

Endogenous Property Rights*

Daniel Diermeier Georgy Egorov
Northwestern University Northwestern University

Konstantin Sonin
Higher School of Economics

December 2013

Abstract

It is often argued that additional checks and balances provide economic agents with better protection from expropriation of their wealth or productive capital. We demonstrate that in a dynamic political economy model this intuition may be flawed. Surprisingly, increasing the number of veto players or the majority requirement for redistribution may reduce property right protection on the equilibrium path. The reason is the existence of two distinct mechanisms of property rights protection. One are formal constraints that allow individuals or groups to block any redistribution which is not in their favor. The other occurs in equilibrium where agents without such powers protect each other from redistribution. Players without formal blocking power anticipate that the expropriation of other similar players will ultimately hurt them and thus combine their influence to prevent redistributions. Yet, such incentives can be undermined by adding formal constraints. The flip-side of this effect is that individual investment efforts might require coordination. The model also predicts that the distribution of wealth in societies with weaker formal institutions (smaller supermajority requirements) among players without veto power will tend to be more homogenous.

Keywords: political economy, legislative bargaining, property rights, institutions.

JEL Classification: D71, D74, C71.

*We are grateful to seminar participants in Princeton, Moscow, Frankfurt, Mannheim, Northwestern, the University of Chicago and to participants of MACIE conference in Marburg and CIFAR meeting in Toronto for valuable comments.

1 Introduction

The protection of property rights is widely viewed as a cornerstone of efficiency and economic growth (e.g., Coase, 1960, Alchian, 1965, Demsetz 1967, Barro 1998).¹ But whether property rights are effectively protected depends on the political economy of the respective society and its institutions. Among political institutions, checks and balances have long been viewed as an essential tool to limit the power of government to expropriate its subjects. The ideas date back at least to the Roman republic (Polybius [2010], Macchiavelli [1984]) and, in modern times, to Montesquieu’s *Spirit of the Laws* (1748 [1989]) and the Federalist papers, the intellectual foundation of the United States Constitution. In Federalist papers #51, James Madison argued for the need to contrive the government “as that its several constituent parts may, by their mutual relations, be the means of keeping each other in their proper places.” Riker (1987) concurs: “For those who believe, with Madison, that freedom depends on countering ambition with ambition, this constancy of federal conflict is a fundamental protection of freedom.”

In modern political economy checks and balances have been associated with various beneficial consequences.² North and Weingast (1989) argued that the checks and balances established by the Glorious Revolution in 1688 provided “the credible commitment by the government to honour its financial agreement [that] was part of a larger commitment to secure private rights”.³ Similarly, Root (1989) has argued that checks and balances on British monarchs led to lower borrowing costs compared to the French Kings. Persson, Roland, and Tabellini (1997, 2000) model checks and balances as the separation of taxing and spending decisions within budgetary decision-makings. They argue that properly designed checks and balances improve the accountability of elected officials and thereby voter utility and limit rent-seeking by politicians. Keefer (2004) argues that “The absence of multiple veto players in countries often means that some groups in society are less represented than they otherwise would be.”⁴

In this paper, we study political mechanisms that ensure protection of individuals’ property. Checks and balances come in different forms such as the separation of powers between the

¹Alchian (2008) defines private property as “the exclusive authority to determine how a resource is used.”

²Recent research in political economy, however, has also pointed out that in some domains check and balances may lead to inefficiencies (e.g., see Diermeier and Myerson, 1999, on inefficiencies of bicameralism due to incentives to create restrictive internal procedures and grid-lock).

³Much of the literature on the political economy of checks and balances has focused on incentives for expropriation by a monarch and the associated commitment problem (e.g. North and Weingast 1989, Root 1989). Our approach allows us to consider the issue of democratic expropriation, e.g., the expropriation of a minority by a majority. This was one of the central concerns of, e.g., Montesquieu and Madison, [[and]] led to various power sharing arrangements from multi-cameralism to Federal political structures.

⁴In a recent paper, Acemoglu, Robinson, and Torvik (2011) have argued that this phenomenon can be used to explain why voters may sometimes dismantle checks and balances. Under checks and balances, politicians receive less rents through the political process which makes them more susceptible to be bribed by elites. Removing checks and balances leads to more rents, but better policy outcomes for the majority.

legislative, executive, and judicial branches of government, multi-cameralism, or federalism. However, at its core are institutional limits on majority rule, usually in the form of veto rights for individuals or groups, and supermajority requirements. Such limits allow individuals or collectives to block any redistribution without their consent. In our model, these serve as abstract representations of checks and balances.⁵ Those include a president with veto powers, a supreme court that can strike down a law as unconstitutional, or the Spartan *Gerousia*, the Council of Elders, that could veto motions passed by the *Apella*, the citizens' assembly (Plutarch [2010]). If we interpret property rights as institutions that sustain allocations unless changed by the legislature, we can formally investigate the effect of checks and balances on the allocation of property rights and prerogatives.⁶

Much of existing work has interpreted veto rights as *constraints* on collective choice processes. They correspond to individual or collective rights to block any undesirable reallocation. Often, though not always, they are formalized in constitutions or codes of law. Formally, they are features of the game-form. But property rights can be protected in other ways. That is, the protection of property rights can also be understood as equilibrium outcomes of interacting economic agents. The property rights of an individual may be respected in equilibrium not because he is entitled or powerful enough to protect them on his own, i.e. has veto power, but because others find it in their respective interest to respect such rights. Specifically, members of a coalition, formed in equilibrium, have an incentive to oppose the expropriation of each other because they know that once a member of the group is expropriated, others will be next in the line for expropriation. Such groups serve as *endogenous* blocking coalitions, i.e. collections of agents that *in equilibrium* can prevent any path that makes any of the members worse off. That is, an agent's incentive to protect a particular property rights system hinges on the rational foresight of what might happen once the redistributive process is initiated. This means that property rights systems can be stable even in the absence of explicit veto powers.⁷

Paradoxically, adding additional exogenous protection (e.g., by increasing the number of veto players) may lead to the break-down of an equilibrium with stable property rights (see Example 2), as the newly empowered agent (e.g., one who was granted or has acquired on his own veto power) now no longer as an incentive to protect others. A similar effect can occur with additional

⁵Some scholars have argued that veto player arrangements are the most important feature of political governance structures (e.g., Tsebelis 2002).

⁶While we use the language of "voting" and "vetos" which suggests a legislative process, our model can also be interpreted as power struggle between individuals and coalitions. Here a "veto" simply means that an individual has the military or economic means to block any redistribution without his consent.

⁷Similar effects can be observed in models of legislative bargaining in dynamic policy environments with a persistent proposer (Diermeier and Fong, 2011). In that model it is the fact that the persistent agenda setter, effectively the sole veto player, can always reconsider a proposal that induces a group of voters to "protect" each other's allocation.

super-majority requirements. Thus, by adding additional constraints, in the form of veto players or super-majority requirements, the protection property rights may inadvertently be *eroded*. In other words, agents' property rights may be well-protected in the absences of formal constraints, while strengthening formal constraints may worsen property rights protection. Consider the following example.

Example 1 *Five legislators decide how to split 15 units of wealth, with the status quo being (1, 2, 3, 4; 5). Legislator #5 is the sole veto player and reallocation requires a majority of votes. For the sake of simplicity we assume that when agents are indifferent, they support the proposer. In a standard legislative bargaining model in the tradition of Baron and Ferejohn (1989a) a proposer makes a proposal to the legislature and the game ends when the proposal is accepted. In this setting a proposer, say legislator #5, would simply build a coalition to expropriate two agents, say agents #3 and #4, and capture the surplus resulting in the allocation (1, 2, 0, 0; 12). But this logic does not hold in a dynamic model where the agreed upon allocation can be reallocated in the subsequent periods. That is, with the new status quo (0, 0, 2, 1; 12), agent #5 might propose to expropriate agents #1 and #2 by proposing (0, 0, 0, 0; 15), which is accepted in equilibrium. Anticipating this, agents #1 and #2 should not agree to the first expropriation, thus becoming the effective guarantors of property rights of agents #3 and #4.⁸*

To demonstrate which allocations are stable when further reallocations are possible, consider the simplest possible case. Allocation (0, 0, 0, 0; 15) is stable as agent #5 has veto power, while allocations where where a single agent among #1 – 4 has some wealth and the rest has zero (e.g., (0, 1, 0, 0; 14), (3, 0, 0, 0; 12), or (1, 1, 0, 0; 13)) are unstable, as the veto player can always use two of the remaining agents to redistribute all wealth to himself. Allocation (1, 1, 1, 0; 12), however, is stable: in order to improve his position, the veto player must either move to (0, 0, 0, 0; 15) or to an unstable allocation (which ultimately leads to the same result). For any such transition, the veto player will not get enough votes from players with sufficient foresight. Notice, however, that (1, 1, 1, 1; 11) is unstable: the veto player can expropriate one of the non-veto ones and gain consent of the remaining three as they know that there will be no further reallocations after the initial expropriation. In general, one can prove that allocation $(x_1, x_2, x_3, x_4; x_5)$ is stable if and only if among the four non-veto agents, three have the same wealth (positive or zero), and the remaining agent has zero.

This simple example points out to an important phenomenon, which is much more general. Protection of property rights may not be based on a formal set of rules, but emerge from

⁸We will see the later that the status quo allocation (4, 3, 2, 1; 5) is not stable either. [[The allocation (0, 3, 3, 3; 6), however, is stable.]] ((For example, there is an equilibrium where it leads to the allocation (0, 3, 3, 3; 6).))

equilibrium behavior of economic agents, whose commitment to protection of property rights hinges on rational foresight. The critical element is the observation that property rights are dynamic in nature. For example, the right to own a parcel of land means that the owner benefits from the income stream generated from the asset. In a political economy context this means that we can capture property rights as equilibrium phenomena that generate a stream of benefits to individuals according to a status quo allocation of assets. A status quo allocation of assets stays in place for the next period unless it is changed by the political decision mechanism in which case the newly chosen allocation becomes the status quo for the next period.

The logic of endogenous veto power explains how a small change in formal political rules might lead to a dramatic reallocation. In a historical context, this might help to explain why rulers often oppose even small concessions to the political opposition. Many monarchs from Charles I of England to Nicholas II of Russia, for instance, would not concede any of their formidable formal power until the very end (Schwarz and Sonin, 2009).

Our next example demonstrates the workings of this logic formally. A small change in political rights, an addition of a veto player, drastically changes the equilibrium: the process of redistribution will involve the majority of agents (with about one half of agents being expropriated) and the most part of the society's wealth.

Example 2 *To see this, let us modify Example 1. There are again five agents, and three votes are required to make a change, but now there are two veto players instead of one, agents #4 and #5. If only these two players may make a proposal, all allocations of the type $(0, 0, 0; x_4, x_5)$ are stable, and those of the type $(x_1, x_2, x_3; x_4, x_5)$, where one or two out of x_1, x_2, x_3 are zeroes, are unstable (the two veto players will get the vote of one agent with zero and redistribute the assets of the remaining two agents). But then any allocation such that $x_1 = x_2 = x_3 = 1$ is stable, because, within the realm of stable states, it is impossible to improve the welfare of the veto players without hurting all of non-veto players. Proceeding inductively, one can prove that an allocation is stable if and only if $x_1 = x_2 = x_3$. Thus, the allocation $(1, 1, 0; 1, 12)$ with two veto players, players #4 and #5, is unstable as $x_1 = x_2 \neq x_3$. The addition of a new veto player (agent #4) leads to expropriation of agents #1 and #2: if there is an offer to move to $(0, 0, 0; 3, 12)$, $(0, 0, 0; 2, 13)$, or $(0, 0, 0; 1, 14)$, there will be the majority of three, including both veto players, in support of this motion.⁹*

We see here an interesting phenomenon. The naive intuition would suggest that giving one extra player (agent #4 in this example) veto power would make it more difficult for agent #5 to

⁹We ignore for the moment the possibility that agent #5 might block moving to $(0, 0, 0; 2, 13)$ in hope that the eventual outcome is $(0, 0, 0; 1, 14)$ which player #5 prefers.

expropriate the rest of the group. Our analysis, however, shows that this is not the case. On the contrary, the introduction of a new veto player breaks the stable coalition of non-veto players, and makes #5, the agenda-setter, more powerful. Beforehand, non-veto players sustained an equal allocation, precisely because they were individually more vulnerable. With only one veto player and an equal allocation for agents #2, #3, and #4, the three non-veto players formed an endogenous veto group, which blocked any transition that hurts the group as a whole (even one of them). Here, an additional veto player makes expropriation more, not less, likely.

The example provides another important observation. With one veto player, #5, allocation $(1, 1, 0, 1; 12)$ is stable. Adding one veto player (say, agent #4) makes allocation $(1, 1, 0; 1, 12)$ unstable as agent #4 would now be willing to join #3 and #5 in expropriating #1–2. The critical role is played by #4: previously, he was opposing expropriation of #1–2 as he would have been vulnerable in the next round. Once he is safe as a veto player, he ceases to oppose the expropriation of #1–2. A resulting stable allocation is $(0, 0, 0; x, y)$, where $x \geq 1$, $y \geq 12$, and $x + y = 15$. Both the amount of wealth being redistributed and the number of agents affected by expropriation is significant. The number of agents who stand to lose is two, close to half the total number of agents, and a fifth of the total wealth is redistributed through voting (this last number would be even greater if the veto player #5 were poorer in the beginning).

Changes to the set of veto players is not the only significant change to consider. We can explore the impact of changing supermajority rules. Like veto players, supermajority rules are usually considered safeguards to make reallocation more difficult. As we will see in the next example, this intuition is flawed. Adding super-majority requirements may lead to additional redistribution.

Example 3 *As above, consider an economy with five agents, which makes redistributive decisions by majority, and one of which (#5) has veto power. With one veto player, #5, allocation $(1, 1, 1, 0; 12)$ is stable. Now, instead of a change in the number of veto agents, consider a change in the supermajority requirements. If a new rule requires four votes, rather than three, the status quo allocation becomes unstable. Instead, $(1, 1, 0, 0; 13)$ becomes stable, and this move is supported by coalition of four agents out of five. (The veto-player, #5, benefits from the move, #4 is indifferent as he gets 0 in both allocations, and #1–2 support this move as they realize that with the new supermajority requirement they form a group which is sufficient to protect its members against any expropriation.) Thus, an increase in supermajority may result in expropriation and redistribution.*

A naive view of supermajority requirements suggests that raising the supermajority requirement should strengthen property rights. As Example 3 demonstrates, this might produce the

opposite: a number of agents are expropriated. Proposition 5 below establishes that this phenomenon, as well the one discussed in Example 2, is generic: adding a veto player or raising the supermajority requirement almost *always* leads to a wave of redistribution.

To provide a general characterization of politically stable allocations of wealth, we use the approach developed in Acemoglu, Egorov, and Sonin (2008) and Acemoglu, Egorov, and Sonin (2012). In particular, we characterize the (Markov perfect) political equilibrium and the stable allocations as outcomes of a non-cooperative voting game. More precisely, we show that stable allocations correspond to the von Neumann-Morgenstern stable set, and may be found using a recursive algorithm. We then use this characterization to obtain the comparative statics results described above.¹⁰

Our results also provide a new perspective on the role of veto players on policy stability. The existing literature has focused on the role of veto players in a static environment. One approach has conceptualized veto players as constraints on majority rule in a social choice theoretic environment (e.g., Tsebelis, 2002). Another approach has adapted the sequential bargaining approach developed by Baron and Ferejohn (1989b). Examples include Romer and Rosenthal (1978), Diermeier and Myerson (1994), McCarthy (2000), Cameron and McCarthy (2004), and Persson, Roland, and Tabellini (1997, 2000). The policy environments in these models, however, are static: once a decision is made it is not revised and the chosen policy does not influence future decisions. (See Bernheim, Rangel, and Rayo, 2006, on the power of the last proposer in a legislative setting.)

Tsebelis (2002) summarizes the main insights of the standard (static) veto players theory on policy consequences as follows: “The veto players theory expects policy stability (impossibility of significant change of the status quo) to be caused by many veto players, by big ideological distances among them, by high qualified majority thresholds (or equivalents) in any collective veto player.”¹¹ Keefer (2004) asserts that “as the number of veto players increases, independent of their preferences, their incentives to offer favors to special interests diminishes,” and corroborates this with empirical evidence.

¹⁰Our analysis contains the results of Diermeier and Fong (2011) and Diermeier and Fong (2012) as a special case. Diermeier and Fong considered a single persistent proposer, who also, by definition, was the single veto player. We separate proposer and veto power, which allows us to study the effects of changing the number of veto players and super-majority rules. [[-]] ((paper hold if we relax the assumption that there is a single proposer.))

¹¹Spiller and Tomassi (2008, in Handbook of NIE) argue that “The main deterrent to stable policies is that government controls both the administrative and the legislative processes. Thus, political changes that bring about a change in government can also bring about legislative changes. By having few institutional checks and balances, such systems have an inherent instability that raises questions about their ability to provide regulatory commitment.” Making a more general point, Allston and Mueller (2008, in Handbook of NIE) state: “A set of universally shared beliefs in a system of checks and balances is what separates populist democracies from democracies with respect for the rule of law.” Gehlbach and Malesky (2010) provide a formal model where more numerous veto players might help a policy reform.

Our analysis shows that in a dynamic policy environment this intuition no longer holds. Examples of continuing policies include entitlements such as social security, monetary policy, but also allocations of land, property and privileges, and many others. In such environments, the policy or allocation remains in effect unless it is changed by the legislature. There is a growing literature in political economy that has investigated dynamic policy environments (Baron, 1996, Battaglini and Coate, 2007, 2008, Battaglini and Palfrey, 2007, Bowen and Zahran, 2012, Dekel, Jackson, and Wolinsky, 2009, Diermeier and Fong, 2011, Dixit, Grossman, and Gul, 2000, Duggan and Kalandrakis, 2011, Kalandrakis, 2004, 2007, Morelli, 1999). Our analysis shows that once we consider a dynamic policy environment policy stability should be viewed as an equilibrium phenomenon that is sustained by the long-run incentives of political agents. Adding additional veto players or strengthening super-majority requirements changes the dynamic equilibrium of the political economy and may lead to significant redistribution and policy change.

The remainder of the paper is organized as follows. Section 2 introduces our general model. In Section 3, we establish the existence of (pure-strategy Markov perfect) equilibrium in a non-cooperative game and provide full characterization of stable wealth allocations. Section 4 focuses on the impact of changes in the number of veto players or supermajority requirements. Section 5 contains an extension, in which collective decisions over wealth distribution are preceded by an investment stage.

2 Setup

Consider a set N of $n = |N|$ political agents who have to choose an outcome out of a finite set \mathbb{A} of feasible allocations. We assume that $\mathbb{A} \subset \mathbb{R}^n$, and we will use lower indices (x_i) to denote the amount that individual i gets under allocation $x \in \mathbb{A}$. Throughout, we focus on the case where agents redistribute b indivisible objects, so

$$\mathbb{A} = \left\{ x \in (\mathbb{N} \cup \{0\})^n : \sum_{i=1}^n x_i = b \right\}.$$

Time is discrete and indexed by $t > 0$, and in each period there is a status quo $x^t \in \mathbb{A}$. The initial status quo x^0 is given exogenously, while x^t for $t \geq 1$ is determined through a voting procedure in period t . More precisely, in period t an alternative x defeats x^{t-1} and becomes x^t if it gains the support of a winning coalition of agents.¹² In each period t , each agent i gets

¹²Acemoglu, Egorov and Sonin (2012) consider an environment where political power may depend on the state. The characterization results in this paper are made possible by keeping power allocation (i.e. the set of all possible winning coalitions in each state) constant, which is natural in [[a]] legislative bargaining context.

instantaneous utility x_i^t and acts as to maximize his continuation utility

$$U_i^t = x_i^t + \sum_{j=1}^{\infty} \beta^j x_i^{t+j},$$

where $\beta \in (0, 1)$ is a common discount factor. We will focus on the case where β is close to 1, i.e. individuals are sufficiently forward-looking.¹³

To define which coalitions are powerful enough to redistribute, we use the language of winning coalitions. Let $V \subset N$ be a non-empty set of veto players (denote $v = |V|$; without loss of generality, let us assume that V corresponds to the last v agents $n - v + 1, \dots, n$), and let $k \in [v, n]$ be a positive integer. A coalition W is *winning* if and only if (a) $V \subset W$ and (b) $|W| \geq k$. The set of winning coalitions is denoted by \mathcal{W} :

$$\mathcal{W} = \{X \in 2^N \setminus \{\emptyset\} : |X| \geq k \text{ and } V \subset X\}.$$

In this case, we say that the society is governed by a k -rule with veto players V , meaning that a transition is successful if it is supported by at least k players and no veto player opposes it. We will compare the results for different k and V . We maintain the assumption that there is at least one veto player—that V is non-empty—throughout the paper; this helps us capture various political institutions, e.g., a supreme court, but it is also helpful in ruling out cycles. We do not require that $k > n/2$, so we allow for minority rules. For example, 1-rule with the set of veto players $\{i\}$ is a dictatorship of legislator i .

The timing of the game below uses the notion of a protocol (e.g., Acemoglu, Egorov, and Sonin, 2012, Ray and Vohra, 2013). A protocol is a sequence (permutation) of legislators who are able to make proposals. We assume that only veto players are able to make proposals,¹⁴ so the sequence is described by the order of veto players, π_1, \dots, π_v (e.g., according to seniority or in alphabetical order). The protocol is the same for all states and defines both the sequence in which legislators make proposals and vote (both assumptions are non-consequential for the results). More precisely, the timing of the game in period $t \geq 1$ is the following.¹⁵

1. For $j = 1$, legislator π_j is recognized as an agenda-setter as described by the protocol (π_1, \dots, π_v) , and either proposes an alternative $z^j \neq x^{t-1}$ or passes (in which case we write $z^j = \emptyset$).

¹³This assumption is common in dynamic bargaining models and allows us to focus on long-run outcomes. See, e.g., Acemoglu, Egorov, and Sonin (2009, 2012) and Diermeier and Fong (2011).

¹⁴This assumption simplifies the analysis and formal statements considerably, but it is not necessary for the main results in this paper.

¹⁵There are many game forms that would yield to identical results. For example, we could have each agenda-setter nominate an alternative and then proceed to choosing one that will be put for a vote against the status-quo. To simplify the exposition and proofs, we opted for a simpler game.

2. If legislator j passed and $j < v$, the game moves to stage 1 with j increased by 1, and if $j = n$, then $x^t = x^{t-1}$ and the game moves to stage 5.
3. If $z_j \neq \emptyset$, then all legislators vote in a sequence given by protocol π , *yes* or *no*.
4. If the set of those who voted *yes*, Y^j , is a winning coalition, i.e. $Y^j \in \mathcal{W}$, then the new allocation is $x^t = z^j$, otherwise $x^t = x^{t-1}$.
5. Each legislator i receives an instantaneous payoff x_i^t .

The equilibrium concept we use in this game of full information is Markov Perfect equilibrium in pure strategies, with an additional requirement: we impose that when a legislator is indifferent between continuation payoffs given by the two subgames, he votes *yes*, and when he is indifferent between making a proposal and not, he does not.¹⁶ One can interpret it as that proposing a redistribution, as well as resisting a redistribution, is slightly costly, but passively approving it is not. We do not model cost of redistribution explicitly to simplify exposition and to save on notation.

3 Analysis

Our strategy is as follows. We start by proving some basic results about equilibria of the non-cooperative game described above. Then, we characterize stable allocations, i.e. allocations with no expropriations, and demonstrate that the stable allocations correspond to equilibria of the non-cooperative game. We then proceed to studying comparative statics with respect to the number of veto players, different voting rules (majority requirements), and equilibrium paths that follow an exogenous shock to some agent's wealth.

3.1 Non-cooperative Characterization

Every pure-strategy Markov Perfect Equilibrium (MPE) σ of the non-cooperative game gives rise to a transition mapping $\phi = \phi_\sigma$ (we will omit σ whenever it leads to no ambiguity). This mapping is defined as follows: If x^t is the allocation in period t , then the allocation in period $t + 1$ determined by the profile of equilibrium strategies σ is $x^{t+1} = \phi(x^t)$. The Markovian property ensures that this transition mapping is well-defined. The usefulness of the mapping ϕ is twofold: first, it allows us to capture equilibrium paths in terms of allocations and transitions rather than individuals' agenda-setting and voting strategies (i.e., more concisely), and second,

¹⁶This assumption is technical and allows for a simpler characterization of stable allocations. Without it, the main insights on stability and the effects of majority requirements would still hold, but would be less transparent.

it allows continuation utilities to be written in a very simple form. If allocation x^t is chosen in period t , then the continuation utility of legislator i from period t onwards is

$$U_i^t(x^t) = x^t + \sum_{j=1}^{\infty} \beta^j [\phi^j(x^t)]_i.$$

Iterating the mapping ϕ gives a sequence of mappings $\phi, \phi^2, \phi^3, \dots$, which must converge if ϕ is acyclic. (Mapping ϕ is *acyclic* if $x \neq \phi(x)$ implies $x \neq \phi^\tau(x)$ for any $\tau > 1$; we will show that every MPE satisfies this property.) Denote this limit by ϕ^∞ , which is simply ϕ^{t_0} for some t_0 as the set \mathbb{A} is finite. We say that mapping ϕ is *one-step* if $\phi = \phi^\infty$ (this is equivalent to $\phi = \phi^2$), and we call an MPE σ *simple* if ϕ_σ is one-step. Given an MPE σ , we call allocation x *stable* if $\phi_\sigma(x) = x$. Naturally, ϕ_σ^∞ maps any allocation into a stable allocation.

Our first result deals with existence of an equilibrium. The following theorem shows that an MPE exists for any protocol when the discount factor β is sufficiently high. More importantly, it demonstrates formally that our approach is essentially protocol-free and justifies our focus on simple MPE: for any two protocols, the fixed point of an equilibrium transition mapping under one protocol is a reshuffling of a fixed point of the equilibrium transition mapping of the other protocol.

Proposition 1 *There exists $\beta_0 < 1$ such that for any discount factor $\beta \in (\beta_0, 1)$:*

1. *For any protocol π , there exists a Markov Perfect Equilibrium σ . Moreover, there is a simple MPE.*
2. *Every MPE σ is acyclic. If a transition mapping χ is the limit of transition iterations under some MPE σ (i.e., $\chi = \phi_\sigma^\infty$ for some σ), then there is a simple MPE σ' such that $\chi = \phi_{\sigma'}$.*

Apart from existence, this result implies that we can, without any loss of generality, focus on simple equilibria. Indeed, they always exist (part 1) and they result in the same ultimate allocations that an MPE can support (part 2); moreover, we show in the Appendix that the set of mappings that may be supported by a simple MVE does not depend on the protocol σ . The proof of Proposition 1 is technically cumbersome and is relegated to the Appendix.

Proposition 1 shows existence of an equilibrium. The following Example 4 demonstrates that the equilibrium is not necessarily unique: there may be multiple equilibria. However, the set of stable allocations that may be reached with these equilibria is the same for any protocol.

Example 4 *Suppose there are $b = 3$ units of wealth, four agents, the required number of votes is $k = 3$, and the set of veto players is $V = \{\#4\}$. In this case, there is a simple equilibrium with*

transition mapping ϕ , under which allocations $(0, 0, 0; 3)$, $(1, 1, 0; 1)$, $(1, 0, 1; 1)$ and $(0, 1, 1; 1)$ are stable. Specifically, we have the following transitions: $\phi(2, 1, 0; 0) = \phi(1, 2, 0; 0) = (1, 1, 0; 1)$; $\phi(0, 2, 1; 0) = \phi(0, 1, 2; 0) = (0, 1, 1; 1)$; $\phi(2, 0, 1; 0) = \phi(1, 0, 2; 0) = \phi(1, 1, 1; 0) = (1, 0, 1; 1)$; and any allocation with $x_4 = 2$ has $\phi(x) = (0, 0, 0; 3)$. However, another mapping ϕ' coinciding with ϕ except that $\phi'(1, 1, 1; 0) = (1, 1, 0; 1)$ may also be supported in equilibrium.

3.2 Stable Allocations

Our next goal is to get a more precise characterization of equilibrium mappings and stable allocations. Let us define a binary relation \triangleright (interpreted as a “dominance” relation) on \mathbb{A} as follows:

$$x \triangleright y \iff \{i \in N : x_i \geq y_i\} \in \mathcal{W} \text{ and } x_j > y_j \text{ for some } j \in V.$$

Intuitively, allocation x dominates allocation y if some agenda-setter strictly prefers x to y so as to be willing to make this motion, and there is a winning coalition that (weakly) prefers x to y . Note that this does not imply that x will be proposed or supported in an actual voting against y because of further changes this move may lead to. Following the standard definition (von Neumann and Morgenstern, 1947, Greenberg, 1990), we call a set of states $\mathbf{S} \subset \mathbb{A}$ *von Neumann-Morgenstern- (vNM-)stable* if the following two conditions hold: (i) For no states $x, y \in \mathbf{S}$ it holds that $y \triangleright x$ (*internal stability*); and (ii) For each $x \notin \mathbf{S}$ there exists $y \in \mathbf{S}$ such that $y \triangleright x$ (*external stability*).

Below, we prove that in our case, the vNM-stable set is unique, and corresponds to the fixed points of transition mappings of non-cooperative equilibria described in Proposition 1. Let us denote $q = k - v$, the number of non-veto players that is required in any winning coalition; $d = m - q + 1 = n - k + 1$, the size of a minimal blocking coalition of non-veto players; and, finally, $r = \lfloor m/d \rfloor$, the maximum number of pair-wise disjoint blocking coalition that non-veto players may be split into.

Proposition 2 1. For the binary relation \triangleright , a vNM-stable set exists and is unique.

2. Each element x of this set \mathbf{S} has the following structure: the set of non-veto players $M = N \setminus V$ may be split into a disjoint union of r groups G_1, \dots, G_r of size d and one (perhaps empty) group G_0 of size $m - rd$, such that inside each group, the distribution of wealth is equal: $x_i = x_j = x_{G_k}$ whenever $i, j \in G_k$ for some $k \geq 1$, and $x_i = 0$ for any $i \in G_0$. In other words, up to a permutation of members in M :

$$x \in \mathbf{S} \iff x = \left(\underbrace{\lambda_1, \dots, \lambda_1}_{d \text{ times}}, \underbrace{\lambda_2, \dots, \lambda_2}_{d \text{ times}}, \dots, \underbrace{\lambda_r, \dots, \lambda_r}_{d \text{ times}}, \underbrace{0, \dots, 0}_{m-rd \text{ times}}; \underbrace{x_{m+1}, \dots, x_n}_{\text{veto players}} \right).$$

3. There exists $\beta_0 < 1$ such that for any discount factor $\beta \in (\beta_0, 1)$ and any MPE σ , $\phi_\sigma(x) = x \Leftrightarrow x \in \mathbf{S}$.

The proof of Parts 1 and 2 is important for understanding the structure of endogenous veto groups, and we prove it in the text (the proof of Part 3 is technical, and we relegate it to the Appendix). We show that starting from any wealth allocation $x \in \mathbf{S}$, it is impossible to redistribute the units between agents without making at least d agents worse off, and thus no redistribution would gain support from a winning coalition. In contrast, starting from any allocation $x \notin \mathbf{S}$, such redistribution is possible. Furthermore, our proof will show that in any transition, the number of individuals who are worse-off as a result is limited to the $d - 1$ richest non-veto players.

Proof of Proposition 2, Parts 1 and 2. We will prove that set \mathbf{S} , as defined in Part 2, is vNM-stable, thus ensuring existence. To show internal stability, suppose that $x, y \in \mathbf{S}$ and $y \triangleright x$, and let the r groups be G_1, \dots, G_r and H_1, \dots, H_r , respectively. Without loss of generality, we can assume that each set of groups is ordered so that x_{G_j} and y_{H_j} are non-increasing in j for $1 \leq j \leq r$. Let us prove, by induction, that $x_{G_j} \leq y_{H_j}$ for all j .

The induction base is as follows. Suppose that the statement is false and $x_{G_1} > y_{H_1}$; then $x_{G_1} > y_s$ for all $s \in M$. This yields that for all agents $i \in G_1$, we have $x_i > y_i$. Since the total number of agents in G_1 is d , G_1 is a blocking coalition, and therefore it cannot be true that $y_j \geq x_j$ for a winning coalition, contradicting that $y \triangleright x$.

For the induction step, suppose that $x_{G_l} \leq y_{H_l}$ for $1 \leq l < j$, and also assume, to obtain a contradiction, that $x_{G_j} > y_{H_j}$. Given the ordering of groups, this means that for any l, s such that $1 \leq l \leq j$ and $j \leq s \leq r$, $x_{G_l} > y_{H_s}$. Consequently, for agent $i \in \bigcup_{l=1}^j G_l$ to have $y_i \geq x_i$, he must belong to $\bigcup_{s=1}^{j-1} H_s$. This implies that for at least $jd - (j-1)d = d$ agents in $\bigcup_{l=1}^j G_l \subset M$, it cannot be the case that $y_i \geq x_i$, which contradicts the assumption that $y \triangleright x$. This establishes that $x_{G_j} \leq y_{H_j}$ for all j , and therefore $\sum_{i \in M} x_i \leq \sum_{i \in M} y_i$. But $y \triangleright x$ would require that $x_i \leq y_i$ for all $i \in V$ with at least one inequality strict, which implies $b = \sum_{i \in N} x_i < \sum_{i \in N} y_i = b$, a contradiction. This proves internal stability of set \mathbf{S} .

Let us now show that the external stability condition holds. To do this, we take any $x \notin \mathbf{S}$ and will show that there is $y \in \mathbf{S}$ such that $y \triangleright x$. Without loss of generality, we can assume that x_i is non-increasing for $1 \leq i \leq m$ (i.e., non-veto players are ordered from richest to poorest). Let us denote $G_j = \{(j-1)d + 1, \dots, jd\}$ for $1 \leq j \leq r$ and $G_0 = M \setminus \left(\bigcup_{j=1}^r G_j\right)$. Since $x \notin \mathbf{S}$, it must be that either for some G_j , $1 \leq j \leq r$, the agents in G_j do not get the same allocation,

or they do, but some individual $i \in G_0$ has $x_i > 0$. In the latter case, we define y by

$$y_i = \begin{cases} x_i & \text{if } i \leq dr \text{ or } i > m + 1; \\ 0 & \text{if } dr < i \leq m; \\ x_i + \sum_{j \in G_0} x_j & \text{if } i = m + 1 \end{cases}$$

(In other words, we take everything possessed by individuals in G_0 and distribute it among veto players, for example, by giving everything to one of them). Obviously, $y \in \mathbf{S}$ and $y \triangleright x$.

If there exists a group G_j such that not all of its members have the same amount of wealth, let j be the smallest such number. For $i \in G_l$ with $l < j$, we let $y_i = x_i$. Take the first $d-1$ members of group G_j , $Z = \{(j-1)d+1, \dots, jd-1\}$. Together, they possess $z = \sum_{i=(j-1)d+1}^{jd-1} x_i > (d-1)x_{jd}$ (the inequality is strict precisely because not all x_i in G_j are equal). Let us now take these z units and redistribute it among all the agents (perhaps including those in Z) in the following way. For each $s : j < s < r$, we let $y_{(s-1)d} = y_{(s-1)d+1} = \dots = y_{sd-1} = x_{(s-1)d}$; this makes these d agents having the same amount of wealth and being weakly better off as the agent with number $(s-1)d$ was the richest among them.

Now, observe that in each group s , we spend at most $(d-1)(x_{(s-1)d} - x_{sd-1}) \leq (d-1)(x_{(s-1)d} - x_{sd})$. For $s = r$, we take d agents as follows: $D = \{(r-1)d, \dots, m\} \cup Z'$, where $Z' \subset Z$ is a subset of the first $d - (m - (r-1)d + 1) = rd - m - 1$ agents needed to make D a collection of exactly d agents (notice that $Z' = \emptyset$ if $|G_0| = d-1$ and $Z' = Z$ if $G_0 = \emptyset$). For all $i \in D$, we let $y_i = x_{(r-1)d}$ (thus making all members of G_0 weakly better off and spending at most $(d-1)x_{(r-1)d}$ units) and we let $y_i = 0$ for each $i \in Z \setminus Z'$. We have thus defined y_i for all $i \in M$ and distributed

$$c \leq (d-1)(x_{jd} - x_{(j+1)d} + \dots + x_{(r-2)d} - x_{(r-1)d} + x_{(r-1)d}) = (d-1)x_{jd},$$

thus having $z - c > 0$ remaining in our disposal. As before, we let $y_{m+1} = x_{m+1} + z - c$ and $y_i = x_i$ for $i > m + 1$. We have thus constructed $y \in \mathbf{S}$ such that $y_{m+1} > x_{m+1}$ and $\{i \in N : y_i < x_i\} \subset Z$. The latter, given $|Z| \leq d-1$, implies $\{i \in N : y_i \geq x_i\} \in \mathcal{W}$, which means $y \triangleright x$. This completes the proof of external stability, and thus \mathbf{S} is vNM-stable.

Let us now show that \mathbf{S} is a unique stable set defined by \triangleright . Suppose not, so there is \mathbf{S}' that is also vNM-stable. Let us prove that $x \in \mathbf{S} \Leftrightarrow x \in \mathbf{S}'$ by induction on $\sum_{i \in M} x_i$. The induction base is trivial: if $x_i = 0$ for all $i \in M$, then $x \in \mathbf{S}$ by definition of \mathbf{S} . If $x \notin \mathbf{S}'$, then there must be some y such that $y \triangleright x$. But for such y ,

$$b = \sum_{i \in N} y_i \geq \sum_{i \in V} y_i > \sum_{i \in V} x_i = \sum_{i \in N} x_i = b,$$

a contradiction.

The induction step is as follows. Suppose that for some x with $\sum_{i \in M} x_i = j > 0$, $x \in \mathbf{S}$ but $x \notin \mathbf{S}'$ (the vice-versa case is treated similarly). By external stability of \mathbf{S}' , $x \notin \mathbf{S}'$ implies that for some $y \in \mathbf{S}'$, $y \triangleright x$, which in turn yields that $\sum_{i \in V} y_i > \sum_{i \in V} x_i$. We have

$$\sum_{i \in M} y_i = b - \sum_{i \in V} y_i < b - \sum_{i \in V} x_i = \sum_{i \in M} x_i = j.$$

For y such that $\sum_{i \in M} y_i < j$ induction yields that $y \in \mathbf{S} \Leftrightarrow y \in \mathbf{S}'$, and thus $y \in \mathbf{S}$. Consequently, there exists some $y \in \mathbf{S}$ such that $y \triangleright x$, but this contradicts $x \in \mathbf{S}$. This contradiction establishes uniqueness of the stable set. ■

The characterization of stable allocations obtained in Proposition 1 gives several important insights. First, the set of stable allocations (fixed points of any transition mapping under any MPE) does not depend on the mapping; it maps into itself when either the veto players V or the non-veto players $N \setminus V$ are reshuffled in any way. Second, the allocation of wealth among veto players does not have any effect on stability of allocations. Third, each stable allocation has a well-defined “class” structure: every non-veto player with a positive allocation is part of a group of size d (or a multiple of d) of equally-endowed individuals who have incentives to protect each other’s interests.¹⁷

To demonstrate how this protection works, consider the following example.

Example 5 *There are $b = 15$ units. Take $n = 5$ individuals with one veto agent (#5) and a supermajority requirement $k = 4$ (i.e., a supermajority of four-fifths is needed for a transition). Then $d = 2$ and $r = 2$, so stable allocations have two groups of size two. In particular, allocation $x = (4, 4, 2, 2; 3)$ is stable. Let ϕ be a transition mapping for some simple MPE σ .*

Suppose that we, exogenously, remove a unit from agent #2 and give it to the veto player; i.e., consider $y = (4, 3, 2, 2; 4)$. This resulting allocation is unstable, and agent #1 will necessarily be expropriated. The way redistribution may take place is not unique; for example, $\phi(y) = (3, 3, 2, 2; 5)$ is possible, but so are $\phi(y) = (2, 3, 3, 2; 5)$ or $\phi(y) = (2, 3, 2, 3; 5)$.

Now suppose that one of the agents possessing two units, say agent 3, was expropriated, i.e., take $z = (4, 4, 1, 2; 4)$. Then it is possible that the other member, agent 4, would be expropriated as well: $\phi(z) = (4, 4, 1, 1; 5)$. But it is also possible that one of the richer agents may be expropriated instead: e.g., a transition to $\phi(z) = (4, 1, 1, 4; 5)$ would be supported by all agents except agent #2.

¹⁷It is permissible that two groups have equal allocations, $x_{G_j} = x_{G_k}$, or that members of some or all groups get zero. In particular, any allocation x where $x_i = 0$ for all $i \in M$ is in \mathbf{S} . Notice that if non-veto players get the same under two allocations x and y , so $x|_M = y|_M$, then $x \in \mathbf{S} \Leftrightarrow y \in \mathbf{S}$; moreover, this is true if $x_i = y_{\pi(i)}$ for all $i \in M$ and some permutation π on M .

Example 5 demonstrates that equilibrium protection that agents provide to each other extends beyond members of the same economic class. In the latter case, agent #2 would oppose a move from $(4, 4, 2, 2; 3)$ to $(4, 4, 1, 2; 4)$ if the subgame the next move is to $(4, 1, 1, 4; 5)$. This corresponds to a coalition among economic classes.

We see that in general, an exogenous shock may lead to expropriation, on the subsequent equilibrium path, of agents belonging to different wealth groups; the particular path depends on the equilibrium mapping, which is not unique. However, if we apply the refinement that only equilibria with a “minimal” (in terms of the number of units that need to be transferred) redistribution along the equilibrium path are allowed, then only the agents with exactly the same wealth endowment would suffer from the redistribution that follows a shock.

Also, Example 5 demonstrates the mechanism of mutual protection among players with the same wealth. If a non-veto player becomes poorer, at least $d - 1$ agents other agents would suffer in the subsequent redistribution. This makes them willing to oppose any redistribution from any of their members. Their number, if we include the initial expropriation target himself, is d , which is sufficient to block a transition. Thus, members of the same economic class have an incentive to act as a politically cohesive coalition, in which its members mutually protect each others’ economic interests.

The next proposition generalizes Example 5 to establish the mutual protection result formally. It also highlights that protection of a non-veto player is sustained, in equilibrium, by equally endowed or richer individuals, rather than by those who has less wealth. Proposition 3 focuses on equilibrium consequences of a negative shock to some agent’s wealth, starting with a stable allocation.

Proposition 3 *Consider any MPE σ and let $\phi = \phi_\sigma$. Suppose that the voting rule is not unanimity ($k < n$), so $d > 1$. Take any stable allocation $x \in \mathbf{S}$, some non-veto player $i \in M$, and let new allocation $y \in \mathbf{A}$ be such $y|_{M \setminus \{i\}} = x|_{M \setminus \{i\}}$ and $y_i < x_i$. Then:*

1. *Player i will never be as well off as before the shock, but he will not get any worse off: $y_i \leq [\phi(y)]_i < x_i$. Furthermore, the number of agents that suffer as a result of a redistribution on the equilibrium path defined by σ is given by:*

$$\left| \left\{ j \in M \setminus \{i\} : [\phi(y)]_j < y_j \right\} \right| = d - 1;$$

2. *Suppose, in addition, that for any $k \in M$ with $x_k < x_i$, $x_k \leq y_i$, i.e., the shock did not make agent i poorer than the agents in the next wealth group. Then $[\phi(y)]_j < y_j$ implies $x_j \geq x_i$, i.e., members of poorer wealth groups do not suffer from redistribution.*

The essence of Proposition 3 is that following a negative shock to some agent's wealth ($y_i < x_i$), at least $d - 1$ other agents are expropriated, and agent i never fully recovers. If the shock is relatively minor so the ranking of agent i relative to other wealth groups did not change (weak inequalities are preserved),¹⁸ then it must be equally endowed or richer people who suffer from subsequent redistribution. Thus, in the initial stable allocation x , they have incentives to protect i from the negative shock. This result may be extended to the case when a negative shock affects more than one (but less than d) non-veto players. The proof is straightforward when all the affected agents belong to the same wealth group. However, this requirement is not necessary. If expropriated agents belong to different groups, then the lower bound of the resulting wealth after redistribution is the amount of wealth that the poorest (post-shock) agent possesses. In this case, the number of agents who suffer as a result of the redistribution following the shock is still limited by $d - 1$.

Our next step is to derive comparative statics with respect to different voting rules given by k and v .

4 Comparing Voting Rules

Suppose that we vary the supermajority requirement, k , and the number of veto players, v . The following result easily follows from the characterization in Proposition 2.

Proposition 4 *Fix the number of individuals n .*

1. *The size of each group G_j , $j \geq 1$, is decreasing as the majority requirement k increases. In particular, for $k = v + 1$, $d = n - v = m$, and thus all the non-veto players form a single group; for $k = n$ (unanimity rule), $d = 1$, and so each agent can veto any change.*
2. *The number of groups is weakly increasing in k , from 1 when $k = v + 1$ to m when $k = n$ (from 0 when $k < v + 1$).*
3. *The size of each group G_j , $j \geq 1$ does not depend on the number of veto players, but as v increases, the number of groups weakly decreases, reaching zero for $v > n - d$.*

The last result is most remarkable. The size of groups does *not* depend on the number of veto players, but only on the majority requirement as it determines the minimal size of blocking coalitions. As the majority requirement increases, groups become smaller. This has a very simple intuition: as redistribution becomes harder (it is necessary to get approval of more agents), it takes fewer non-veto players to defend themselves; as such, smaller groups are

¹⁸Note that this will always be the case if, e.g., $y_i = x_i - 1$.

sufficient. Conversely, the largest group (all non-veto players together) is formed when a single vote from a non-veto player is sufficient for veto players to accept a redistribution; in this case, non-veto players can only keep a positive payoff by holding equal amounts.

Example 6 *Suppose that $k = v + 1$; as before, $d = n - v$. In this case, an allocation x is stable if $x_i = x_j$ for all non-veto players i and j , i.e., if all non-veto players hold the same amount. This is consistent with Proposition 1 of Diermeier and Fong (2011), which shows this result for the special case $n = 3$, $v = 1$. More generally, a single group of non-veto players with positive amount of wealth may be formed if and only if $k - v \equiv q \leq (m + 1) / 2$. In this case, some $n - k + 1$ non-veto players belong to the group and get the same amount, and the rest get zero.*

One important implication of Proposition 4 is that k and v affect the economic heterogeneity of the society in equilibrium, at least for the non-veto players. Indeed, take n large and v small (so that m is large enough to be interesting) and start with the smallest possible value of $k = v + 1$. Then all the non-veto players possess the same allocation in any equilibrium. In other words, all agents, except perhaps those explicitly endowed with veto power, must be equal. If we increase k , then two groups will form, one of which may possess a positive amount, while the rest possesses zero, which is clearly more heterogenous than for $k = v + 1$. If we increase k further beyond $v + (m + 1) / 2$, then both groups may possess positive amounts and a third group will form further, etc. In other words, as k increases, so does the number of groups, which implies that the society becomes less and less homogenous and can support more and more groups of smaller size. We see that in this model, heterogeneity of the society is directly linked to difficulty of expropriation, measured by the degree of majority needed for expropriation or, equivalently, by the minimal size of a coalition that is able to resist attempts to expropriate. If we interpret the equally-endowed groups as economic classes, then we have the following result: *the more politically difficult it is to expropriate, the larger is the number of classes in a society.*

Proposition 4 dealt with comparing stable allocations for different k and v . We now study whether or not an allocation that was stable under some rules k and v remains stable if these rules change. For example, suppose that we make an extra individual a veto player (increase v), or increase the majority rule requirement (increase k). A naive intuition would say that in both these cases, individuals would not be worse off from better property rights protection. As the next proposition shows, in general, the opposite is likely to be true. Let $\mathbf{S}_{k,V}$ denote the set of stable allocations under the supermajority requirement k and the set of veto players V .

Proposition 5 *Suppose that allocation x is stable for k ($k < n$) and V ($x \in \mathbf{S}_{k,V}$). Then:*

1. *If we increase the number of veto players by granting an individual $i \notin V$ veto power so that the new veto set is $V \cup \{i\}$, then allocation $x \in \mathbf{S}_{k,V \cup \{i\}}$ if and only if $x_i = 0$;*

2. Suppose $k + 1 < n$ and all groups G_j , $j \geq 0$, had different amounts of wealth under x : $x_{G_j} \neq x_{G_{j'}}$, for $j' \neq j$ (and $x|_M \neq 0$). If we increase the majority requirement from k to $k' = k + 1$, and $k' < n$, then $x \notin \mathbf{S}_{k+1,V}$.

The first part of this proposition suggests that adding a veto player makes an allocation unstable, and therefore will lead to a redistribution hurting some individual. There is only one exception to this rule: if the new veto player had nothing to begin with, then the allocation will remain stable. On the other hand, if the new veto player had a positive amount of the good, then, while he will be weakly better off from becoming a veto player, there will be at least one other non-veto player who will be worse off. Indeed, removing a member of one of the groups G_j without changing the required sizes of the groups *must* lead to redistribution. This logic would not apply if $V' = N$, when all agents become veto players; however, the proposition is still true in this case because then i would have to be the last non-veto player, and under $k < n$ he would have to get $x_i = 0$ in a stable allocation x . Interestingly, removing a veto player i (making him non-veto) will also make x unstable as long as $x_i > 0$. This is, of course, less surprising, as this individual may be expected to be worse off.

The second part says that if all groups got different allocations (which is the typical case), then an increase in k would decrease the required group sizes, leading to redistribution. When some groups have equal amounts of wealth in a stable allocation, then allocation x may, in principle, remain stable. This is trivially true when all non-veto players get zero ($x_i = 0$ for all $i \notin V$), but, as the following Example 7 demonstrates, this is possible in other cases as well.

Example 7 Suppose $n = 7$, $V = \{\#7\}$, $b = 6$ and the supermajority requirement is $k = 5$. Then $x = (1, 1, 1, 1, 1, 1; 0)$ is a stable allocation, because $d = 3$ and the non-veto players form two groups of size three. If we increase k to $k' = 6$, then x remains stable, as then $d' = 2$ and x has three groups of size two.

5 Veto Power and Incentives to Invest

Our analysis so far has focused on the game of pure redistribution. This allowed us to identify two distinctive sources of property rights: legal rights (here modeled as exogenous veto power) and equilibrium property rights.¹⁹ In Section 3, we showed that the consequences of these rights might be different: in particular, in response to a shock, agents with equilibrium property rights may suffer, and this makes them protect each other on the equilibrium path. A natural

¹⁹Recall that all we mean by "legal rights" here is that they are modeled as exogenous veto power. No reallocation is possible without the consent of the current asset holder. As we discussed above such exogenous veto power can rest in explicit legal protections or in the ability to defend allocations unilaterally by force.

question is whether equilibrium property rights are different from legal ones when it comes to important economic activities such as production, investment, and, ultimately, the implications for economic growth. While a comprehensive analysis of these issues is, obviously, beyond the scope of the paper, we argue below that the distinction between two types of property rights has very real implications. To show why, we consider a very simple extension to the game of Section 2. Consider first the following example.

Example 8 *Take four agents $n = 4$ and assume that one of them is veto; $V = \{\#4\}$; let $b = 12$. The supermajority requirement is $k = 2$, i.e., it takes the veto player and one non-veto player to redistribute successfully. In this case, the size of the minimal blocking coalition of non-veto players is $d = 3$. Start with a stable allocation $x = (3, 3, 3, 3)$.*

Suppose that before the redistribution game is played, in period 0, some agents (a subset L) get an opportunity to work and produce: if an individual i works, he gets a disutility $c < 1$ from labor, but creates 1 unit of wealth that initially goes to him: $x_i^0 = x_i + 1$. Assume also that agents know who has a production opportunity and who does not.

The incentives of the veto player #4 are clear: he produces whenever he is given an opportunity to, because he will keep the product of his labor. Now, consider the incentives that agent #1 face. He is only going to keep the product of his labor if the other two non-veto players produce successfully, so that each of them gets 4 units. Therefore, he will not invest unless $L \subset \{1, 2, 3\}$, and even in that case, multiple equilibria are possible: with the three non-veto players investing and with the three of them not investing. The reason for this multiplicity is that under the threat of redistribution, production is a coordination game: a agent can retain what he produces only if all the agents in his group get richer; otherwise he gets expropriated. For example, if #1 is the only one who invested, the allocation is $x_i^0 = (4, 3, 3, 3)$, which will lead, after redistribution, to $x_i^t = (3, 3, 3, 4)$ from the next period on. This means that if the opportunity to produce comes to all the agents with equal probability and independently, then a veto player is ex ante strictly more likely to invest than a non-veto player – even though they may be equal in terms of the initial endowment.

Now let us modify the example by increasing the supermajority requirement to $k = 3$. In that case, $d = 2$; let us start with a stable allocation $\tilde{x} = (4, 4, 0, 4)$. Again, the veto player makes the efficient decision to produce: he does so whenever he has an opportunity. Agents #1 and #2 produce only if both of them have an opportunity to do so: indeed, if only one agent produces, he is going to be expropriated: e.g., if $\tilde{x}^0 = (5, 4, 0, 4)$, then redistribution may lead to $(4, 4, 0, 5)$ or $(0, 4, 4, 5)$, but in any case agent #1 would not keep the extra unit that he produces. In other words, agents #1 and #2 are again less likely to produce than the veto player. However, in this case, they do not need to have agent #3 producing in order to coordinate on production. This

implies that production by non-veto players is a more likely event when property rights are better defended: when it is harder to expropriate, coordination between fewer players is needed for a production decision to be undertaken.

The intuition developed in Example 8 holds more generally. To study the consequences of different types of property rights, we develop a simple growth model below.

For simplicity and tractability, let us focus on the case where $q \leq (m + 1)/2$, so that in any stable allocation, there is only one group of non-veto agents with positive allocation (see Example 6); in addition, assume that there is only one veto player. In this case, the condition $q \leq (m + 1)/2$ may then be rewritten as $k \leq n/2 + 1$. These assumptions ensure that the distribution of wealth is characterized by only two variables: the amount that the veto player possesses and the amount that non-veto players possess.

Assume that in the beginning every period (which starts with allocation x^{t-1} inherited from the previous period) each agent gets an opportunity to produce with probability p , independently across agents and periods; production requires effort $c > 0$. Let L denote the set of agents who have the opportunity to produce. All agents observe who belongs to L and make production decisions ($e_i^t \in \{0, 1\}$). After producing and observing the intermediate outcome \tilde{x}^t given by

$$\tilde{x}_i^t = x_i^{t-1} + e_i,$$

they play a within-period redistribution game as in Section 2, with the additional assumption that voting against a proposal costs an agent $\varepsilon \in (0, 1)$.²⁰ We are interested in Markov Perfect equilibria of the game with pure production strategies along the equilibrium path.

To obtain a precise characterization, let us furthermore assume that the initial allocation, x^0 , is stable in the corresponding game without production, i.e., $x^0 \in \mathbf{S}$, and the group structure among non-veto players is already established, namely, $x_i^0 > 1$ for some $i \in n \setminus V$ (let the set of such non-veto players be G). Finally, let us focus on MPEs with minimal redistribution on the equilibrium path: for every x , the difference $\|\phi_\sigma(x) - x\|$ achieves its minimum among all possible MPEs σ . The next example illustrates this concept.

Example 9 *Consider again Example 4: there are four agents, one of them (#4) has veto power, with $k = 3$ and $b = 3$. Then the equilibrium ϕ described there is minimal: in particular, $\phi((2, 1, 0; 0)) = (1, 1, 0; 1)$. On the other hand, if we take ψ equal to ϕ except that $\psi((2, 1, 0; 0)) = (0, 1, 1; 1)$, then ψ may be supported in a simple MPE, but is not minimal.*

²⁰In the basic setup, we assume that agents weakly prefer to agree with proposals, now we assume that they strictly prefer to do so.

Focusing on minimal equilibria appears to be a reasonable equilibrium selection criterion as the actual process of transferring units of wealth might involve some costs. Under the assumptions discussed above, we get the following results.

Proposition 6 *Suppose that the status-quo x^0 is a stable allocation, and let G be the subset of non-veto players with $x_i^0 > 1$. There exists $\beta_0 > 0$ such that for any discount factor $\beta \in (\beta_0, 1)$, there exists a threshold $p_0 > 0$ such that for any $p \in (0, p_0)$:*

- (i) *the veto player ($i = \#n$) produces whenever he has the opportunity ($i \in L$);*
- (ii) *there always exists a minimal MPE where non-veto players never produce. In any minimal MPE, non-veto players with zero endowment produce nothing: for $i \notin V$, $x_i^{t-1} = 0$ implies $e_i = 0$.*
- (iii) *in the equilibrium maximizing social welfare, non-veto players from group G produce whenever all of them have this opportunity, i.e., $G \subset L$ implies $e_i = 1$ for $i \in G$.*

In other words, veto players make an efficient decision to produce. As for non-veto players, there always exists an equilibrium where they do not invest at all. They may only invest if all members of their group have an opportunity to do so; only then may they coordinate on investment.

Proposition 6 yields an important conclusion: equilibrium property rights enjoyed by non-veto players create less incentives to invest, and ultimately lead to inefficient outcomes. For non-veto players whose property rights are just an equilibrium phenomenon, production effectively becomes a game of coordination. Producing alone does not make sense, as the successful producer would be expropriated along the equilibrium path. It is only worth the effort if there are multiple agents that will produce and will be able (and willing) to protect each other (and obviously, if the result of production was uncertain, the decision to produce would be even harder to coordinate on). As a result, non-veto players produce only if some other players produce, and therefore they do so less frequently than veto players do. It is straightforward to show that in the absence of redistribution, they would be more investment by non-veto players.

Let us now study comparative statics, focusing on the following variable of interest: $g = \max_{\sigma} \mathbb{E} \sum_{i \in N} e_i$, where the maximum is taken over all minimal MPEs (and this maximum does not depend on the period). This g has a natural interpretation as (average) economic growth.

Proposition 7 *Under the assumptions of Proposition 6, in the minimal MPE that maximizes social welfare:*

1. *the growth rate $g = \max_{\sigma} \mathbb{E} \sum_{i \in N} e_i$ is increasing in p , the rate at which production opportunities arrive;*

2. the growth rate g is increasing in k , the degree of property rights protection.

These comparative statics results are intuitive: the more often individuals have a chance to produce, the faster wealth accumulates. The smaller is the group of agents that needs to coordinate, the more likely they are to succeed in coordinating. This finding has important implications: the harder it is to expropriate, the more likely we have production or investment decisions, resulting in faster economic growth. Remarkably, this effect does not result from the actions of agents whose property rights are legally protected (veto players), but rather from those who only enjoy equilibrium property rights.

This analysis yields the following important insights. First, property rights have different implications for growth: the individuals who have legal rights are more likely to produce and to accumulate wealth than other agents. Non-veto players who possess property rights in equilibrium also can invest, but do so less frequently and only if they are able to coordinate. In other words, while the equilibrium effects we study in this paper are able to maintain a status quo, they are less effective in promoting growth and investment, especially if expropriation is relatively easy. Second, production decisions by individuals or groups of individuals may be different even if their observed wealth is the same: these decisions depend on whether their property rights are protected by law or are sustained in equilibrium by others' fear of being expropriated. These insights may be helpful for understanding allocations of wealth and means of production, economic growth, and social structure in a historical perspective.

6 Conclusion

The modern political economy literature on checks and balances has mainly concentrated on public spending with less attention being paid to the original motivation of protecting property rights. The reason may lie in the intuition that the relationship between additional checks and balances and better protection of rights seems so obvious that there is little left to explain. Yet, from a political economy perspective property rights systems are to be understood as equilibrium outcomes rather than exogenous fixed constraints. Legislators cannot commit to entitlements, prerogatives, and rights. Rather, any allocation must be maintained in equilibrium. By varying the characteristic of the political institutions (here modeled as checks and balances) one can assess the consequences for economic institutions (property rights).

Our approach suggests shows that a dynamic perspective may lead to a more subtle understanding of the effects of checks and balances such as veto players and super-majority rules. Traditionally, such features are viewed as constraints on collective choice; they are exogenous to political behavior. Our model shows that a dynamic environment may lead to the emergence of

endogenous veto groups: groups of players that sustain a stable allocation in equilibrium. Our analysis also shows that stable allocations give rise to an “economic classes” structure. That is, non-veto players with a positive allocation are part of a group of equally-endowed individuals who have incentives to protect each other’s interests. The effect of exogenous constraints on endogenous veto groups is complex. On the one hand, endogenous veto groups may protect each other in equilibrium even in the absence of formal veto rights. On the other hand, adding more veto players may lead to more instability and policy change if such additions upset dynamic equilibria where agents were mutually protecting each other.

Our results point to the importance of looking beyond formally defined property rights, and more, generally, beyond formal institutions. While formal institutions provide better incentives for investment and production, the incentives provided by informal equilibrium institutions are substantial as well. Thus, a change in formal institutions might strengthen protection of property rights of designated agents, yet have negative consequences for protection of property rights of the others, and, as a result, a negative overall effect.

References

- [1] Acemoglu, Daron, Georgy Egorov, and Konstantin Sonin. 2008. "Coalition Formation in Nondemocracies." *Review of Economic Studies*. 75(4): 987-1009.
- [2] Acemoglu, Daron, Georgy Egorov, and Konstantin Sonin. 2012. "Dynamics and Stability of Constitutions, Coalitions, and Clubs" *American Economic Review*. 102(4): 1446-1476.
- [3] Acemoglu, Daron, James Robinson, and Ragnar Torvik. 2013. Political Economy of Checks and Balances. *Review of Economic Studies*, forthcoming.
- [4] Baron, David P. 1996. "A Dynamic Theory of Collective Goods Programs." *American Political Science Review*, 90(2): 316-30.
- [5] Baron, David P. and John Ferejohn. 1989a. "Bargaining in Legislatures." *American Political Science Review*, 83(4): 1181-1206.
- [6] Baron, David P. and John Ferejohn. 1989b. "The Power to Propose." In *Models of Strategic Choice in Politics*, ed. Peter C. Ordeshook. Ann Arbor, MI: University of Michigan Press, pp.343-366.
- [7] Battaglini, Marco and Stephen Coate. 2007. "Inefficiency in Legislative Policy-Making: A Dynamic Analysis." *American Economic Review*, 97(1): 118-149.
- [8] Battaglini, Marco and Stephen Coate. 2008. "A Dynamic Theory of Public Spending, Taxation and Debt." *American Economic Review*, 98(1): 201-236.
- [9] Battaglini, Marco and Thomas R. Palfrey. 2007. "The Dynamics of Distributive Politics." Unpublished.
- [10] Bernheim, Douglas, Antonio Rangel, and Luis Rayo. 2006. "The Power of the Last Word in Legislative Policy Making." *Econometrica*, 74(5): 1161-1190.
- [11] Bowen, T. Renee and Zaki Zahran. 2012. "On Dynamic Compromise." *Games and Economic Behavior*, 76(2):391-419.
- [12] Dekel, Eddie, Matthew O. Jackson, and Asher Wolinsky. 2009. "Vote Buying: Legislatures and Lobbying." *Quarterly Journal of Political Science*, 4:103-128.
- [13] Diermeier, Daniel and Pohan Fong. 2011. "Legislative Bargaining with Reconsideration." *Quarterly Journal of Economics*, 126 (2): 947-985.

- [14] Diermeier, Daniel and Pohan Fong. 2012. "A Dynamic Theory of Parliamentary Democracy." *Economic Theory*, 49(3): 703-738.
- [15] Diermeier, Daniel and Roger B. Myerson. 1994. "Bargaining, Veto Power, and Legislative Committees," Northwestern University CMSEMS Working Paper No.1089.
- [16] Diermeier, Daniel and Roger B. Myerson. 1999. "Bicameralism and Its Consequences for the Internal Organization of Legislatures." *American Economic Review*, 89(5):1182-1196.
- [17] Dixit, Avinash, Gene M. Grossman, and Faruk Gul. 2000. "The Dynamics of Political Compromise." *Journal of Political Economy*, 108(3): 531-568.
- [18] Duggan, John and Tasos Kalandrakis. 2007. "Dynamic Legislative Policy Making." Wallis Institute of Political Economy Working Paper No.45.
- [19] Gehlbach, Scott and Edmund J. Malesky. 2010. "The Contribution of Veto Players to Economic Reform." *Journal of Politics* 72(4): 957-975.
- [20] Greenberg, Joseph. 1990. *The Theory of Social Situations: An Alternative Game-Theoretic Approach*, Cambridge Univ. Press, Cambridge, UK.
- [21] Hassler, John, Kjetil Storesletten, José V. Rodríguez Mora, and Fabrizio Zilibotti. 2003. "The Survival of the Welfare State." *American Economic Review* 93(1): 87-112.
- [22] Kalandrakis, Tasos. 2004. "A Three-Player Dynamic Majoritarian Bargaining Game." *Journal of Economic Theory*, 16(2): 294-322.
- [23] Kalandrakis, Tasos. 2007. "Majority Rule Dynamics with Endogenous Status Quo." Wallis Institute of Political Economy Working Paper No.46.
- [24] Maskin, Eric and Jean Tirole. 2001. "Markov Perfect Equilibrium I. Observable Actions." *Journal of Economic Theory*, 100(2): 191-219.
- [25] Morelli, Massimo. 1999. "Demand Competition and Policy Compromise in Legislative Bargaining." *American Political Science Review*, 93(4): 809-820.
- [26] Riker, William. 1987. *The Development of American Federalism*, Boston: Kluwer Academic Publishers.
- [27] Root, Hilton L. 1989. "Tying the King's Hands: Credible Commitments and Royal Fiscal Policy during the Old Regime." *Rationality and Society*. 1(2): 240-258.

- [28] Romer, Thomas and Howard Rosenthal. 1978. "Political Resource Allocation, Controlled Agendas, and the Status Quo." *Public Choice*, 33(4): 27-43.
- [29] Schwarz Michael and Konstantin Sonin. 2008. "A Theory of Brinkmanship, Conflicts, and Commitment." *Journal of Law, Economics, and Organization*. 24(1):163-183.
- [30] Tsebelis, George. 2002. *Veto Players: How Political Institutions Work*, Princeton Univ. Press.
- [31] von Neumann, John and Oskar Morgenstern. 1947. *Theory of Games and Economic Behavior*, Princeton University Press, Princeton.

Appendix

Proofs

Our strategy of proof is the following. First, we prove three auxiliary lemmas; then, we complete the proof of Proposition 2. After that, we prove Proposition 1.

Let $\beta_0 = \left(\frac{|\mathbf{A}|}{|\mathbf{A}|+1}\right)^{\frac{1}{|\mathbf{A}|}}$. We show that for $\beta \in (\beta_0, 1)$, and any acyclic mapping ϕ , if an individual prefers the ultimate allocation $\phi^\infty(x)$ to $\phi^\infty(y)$, then he prefers the continuation utility starting from x to one starting from y .

Lemma 1 *Suppose $\beta \in (\beta_0, 1)$. Then for any acyclic mapping ϕ , any agent $i \in N$ and any allocations $x, y \in \mathbf{A}$, $[\phi^\infty(x)]_i > [\phi^\infty(y)]_i$ implies $U_i(x) > U_i(y)$.*

Proof of Lemma 1. Simple algebra shows that β satisfies $(1 - \beta^{|\mathbf{A}|})|\mathbf{A}| > \beta^{|\mathbf{A}|}$. Now, since ϕ is acyclic, we have $\phi^\infty = \phi^t$ for $t \geq |\mathbf{A}|$. Using these two results, we get

$$\begin{aligned} U_i(x) &= \left(x_i + [\phi(x)]_i + \dots + [\phi^{|\mathbf{A}|-1}(x)]_i\right) + \frac{\beta^{|\mathbf{A}|}}{1 - \beta} [\phi^\infty(x)]_i \\ &\geq \frac{\beta^{|\mathbf{A}|}}{1 - \beta} [\phi^\infty(x)]_i > \frac{1 - \beta^{|\mathbf{A}|}}{1 - \beta} |\mathbf{A}| + \frac{\beta^{|\mathbf{A}|}}{1 - \beta} ([\phi^\infty(x)]_i - 1) \\ &\geq \frac{1 - \beta^{|\mathbf{A}|}}{1 - \beta} |\mathbf{A}| + \frac{\beta^{|\mathbf{A}|}}{1 - \beta} [\phi^\infty(y)]_i \\ &\geq \left(y_i + [\phi(y)]_i + \dots + [\phi^{|\mathbf{A}|-1}(y)]_i\right) + \frac{\beta^{|\mathbf{A}|}}{1 - \beta} [\phi^\infty(y)]_i = U_i(y). \end{aligned}$$

■

To formulate the next lemma, we introduce the following notation. Take any (not necessarily simple) MPE σ ; then for each status quo x there is a set of stages $J_x \subset \{1, \dots, v\}$ such that if stage $j \in J_x$ is reached, agent $\pi_{x,j}$ proposes an alternative $z_{x,j} \neq \emptyset$, and it is accepted in subsequent voting.

Lemma 2 *Suppose $\beta > \beta_0 = \left(\frac{|\mathbf{A}|}{|\mathbf{A}|+1}\right)^{\frac{1}{|\mathbf{A}|}}$. For any (acyclic) MPE σ and any $x \in \mathbf{A}$:*

1. if proposal y is accepted at some stage j , then the set of agents $\{i \in N : [\phi_\sigma^\infty(y)]_i \geq [\phi_\sigma^\infty(x)]_i\} \in \mathcal{W}$;
2. if proposal y is accepted at some stage j , then for any $i \in V$, $U_i(y) \geq U_i(x)$;
3. the set of agents $\{i \in N : U_i(\phi_\sigma(x)) \geq U_i(x)\} \in \mathcal{W}$;
4. the set of agents $\{i \in N : [\phi_\sigma^\infty(x)]_i \geq x_i\} \in \mathcal{W}$;

5. either $\phi_\sigma^\infty(x) = x$ or $\phi_\sigma^\infty(x) \triangleright x$.

Proof of Lemma 2. Part 1. Suppose not; then take any $i \in N$ for whom the opposite holds: $[\phi_\sigma^\infty(y)]_i < [\phi_\sigma^\infty(x)]_i$. By Lemma 1, this must imply $U_i(y) < U_i(x)$. Hence, the set $\{i \in N : U_i(y) \geq U_i(x)\} \notin \mathcal{W}$. But this implies that y cannot be the outcome of the voting at that stage, which contradicts the assumption.

Part 2. This is trivial: if $U_i(y) < U_i(x)$ and y is accepted in equilibrium, it means that agent i votes *Yes*, but it would be a best response to vote *No* and thus veto the proposal, which cannot be true in an equilibrium.

Part 3. This statement is trivial if $\phi_\sigma(x) = x$. Otherwise, $J_x \neq \emptyset$ and at the stage $j = \min J_x$, the proposal $z_{x,j} = \phi_\sigma(x)$ is made and is accepted. Each agent i gets utility $U_i(\phi_\sigma(x))$ if it is accepted and utility $U_i(x)$ if it is not. Assuming, to obtain a contradiction, that $\{i \in N : U_i(\phi_\sigma(x)) \geq U_i(x)\} \notin \mathcal{W}$, would imply that the proposal cannot be accepted in equilibrium; this proves the result.

Part 4. The statement is trivial if $\phi_\sigma(x) = x$, so consider the case $\phi_\sigma(x) \neq x$. Consider a mapping $\phi_0 : \mathbf{A} \rightarrow \mathbf{A}$ given by $\phi_0(y) = \phi(y)$ if $y \neq x$ and $\phi_0(x) = x$; since ϕ is acyclic, ϕ_0 is acyclic, too. Consider an individual $i \in N$ such that $[\phi_\sigma^\infty(x)]_i < x_i$. Since $\phi_0^\infty(\phi_\sigma(x)) = \phi_\sigma^\infty(x)$ and $\phi_0^\infty(x) = x$, then applying Lemma 1 to mapping ϕ_0 yields $U_i(\phi_\sigma(x)) < x_i / (1 - \beta)$. Rearranging, we have $U_i(\phi_\sigma(x)) < x_i + \beta U_i(\phi_\sigma(x))$, which is equivalent to $U_i(\phi_\sigma(x)) < U_i(x)$. Thus, $\{i \in N : [\phi_\sigma^\infty(x)]_i < x_i\} \subset \{i \in N : U_i(\phi_\sigma(x)) < U_i(x)\}$, and hence $\{i \in N : [\phi_\sigma^\infty(x)]_i \geq x_i\} \supset \{i \in N : U_i(\phi_\sigma(x)) \geq U_i(x)\}$. By Part 3, the latter is winning, which, coupled with monotonicity, completes the proof.

Part 5. If $\phi_\sigma^\infty(x) \neq x$, then the set $J_x \neq \emptyset$. Assume, to obtain a contradiction, that $\phi_\sigma^\infty(x) \not\triangleright x$. Since $\{i \in N : [\phi_\sigma^\infty(x)]_i \geq x_i\} \in \mathcal{W}$ by Part 4, it must be that for all $i \in V : [\phi_\sigma^\infty(x)]_i = x_i$. Consider the agent $k = \pi_{\min J_x} \in V$ who makes the proposal at the stage $\min J_x$. Let us prove that $[\phi_\sigma^l(x)]_k = x_k$ for all $l \geq 1$. Indeed, suppose first that some l , $[\phi_\sigma^l(x)]_k > x_k$. But then Part 4 (given that $k \in V$) would imply that $[\phi_\sigma^\infty(x)]_k = [\phi_\sigma^\infty(\phi_\sigma^l(x))]_k \geq [\phi_\sigma^l(x)]_k > x_k$, contradicting the earlier result. Now suppose that $[\phi_\sigma^l(x)]_k \leq x_k$ for all l , but $[\phi_\sigma^l(x)]_k < x_k$ for some l . But in that case, $U_k(\phi_\sigma(x)) < x_k / (1 - \beta)$, implying $U_k(\phi_\sigma(x)) < U_k(x)$, which contradicts the result of Part 2. Hence, $[\phi_\sigma^l(x)]_k = x_k$ for all $l \geq 1$. This implies that $U_k(\phi_\sigma(x)) = U_k(x) = x_k / (1 - \beta)$.

Consider the utility of agent k if he passes on his chance to propose. If $|J_x| = 1$, then in that stage x remains the status quo, and k gets continuation utility $U'_k = x_k + \beta U_k(x) = x_k + \beta x_k / (1 - \beta) = x_k / (1 - \beta) = U_k(x)$. If $|J_x| > 1$, then some proposal $y \neq x$ is accepted, and from Part 2 we know that in this case, $U'_k = U_k(y) \geq U_k(x)$. In either case, $U'_k \geq U_k(x)$, so his out-of-equilibrium continuation utility if he passes is at least as high as his utility from

making the proposal. In other words, he does not strictly prefer to make the proposal, which is impossible in the set of equilibria that we consider. This contradiction completes the proof. ■

Lemma 3 *Any MPE σ is acyclic.*

Proof of Lemma 3. Part 1. Suppose that there is a cycle starting from x : $\phi_\sigma(x) \neq x$, but $\phi_\sigma^l(x) = x$ for some $l > 1$. Similarly to Part 2 of Lemma 2, we can prove that for any $i \in V$, $U_i(\phi_\sigma^{j+1}(x)) \geq U_i(\phi_\sigma^j(x))$ for any j , which means that $U_i(x) = U_i(\phi_\sigma(x)) = \dots = U_i(\phi_\sigma^{l-1}(x))$, and therefore $x_i = [\phi_\sigma(x)]_i = \dots = [\phi_\sigma^{l-1}(x)]_i$. Consider the agent $k = \pi_{\min J_x}$; as in the proof of Part 5 of Lemma 2, we can show that by passing on the opportunity to make his proposal, he can guarantee $U'_k \geq x_k/(1-\beta) = U_k(\phi_\sigma(x)) = U_k(x)$ which he gets if he follows the equilibrium play. But this means that he is not strictly better off by making a proposal, which is impossible. This contradiction completes the proof. ■

Proof of Proposition 2. Part 1, Part 2: Proven in the text.

Part 3. We let $\beta \in (\beta_0, 1)$, with β_0 defined above. Let $D = \{x \in \mathbf{A} : \phi_\sigma(x) = x\}$; we then must prove that $D = S$. To do so, we prove that D is vNM-stable with respect to the binary relation \triangleright ; given the results of Part 1 and Part 2, this will imply that $D = S$. We need to show that D satisfies internal stability and external stability. We also know, by Lemma 3, that ϕ_σ is acyclic, and thus mapping ϕ_σ^∞ is well-defined and D is nonempty.

External stability: Take any $x \notin D$. Let $y = \phi_\sigma^\infty(x) \neq x$ (because ϕ_σ is acyclic). By Part 5 of Lemma 2, $y \triangleright x$, which proves external stability.

Internal stability: Suppose, to obtain a contradiction, that for some $x, y \in D$, $y \triangleright x$. We know that if the current state is x , no alternative is accepted (because $x \in D$), and thus $|J_x| = \emptyset$. Let us show that this means that at no stage any agent makes a proposal. Indeed, suppose that at stage j , agent π_j makes a proposal that is rejected. Then equilibrium play gives him $U_{\pi_j}(x) = x_{\pi_j}/(1-\beta)$, whereas if he passes, then, by Part 2 of Lemma 2, he will get $U'_{\pi_j} \geq U_{\pi_j}(x)$, which means that proposing is not strictly better than passing. Given our equilibrium refinement, this means that no proposal is made at any stage when the state is x .

Now take $k \in V$ that satisfies $y_k > x_k$ and consider stage j with $\pi_j = k$. If, at this stage, agent k made proposal y , then it would be accepted, because $y \triangleright x$ means, in particular, that $\{i \in N : y_i \geq x_i\} \in \mathcal{W}$, and thus $\{i \in N : U_i(y) \geq U_i(x)\} \in \mathcal{W}$ (as, of course, $U_i(y) = y_i/(1-\beta)$ and $U_i(x) = x_i/(1-\beta)$). We showed that in equilibrium, k does not make any proposal in this subgame, and gets $U_k(x)$. But then he would be able to get more by proposing y , as $U_k(y) = y_k/(1-\beta) > x_k/(1-\beta) = U_k(x)$. Hence, he has a profitable deviation in this

subgame, which is impossible. This contradiction proves that D is also internally stable. Thus, it is vNM-stable, and given uniqueness, coincides with S . This completes the proof. ■

Proof of Proposition 1. Part 1. Let us prove a stronger result: for any mapping ϕ such that $\phi(x) = x$ for any $x \in \mathbf{S}$ and for any $x \notin \mathbf{S}$, $\phi(x) \in \mathbf{S}$ and $\phi(x) \triangleright x$. (Existence of such a mapping follows from external stability of mapping \mathbf{S} implying that for any \mathbf{S} we can pick such $\phi(x) \in \mathbf{S}$; clearly, equilibrium σ with $\phi_\sigma = \phi$ will be simple because $\phi^2 = \phi$.)

To construct σ , take the following strategy profile. First, define continuation utilities of each agent i if the current period ends with state x , $U_i(x) = x_i + \sum_{t=1}^{\infty} [\phi^t(x)]_i$. If the current state is $x \in \mathbf{A} \setminus \mathbf{S}$, the last agenda-setter i with $[\phi(x)]_i > x_i$ proposes a transition to $\phi(x)$, and all agents $[\phi(x)]_j \geq x_j$ support this proposal while the rest vote against it, so it is accepted in equilibrium. Other agenda-setters — earlier and later — do not propose. (We do not need to define voting strategies for these proposals if they are made, or for any other proposal made by agent i , explicitly: the continuation equilibrium play is defined for any outcome of any given voting, either the proposal y or the status quo x , and thus the rule that agents support the proposal y whenever $U_i(y) \geq U_i(x)$ gives a unique set of strategies.) If the current state is $x \in \mathbf{S}$, then no agenda-setter makes a proposal; again, this defines voting strategies if some proposal is made. This set of strategies (call it σ) is clearly Markovian. We need to check that this is indeed an equilibrium, i.e., that there is no one-step deviation, and that it satisfies the requirements we imposed on agents' actions when they are indifferent.

If strategies in σ are played, mapping $\phi_\sigma = \phi$ is implemented, and therefore continuation utilities are indeed given by $U_i(x)$. This automatically implies that all voting strategies that we specified are best responses; moreover, we required agents to vote *Yes* whenever they are indifferent. We only need to consider agenda-setting strategies. First, suppose $x \in \mathbf{A} \setminus \mathbf{S}$, and agent j gets a chance to propose after i . By the choice of agent i , it must be that $[\phi(x)]_j \leq x_j$ (in fact, equality must hold), so $U_j(\phi(x)) \leq U_j(x_j)$, and the agent i is not better off making the proposal $\phi(x)$. Suppose he makes some other proposal y ; this is a profitable deviation only if y is to be accepted (otherwise, passing yields the same utility). If y is accepted, Part 1 of Lemma 2 also implies that $\{k \in N : [\phi(y)]_k \geq [\phi(x)]_k\} \in \mathcal{W}$ (because mapping ϕ is simple). Since j is strictly better off from this deviation, it must be that $U_j(y) > U_j(x) = x_j / (1 - \beta)$, which means that either $y_j > x_j$ or $[\phi(y)]_j > x_j$; in either case, $[\phi(y)]_j > x_j = [\phi(x)]_j$. Now, we have shown that $\phi(y) \triangleright \phi(x)$, but this is impossible for $\phi(x), \phi(y) \in \mathbf{S}$.

Let us now suppose that agent j gets a chance to propose before agent i . Then he is not strictly better off proposing $\phi(x)$, because i would do so along the equilibrium path if j passes. Suppose he proposes some $y \neq \phi(x)$; then again it is only profitable if y passes. For this to be

true, it must be that $\{k \in N : [\phi(y)]_k \geq [\phi(x)]_k\} \in \mathcal{W}$. But if j is strictly better off, it must be that $[\phi(y)]_j > [\phi(x)]_j$, again implying $\phi(y) \triangleright \phi(x)$, which cannot be true.

Consider the possibility that agent i deviates. He does not want to deviate by passing, since $[\phi(x)]_i > x_i$ and thus $U_i(\phi(x)) > U_i(x)$. Suppose that he deviates to proposing some $y \neq \phi(x)$; again, $\phi(x)$ must be accepted. We analogously get that $\{k \in N : [\phi(y)]_k \geq [\phi(x)]_k\} \in \mathcal{W}$ and $[\phi(y)]_j > [\phi(x)]_j$, leading to the same contradiction. Thus, there is no profitable deviation if $x \in \mathbf{A} \setminus \mathbf{S}$.

Finally, suppose that for some $x \in \mathbf{S}$, some agent $j \in V$ deviates and proposes some $y \neq x$. For this to be profitable, y must be accepted, which would imply, again by Part 1 of Lemma 2, that $\{k \in N : [\phi(y)]_k \geq x_k\} \in \mathcal{W}$. Again, similarly to earlier cases, we must have $[\phi(y)]_j \geq x_j$ if the deviation is to be profitable. But then $\phi(y) \triangleright x$ for $x, \phi(y) \in \mathbf{S}$, which is impossible. This contradiction completes the proof that σ is a MPE and that it satisfies the refinements.

Part 2. Acyclicity of any MPE was proved in Lemma 3. Now take mapping $\chi = \phi_\sigma^\infty$ for some σ . From Part 3 of Proposition 2 it follows that $\phi_\sigma(x) = x \Leftrightarrow x \in \mathbf{S}$; since σ is acyclic, $\chi(x) = x \Leftrightarrow x \in \mathbf{S}$. By Part 5 of Lemma 2, $\phi_\sigma^\infty(x) = x$ or $\phi_\sigma^\infty(x) \triangleright x$, which implies that for any $x \notin \mathbf{S}$, $\chi(x) \triangleright x$. Consequently, mapping χ satisfies the conditions specified in the proof of Part 1, and there is a simple equilibrium σ' with $\phi_{\sigma'} = \chi = \phi_\sigma^\infty$. This proves Part 2. ■

Proof of Proposition 3. Part 1. Lemma 2 implies that $\phi(y) \triangleright y$; in particular, for each $j \in V$, $[\phi(y)]_j \geq y_j$ and for at least one of them the inequality is strict. Suppose, to obtain a contradiction, that $\left| \left\{ j \in M \setminus \{i\} : [\phi(y)]_j < y_j \right\} \right| < d - 1$; then $\left| \left\{ j \in M : [\phi(y)]_j < x_j \right\} \right| < d$. But we also have that for each $j \in V$, $[\phi(y)]_j \geq x_j$, with at least inequality strict. This means $\phi(y) \triangleright x$, which is impossible, given that $x, \phi(y) \in \mathbf{S}$. Now suppose, to obtain a contradiction, that $\left| \left\{ j \in M \setminus \{i\} : [\phi(y)]_j < y_j \right\} \right| > d - 1$. But then for at least d agents $[\phi(y)]_j < y_j$, which contradicts $\phi(y) \triangleright y$. This contradiction proves that $\left| \left\{ j \in M \setminus \{i\} : [\phi(y)]_j < y_j \right\} \right| = d - 1$. It remains to prove that $y_i \leq [\phi(y)]_i < x_i$. Suppose not, i.e., either $[\phi(y)]_i < y_i$ or $[\phi(y)]_i \geq x_i$. In the first case, we would have that at least d agents have $[\phi(y)]_j < y_j$, contradicting $\phi(y) \triangleright y$. In the second case, $[\phi(y)]_i \geq x_i$, coupled with the already established $\left| \left\{ j \in M \setminus \{i\} : [\phi(y)]_j < y_j \right\} \right| = d - 1$, would mean $\left| \left\{ j \in M : [\phi(y)]_j < x_j \right\} \right| = d - 1$, and therefore $\phi(y) \triangleright x$. This is impossible, and this contradiction completes the proof.

Part 2. This proof is similar to the proof of internal stability in the proof of Proposition 2. Denote $\phi(y) = z$; then $z \triangleright y$ and $x, z \in \mathbf{S}$. We know that x and z have the group structure by Part 2 of Proposition 2; then let the r groups be G_1, \dots, G_r for x and H_1, \dots, H_r for z , respectively. Without loss of generality, we can assume that each set of groups are ordered so that x_{G_j} and z_{H_j} are nonincreasing in j for $1 \leq j \leq r$. Suppose, to obtain a contradiction, that

for some agent $i' \in M$ with $x_{i'} \leq y_i < x_i$, $z_{i'} < y_{i'}$. In that case, among the set $\{j \in M : x_j \geq x_i\}$ there are at most $d - 1$ agents with $z_j < y_j$; similarly, among the set $\{j \in M : x_j < x_i\}$ there are at most $d - 1$ agents with $z_j < y_j$.

We can now proceed by induction, similarly to the proof of Proposition 2, and show that $x_{G_j} \leq z_{H_j}$ for all j . Base: suppose not, then $x_{G_1} > z_{H_1}$; then $x_{G_1} > z_s$ for all $s \in M$. But this mean that for all agents $l \in G_1$ have $x_l > z_l$; since their total number is d , we get a contradiction. Step: suppose $x_{G_l} \leq z_{H_l}$ for $1 \leq l < j$, and suppose, to obtain a contradiction, that $x_{G_j} > z_{H_j}$. Given the ordering of groups, this means that for any l, s such that $1 \leq l \leq j$ and $j \leq s \leq r$, $x_{G_l} > z_{H_s}$. Consequently, for a agent $i'' \in \bigcup_{l=1}^j G_l$ to have $z_{i''} \geq x_{i''}$, he must belong to $\bigcup_{s=1}^{j-1} H_s$. This implies that for at least $jd - (j - 1)d = d$ agents in $\bigcup_{l=1}^j G_l \subset M$, $z_{i''} \geq x_{i''}$ does not hold (denote this set by D . If that is true, it must be that $\bigcup_{l=1}^j G_l$ includes all the agents in D , including agents i and i' found earlier, and in particular, $x_{G_j} \leq y_i < x_i$. But on the other hand, these d agents are not in $\bigcup_{s=1}^{j-1} H_s$. In particular, this implies that for any $i'' \in D$, $z_{i''} < x_{G_j}$, but $x_{i''} \geq x_{G_j}$, which means $z_{i''} < x_{i''}$. But $z_{i''} \geq y_{i''}$ by Part 1 of this Proposition, so $y_{i''} < x_{i''}$. But this contradicts with the way we chose i' to satisfy $x_{i'} \leq y_i < x_i$. This proves that such i'' cannot exist, and thus the $d - 1$ agents other than i who are made worse off satisfy $x_j \geq x_i$. ■

Proof of Proposition 4. This result immediately follows from the formulas $m = n - v$, $d = n - k + 1$, $r = \lfloor m/d \rfloor$ and from Proposition 2. ■

Proof of Proposition 5. Part 1. If $k < n$, then $d > 1$. An allocation x is stable only if $|\{j \in M : x_j > 0\}|$ is divisible by d . If x is stable and some agent i with $x_i > 0$ is made a veto agent, then the set $|\{j \in M' : x_j > 0\}| = |\{j \in M : x_j > 0\}| - 1$ and is not divisible by d , thus x becomes unstable. At the same time, if $x_i = 0$, then the group structure for all groups with a positive amount is preserved; thus x remains a stable allocation.

Part 2. In this case, the size of each group in x is $d > 2$, and every positive amount is possessed by either none or d non-veto players. If k increases by 1, d decreases by 2. Then allocation x becomes unstable, except for the case $x|_M = 0$. ■

Proof of Proposition 6. We first show that a minimal equilibrium exists; moreover, we can construct on-equilibrium production strategies explicitly. We will use the following notation: for a vector of payoffs x and a coalition X , we write x^{+X} to denote a vector defined by

$$[x^{+X}]_i = \begin{cases} x_i + 1 & \text{if } i \in X, \\ x_i & \text{if } i \notin X. \end{cases}$$

Fix for a moment the number of units available in a society as b . For every such b let us construct, for every $x \notin \mathbf{S}$, a set $Z_x \subset \mathbf{S}$ consisting of all $y \in \mathbf{S}$ such that $y \triangleright x$. Consider a

subset $Z'_x \subset Z_x$ consisting of all $y \in Z_x$ that achieve $\min_{y \in Z_x} \|y - x\|_1$. Let us now order all elements of Z'_x lexicographically according to the preferences of player 1 (from high to low), then player 2, etc, and let $\phi(x) \in Z'_x$ be the first element. In other words, $\phi(x)$ maximizes z_1 among $z \in Z'_x$; among those that also maximize z_1 , $\phi(x)$ maximizes z_2 , etc. This element $\phi(x)$ is uniquely defined for every $x \notin \mathbf{S}$. Let us define $\phi(x) = x$ if $x \in \mathbf{S}$, and let us repeat this construction and definition for all values of $b \geq 0$.

We construct an equilibrium where in each stage of redistribution which starts with \tilde{x}^t , an immediate transition to $x_t = z(\tilde{x}^t)$ is achieved. Since p is assumed to be small, this is possible. Notice that in this way, the only veto player who benefits from a redistribution is the first one, player $n - v + 1$; moreover, if $x|_M = y|_M$, then $[\phi(x)]|_M = [\phi(y)]|_M$, i.e., the allocations that non-veto players get only depend on the allocations that they possess before a redistribution.

Next, let us define production strategies. Suppose that the current state is $x = x^{t-1}$. In all cases, we will require that veto players make the efficient investment, $e_i = 1$ whenever $i \in L \cap V$. To define strategies of other players, consider the case where $x \in \mathbf{S}$. If $\|x|_M\| = 0$ (i.e., if non-veto players possess nothing), then they choose $e_i = 0$. Otherwise, for $x \in \mathbf{S}$, there is a group of non-veto players G of size d which possesses a positive amount x_G . If $x_G > 1$, then let us again require that all non-veto players choose $e_i = 0$. If, however, $x_G = 1$, we require that a player $i \in L \cap M$ invests if and only if $x_i = 0$ and $|\{j < i : j \in G \cup L\}| < d$. This specifies strategies for any $x \in \mathbf{S}$, and therefore gives continuation utilities for any combination of investment decisions that players make if $x \notin \mathbf{S}$; for such x , we can therefore pick (possibly mixed) strategies that form an equilibrium at period t if the continuation play is given by the strategies above. Finally, it is easy to verify that given the redistribution mapping ϕ , investment strategies that we defined for $x \in \mathbf{S}$ are part of equilibrium, i.e., no player has an incentive to deviate. We have thus proved existence of an equilibrium with the properties required by Part (ii) of the Proposition.

To prove Part (i), suppose that player n (the veto player) considers whether to invest. If without him investing the (temporary) allocation would be \tilde{x} , then if he invests, it would be $\tilde{x}^{+\{n\}}$. If \tilde{x} is stable then so is $\tilde{x}^{+\{n\}}$, and if they are not, then any minimal mapping would transfer exactly the same amount to player n during the redistribution phase. Consequently, player n will invest whenever he has the opportunity to do so and for any strategy profile that other players play.

To prove Part (iii), note that on the equilibrium path, we only need to study production decisions for $x \in \mathbf{S}$ and where $x_G > 2$. Notice that it is never a best response to produce if $|L \cap M| < d$, because all new units will be expropriated immediately. It is also never a best response for $i \in G$ to produce if $|L \cap M| = d$, but $L \cap M \neq G$, because d units will be expropriated (so the amount possessed by i will not increase), and moreover, in a minimal equilibrium,

possessing exactly $x_i = x_G$ guarantees that i will not be expropriated, while possessing $x_i + 1$ need not guarantee that. Hence, $i \in G$ does not invest, and therefore the rest do not invest, since a minimal equilibrium would require that the newly created units (by less than d players) will be moved to the veto player.

If $|L \cap M| = d$ and $L \cap M = G$, it is straightforward to see that having players in group G produce is an equilibrium. Moreover, if $|L \cap M| > d$ and $G \subset L \cap M$, it is an equilibrium for these players in G to produce and for the rest not to (because the units created by $i \in M \setminus G$ will be transferred to player n). Finally, let us prove that there cannot be an equilibrium where more than d units are produced. Indeed, if so, then some of these units must be expropriated. Moreover, in a minimal equilibrium, all units produced by players in $M \setminus G$ will be taken, and thus such players do not have incentives to produce. This contradicts the assertion that more than d units are produced, which shows that exactly d units are produced in the best equilibrium where only pure strategies are played along the equilibrium path. ■

Proof of Proposition 7. From Proposition 6, it follows that

$$g = p + dp^d = p + (n - k + 1)p^{n-k+1}.$$

Therefore, g is increasing in p . Consider

$$\frac{(d+1)p^{d+1}}{dp^d} = \frac{d+1}{d}p \geq \frac{3}{2}p,$$

because $d \geq 2$. Since we are considering small p , this ratio is less than 1. Consequently, g is decreasing in d , and thus is increasing in k . This completes the proof. ■