi_\ell \rightarrow -\infty$ ), the production technology takes a Leontief form such that capital and both types of labor become perfect compliments. When  $\sigma_k \rightarrow \infty$  and  $\sigma_\ell \rightarrow \infty$  (or  $\psi_k \rightarrow 1$  and  $\psi_\ell \rightarrow 1$ ), capital and both types of labor become perfect substitutes.

The OECD (2011) argues that technological progress and globalization have shifted firms’ production technologies in favor of highly-skilled workers, yielding these workers higher rewards from labor at the expense of others who lack unique skills. We capture this idea by assuming that the representative firm’s production technology (13) shifts over time, as governed by equations (15) through (17). Specifically, the diffusion process shifts the parameters  $\theta_t$  and  $\rho_t$  along an S-shaped trajectory as the new technology is gradually adopted by firms. The state variable  $\pi_t$  can be interpreted as the share of firms employing the new technology. Our setup can also be viewed as capturing a process whereby old firms using obsolete technology die off over time and are replaced by new firms using the latest technology. Along these lines, Hobijn and Jovanovic (2001, p. 1219) argue that “major technological change—like the IT [information technology] revolution—destroys old firms. It does so by making machines, workers, and managers obsolete.”

Goldin and Katz (2007) develop an analytical framework that allows CES production function share parameters to shift over time as a way of capturing skill-biased technological change. Our setup can be interpreted in the same way. To see this, we can rewrite the

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<sup>13</sup>Philippon and Reshef (2013, p. 1591) find that the increase in financial-sector wages between the mid-1990s and 2006 is much larger than the benchmark change predicted on the basis of relative education levels alone.

production function (13) as follows

$$y_t = A \exp(z_t) \left\{ \left[ \left( \theta_t^{\frac{1}{\psi_\ell}} (1 - \rho_t)^{\frac{1}{\psi_k}} k_{n,t} \right)^{\psi_k} + \left( \theta_t^{\frac{1}{\psi_\ell}} \rho_t^{\frac{1}{\psi_k}} \ell_t^c \right)^{\psi_k} \right]^{\frac{\psi_\ell}{\psi_k}} + \left[ (1 - \theta_t)^{\frac{1}{\psi_\ell}} n \ell_t^w \right]^{\psi_\ell} \right\}^{\frac{1}{\psi_\ell}}, \quad (18)$$

where we define  $k_{n,t} \equiv k_t / \exp(z_t)$  as the normalized capital stock (a stationary variable). In the above formulation, shifts in  $z_t$  represent a form of “neutral” technology change that affects output generally, whereas shifts in  $\theta_t$  or  $\rho_t$  represent “biased” technology changes that affect the relative demand for the different productive inputs. Equation (18) shows that the quantitative impact of a given shift in either  $\theta_t$  or  $\rho_t$  on input demand will depend on the values the substitution elasticity parameters  $\sigma_k$  and  $\sigma_\ell$  which govern the values of  $\psi_k$  and  $\psi_\ell$ .

Equation (17) has two steady states at  $\pi_t = 0$  and  $\pi_t = 1$ . At the initial steady state, we have  $\theta_t = \theta_0$  and  $\rho_t = \rho_0$ . At date  $t = 1$ , corresponding to the year 1981, we assume that 1% of firms unilaterally adopt the new technology (or, alternatively, that 1% of existing firms die and are replaced by new firms using the new technology). Given this initial impulse, the diffusion law of motion implies  $\pi_t \rightarrow 1$  as  $t \rightarrow \infty$ . The response parameters  $\delta_\theta$  and  $\delta_\rho$  govern the degree to which the technology diffusion shifts the production function parameters  $\theta_t$  and  $\rho_t$ , which in turn govern the shares of labor and capital income going to the top decile of households. When  $\delta_\theta = \delta_\rho = 0$ , the model economy grows along the “no-change trend,” such that the top decile income share does not increase over time, but instead remains constant at the level observed in 1980.

Resources devoted to investment augment the stock of physical capital according to the law of motion

$$k_{t+1} = B k_t^{1-\lambda} i_t^\lambda, \quad (19)$$

with  $k_0$  given. The parameter  $\lambda \in (0, 1]$  is the elasticity of new capital with respect to new investment. When  $\lambda < 1$ , equation (19) reflects the presence of capital adjustment costs.<sup>14</sup>

Under the assumption that the labor market is competitive, firms take wages as given and choose sequences of  $n \ell_{t+j}^w$ ,  $\ell_{t+j}^c$ , and  $k_{t+1+j}$  to maximize the following discounted stream of expected dividends:

$$\widehat{E}_0 \sum_{j=0}^{\infty} M_{t+j}^c \underbrace{\left[ y_{t+j} - w_{t+j}^w n \ell_{t+j}^w - w_{t+j}^c \ell_{t+j}^c - i_{t+j} \right]}_{d_{t+j}}, \quad (20)$$

<sup>14</sup>Equation (19) can be interpreted as a power-function approximation of the following specification employed by Jermann (1998):  $k_{t+1} = k_t [1 - \delta + \psi_0 (i_t / k_t)^{\psi_1}]$ .

subject to the production function (13) and the law of motion for capital (19). Firms act in the best interests of their owners such that dividends in period  $t + j$  are discounted using the capital owner's stochastic discount factor  $M_{t+j}^c \equiv \beta^j \lambda_{t+j}^c / \lambda_t^c$ , where  $\lambda_t^c$  is given by equation (9).

The firm's first-order conditions with respect to  $n \ell_t^w$ ,  $\ell_t^c$ , and  $k_{t+1}$  are given by:

$$w_t^w = (1 - s_t^c) y_t / (n \ell_t^w), \quad (21)$$

$$w_t^c = (s_t^c - s_t^k) y_t / \ell_t^c \quad (22)$$

$$i_t / \lambda = E_t M_{t+1}^c [s_{t+1}^k y_{t+1} - i_{t+1} + i_{t+1} / \lambda], \quad (23)$$

where  $s_t^c$  represents the share of pre-tax income going to capital owners and  $s_t^k$  represents capital's share of total income. The share of pre-tax income going to workers is  $1 - s_t^c$ , while labor's share of total income is  $1 - s_t^k$ . The share of pre-tax income going to entrepreneurial labor is  $s_t^c - s_t^k$ .

Equations (21) and (22) show that each type of labor is paid its marginal product. Comparing the firm's intertemporal first-order condition (23) to the equity pricing equation (11) shows that the ex-dividend price of an equity share is given by  $p_t = i_t / \lambda$ . The equity share is a claim to a perpetual stream of dividends  $d_{t+1} = s_{t+1}^k y_{t+1} - i_{t+1}$  starting in period  $t + 1$ .<sup>15</sup> The model's adjustment cost specification (19) implies a direct link between the equity price and investment, consistent with a standard Tobin's  $q$  framework. This feature is also consistent with the observed low-frequency comovement between the S&P 500 stock price index and business investment in recent decades, as documented by Lansing (2012).

Given the form of the production function (13), we have

$$s_t^c = \frac{\partial y_t}{\partial \ell_t^c} \frac{\ell_t^c}{y_t} + \frac{\partial y_t}{\partial k_t} \frac{k_t}{y_t} = \frac{\theta_t \left[ (1 - \rho_t) k_{n,t}^{\psi_k} + \rho_t (\ell_t^c)^{\psi_k} \right]^{\frac{\psi_\ell}{\psi_k}}}{\theta_t \left[ (1 - \rho_t) k_{n,t}^{\psi_k} + \rho_t (\ell_t^c)^{\psi_k} \right]^{\frac{\psi_\ell}{\psi_k}} + (1 - \theta_t) (n \ell_t^w)^{\psi_\ell}}, \quad (24)$$

$$s_t^k = \frac{\partial y_t}{\partial k_t} \frac{k_t}{y_t} = \frac{\theta_t (1 - \rho_t) \left[ (1 - \rho_t) k_{n,t}^{\psi_k} + \rho_t (\ell_t^c)^{\psi_k} \right]^{\frac{\psi_\ell - \psi_k}{\psi_k}} k_{n,t}^{\psi_k}}{\theta_t \left[ (1 - \rho_t) k_{n,t}^{\psi_k} + \rho_t (\ell_t^c)^{\psi_k} \right]^{\frac{\psi_\ell}{\psi_k}} + (1 - \theta_t) (n \ell_t^w)^{\psi_\ell}}, \quad (25)$$

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<sup>15</sup>After taking the derivative of the profit function (20) with respect to  $k_{t+1}$ , we have multiplied both sides of the resulting first-order condition by  $k_{t+1}$  (which is known at time  $t$ ) to obtain equation (23)

where  $k_{n,t} \equiv k_t / \exp(z_t)$ . In the Cobb-Douglas case when  $\psi_k = \psi_\ell = 0$ , the above equations simplify to  $s_t^c = \theta_t$  and  $s_t^k = \theta_t (1 - \rho_t)$ .

### 3.4 Government

The government redistributes income from capital owners to workers by means of a lump-sum tax and transfer scheme. In the U.S. economy, the top decile of taxpayers accounted for about 70% of federal individual income taxes paid in 2009, according to the Internal Revenue Service.<sup>16</sup> We abstract from distortionary taxation given that most of the revenue collected by distortionary taxes in the U.S. economy is used for either direct government purchases of goods and services or debt service—two features which are absent from our model. Moreover, in the case of the OASDI program, transfers are financed by a tax on income up to a given threshold, so there is no marginal tax distortion for income earned above the threshold.

We assume that the ratio of aggregate transfer payments to output in the model is governed by the following law of motion:

$$\tau_t \equiv T_t/y_t = \tau_0 \exp[\delta_\tau (\pi_t - \pi_0)], \quad (26)$$

where  $\tau_t$  represents the lump-sum tax rate and  $\tau_0$  is given. The response parameter  $\delta_\tau$  governs the path of transfers during the transition period. Along the economy's no-change trend, we have  $\delta_\tau = 0$  such that the ratio of transfers to GDP remains constant. We link  $\tau_t$  to the technology adoption share  $\pi_t$  as a way of capturing the rising trend of U.S. transfer payments relative to GDP plotted earlier in Figure 3. The underlying assumption is that the rapid growth in various types of means-tested transfers and income security programs from 1980 to 2010 reflects a deliberate effort by the government to try to offset the trend of rising pre-tax income inequality. In our quantitative analysis, we show that the model's simulated path for the ratio of transfers to GDP tracks well with the data trend plotted in Figure 3.

### 3.5 Expectations

Following Heathcote, et al. (2010), we consider different assumptions about the way in which agents form expectations about future variables that will affect their permanent income. Here, only firms and capital owners make forecasts about future variables; workers simply consume their wage income plus transfers each period. In the appendix, we show that the firm's intertemporal first-order condition (23) can be written in terms of stationary variables as follows:

$$f(x_t, \ell_t^c, n \ell_t^w, k_{n,t}, \pi_t) = \widehat{E}_t h(x_{t+1}, \ell_{t+1}^c, n \ell_{t+1}^w, k_{n,t+1}, \pi_{t+1}), \quad (27)$$

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<sup>16</sup>Tax share data are from [www.irs.gov/pub/irs-soi/09inratesnap.pdf](http://www.irs.gov/pub/irs-soi/09inratesnap.pdf).

where  $x_t \equiv i_t/y_t$  is the investment-output ratio and  $k_{n,t} \equiv k_t/\exp(z_t)$  is the normalized capital stock.

To establish a benchmark, we first consider the standard case of rational expectations where agents are assumed to know the laws of motion governing the evolution of future variables. In our setting, rational expectations corresponds to perfect foresight because the laws of motion that govern trend growth and the diffusion of new technology abstract from stochastic variation. Under perfect foresight, we drop the subjective expectation operator  $\widehat{E}_t$  in equation (27), yielding a set of deterministic nonlinear difference equations that can be solved numerically, as described in the appendix.

The notion that agents have perfect foresight about the process governing their future income is obviously an extreme assumption. This is especially true in our setting, where the economy is undergoing a never-before-seen shift in technology that significantly alters firms' production processes. Towards the other end of the information spectrum, we might assume that agents are myopic, i.e., their forecast about a future variable is given by the most recently-observed value of the same variable. This type of forecast rule is optimal when the variable in question evolves as a random walk. But even if this is not the case, a random walk forecast can be viewed as boundedly-rational because it economizes on the costs of collecting and processing information. As noted by Nerlove (1983, p. 1255): "Purposeful economic agents have incentives to eliminate errors up to a point justified by the costs of obtaining the information necessary to do so...The most readily available and least costly information about the future value of a variable is its past value." To implement myopic expectations in equation (27), we assume  $\widehat{E}_t h(t+1) = h(t-1)$ , which implies that agents do not observe the realized value  $h(t)$  at the time they construct their forecast.<sup>17</sup>

According to Heathcote et al. (2010, p. 717) "Myopic beliefs and perfect foresight represent polar extreme models for expectations, and presumably the truth lies somewhere in between the two." Along these lines, we consider an intermediate case labeled "learning" in which the share of firms and capital owners with knowledge about the future transition path increases gradually over time as the new technology is adopted. Put differently, we assume that agents who adopt the new technology acquire knowledge about its speed of diffusion and its implications for their future income. To implement learning in equation (27), we assume  $\widehat{E}_t h(t+1) = \omega_t h(t+1) + (1 - \omega_t) h(t-1)$ , where  $\omega_t$  represents the fraction of entrepreneurial agents with knowledge about the laws of motion governing the transition path. Intuitively,

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<sup>17</sup> Alternatively, we could assume  $\widehat{E}_t h(t+1) = h(t)$  which would allow for simultaneity in the observed and expected values of the forecast variables. For our setting, the solution turns out to be nearly identical to the case where  $\widehat{E}_t h(t+1) = h(t-1)$ . This result may not hold for others settings, however. See, for example, Van Zandt and Lettau (2003).

one might expect the fraction of knowledgeable agents to start at zero and then increase gradually over time, eventually reaching unity when the new technology has been fully adopted. We can achieve such a trajectory very simply by linking the fraction of knowledgeable agents to the diffusion process itself, i.e., by imposing  $\omega_t = \pi_t$ .

It should be noted that the learning regime might be interpreted as imposing an even higher level of sophistication on the part of knowledgeable capital owner/entrepreneurs. Not only do these knowledgeable agents need to understand the dynamics of the exogenous technology diffusion process, but now they must also take into account the influence of the remaining myopic capital owners on the future transition path of the economy. For this reason, and given the assumed one-time shift in the economy’s production technology, one might argue that the myopic expectations regime is the more plausible setup.

## 4 Model Calibration

Table 1 summarizes our choice of parameter values for the baseline model. Some parameters are set to achieve target values for steady-state variables while others are set to commonly-used values in the literature.

Table 1: Baseline Model Parameter Values

Parameter	Value	Description/Target
$n$	9	Capital owners = top income decile.
$\mu$	0.02	Per capita trend growth = 2%.
$\alpha$	2	EIS = $1/\alpha = 0.5$ .
$\beta$	0.964	Real equity return = 8%.
$\gamma$	3	Labor supply elasticity = 0.5.
$D^w$	0.65	Initial worker labor supply $\ell^w = 1$ .
$D^c$	5.54	Initial relative wage $w^c/w^w = 2$ .
$\sigma_k$	0.4	Empirical estimates.
$\sigma_\ell$	1.0	Empirical estimates.
$A$	0.816	Match Cobb-Douglas initial steady state.
$B$	1.273	Initial steady-state $k/y = 2.6 \times 0.8$ .
$\lambda$	0.088	Initial steady-state $i/y = 0.21 \times 0.8$ .
$\kappa$	0.25	Match ICT diffusion path for U.S. economy.
$\pi_0$	0	Initial steady state $\pi = 0$ .
$\theta_0$	0.350	Initial steady-state $s^c = 0.35$ .
$\rho_0$	0.001	Initial steady-state $s^k = 0.35 \times 0.8 = 0.28$ .
$\tau_0$	0.100	Initial steady-state transfers/GDP = 10%.
$\delta_\theta$	0.336	Final steady-state $s^c = 0.49$ .
$\delta_\rho$	0.685	Final steady-state $s^k = 0.41 \times 0.8 = 0.328$ .
$\delta_\tau$	0.405	Final steady-state transfers/GDP = 15%.

The time period in the model is one year. The number of workers per capital owner is  $n = 9$  so that capital owners represent the top decile of households. In the model, capital owners possess 100% of the physical capital wealth, whereas the top decile of U.S. households owns approximately 80% of financial wealth. Our setup implies a Gini coefficient for physical capital wealth of 0.90. The Gini coefficient for financial wealth in U.S. data has ranged between 0.89 and 0.93 over the period 1983 to 2001.<sup>18</sup>

The parameter  $\mu = 0.02$  implies a per capita trend growth rate of 2%, consistent with the long-run U.S. average. The value  $\alpha = 2$  implies an EIS of  $1/\alpha = 0.5$  for the composite consumption basket of each agent—a typical value.<sup>19</sup> In the sensitivity analysis, we also consider the values  $1/\alpha = 1$  and  $1/\alpha = 0.33$ . Given the values for  $\alpha$  and  $\mu$ , we choose  $\beta$  such that the steady-state net equity return is  $r^s = \beta^{-1} \exp(\alpha \mu) - 1 = 8\%$ , consistent with the long-run real return on the S&P 500 stock price index.

We choose  $\gamma = 3$  to achieve an intertemporal elasticity of substitution in labor supply of  $(\gamma - 1)^{-1} = 0.5$ , consistent with the range of estimates obtained by Eissa (1996) and Mulligan (1999), among others. In the sensitivity analysis, we also examine the effects of a more-elastic labor supply with  $(\gamma - 1)^{-1} = 1.5$  and a less-elastic labor supply with  $(\gamma - 1)^{-1} = 0.1$ . We choose the labor supply disutility parameter  $D^w$  to normalize  $\ell^w = 1$  at the initial steady state. Given this value, we choose  $D^c$  to achieve a target relative wage at the initial steady state of  $w^c/w^w = 2$ . For comparison, Heathcote, et al. (2010, p. 686) report a male college wage premium of about 1.4 in 1980, whereas Gottschalk and Danziger (2005, p. 238) report a male wage ratio of about 4 when comparing the top decile to the bottom decile. The wage ratio  $w^c/w^w$  in our model compares the top decile to the remainder of households, so we would expect it to fall somewhere in between the values reported by the two studies, but likely closer to the value reported by Heathcote, et al. (2010). The welfare results are not sensitive to the initial wage ratio.

The baseline values for the production function curvature parameters  $\sigma_k$  and  $\sigma_\ell$  strike a balance between various empirical estimates. Using data on the observed wage premium of college-educated workers in the U.S. economy from 1963 to 1992, Krusell, et al. (2000, p. 1041) estimate a substitution elasticity of 0.67 between equipment capital and skilled labor. They estimate a substitution elasticity of 1.67 between equipment capital and unskilled labor. There is also a large literature that estimates the elasticity of substitution between aggregate physical capital and aggregate labor, without distinguishing between skilled versus unskilled labor. In a review of this literature, Chirinko (2008) concludes that the evidence suggests a range of 0.4 to 0.6 for the aggregate capital-labor substitution elasticity. The capital-entrepreneurial skill

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<sup>18</sup>See Wolff (2006), Table 4.2, p. 113.

<sup>19</sup>See, for example, Mendoza (2010).

complementarity effect considered here applies to the top decile which is a more exclusive group than the pool of college-educated workers. Workers comprise nine-tenths of the population in our model, and thus represent a broader group than the pool of unskilled (non-college) workers. Based on this reasoning, we set  $\sigma_k = 0.4$  and  $\sigma_\ell = 1$ , which imply that both types of labor in our model exhibit stronger complementarity to physical capital than the college versus non-college workers considered by Krusell, et al. (2000). In the sensitivity analysis, we consider different combinations of values for  $\sigma_k$  and  $\sigma_\ell$ , including the Cobb-Douglas case when  $\sigma_k = \sigma_\ell = 1$ .

We normalize the production function parameter  $A$  to unity in the Cobb-Douglas case. When  $\sigma_k \neq 1$  or  $\sigma_\ell \neq 1$ , we choose the value of  $A$  to maintain the same initial steady-state value of  $k_n$  as in the Cobb-Douglas model. In this way, changes in either  $\sigma_k$  or  $\sigma_\ell$  identify a family of CES production functions that are distinguished only by the elasticity parameters, and not by their initial steady-state allocations.<sup>20</sup> The parameter  $B$  in the capital law of motion (19) is chosen to be consistent with the long-run average capital-output ratio in the U.S. economy. The average ratio from annual data is about 2.6, but this figure includes all physical capital whereas the top decile of U.S. households owns about 80% of financial wealth, as represented by claims to physical capital, i.e., equity shares. We therefore apply a scale factor of 0.8 to the U.S. capital-output ratio to arrive at a target capital-output ratio of 2.08 for the model. The parameter  $\lambda$  in the capital law of motion (19) is chosen to be consistent with the U.S. average investment-output ratio of about 0.21 (including business investment and purchases of consumer durables). We again apply a scale factor of 0.8 to the U.S. ratio to arrive at a target investment-output ratio of 0.168 for the model.

The initial share parameter  $\theta_0 = 0.35$  is chosen to match the 35% income share of the top decile of U.S. households in 1980, as plotted earlier in Figure 1. Similarly, we choose  $\rho_0$  to match capital's share of total income in the U.S. economy in 1980, also plotted in Figure 1. Similar to the other capital-related parameters, we apply a scale factor of 0.8 to the 1980 capital income share of 0.35, resulting in an initial steady-state capital share in the model of 0.28. The technology diffusion speed is set to  $\kappa = 0.25$ , as noted earlier in the discussion of Figure 4. Given  $\theta_0$ ,  $\rho_0$  and  $\kappa$ , we choose  $\delta_\theta$  and  $\delta_\rho$  to achieve target values for the top decile income share  $s^c$  and the capital share  $s^k$  at the final steady state. The target values at the final steady state are slightly above the (scaled) end-of-sample values plotted in Figure 1. The model diffusion speed implies that technology adoption is about 92% complete after three decades. Finally, we choose  $\tau_0 = 0.10$  to match the 10% ratio of U.S. transfers to GDP in 1980, as shown in Figure 3. Based on the trend plotted in Figure 3, we choose  $\delta_\tau$  to achieve

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<sup>20</sup>Klump and Saam (2008) emphasize that such a normalization procedure is necessary to avoid “arbitrary and inconsistent results” when comparing CES production models with different parameterizations.

a target ratio of 15% at the final steady state.

## 5 Quantitative Results

In this section, we examine the quantitative implications of the model via numerical simulations. We first consider the baseline model’s dynamic response to shifting income shares under different expectation regimes. Next, we examine the implications of departing from the baseline assumptions regarding the path for redistributive transfers, the path for capital’s share of total income, and the degree of capital-entrepreneurial skill complementarity. Finally, we consider the welfare consequences of rising income inequality and its sensitivity to different model specifications and parameter values. Details regarding the model solution procedure and the welfare computation are contained in the appendix.

### 5.1 Dynamic Response to Shifting Income Shares: Baseline Model

Figure 5 compares the model’s simulated paths for the top decile income share  $s_t^c$  and the ratio of redistributive transfers to GDP  $T_t/y_t$  to the corresponding trajectories in U.S. data. Recall that we calibrated the baseline technology diffusion speed  $\kappa = 0.25$  using independent data on the U.S. adoption share of ICT (see Figure 4). The model’s simulated paths for  $s_t^c$  and  $T_t/y_t$  both exhibit S-shaped trajectories which are similar to those in the data. The baseline path for  $s_t^c$  tracks a bit below the U.S. top decile income share for the first twenty years of the sample starting in 1980, but the baseline path for  $T_t/y_t$  provides an excellent fit of the trend in the U.S. transfer-to-GDP ratio since 1980. Figure 5 also plots the model’s simulated paths for an alternative model calibration with  $\kappa = 0.35$ , which implies a faster diffusion speed. In this case, the model simulated path for  $s_t^c$  tracks closer to the U.S. data. In our sensitivity analysis, we show that the welfare effects of the alternative calibration with  $\kappa = 0.35$  are quantitatively similar to those with  $\kappa = 0.25$ . It should be noted, of course, that the U.S. data may be influenced by a number of transitory factors, including business cycle shocks, which fall outside the scope of our analysis.

Figure 6 plots the baseline transition paths for selected model variables starting from the initial steady state with  $\pi_0 = 0$ .<sup>21</sup> At date  $t = 1$ , we assume that 1% of firms unilaterally adopt the new technology. For  $t > 1$ , the technology diffusion process is governed by equations (15) through (17). For each variable, we plot the equilibrium trajectory for three different expectation regimes: perfect foresight (solid blue line), myopic expectations (dashed red line), and learning (dash-dotted green line).

<sup>21</sup>Note that the horizontal scale in Figure 6 is now much longer than in Figure 5.

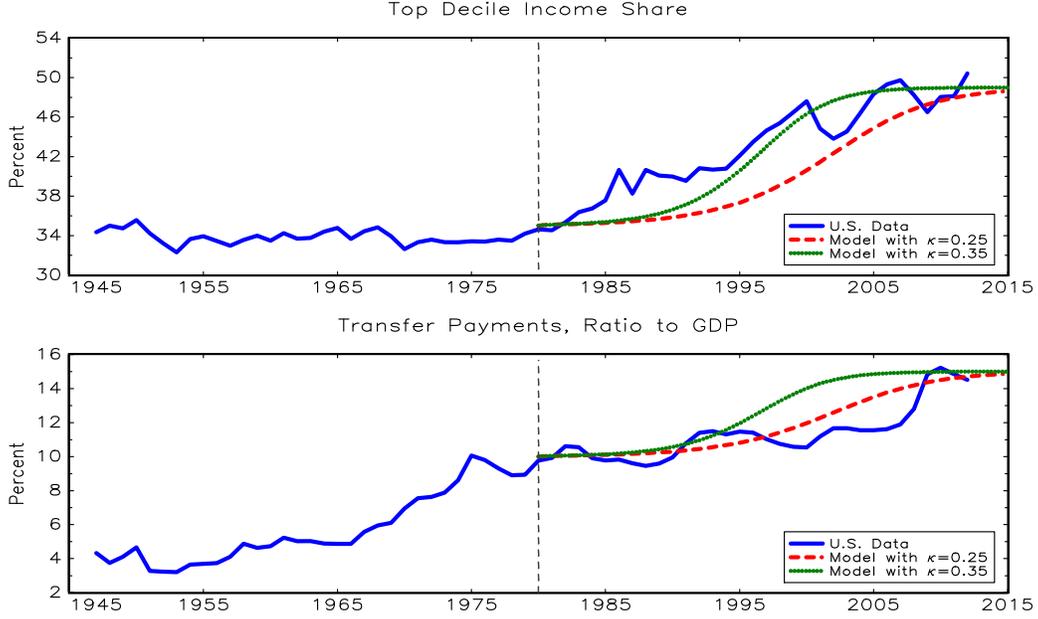


Figure 5: The model’s simulated paths for the top decile income share  $s_t^c$  and the ratio of redistributive transfers to GDP  $T_t/y_t$  both exhibit S-shaped trajectories which are similar to those in the data.

The top left panel of Figure 6 plots the transition path for the top decile income share  $s_t^c$ . The model path starts at 35% and then increases to about 48% at  $t = 30$ , corresponding to the year 2010. Our baseline calibration with  $\sigma_\ell = 1$  implies  $\psi_\ell = 0$  such that  $s_t^c = \theta_t$  from equation (24). Since  $\theta_t$  follows an exogenous law of motion, expectations do not influence the trajectory of  $s_t^c$  in the baseline calibration, unlike the other variables in the figure. Capital’s share of total income  $s_t^k$  (top right panel) starts from an initial steady state of 28% and eventually reaches a final steady state of 32.8%. In between, the trajectory is governed by equation (25) which depends on the endogenous variables  $k_{n,t}$  and  $\ell_t^c$  even when  $\psi_\ell = 0$ . Under all three expectation regimes, the transition path for  $s_t^k$  exhibits some overshooting such that value at  $t = 30$  is somewhat above the final steady state value.

The role of expectations is most clearly illustrated in the middle left panel of Figure 6, which plots the equilibrium investment-output ratio  $i_t/y_t$ . Under perfect foresight, the investment-output ratio drops sharply at  $t = 1$ . This is because capital owners foresee the large increase in their permanent income over the future transition period. As a result, they immediately increase their consumption at the expense of investment. While such dynamics do not seem very plausible, it must be remembered that our model abstracts from stochastic shocks which would introduce a precautionary saving motive, thus limiting the sharp drop in

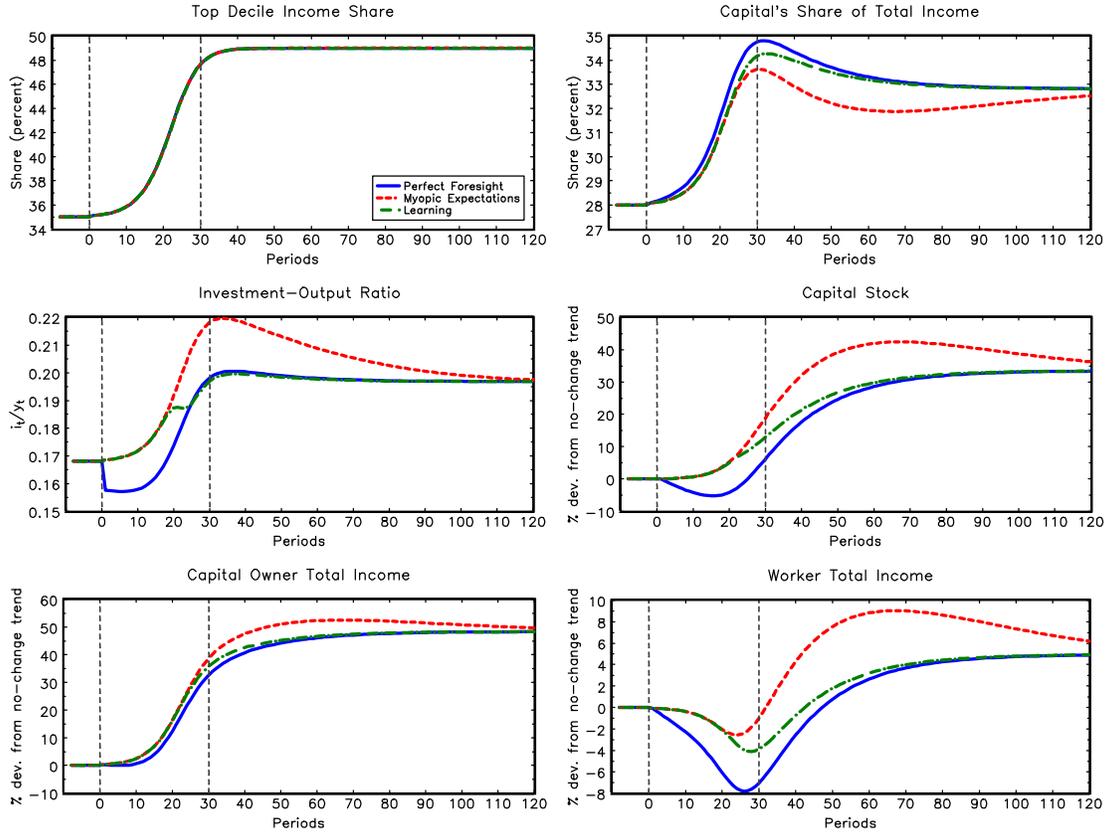


Figure 6: Under perfect foresight, the investment-output ratio drops sharply at  $t = 1$  because capital owners foresee the increase in their permanent income. The drop in investment slows capital accumulation, thereby hindering the growth of wages and total income relative to the model with either myopic expectations or learning.

the investment-output ratio.<sup>22</sup>

Under myopic expectations, capital owners do not foresee the increase in their permanent income. Consequently, their consumption at  $t = 1$  does not jump (investment at  $t = 1$  does not fall), but rather the capital owner's consumption and investment both increase gradually along with current income. Under learning, the trajectories for all variables initially mimic those under myopic expectations, but the paths eventually catch-up and merge with the perfect foresight trajectories.

The middle right panel of Figure 6 plots the evolution of the capital stock expressed as a percent deviation from the no-change trend (which holds income shares constant at their initial levels). The capital stock increases fastest under myopic expectations due to the higher

<sup>22</sup>Our closed economy model also abstracts from foreign capital inflows. Such inflows could finance an increase in domestic investment even if there were a sharp drop in domestic saving.

investment trajectory, which boosts capital accumulation. In contrast, the perfect foresight path for the capital stock initially drops below the no-change trend due to the sharp drop in the investment-output ratio at  $t = 1$ . Later, however, the rising marginal product of capital from the technology diffusion process (as summarized by the shifts in  $\theta_t$  and  $\rho_t$ ) stimulates an increase in investment which allows the capital stock to surpass the no-change trend.

The bottom panels in Figure 6 plot the agents' total income after taxes and transfers, again expressed as percent deviations from the no-change trend. These two panels provide insight into the welfare effects to be discussed later. In the bottom left panel, the capital owner's total income increases fastest under myopic expectations and slowest under perfect foresight. This is due to the faster rate of capital accumulation under myopic expectations which contributes to faster wage growth for capital owners. Workers also receive wage benefits from faster capital accumulation. The bottom right panel shows that the worker's total income is highest under myopic expectations and lowest under perfect foresight. For workers, more income translates directly into more consumption, which in turn contributes to higher welfare. For capital owners, more income under myopic expectations translates into more investment, thus postponing consumption and reducing welfare relative to the perfect foresight case. Hence, as we shall see, myopia on the part of capital owners is harmful for their own welfare but beneficial for workers' welfare.

Figure 7 plots the paths of some additional model variables as percent deviations from the no-change trend. The top left panel shows the immediate jump in the capital owner's consumption that occurs under perfect foresight. This is the flip-side to the sharp drop in the investment-output ratio shown in Figure 6. The immediate jump in the capital owner's consumption hinders capital accumulation, which lowers the wage paths for both capital owners and workers, as shown in the two middle panels. The top right panel of the figure shows that myopic expectations delivers the most favorable consumption path for workers, again because faster capital accumulation boosts wages relative to the other two expectation regimes. Notice that the path for the worker's consumption in Figure 7 is identical to the path for the worker's total income (including transfers) shown in the bottom right panel of Figure 6. The worker's consumption under myopic expectations initially declines relative to the no-change trend as the technology shift relentlessly shrinks the pre-tax income share of workers. Eventually, however, when  $t \gtrsim 30$ , recovering wages for workers (from capital accumulation) together with rising transfer payments from the government lead to an increase in the worker's consumption relative to the no-change trend. As a result, the myopic expectations regime can deliver welfare gains to workers.

To better understand the behavior of wages during the transition, we can combine the

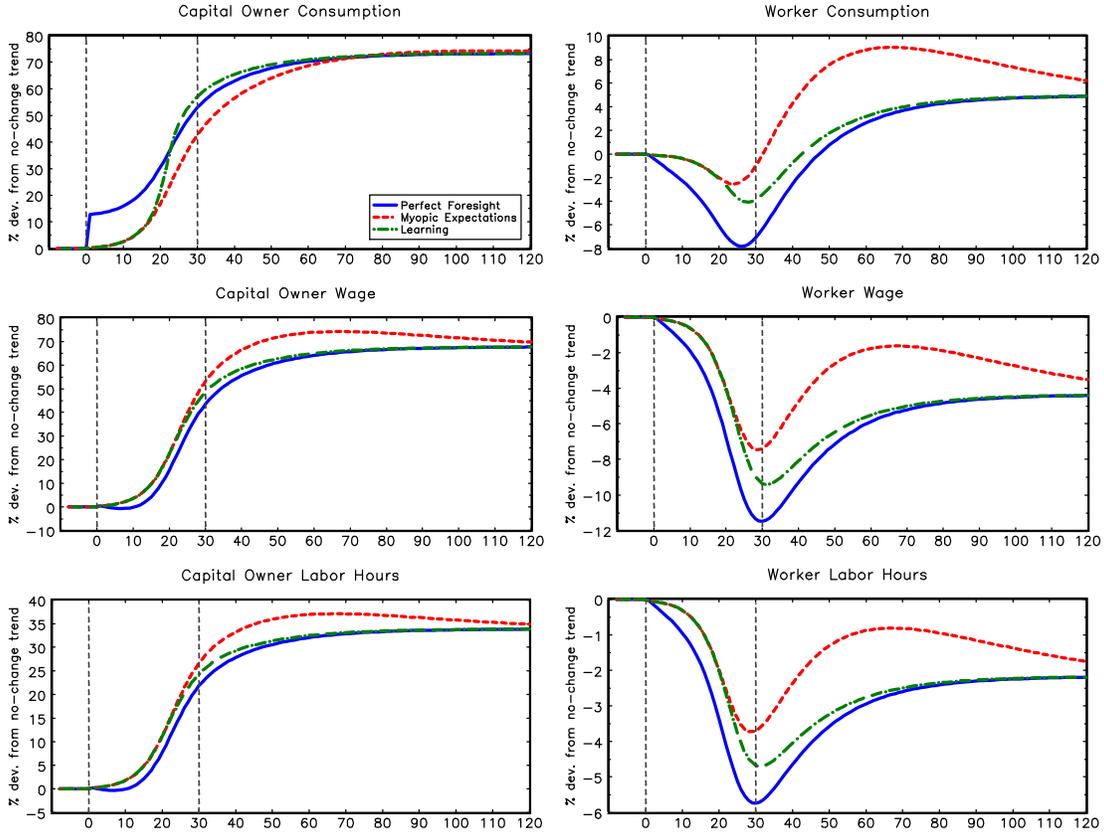


Figure 7: The capital owner’s consumption jumps immediately at  $t = 1$  under perfect foresight. This hinders capital accumulation and lowers the wage trajectories for both capital owners and workers. The myopic expectations regime delivers the most favorable consumption trajectory for workers because faster capital accumulation boosts wages relative to the other two expectation regimes. The transition paths for labor hours mimic the patterns for wages.

firm’s first-order conditions (21) and (22) with the labor supply equations (6) and (12) to obtain the following equilibrium relationship

$$\begin{aligned}
 w_t^w &= w_t^c \left[ \frac{1 - s_t^c}{s_t^c - s_t^k} \right] \frac{\ell_t^c}{n \ell_t^w}, \\
 &= w_t^c \left[ \frac{1 - s_t^c}{s_t^c - s_t^k} \right]^{\frac{\gamma-1}{\gamma}} \left( \frac{D^w}{D^c} \right)^{\frac{1}{\gamma}}, \tag{28}
 \end{aligned}$$

which is a rearranged version of the standard skill premium equation estimated by numerous empirical studies.<sup>23</sup> The term in square brackets summarizes the effects of “skill-biased” or “skill-replacing” changes in technology. Changes in the ratio  $\ell_t^c / (n \ell_t^w)$  capture shifts the

<sup>23</sup>See, for example, Goldin and Katz (2007, p. 7) and Acemoglu and Autor (2012, p. 434).

relative supplies of the two types of labor.

Equation (28) shows that the worker's wage  $w_t^w$  is influenced by several variables. An increase in the capital owner's wage  $w_t^c$  (due to technology diffusion or ordinary trend growth) will serve to increase the worker's wage. In contrast, an increase in the top decile income share  $s_t^c$  or an increase in the wage income share of the top decile  $s_t^c - s_t^k$  will both serve to decrease the worker's wage. All else equal, an increase in capital's share of total income  $s_t^k$  will serve to increase the worker's wage.<sup>24</sup> The strength of these various opposing effects depends strongly on the degree of capital-entrepreneurial skill complementarity. In the baseline model with  $\sigma_k < \sigma_\ell$ , capital owners enjoy a large increase in  $w_t^c$  as the technology diffusion increases the productivity of both capital and entrepreneurial labor which are tightly coupled when  $\sigma_k = 0.4$ . The increase in  $w_t^c$  helps to offset the upward shifts in  $s_t^c$  and  $s_t^c - s_t^k$  such that the equilibrium path for  $w_t^w$  is higher than otherwise. As evidence, the middle panels of Figure 7 show that the largest increase in  $w_t^c$  occurs under myopic expectations, which also delivers the most favorable path for  $w_t^w$ .

The bottom panels of Figure 7 show that the transition paths for labor hours mimic the patterns for wages. This is a direct consequence of the labor supply equations (6) and (12) which show that movements in  $\ell_t^w$  and  $\ell_t^c$  are directly proportional to movements in  $w_t^w$  and  $w_t^c$ . The increase in labor hours for capital owners, together with the increase in the productivity of the two entrepreneurial inputs ( $k_t$  and  $\ell_t^c$ ) is more than enough to offset the decline in the worker labor hours. As a result, aggregate output eventually surpasses the no-change trend under all expectation regimes (top left panel of Figure 10). The higher level of aggregate output boosts the amount of redistributive transfers received by workers each period since transfers are computed as a fraction of GDP.

## 5.2 Departures from the Baseline Model

We now consider three experiments that depart from the baseline model.<sup>25</sup> The results will prove helpful for understanding the welfare effects to be discussed later. The first experiment imposes  $\delta_\tau = 0$  in equation (26) such that the ratio of redistributive government transfers to GDP remains constant at the 1980 level of  $T_t/y_t = 10\%$ , rather than increasing to 15% as in the data. The second experiment holds capital's share of total income constant at the initial calibrated level of  $s_0^k = 0.35 \times 0.8 = 0.28$ , rather than increasing to a final share of

<sup>24</sup>A simple qualitative example along the lines of Krusell et al. (2000, p. 1030) helps to illustrate the point. Suppose the production function is  $y_t = (k_t + n\ell_t^w)^{\theta_t} (\ell_t^c)^{1-\theta_t}$ . The equilibrium skill premium is  $w_t^c/w_t^w = (1/\theta_t - 1)(k_t + n\ell_t^w)/\ell_t^c$ . Hence, an increase in  $\theta_t$  can benefit workers by shrinking the gap between  $w_t^c$  and  $w_t^w$ .

<sup>25</sup>Whenever a parameter value is changed from the baseline value shown in Table 1, we recalibrate the remaining parameters, where applicable, to achieve the same empirical targets as the baseline model.

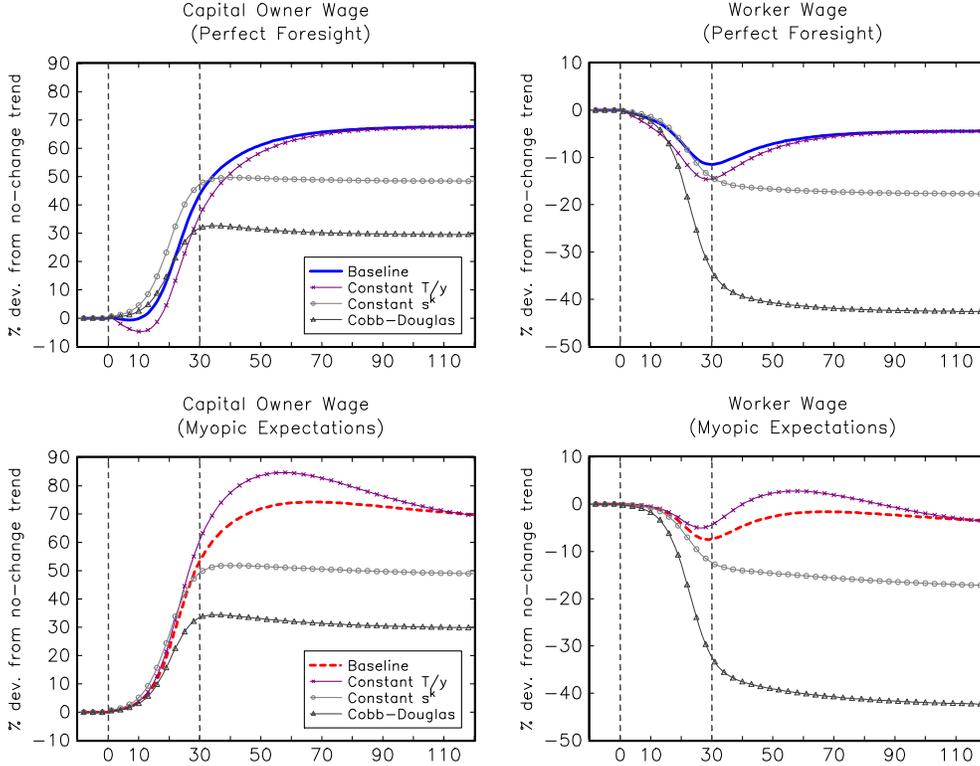


Figure 8: When the ratio of redistributive transfers to GDP is held constant at its initial level, wage paths are lower than the baseline paths under perfect foresight but higher than the baseline paths under myopic expectations. Holding capital’s share of income constant at its initial level lowers the wage paths of both types of agents relative to the baseline paths. The results for the Cobb-Douglas model are qualitatively similar to those for holding  $s_t^k$  constant, but the quantitative effects on the wage paths are now much larger.

$0.41 \times 0.8 = 0.328$ .<sup>26</sup> The third experiment imposes  $\sigma_k = \sigma_\ell = 1$  in equation (13) to recover a standard Cobb-Douglas production function which omits the feature of capital-entrepreneurial skill complementarity. Figure 8 shows how each experiment influences the path of wages, as expressed in percent deviations from the no-change trend. Figures 9 and 10 show the effects on the actual consumption trajectories of capital owners and workers. Figure 11 shows the effects on aggregate output.<sup>27</sup>

In the baseline model, the capital owner’s consumption rises faster than the no-change trend under all expectation regimes (top left panel of Figure 9). The worker’s consumption in

<sup>26</sup>For this experiment, the target top decile income share at the final steady state is adjusted downward from the baseline value of  $s^c = 0.49$  to  $s^c = 0.442$  in order to maintain the same absolute change in the top decile wage income share as in the baseline model. We then solve for a sequence of values for  $\rho_t$  from  $t = 1$  to  $t = 1500$  such that  $s_t^k = s_0^k$  while  $\theta_t$  is governed by equation (12) using the re-calibrated value  $\delta_\theta = 0.234$ .

<sup>27</sup>For clarity, we omit the learning regime plots in Figures 8 through 11 because these always track in between the plots for the other two expectation regimes.

the baseline model initially falls below the no-change trend as the top decile income share shifts upward in favor of capital owners (top left panel of Figure 10). But under myopic expectations, the worker's consumption later starts catching up and can even surpass the no-change trend as rising wages (from capital accumulation) and rising transfer payments from the government increase the worker's total income.

Under perfect foresight, aggregate output in the baseline model initially experiences a slowdown relative to the no-change trend, but growth later accelerates to allow output to surpass the no-change trend for  $t > 25$  (top left panel of Figure 11). This type of trajectory is consistent with the narratives emphasized by Hornstein and Krusell (1996) and Greenwood and Yorükoğlu (1997) whereby a skill-biased technology improvement initially leads to a measured slowdown in total factor productivity. The empirical evidence on the links between income inequality and growth remains inconclusive. A panel data study by Forbes (2000) finds a positive relationship between inequality and growth. She also demonstrates that data quality, sample periods, and the choice of estimation method can affect the sign and significance of the estimated coefficient on income inequality in typical growth regressions. A recent cross-country study by Berg and Ostry (2011) finds that higher levels of income inequality are often (but not always) associated with shorter growth spells, such that higher inequality tends to reduce an economy's average long-run growth rate. Figure 11 shows that, depending on assumptions, the model can produce simulations in which rising income inequality is associated with either faster or slower output growth in comparison to the no-change trend. This theoretical ambiguity may provide some basis for the conflicting findings in the empirical literature. In the baseline simulation with a strong degree of capital-entrepreneurial skill complementarity, rising income inequality is associated with faster growth during the transition period. The reverse is true for the Cobb-Douglas simulation.

#### *Effect of Redistributive Government Transfers*

Under perfect foresight, holding  $T_t/y_t$  constant lowers the wage paths for both types of agents relative to the baseline paths (top panels of Figure 8). In contrast, the wage paths for both types of agents are *raised* relative to the baseline paths under myopic expectations (bottom panels of Figure 8). Holding  $T_t/y_t$  constant leads to a larger initial jump in the capital owner's consumption under perfect foresight because the agent foresees that future lump-sum tax rates will not be increasing, thus implying higher permanent income relative to the baseline model. While beneficial for the welfare of capital owners, the larger initial jump in consumption slows capital accumulation which depresses the wage paths of both types of agents relative to the baseline model. In the case of myopic expectations, holding  $T_t/y_t$  constant allows the capital owner's consumption and investment to both increase faster

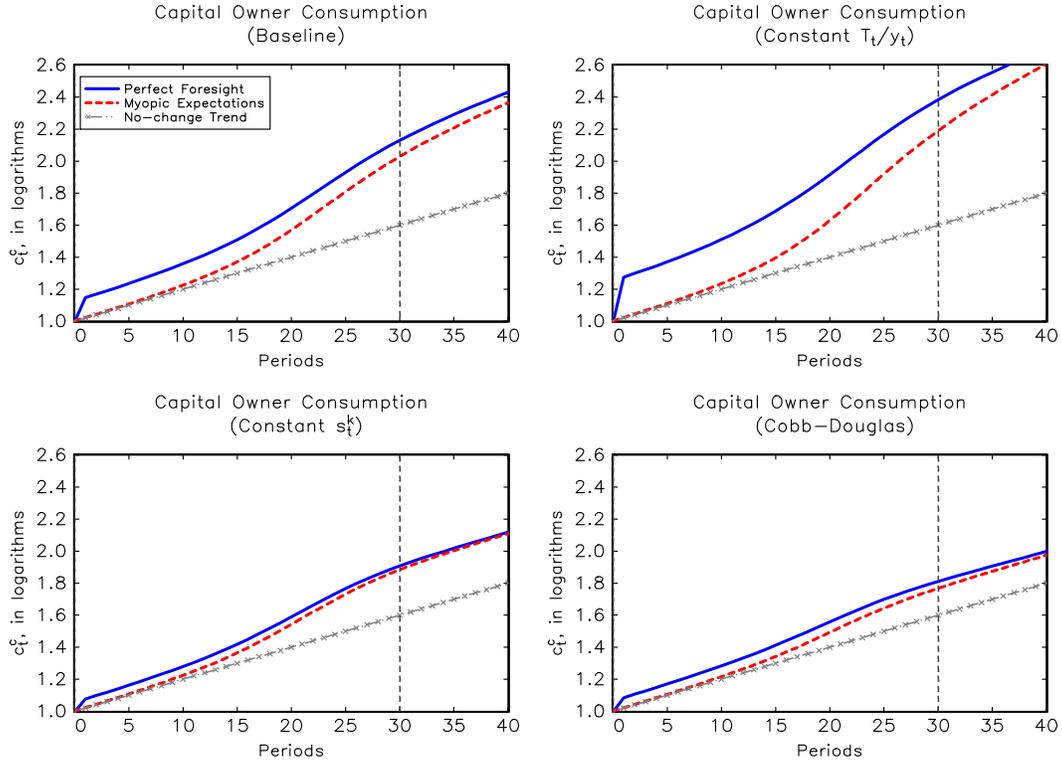


Figure 9: Holding transfers to GDP constant boosts the capital owner’s consumption trajectory relative to the baseline model. The opposite is true when either capital’s share of total income is held constant or when the production function is Cobb-Douglas. In all cases, however, the capital owner’s consumption trajectory surpasses the no-change trend.

than the baseline paths because after-tax income is now higher in each period. The resulting boost in capital accumulation raises the wage paths of both types of agents relative to the baseline wage paths. In the long-run, the ratio of lump-sum transfers to GDP has no effect on the marginal products of labor so the wage paths eventually converge to the baseline paths, regardless of the expectation regime.

Under perfect foresight, holding  $T_t/y_t$  constant leads to a larger initial jump in the capital owner’s consumption (top right panel of Figure 9). The larger initial jump is detrimental to the worker’s wage and consumption paths. But under myopic expectations, the higher after-tax income for capital owners induces higher investment and hence a higher wage path for workers relative to the baseline model. Consequently, the worker’s consumption path can still catch up and surpass the no-change trend, despite the constant transfer ratio (top right panel of Figure 10). Aggregate output surpasses the no-change trend under both expectations regimes (top right panel of Figure 11).

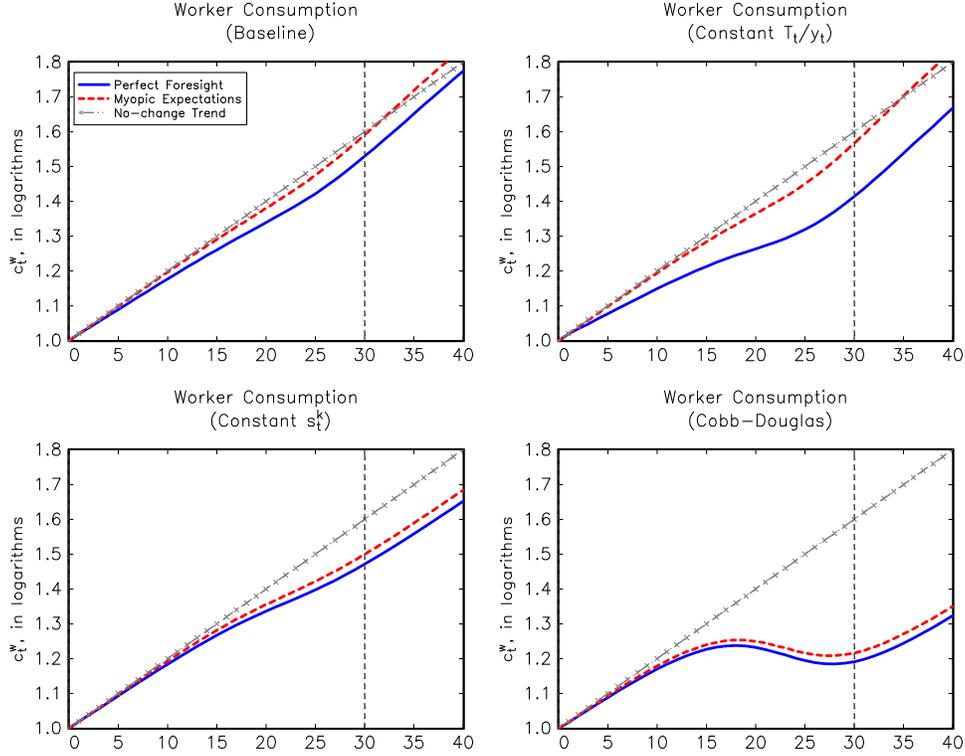


Figure 10: Under myopic expectations, the worker’s consumption trajectory can surpass the no-change trend for  $t \gtrsim 35$  in the baseline model and when transfers to GDP are held constant. However, the worker’s consumption trajectory remains below the no-change trend when capital’s share of total income is held constant or when the production function is Cobb–Douglas.

### *Effect of Capital’s Share of Total Income*

Figure 8 shows that holding  $s_t^k$  constant lowers the wage paths for both types of agents relative to the baseline paths, regardless of the expectation regime. The capital owner’s wage path continues to significantly exceed the no-change trend (i.e., the percent deviation remains in positive territory) but the worker’s wage path now drops below the no-change trend and stays there—representing a permanent downward level shift. This experiment shows that both types of agents derive wage benefits from a rise in capital’s share of total income even though capital ownership is concentrated in the hands of the top decile. The intuition for this result is straightforward. Since factor markets are competitive, any increase in  $s_t^k$  reflects an increase in the productivity of physical capital. In the presence of capital-entrepreneurial skill complementarity, a more productive capital stock also raises the marginal product of entrepreneurial labor, thus bestowing wage benefits on capital owners. The equilibrium conditions of the labor market, as summarized by equation (28), imply that workers can also receive wage benefits,

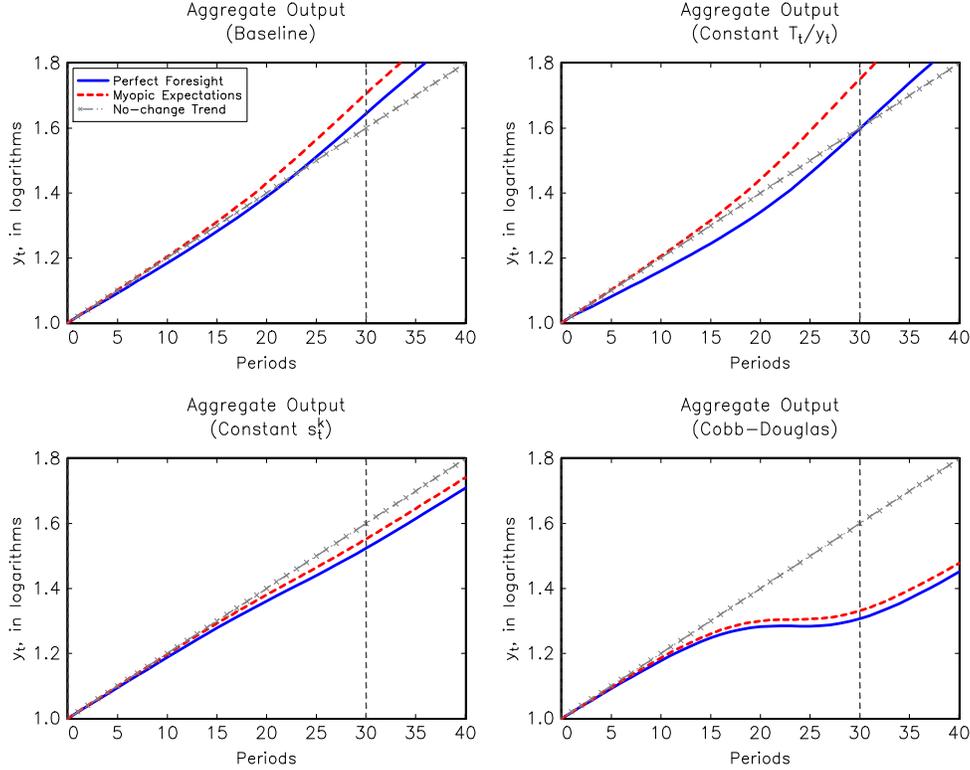


Figure 11: Depending on assumptions, the model can produce simulations in which rising income inequality is associated with either faster or slower output growth in comparison to the no-change trend. This result may provide some basis for the conflicting findings in the empirical literature on the links between inequality and growth.

since the marginal products of both types of labor are positively linked along the model's balanced growth path.

In Figures 9 and 10, we see that holding  $s_t^k$  constant leads to less-favorable consumption trajectories for both types of agents relative to the baseline model. This result is due to the less-favorable income paths for both types of agents. The capital owner's consumption trajectory still exceeds the no-change (bottom left panel of Figure 9) but the worker's consumption trajectory now drops below the no-change trend and remains there (bottom left panel of Figure 10). Recall that in the baseline model, the worker's consumption trajectory was able to eventually surpass the no-change trend, particularly under myopic expectations. The bottom left panel of Figure 11 shows that aggregate output grows slower than the no-change trend when  $s_t^k$  is held constant. This is because the technology change now omits an important feature that serves to increase the productivity of both physical capital and entrepreneurial labor (which are strong compliments in production). Holding  $s_t^k$  constant serves to reduce

income inequality relative to the baseline model. At the same time, output growth is now slower than in the baseline simulation—once again illustrating the ambiguous theoretical links between inequality and growth.

### *Effect of Capital-Entrepreneurial Skill Complementarity*

The Cobb-Douglas experiment can be viewed as a more extreme version of the previous experiment that holds  $s_t^k$  constant. The absence of capital-entrepreneurial skill complementarity means that a technology change which raises the productivity of physical capital yields lower wage paths than otherwise for both types of agents. Figure 8 shows that the wage paths in the Cobb-Douglas model are significantly lower than the baseline paths, regardless of the expectation regime. Although  $w_t^c$  continues to exceed the no-change trend, the magnitude of the increase is now much smaller than in the baseline model. The behavior of the worker’s wage can once again be understood from the labor market equilibrium relationship (28). The smaller net increase in  $w_t^c$  over the transition means that the dynamics of  $w_t^w$  now tend to be dominated by shifts in the income shares  $s_t^c$  and  $s_t^c - s_t^k$ , which transfer resources away from workers. Accordingly, the permanent shifts in the income shares now push  $w_t^w$  well below the no-change trend.

The lower wage path for workers reduces their labor supply by enough to keep aggregate output well-below the no-change trend (bottom right panel of Figure 11). Lower output during the transition implies lower transfer payments for workers since transfers are computed as a fraction of aggregate output. Consequently, the worker’s total income takes a hit from two sides: lower wages and a lower level of transfers than otherwise, resulting in a severe drop in consumption relative to both the baseline model and the no-change trend (bottom right panel of Figure 10).

The capital owner’s consumption trajectory still exceeds the no-change trend, but the gains are much smaller than in the baseline model (bottom right panel of Figure 9). Although capital owners receive a lower wage path relative to the baseline model, the effect on their consumption trajectory is mitigated by a lower level of lump sum taxes each period that must be paid to the government.

### **5.3 Welfare Analysis**

We use model simulations to gauge the welfare effects of rising income inequality. An advantage of our quantitative modeling approach (as opposed to an empirical estimation approach) is that we have full knowledge of the counterfactual scenario in which the income shares do not change. In the data, we see only one path for economic variables such as real GDP, so there is no way know how much the observed shifts in the income shares influenced the actual

trajectory of real GDP from 1980 to the present. The model predicts that the feedback effects from rising income inequality to output growth can be significant (Figure 11).

Changes in the distribution of U.S. pre-tax income do not translate directly into changes in welfare. This is because households and the government can respond in various ways to changes in the economic environment. Our model allows for several types of responses, including household adjustments along the consumption-saving margin and labor-leisure margin, and government adjustments along the redistribution margin. While no model can ever hope to incorporate the many details surrounding the real-world economy, we think that our simple capitalist-worker framework captures an essential dichotomy that governs the welfare consequences of rising income inequality. Welfare effects are measured by the constant percentage amount by which the agent's composite consumption basket in the no-change economy must be adjusted upward or downward each period to make lifetime utility equal to that obtained in the transition economy. In Appendix B, we show that the composite consumption baskets for each agent type can be written as follows

$$c_t^w - \frac{D^w}{\gamma} H_t (\ell_t^w)^\gamma = \frac{y_t}{n} \left[ \frac{\gamma - 1}{\gamma} (1 - s_t^c) + \tau_t \right], \quad (29)$$

$$c_t^c - \frac{D^c}{\gamma} H_t (\ell_t^c)^\gamma = y_t \left[ \frac{\gamma - 1}{\gamma} s_t^c + \frac{1}{\gamma} s_t^k - \tau_t - \frac{i_t}{y_t} \right], \quad (30)$$

where the right-side expressions make use of equilibrium relationships. The above equations show that the welfare results can be influenced by the trajectories of five separate variables, namely: real output  $y_t$ , the top decile income share  $s_t^c$ , capital's share of income  $s_t^k$ , the transfer-output ratio  $\tau_t$ , and the investment-output ratio  $i_t/y_t$ . Notably, the trajectory of  $i_t/y_t$  is strongly influenced by the nature of capital-owner expectations (Figure 6). Moreover, we see that the value of the labor supply elasticity parameter  $\gamma$  will influence the relative magnitude of various terms in the consumption basket.

Table 2 summarizes the welfare effects of rising income inequality for a variety of different model specifications. Going from left to right in the table, the three expectation regimes postulate successively higher degrees of knowledge about the economy's future transition path on the part of capital owners. The boxed entries in the table represent the best welfare outcome for each type of agent in a given expectation regime.

Table 2: Welfare Effects of Rising Income Inequality

Model Specification	Myopic Expectations		Learning		Perfect Foresight	
	Capital		Capital		Capital	
	Owners	Workers	Owners	Workers	Owners	Workers
Baseline	9.09	1.51	14.9	0.56	31.7	-1.28
$\kappa = 0.35$	12.0	1.96	20.6	0.64	37.4	-1.13
Constant $T_t/y_t$	<span style="border: 1px solid black;">16.2</span>	-0.15	<span style="border: 1px solid black;">25.3</span>	-2.38	<span style="border: 1px solid black;">66.2</span>	-6.78
Constant $s_t^k$	1.13	-2.58	2.93	-3.08	8.18	-3.88
Cobb-Douglas, $\sigma_k = \sigma_\ell = 1$	0.37	-12.5	2.62	-12.9	9.01	-13.6
$\sigma_k = 0.8$	2.62	-9.08	5.80	-9.64	14.9	-10.6
$\sigma_\ell = 1.4$	8.23	0.78	13.4	-0.05	26.5	-1.47
$1/\alpha = 1$	14.4	1.05	17.5	0.62	23.1	-0.01
$1/\alpha = 0.33$	6.48	1.67	12.5	0.52	36.8	-2.16
$(\gamma - 1)^{-1} = 1.5$	4.45	0.92	6.74	0.63	11.9	<span style="border: 1px solid black;">0.04</span>
$(\gamma - 1)^{-1} = 0.1$	11.8	1.65	17.8	0.60	35.0	-1.26
$\beta = 0.982$	13.3	<span style="border: 1px solid black;">2.14</span>	23.0	<span style="border: 1px solid black;">0.68</span>	40.0	-1.06

Notes: Baseline model uses  $\sigma_k = 0.4$  and  $\sigma_\ell = 1$ . Cobb-Douglas model uses  $\sigma_k = \sigma_\ell = 1$ . Welfare effects are measured by the percentage change in the per-period consumption basket to make the agent indifferent between the no-change economy (which holds income shares constant) and the transition economy. Boxed entries represent the best welfare outcome for each type of agent in a given expectation regime.

All of the various model specifications in Table 2 deliver positive welfare gains for the capital owners. The gains increase monotonically from left to right along with capital owners' knowledge about the future transition path. Conversely, the welfare outcomes for workers decline monotonically from left to right. At the extreme right under perfect foresight, the welfare outcomes for workers are almost always negative. The sole exception is when both types of agents have a more elastic labor supply, i.e., when  $(\gamma - 1)^{-1} = 1.5$ . This case is discussed in more detail below.

For the baseline model, the welfare gains for capital owners range from 9% under myopic expectations to about 32% under perfect foresight. The huge gain for capital owners under perfect foresight derives from the initial consumption jump at  $t = 1$ . Workers achieve a welfare gain of 1.5% under myopic expectations but suffer a welfare *loss* of about 1.3% under perfect foresight. The workers' loss under perfect foresight derives from the negative wage impacts induced by slower capital accumulation when the investment-output ratio drops sharply at  $t = 1$ . The welfare results under learning fall in between those for the other two expectation regimes. In the baseline learning regime, workers still manage to achieve a welfare gain of 0.6% while the welfare gain for capital owners is now about 15%.

The second row of Table 2 shows the effects of a faster diffusion speed for new technology with  $\kappa = 0.35$ . As noted earlier in the discussion of Figure 5, this alternative calibration provides a somewhat closer fit of the U.S. data for the top decile income share. When  $\kappa = 0.35$ ,

the diffusion process is 99.7% complete by the year 2010 versus 92% in the baseline model. The movement from a 10% adoption share to 90% now takes 13 years versus 18 years in the baseline model. Under each expectation regime, both capital owners and workers are made better off relative to the baseline model, with the effect on capital owners being more pronounced. This experiment shows that a more-rapid technological change (which results in a more-rapid increase in income inequality) can still yield benefits to all agents, even when the technology change is biased in favor of highly-skilled workers.

The third row of Table 2 shows the effect of holding the ratio of redistributive transfers constant at the 1980 value of 10%. As would be expected, holding  $T_t/y_t$  constant is beneficial for capital owners but detrimental to workers. In the absence of a rising ratio of redistributive transfers to GDP, the workers always suffer a welfare loss that ranges from  $-0.15\%$  under myopic expectations to  $-6.8\%$  under perfect foresight. The boxed entries show that this particular model specification delivers the most favorable welfare outcomes for capital owners, regardless of the expectation regime. Interestingly, however, this specification does not deliver the worst welfare outcome for workers. Holding  $T_t/y_t$  constant boosts the after-tax income of capital owners which leads to higher investment than otherwise. The resulting faster rate of capital accumulation delivers wage benefits to workers which helps to mitigate the loss of some transfer payments. Recall that the workers' consumption trajectory can still surpass the no-change trend even when transfer-to-GDP ratio is held constant at 10% (top right panel of Figure 10). Experiments with the myopic expectations model show that the welfare change for workers is approximately zero for a scenario where  $T_t/y_t$  increases from 10% at the initial steady state to 10.38% at the final steady state.

As noted previously, the Cobb-Douglas experiment can be viewed as a more extreme version of the experiment that holds  $s_t^k$  constant. Table 2 shows that both of these experiments deliver less favorable welfare outcomes in each cell when compared to the baseline model. This result is due to the less favorable wage paths obtained in these experiments, as shown earlier in Figure 8. The less favorable wage paths reduce agents' labor supply relative to the baseline model, leading to slower growth in aggregate output during the transition (bottom panels of Figure 11). Of all the different specifications reported in Table 2, the Cobb-Douglas model delivers the worst welfare outcomes for workers, regardless of the expectation regime. This result is striking, particularly since Cobb-Douglas production functions are commonly used in the theoretical and empirical literature on income inequality. Our results show that the use of a Cobb-Douglas specification can lead to a downward bias when gauging the welfare or growth consequences of shifting income shares.

We also experimented with changing either  $\sigma_k$  and  $\sigma_\ell$  individually. When  $\sigma_k = 0.8$  (with

$\sigma_\ell$  maintained at the baseline value of 1), the degree of capital-entrepreneurial skill complementarity is weaker than in the baseline model but stronger than in the Cobb-Douglas model. Table 2 shows that this experiment delivers better welfare outcomes than the Cobb-Douglas model, but both types of agents are still worse-off relative to the baseline model which has  $\sigma_k = 0.4$ . When  $\sigma_\ell = 1.4$  (with  $\sigma_k$  maintained at the baseline value of 0.4), both types of agents are again worse-off relative to the baseline model, but the decline in welfare outcomes is less severe than in the previous experiment with  $\sigma_k = 0.8$ . Hence, in the presence of a technological change that makes physical capital more productive, both types of agents will benefit if either type's labor supply becomes more complementary with physical capital.

Variations in the parameter  $\alpha$  affect the EIS for the agents' composite consumption baskets. Recall that the baseline EIS for both types of agents is  $1/\alpha = 0.5$ . We experimented with setting  $1/\alpha = 1$  or  $1/\alpha = 0.33$ , which allow for a higher or lower EIS than the baseline model. For capital owners, the EIS governs the relative size of the income and substitution effects of the technology change which, in turn, pin down the optimal split between consumption and investment along the transition path. Under perfect foresight, an EIS closer to unity implies a weaker income effect which implies a smaller jump in the capital owner's consumption at  $t = 1$ . This situation lowers the capital owner's welfare relative to the baseline model, but benefits the worker's welfare. However, under myopic expectations and learning, an EIS closer to unity implies a stronger income effect because capital owners now react to current income. A stronger income effect raises the capital owner's consumption trajectory relative to the baseline model. This is beneficial for the capital owner's welfare but since capital accumulation is now slower, the welfare of workers declines relative to the baseline model. All of these effects are reversed when the EIS is further away from unity than the baseline value. For both types of agents, the EIS also influences the lifetime utility evaluation of a given consumption trajectory. But this effect is of second-order importance when compared to effect of the EIS on the level and slope of the consumption trajectory itself.

Our baseline calibration assumed a labor supply elasticity of  $(\gamma - 1)^{-1} = 0.5$  for both types of agents. Keane and Rogerson (2012) argue that the intertemporal elasticity of substitution for labor supply at the macro level is in the range of 1 to 2. Consistent with this view, we set  $(\gamma - 1)^{-1} = 1.5$ . The results of this experiment (shown in the second to last row of Table 2) are mixed. Capital owners are made worse-off relative to the baseline model under all three expectation regimes. Workers are made worse-off under myopic expectations, but their welfare outcomes are improved under learning and perfect foresight. In the case of capital owners, a more-elastic labor supply moderates the increase in their equilibrium wage path, since an increase in the price of their labor now brings forth more supply. This effect, together with

the associated reduction in leisure time, moderates their welfare gains in comparison to the baseline model. Workers benefit from a higher aggregate labor supply because it raises the level of aggregate output and hence transfers. Recall, however, that the technology change causes the workers' own labor supply to initially decline relative to the no-change trend, particularly under learning or perfect foresight (bottom right panel of Figure 7). The decrease in their own labor supply results in more leisure time which, all else equal, is beneficial for their welfare. Relative to the baseline model, the positive effects on workers' welfare outweigh the negative effects under learning and perfect foresight. Table 2 shows that the calibration with  $(\gamma - 1)^{-1} = 1.5$  delivers positive welfare gains for workers under all three expectation regimes. For completeness, we also show the results for the case  $(\gamma - 1)^{-1} = 0.1$ , which implies a very inelastic labor supply for both types of agents. The results for this version of the model are close to the baseline model.

The last row of Table 2 shows the effect of more patience. When  $\beta = 0.982$ , the steady-state net equity return is 6% versus 8% in the baseline model. As with the EIS for consumption, a change in  $\beta$  has a first-order effect on the level and slope of the agents' consumption trajectories and a second-order effect on the lifetime utility evaluation of a given consumption trajectory. A higher value for  $\beta$  improves the welfare outcomes for both types of agents relative to the baseline model. In the case of capital owners, increased patience yields more investment which, in turn, boosts the wage paths of both types of agents via faster capital accumulation. For workers, a higher wage path allows more consumption than otherwise. Moreover, the recovery in the worker's consumption trajectory that occurs later in the transition (top left panel of Figure 9) is now given more weight when computing lifetime utility.

Overall, we find that the range of possible welfare outcomes for both types of agents is enormous. The range of results presented in Table 2 might be viewed as something akin to a confidence interval for the potential welfare effects of rising U.S. income inequality over the past three decades. The welfare gains for capital owners range from a low of 0.37% (Cobb-Douglas, myopic expectations) to a high of 66.2% (constant  $T_t/y_t$ , perfect foresight). The welfare outcomes for workers range from a low of -13.6% (Cobb-Douglas, perfect foresight) to a high of 2.62% ( $\beta = 0.982$ , myopic expectations). We acknowledge that some of the model specifications are more relevant than others for comparison with the U.S. experience. In particular, the perfect foresight regime could be viewed as implausible while the specifications that hold either  $T_t/y_t$  or  $s_t^k$  constant are counterfactual. It should also be noted that the welfare outcomes for both types of agents would be scaled downward if we had assumed that redistributive transfers were financed by a distortionary tax on capital owners' income. Nevertheless, the main point to be taken away from Table 2 is that the welfare consequences of rising income

inequality are highly uncertain, even in the relatively simple framework considered here with only types of agents. This finding is likely extend to more complex model environments that include the basic elements observed in the data, namely, rising income inequality and a stable distribution of financial wealth.

## 6 Conclusion

The U.S. economy experienced a profound upward shift in the share of income going to the top decile of households over the past three decades. Evidence suggests that some form of skill-biased technological change played an important role in this trend. We developed a model of skill-biased technological change in which the share parameters of a CES production function shift over time, similar to the framework of Goldin and Katz (2007). But in contrast to much of the literature in this area, our approach focused on a technology-induced shift in the demand for entrepreneurial labor, representing top incomes, as opposed to the broader pool of college-educated labor. Empirical evidence shows that even among college-educated workers, the income gains of the highest earners is the primary driving force for rising U.S. income inequality.

Our analysis shows that the top decile of agents in the model always benefit from the technology change, but their degree of foresight influences the size of their welfare gains. Workers outside the top decile can also benefit when three elements are in place, namely, a rising ratio of redistributive transfers to GDP, an increase in capital's share of total income, and a strong degree of complementarity between physical capital and entrepreneurial labor. If any one of these three elements are absent from the model, then workers suffer a welfare loss from the technology change. Further, we showed that the effects rising income inequality on growth are theoretically ambiguous, also depending on the degree of capital-entrepreneurial skill complementarity.

Two important caveats of our findings are in order. First, our framework does not allow us to say anything about changes in income inequality among agents in the lower nine-tenths of the U.S. income distribution. This group encompasses individuals with a wide range of skills and education levels. A framework with more than two types of agents is needed to study the consequences of such developments. Second, we abstracted from endogenous human capital investment which could help spread the benefits of skill-biased technological change to agents who fall outside the top decile. Still, the inclusion of such features would not eliminate the large fundamental uncertainty that surrounds the economic consequences of a technology-driven rise in pre-tax income inequality.

## A Appendix: Model Solution

### A.1 First-Order Conditions in Stationary Variables

Combining the agents' labor supply equations (6) and (12) with the firm's labor demand equations (21) and (22) yields the following pair of nonlinear equations that pin down the values of  $n \ell_t^w$  and  $\ell_t^c$  as functions of the two state variables  $k_{n,t} \equiv k_t / \exp(z_t)$  and  $\pi_t$  :

$$n \ell_t^w = \left[ \frac{A(1-\theta_t) n^{\gamma-1}}{D^w \left\{ \theta_t \left[ (1-\rho_t) k_{n,t}^{\psi_k} + \rho_t (\ell_t^c)^{\psi_k} \right]^{\frac{\psi_\ell}{\psi_k}} + (1-\theta_t) (n \ell_t^w)^{\psi_\ell} \right\}^{\frac{\psi_\ell-1}{\psi_\ell}}} \right]^{\frac{1}{\gamma-\psi_\ell}}, \quad (\text{A.1})$$

$$\ell_t^c = \left[ \frac{A \theta_t \rho_t \left[ (1-\rho_t) k_{n,t}^{\psi_k} + \rho_t (\ell_t^c)^{\psi_k} \right]^{\frac{\psi_\ell-\psi_k}{\psi_k}}}{D^c \left\{ \theta_t \left[ (1-\rho_t) k_{n,t}^{\psi_k} + \rho_t (\ell_t^c)^{\psi_k} \right]^{\frac{\psi_\ell}{\psi_k}} + (1-\theta_t) (n \ell_t^w)^{\psi_\ell} \right\}^{\frac{\psi_\ell-1}{\psi_\ell}}} \right]^{\frac{1}{\gamma-\psi_k}}, \quad (\text{A.2})$$

where we have made use of  $H_t = \exp(z_t)$  and the expressions for the income share variables  $s_t^c$  and  $s_t^k$  are given by equations (24) and (25). Recall that  $\theta_t$  and  $\rho_t$  are functions of the state variable  $\pi_t$ , as given by equations (15) and (16).

To facilitate a numerical solution, the firm's intertemporal first-order condition (23) can be rewritten in terms of stationary variables. Dividing both sides of equation (23) by  $y_t$  and defining the firm's intertemporal decision variable as the investment-output ratio  $x_t \equiv i_t / y_t$  yields

$$x_t = \widehat{E}_t \beta \left[ \frac{\lambda_{t+1}^c y_{t+1}}{\lambda_t^c y_t} \right] [\lambda s_{t+1}^k + (1-\lambda) x_{t+1}], \quad (\text{A.3})$$

where we have substituted in  $M_{t+1}^c \equiv \beta \lambda_{t+1}^c / \lambda_t^c$ .

From the capital owner's first-order condition (9), we have

$$\frac{\lambda_{t+1}^c y_{t+1}}{\lambda_t^c y_t} = \left[ \frac{c_{t+1}^c / y_{t+1} - \frac{D^c}{\gamma} H_{t+1} (\ell_{t+1}^c)^\gamma / y_{t+1}}{c_t^c / y_t - \frac{D^c}{\gamma} H_t (\ell_t^c)^\gamma / y_t} \right]^{-\alpha} \left[ \frac{y_{t+1}}{y_t} \right]^{1-\alpha}. \quad (\text{A.4})$$

The above equation can be further transformed by substituting in the following expressions that derive from the capital owner's budget constraint (8), the capital owner's labor supply

equation (12), and the production function (13):

$$c_t^c/y_t = s_t^c - x_t - \tau_t, \quad (\text{A.5})$$

$$\frac{D^c}{\gamma} H_t (\ell_t^c)^\gamma / y_t = \frac{1}{\gamma} w_t^c \ell_t^c / y_t = \frac{1}{\gamma} (s_t^c - s_t^k), \quad (\text{A.6})$$

$$\frac{y_{t+1}}{y_t} = \exp(\mu) \left\{ \frac{\theta_{t+1} \left[ (1 - \rho_t) k_{n,t+1}^{\psi_k} + \rho_{t+1} (\ell_{t+1}^c)^{\psi_k} \right]^{\frac{\psi_\ell}{\psi_k}} + (1 - \theta_{t+1}) (n \ell_{t+1}^w)^{\psi_\ell}}{\theta_t \left[ (1 - \rho_t) k_{n,t}^{\psi_k} + \rho_t (\ell_t^c)^{\psi_k} \right]^{\frac{\psi_\ell}{\psi_k}} + (1 - \theta_t) (n \ell_t^w)^{\psi_\ell}} \right\}^{\frac{1}{\psi_\ell}}, \quad (\text{A.7})$$

where  $\tau_t \equiv T_t/y_t$  is the lump-sum tax rate and we have made use of  $z_{t+1} - z_t = \mu$ . The tax rate is a function of the state variable  $\pi_t$ , as given by equation (26). The stationary endogenous variables  $n \ell_t^w$ ,  $\ell_t^c$ ,  $s_t^c$ , and  $s_t^k$ , are governed by equations (A.1), (A.2), (24) and (25), respectively.

The upshot of all this is that the firm's intertemporal first order condition (A.3) can now be written as the following nonlinear stochastic difference equation involving only stationary variables:

$$x_t \left\{ \theta_t \left[ (1 - \rho_t) k_{n,t}^{\psi_k} + \rho_t (\ell_t^c)^{\psi_k} \right]^{\frac{\psi_\ell}{\psi_k}} + (1 - \theta_t) (n \ell_t^w)^{\psi_\ell} \right\}^{\frac{1-\alpha}{\psi_\ell}} = \frac{\beta' \widehat{E}_t \left\{ \theta_{t+1} \left[ (1 - \rho_{t+1}) k_{n,t+1}^{\psi_k} + \rho_{t+1} (\ell_{t+1}^c)^{\psi_k} \right]^{\frac{\psi_\ell}{\psi_k}} + (1 - \theta_{t+1}) (n \ell_{t+1}^w)^{\psi_\ell} \right\}^{\frac{1-\alpha}{\psi_\ell}}}{\left[ \frac{\gamma-1}{\gamma} s_t^c + \frac{1}{\gamma} s_t^k - x_t - \tau_t \right]^\alpha} \quad (\text{A.8})$$

where  $\beta' \equiv \beta \exp[(1 - \alpha)\mu]$  and we have collected variables dated  $t + 1$  on the right side. The object to be forecasted involves three future decision variables  $x_{t+1}$ ,  $\ell_{t+1}^c$ , and  $n \ell_{t+1}^w$  and two future state variables  $k_{n,t+1}$  and  $\pi_{t+1}$ . Since the law of motion for  $\pi_{t+1}$  is exogenous, the only remaining element needed for a solution is the endogenous law of motion for  $k_{n,t+1}$ , which is derived next.

Starting from the definitional relationship  $k_{n,t+1} \equiv k_{t+1}/\exp(z_{t+1})$ , we have

$$\begin{aligned} k_{n,t+1} &= k_{n,t} \exp(-z_{t+1} + z_t) \frac{k_{t+1}}{k_t}, \\ &= k_{n,t} \exp(-\mu) B \left[ \frac{i_t}{y_t} \frac{y_t}{k_t} \right]^\lambda, \end{aligned} \quad (\text{A.9})$$

where we have substituted in the laws of motion for  $z_{t+1}$  and  $k_{t+1}$ . From the production

function (13), we have

$$\frac{y_t}{k_t} = \frac{A}{k_{n,t}} \left\{ \theta_t \left[ (1 - \rho_t) k_{n,t}^{\psi_k} + \rho_t (\ell_t^c)^{\psi_k} \right]^{\frac{\psi_\ell}{\psi_k}} + (1 - \theta_t) (n \ell_t^w)^{\psi_\ell} \right\}^{\frac{1}{\psi_\ell}}. \quad (\text{A.10})$$

Substituting equation (A.10) into (A.9) together with  $x_t \equiv i_t/y_t$  yields the following law of motion for the normalized capital stock:

$$k_{n,t+1} = A^\lambda B \exp(-\mu) k_{n,t}^{1-\lambda} x_t^\lambda \left\{ \theta_t \left[ (1 - \rho_t) k_{n,t}^{\psi_k} + \rho_t (\ell_t^c)^{\psi_k} \right]^{\frac{\psi_\ell}{\psi_k}} + (1 - \theta_t) (n \ell_t^w)^{\psi_\ell} \right\}^{\frac{\lambda}{\psi_\ell}}. \quad (\text{A.11})$$

## A.2 Perfect Foresight

Under perfect foresight, the transformed intertemporal first-order condition (A.8) becomes

$$f(x_t, \ell_t^c, n \ell_t^w, k_{n,t}, \pi_t) = h(x_{t+1}, \ell_{t+1}^c, n \ell_{t+1}^w, k_{n,t+1}, \pi_{t+1}), \quad (\text{A.12})$$

where we have eliminated  $s_t^c$ ,  $s_t^k$ ,  $s_{t+1}^c$ ,  $s_{t+1}^k$  using equations (24) and (25). The decision variables  $n \ell_t^w$  and  $\ell_t^c$  must satisfy equations (A.1) and (A.2) each period.

Two approximate solutions of the model can be obtained by log-linearizing equations (A.1), (A.2), (A.8), and (A.11) around each of the two steady states corresponding to  $\pi_t = 0$  and  $\pi_t = 1$ . We use the  $\pi_t$ -weighted average from the two sets of log-linear decision rules to construct an initial conjectured sequence of values for the nonlinear function  $h(\cdot)$  from  $t = 0$  (the initial steady state) to  $t = 1500$  (the final steady state). At each time  $t$ , the conjectured value for  $h(t + 1)$  is substituted into the right side of equation (A.12). Given  $h(t + 1)$  equations (A.1), (A.2) and (A.12) can be solved simultaneously for  $x_t$ ,  $\ell_t^c$ , and  $n \ell_t^w$  using a nonlinear equation solver. The resulting values are used to compute  $k_{n,t+1}$  from equation (A.11) with  $\pi_{t+1}$  given by the exogenous law of motion (17). This procedure is repeated each time period, yielding a new conjectured sequence for  $h(\cdot)$  from  $t = 0$  to  $t = 1500$ . The perfect foresight solution is obtained when the conjectured sequence for  $h(\cdot)$  does not change (to an accuracy of 0.0001) from one simulation to the next. In practice, convergence is obtained after about 70 simulations.

## A.3 Myopic Expectations

Under myopic expectations, we assume  $\widehat{E}_t h(t + 1) = h(t - 1)$ . The transformed intertemporal first-order condition (A.8) becomes

$$f(x_t, \ell_t^c, n \ell_t^w, k_{n,t}, \pi_t) = h(x_{t-1}, \ell_{t-1}^c, n \ell_{t-1}^w, k_{n,t-1}, \pi_{t-1}). \quad (\text{A.13})$$

At each date  $t$ , equations (A.1), (A.2) and (A.13) can be solved simultaneously for  $x_t$ ,  $\ell_t^c$ , and  $n \ell_t^w$  using a nonlinear equation solver. The resulting values are used to compute  $k_{n,t+1}$  from equation (A.11) with  $\pi_{t+1}$  computed using the exogenous law of motion (17).

## A.4 Learning

Under learning, we assume  $\widehat{E}_t h(t+1) = \omega_t h(t+1) + (1 - \omega_t) h(t-1)$ , where  $\omega_t = \pi_t$ . The transformed intertemporal first-order condition (A.8) becomes

$$\begin{aligned} f(x_t, \ell_t^c, n \ell_t^w, k_{n,t}, \pi_t) &= \pi_t h(x_{t+1}, \ell_{t+1}^c, n \ell_{t+1}^w, k_{n,t+1}, \pi_{t+1}) \\ &+ (1 - \pi_t) h(x_{t-1}, \ell_{t-1}^c, n \ell_{t-1}^w, k_{n,t-1}, \pi_{t-1}). \end{aligned} \quad (\text{A.14})$$

Similar to case of perfect foresight, the solution under learning requires an initial conjectured sequence of values for the nonlinear function  $h(\cdot)$  from  $t = 0$  to  $t = 1500$ . As before, we construct the initial conjectured sequence using a  $\pi_t$ -weighted average of the two sets of decision rules from the log-linearized learning model. At each time  $t$ , the conjectured value for  $h(t+1)$  and the realized lagged value  $h(t-1)$  are both substituted into the right side of equation (A.14), thus allowing equations (A.1), (A.2) and (A.14) to be solved simultaneously for  $x_t$ ,  $\ell_t^c$ , and  $n \ell_t^w$ . This procedure is repeated each time period, yielding a new conjectured sequence for  $h(\cdot)$ . The learning solution is obtained when the conjectured sequence for  $h(\cdot)$  does not change from one simulation to the next.

## B Appendix: Welfare Computation

An individual worker's lifetime utility can be written as

$$\begin{aligned} V^w &= \sum_{t=0}^{\infty} \beta^t \frac{\left[ c_t^w - \frac{D^w}{\gamma} H_t (\ell_t^w)^\gamma \right]^{1-\alpha} - 1}{1-\alpha} \\ &= \sum_{t=0}^{\infty} \beta^t \frac{\left[ \frac{\gamma-1}{\gamma} c_t^w + \frac{1}{\gamma} T_t/n \right]^{1-\alpha} - 1}{1-\alpha}, \end{aligned} \quad (\text{B.1})$$

where we have substituted in  $D^w H_t (\ell_t^w)^\gamma = w_t^w \ell_t^w$  from the labor supply equation (6) and  $w_t^w \ell_t^w = c_t^w + T_t/n$  from the budget equation (3). Similarly, an individual capital owner's lifetime utility can be written as

$$V^c = \sum_{t=0}^{\infty} \beta^t \frac{\left[ \frac{\gamma-1}{\gamma} c_t^c + \frac{1}{\gamma} (d_t - T_t) \right]^{1-\alpha} - 1}{1-\alpha}, \quad (\text{B.2})$$

where the terms in square brackets in (B.1) and (B.2) are the agents' composite consumption baskets. Both (B.1) and (B.2) show the direct influence of transfers  $T_t$  on lifetime utility. The expressions for the equilibrium consumption baskets can be further simplified by substituting in the relationships  $c_t^w = (1 - s_t^c) y_t/n + T_t/n$ ,  $c_t^c = (s_t^c - s_t^k) y_t + d_t - T_t$ , and  $d_t = s_t^k y_t - i_t$

to obtain

$$\begin{aligned}
C_t^w &\equiv c_t^w - \frac{D^w}{\gamma} H_t (\ell_t^w)^\gamma, \\
&= \frac{y_t}{n} \left[ \frac{\gamma-1}{\gamma} (1 - s_t^c) + \tau_t \right], \tag{B.3}
\end{aligned}$$

$$\begin{aligned}
C_t^c &\equiv c_t^c - \frac{D^c}{\gamma} H_t (\ell_t^c)^\gamma, \\
&= y_t \left[ \frac{\gamma-1}{\gamma} s_t^c + \frac{1}{\gamma} s_t^k - \tau_t - \frac{i_t}{y_t} \right]. \tag{B.4}
\end{aligned}$$

The welfare effect of the technology change is calculated as the constant percentage amount by which the agent's composite consumption basket in the no-change economy (which holds income shares constant at their initial levels) must be adjusted upward or downward each period to make lifetime utility equal to that in the transition economy. Specifically, we find  $\phi^w$  and  $\phi^c$  that solve the following two equations

$$\sum_{t=0}^{\infty} \beta^t \frac{[C_t^w]^{1-\alpha} - 1}{1-\alpha} = \sum_{t=0}^{\infty} \beta^t \frac{[\bar{C}_t^w (1 + \phi^w)]^{1-\alpha} - 1}{1-\alpha}, \tag{B.5}$$

$$\sum_{t=0}^{\infty} \beta^t \frac{[C_t^c]^{1-\alpha} - 1}{1-\alpha} = \sum_{t=0}^{\infty} \beta^t \frac{[\bar{C}_t^c (1 + \phi^c)]^{1-\alpha} - 1}{1-\alpha}, \tag{B.6}$$

where  $C_t^w$  and  $C_t^c$  are the composite consumption baskets in the transition economy and  $\bar{C}_t^w$  and  $\bar{C}_t^c$  are the composite consumption baskets obtained along the no-change trend. The infinite sums in (B.5) and (B.6) are approximated by sums over a 1500 period simulation, after which the results are not changed. The initial conditions at  $t = 0$  correspond to the steady state with  $\pi_t = 0$ .

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