Market Power, Adjustment Costs, and Risk in the Measurement of Banking Services*

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Abstract

We develop a dynamic model of bank pricing where banks have market power and consumers find it costly to adjust deposits. Through numerical simulation and solution of a calibrated version of the model, we construct a measure of the value of services that banks furnish to depositors without explicit payments. Our simulation results show that bank market power and adjustment costs result in an estimated service margin that is roughly 3 times higher than in a model without those features. Additionally, we find that market power and adjustment costs lower the volatility of estimated margins by 27 percent.

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1 Introduction

This paper develops a forward-looking model of bank and depositor behavior in which depositors find it costly to adjust their holdings and banks have market power. Through numerical solution and simulation of the model, we add two contributions to the literature on banking. The first contribution is that our model informs the question of how to measure the value of services that banks furnish to depositors without explicit payment. In our framework, the implicit price of depositor services is equal to the spread between the rate of interest on deposits and a reference rate of interest. This reference rate is equal to the risk-free rate plus a term which accounts for the impact of the bank's current decisions on future deposits. The second contribution is that we have built a model of banking and depositor behavior that captures aspects of the market for deposits that have not be well explored. Such a model can be used to assess, for example, competition and bank behavior within deposit markets.

Our model of bank and consumer behavior has three unique features: (1) banks have market power in deposit markets, (2) both banks and consumers are forward-looking, and (3) consumers face adjustment costs in deposit markets. Consumers in our model also value services associated with deposits, which we incorporate using a money-in-utility (MIU) model (Sidrauski 1967). By deriving the Euler equation associated with the optimal solution to the bank's problem, we can show the key intuition which drives our findings. The bank's optimal policy is to set the deposit rate as the sum of a monopoly markup, a risk-free rate, and a term that accounts for the impact of changes in current deposits on future profits. We denote the sum of the latter two terms as the reference rate for deposits; this reference rate measures the opportunity cost of deposits from the bank's perspective. Through numerical simulation of the model, we show that the reference rate (1) is higher than the risk-free rate, and (2) results in services margins that are less volatile than does the risk-free rate. The intuition behind this result is that deposits provide a stable source of funding that banks are

willing to pay extra for. Without bank market power, adjustment costs, or forward-looking behavior the appropriate reference rate would simply be the risk-free rate.[1]

Because our model provides a reference rate for deposits, we consider its implications for measurement of implicitly priced services provided to depositors, the price of which is the difference between the deposit rate and the reference rate. Measuring the implicitly-priced output of banks has been recognized as a difficult question from the early days of national accounting (Fixler, Reinsdorf, and Smith 2003). For depositor customers, banks are willing to offer a substantial array of services often with no associated explicit fees. These "implicitly priced" services may include bookkeeping, liquidity provision, ATM transaction services, check clearing, convenient branch locations, safekeeping, electronic payments, and others. At present, most national accounting agencies impute the price of banking services to depositors as the difference between some rate of interest that a similar, service-free instrument might earn, and the rate of interest paid on deposits. The intuition behind their methodology is that the rate of interest on the similar, service-free instrument represents an opportunity cost to the bank of funding its lending activities through deposits, often referred to as the "user cost" of deposits. An important question–and a current area of debate in the literature–is what interest rate to use as the user cost of deposits. As we noted above, our model provides an answer to this question: The user cost of deposits is equal to the reference rate derived from the first order conditions and envelope conditions of the bank's optimization problem.

While banks furnish implicitly priced services to both borrowers and depositors, we choose to look at depositors in isolation. The reason is that deposits, and in particular transaction accounts, have a special role in the financial system. Others have provided

[1]Reinsdorf (2011), pgs 5-6, has previously argued that the stability of retail deposits implies that the appropriate reference rate for depositor services is not likely to be a maturity-matched market rate. As we will discuss further in Section 3, one method to obtain a reference rate for depositor services that is used in European countries is to find a service-free security with similar maturity to deposits, such as a LIBOR rate. In our framework, this would correspond to using the risk-free rate.

evidence that switching costs in deposit markets can provide banks with substantial effective monopoly power in these markets (see below), and that customers find it difficult not only to switch between banks, but to adjust the level of their deposit holdings in response to changes in the interest rate environment. For this reason–and because bank runs for retail deposits have been relegated to history books by the existence of mandated Federal deposit insurance–banks view deposits as a particularly safe form of borrowing. If banks view deposits as being particularly safe, then they would be willing to switch to another form of borrowing only if such borrowing offered a very stable funding source, which undoubtedly would come at a high price. If banks are willing to pay a lot for deposits, but end up paying only very little in explicit payments of interest, then this would imply a large value for unpriced services provided as a payment in kind to depositors.

Before turning to the body of the paper, we make some final notes on our modeling assumptions. First, we assume a monopoly bank, which can be justified in a model of banking in which there are large costs of switching between banks. A model with perfect information and no consumer preference shocks can produce an equilibrium in which banks act as monopolists and no consumers ever switch banks (the intuition behind this is laid out in Klemperer (1995)). Sharpe (1997) provides empirical evidence that switching costs can play a large role in the deposit rates that bank customers face.[2] We also must assume that customers value deposits more than they value savings, as deposits directly enter depositors' utility function. This leads to the money-in-utility (MIU) model (Sidrauski 1967); the MIU model has a long tradition in economic theory, and is functionally equivalent to a number of other ways of modeling money demand (Feenstra 1986). With pricing power and armed with the knowledge that consumers face difficult in rapidly adjusting their deposit holdings, a bank sees deposits as being a very safe form of borrowing: As interest rates fluctuate, the supply of deposits remains relatively steady.

[2]See also Zephirin (1994), "[switching costs] encourage collusive pricing by banks," and Kiser (2002).

An outline of the rest of our paper is as follows. Section 2 discusses prior work and evidence for consumer adjustment costs. Section 3 provides an overview of how banking services are treated in the National Accounts. We introduce our model in Section 4, and describe our solution method in Section 5. We outline our results in Section 6.

2 Consumer Adjustment Costs: Literature and Evidence

Our model of bank and consumer behavior draws elements from other models in the literature such as those of Wang, Basu, and Fernald (2009) and Kiser (2004), and adds to the literature by introducing consumer adjustment costs. If banks enjoy market power over their customers and if customers are inattentive or face reoptimiziation, physical, or psychic costs with regard to adjusting their deposit holdings, then the bank will view deposits as being less risky than other potential sources of funding.

We contribute to the literature on banking by building a dynamic model of bank behavior as it relates to demand deposits. While there is a large literature on the value of retail deposits as a hedge against bank runs (for early work on this topic, see Diamond and Dybvig (1983); for more recent evidence, see Shin (2009), Gorton and Metrick (2012) and references therein), to our knowledge no work has been done on modeling the behavior of a bank with market power where depositors are slow to react to market conditions. There is some evidence, however, that the market reflects this phenomenon. In a May 12, 2011 special report, for example, the Economist[3] suggests that deposits are "sticky" and that ordinary savers don't tend to move their funds from asset to asset. Studies of depositor behavior in the U.K. have also found that consumers are unlikely to move their funds between different products or banks in response to interest rate changes, and attributed this to consumers'

[3] The Economist (2011). Accessed online on 12/12/2012 at http://www.economist.com/node/18654578.

lack of awareness about market conditions (see Office of Fair Trading (2008), CruickShank (2000), Competition Commission (2007)).

We provide some evidence consistent with the existence of consumer adjustment costs in Figure 2. The existence of consumer adjustment costs should imply that retail deposits interest rates are stable over time, relative to other forms of deposits. This can be seen in the figure, as rates on both savings and interest-bearing checkable deposits are significantly less volatile than interest rates on time deposits and Federal funds purchased/securities repurchase agreements, even though time deposits have longer maturities[4]. We also find that retail deposits are significantly less responsive to the interest rates of potential substitutes than non-retail deposits.

3 Banking services in the US National Accounts

Correctly measuring the output of banks has been an important question in national income accounting for many years. Banks do not levy explicit charges for many of the financial services they provide, instead relying on (net) interest margins for revenue. In national income accounting, interest flows are usually considered to be distributions of income to providers of funds, and not payments for services. If this concept of interest were directly applied to bank interest, banks would appear to provide a negative contribution to national income, with fee income insufficient to cover expenditures on labor and intermediate inputs. To avoid such an outcome, the US National Income and Product Accounts (NIPAs) contain an imputation for implicitly priced bank services. Before the comprehensive revision of the NIPAs of 2003, all of banks' net interest margins (interest receipts less interest payments) were considered to be payments by depositors to banks, for the intermediation services that banks were providing to them. This approach reflects a view of bank interest that

[4]Figures are based on aggregated commercial bank reports on condition and income from the FDIC.

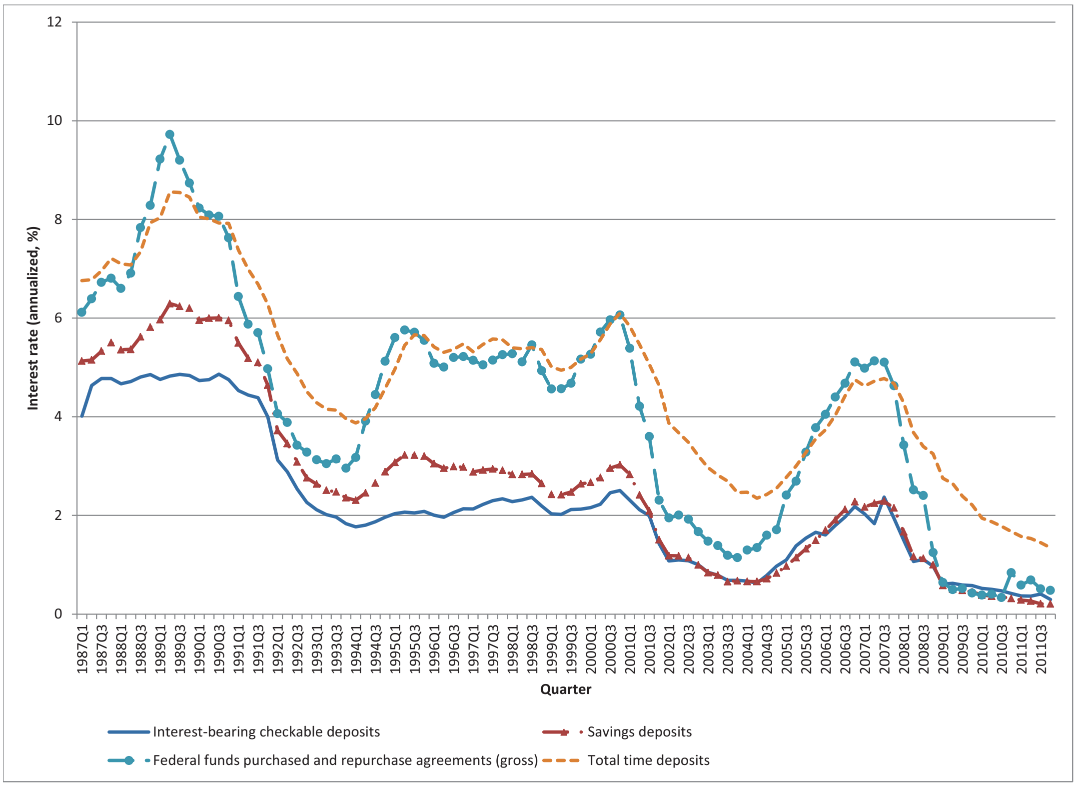

Figure 1: Selected Interest Rates - Bank Liabilities

ultimately belongs to depositors, who provide funding. Such a method, however, ignores the substantial resources that banks employ in providing services to borrowers. These may include, for example, risk management, bookkeeping, monitoring, underwriting, convenient retail locations, technical assistance, and other financial advice. These services are not costless–they require banks to devote time, workforce, and other resources–nor are they unvaluable to borrowers. Based on these considerations, the NIPAs adopted a new approach to estimating bank output in 2003 (Fixler, Reinsdorf, and Smith 2003). This approach uses a single "reference rate" that lies in between the interest rates earned on loans and paid on deposits. Interest margins charged for services to borrowers and depositors are measured

as the difference between this reference rate and loan or deposit rates, respectively. Thus, the reference rate acts as an opportunity cost–the rate that borrowers might pay were they to require no services, and the rate that depositors might earn were they to invest their money in a service-free instrument. To some extent this method lowers total measured bank output, as loans funded by equity and not deposits no longer produce any services to depositors.

In the approach of Fixler, Reinsdorf, and Smith (2003), the reference rate used for implicitly priced depositor services is calculated using the book values of bank assets. The national accounting agencies of non-US countries have taken a different approach, using market rates as reference rates. Typically, the rates used are short-term interbank lending rates (see Organisation for Economic Co-operation and Development (2012), pg 5). The intuition behind the market rate approach is that the appropriate reference rate is the rate of an instrument that has a maturity similar to a deposit, but provides no service.[5] Although this approach is intuitively appealing, as explained in Organisation for Economic Co-operation and Development (2012) approaches that have used market rates have resulted in measures of depositor services that are unrealistically volatile, and that are sometimes negative. (Fixler, Reinsdorf, and Smith 2003) also argue that short-term market rates are inappropriate as reference rates for depositor services, as they produce measures of the value of services to depositors that are too low and too volatile. Using book values rather than market values ameliorates this issue. Our work provides a theoretical justification for using a less volatile reference rate for depositor services. Even though our model produces a relatively volatile risk-free rate, using the reference rate that is suggested by our theory rather than the risk-free rate reduces the volatility and raises the overall level of measured

[5] Wang (2003) and later Wang, Basu, and Fernald (2009) also argue for using reference rates for both loans and deposits. Their analysis however is primarily focused on the measurement of borrower services. In particular, they argue the approach to measuring borrower services taken in the NIPAs ignores the fact that banks assume some risk when performing intermediation activities, and that assumption of risk should not be considered to be a service.

services to depositors.

4 Model Setup

4.1 Consumers

We assume a representative consumer model where in every period consumers face a consumption/savings tradeoff. Consumers in our model have two avenues for savings: they can purchase service-free bonds (s_t), or put money in deposits that provide services (x_t). Consumers receive an endowment d_C each period, which can be allocated to consumption (c_t) or one of the two previously mentioned forms of savings. Unlike savings put into bonds, savings put into deposits enters consumer utility directly. We see this as a reduced form way of capturing the fact that interacting with the bank may have lower transaction costs than the bond market, and that the bank provides services such as storage and liquidity to the consumer. Consumers also face a quadratic cost of adjusting deposits, which could be interpreted as a cost of re-optimizing deposits every period. Consumers take the interest rates on deposits, r_t^x, and on bond savings, r_t^s, as given. For tractability, we assume that the bank and the bond savings market offer a discount rate to consumers, $\delta_t^k = 1/(1 + r_t^k)$, $k \in \{r, s\}$. We express consumer flow utility as

$$u(x_t, c_t) - \frac{\eta}{2}(x_t - x_{t-1})^2 = \frac{(c_t x_t^{\phi})^{(1-\psi)}}{1-\psi} - \frac{\eta}{2}(x_t - x_{t-1})^2, \tag{1}$$

where η is a parameter that reflects the adjustment cost in utility terms. Consumers are assumed to be forward-looking with a discount rate β. In period t, consumers maximize their expected present discounted utility subject to the budget constraint

$$c_t = d_C - \delta_t^x x_t - \delta_t^s s_t + x_{t-1} + s_{t-1} \tag{2}$$

We assume that the consumer problem is finite horizon, with a terminal period T. In period T, we assume that consumers eat all of their savings, and incur no adjustment cost, so that consumer utility is

$$V_T(x_{T-1}) = \frac{(d_C + x_{T-1} + s_{T-1})^{(1-\psi)}}{1-\psi}.$$

In period t, we express the consumer's problem as

$$\begin{aligned} V_t(s_{t-1}, x_{t-1}, \delta_t^s) \quad &= \quad \max_{c_t, s_t, x_t} u(c_t, x_t) - \frac{\eta}{2}(x_t - x_{t-1})^2 + \beta E_{\delta_{t+1}^s | \delta_t^s} V_{t+1}(s_t, x_t, \delta_{t+1}^s) \qquad (3) \\ \text{s.t.} \quad & \quad c_t = d_C - \delta_t^x x_t - \delta_t^s s_t + x_{t-1} + s_{t-1} \\ & \quad c_t > 0, \quad x_t > 0. \end{aligned}$$

We assume that δ_t^s follows a Markov process, which implies it will enter the consumer's value function as a state variable. We denote the CDF of δ_t^s as $F_{\delta^s}(\delta_t^s | \delta_{t-1}^s)$ and denote its support as $[\underline{\delta}^s, \overline{\delta}^s]$. The value of δ_t^x will be endogenously determined as the solution to the bank's optimization problem which we lay out in the next section. The δ_t^x will be a function of the consumer state $(s_{t-1}, x_{t-1}, \delta_t^s)$, and consumers know the functional relationship between δ_t^x and the state.[6] [7]

When solving the consumer problem, it is helpful to use the problem's first order conditions to derive the Euler equations which describes the evolution of the state variables:

[6]We assume that an individual consumer cannot affect the market state, so consumers cannot game the bank's policy rule.

[7]Note that the consumer value function will implicitly also depend on δ_t^x. However, because δ_t^x is a function of the current state, and consumers know both this function and the evolution of the state, it would be redundant to put it explicitly in the value function.

$$
\begin{aligned}
-\delta_t^x u_c(c_t, x_t) - \eta(x_t - x_{t-1}) + u_x(c_t, x_t) + \beta E_{\delta_{t+1}^s|\delta_t^s} \frac{\partial V_{t+1}(s_t, x_t, \delta_{t+1}^s)}{\partial x_t} &= 0 \\
-\delta_t^s u_c(c_t, x_t) + \beta E_{\delta_{t+1}^s|\delta_t^s} \frac{\partial V_{t+1}(s_t, x_t, \delta_{t+1}^s)}{\partial c} &= 0
\end{aligned}
$$

The two envelope conditions associated with this problem are

$$
\begin{aligned}
\frac{\partial V_{t+1}(s_t, x_t, \delta_{t+1}^s)}{\partial x_t} &= u_c(c_{t+1}, x_{t+1}) + \eta(x_{t+1} - x_t) \\
\frac{\partial V_{t+1}(s_t, x_t, \delta_{t+1}^s)}{\partial s_t} &= u_c(c_{t+1}, x_{t+1})
\end{aligned}
$$

Combining the first order conditions with the envelope conditions, we get two Euler equations

$$
\begin{aligned}
\delta_t^x &= -\frac{\eta(x_t - x_{t-1})}{u_c(c_t, x_t)} + \frac{u_x(c_t, x_t)}{u_c(c_t, x_t)} + \beta E_{\delta_{t+1}^s|\delta_t^s} \frac{u_c(c_{t+1}, x_{t+1})}{u_c(c_t, x_t)} + \\
&\quad \beta\eta E_{\delta_{t+1}^s|\delta_t^s} \frac{u_c(c_{t+1}, x_{t+1})}{u_c(c_t, x_t)}(x_{t+1} - x_t) \\
\delta_t^s &= \beta E_{\delta_{t+1}^s|\delta_t^s} \frac{u_c(c_{t+1}, x_{t+1})}{u_c(c_t, x_t)}
\end{aligned} \tag{4}
$$

The system is determined by these two Euler equations, the budget constraint above, and the set of initial conditions. Note that in the system above, the variables c_{t+1} and x_{t+1} are functions of δ_{t+1}^s, as they are chosen optimally by the consumer in period $t+1$. We may re-write the first Euler equation with a substitution from the second to get some insight into

demand,

$$\delta_t^x = \delta_t^s + \frac{u_x(c_t, x_t)}{u_c(c_t, x_t)} - \frac{\eta(x_t - x_{t-1})}{u_c(c_t, x_t)} + \beta\eta E_{\delta_{t+1}^s|\delta_t^s} \frac{u_c(c_{t+1}, x_{t+1})}{u_c(c_t, x_t)}(x_{t+1} - x_t)$$

The equation states that the rate consumers are willing to pay on deposits is the sum of the rate on savings, the marginal rate of substitution between deposits and consumption goods, the marginal adjustment cost this period, and the expected marginal adjustment cost next period times the ratio of marginal utilities.

4.2 The Bank

Assume a monopolist bank that has a fixed number of loans, L, lent out at a rate R^L plus other income from lending activities totalling d_B. The loans are paid back to the bank in period T. Every period the bank chooses a discount rate δ_t^x on deposits, and the marginal cost of servicing deposits is γ. The amount of deposits held by consumers, x_t, will be a function of δ_t^x, δ_t^s, x_{t-1}, and s_{t-1}, and will solve the consumer's optimization problem in equation (3). In period T, consumers hold no deposits so the bank's cash flow is $L + d_B$. For period t, we write the bank's value function as

$$\begin{aligned} \pi_t(s_{t-1}, x_{t-1}, \delta_t^s) = \max_{\delta^x} \quad \{ \quad & g\left(d_B + \delta_t^x x_t(\delta_t^x, s_{t-1}, x_{t-1}, \delta_t^s) - x_{t-1} - \gamma x_t(\delta_t^x, s_{t-1}, x_{t-1}, \delta_t^s)\right) + \\ & \beta E_{\delta_{t+1}^s|\delta_t^s} \pi_{t+1}(s_t(\delta_t^x, s_{t-1}, x_{t-1}, \delta_t^s), x_t(\delta_t^x, s_{t-1}, x_{t-1}, \delta_t^s), \delta_{t+1}^s)\}. \end{aligned} \tag{5}$$

(Profits are a function of δ_{t+1}^s directly because next period's x is a function of δ_{t+1}^s). The function g is convex and increasing, and allows the bank to be risk-averse. This function captures the idea that the bank's shareholders may be risk-averse.[8] We specify g as

[8] We are investigating closing the model and allowing consumers to own shares in the bank. One way to do this is to assume that there exist an infinite number of markets, each monopolized by a bank, and that all

$$g(y) = \frac{y^{(1-\sigma)}}{1-\sigma}.$$

Note that in the bank's problem (5), because the bank is forward-looking it will be choosing δ^x to maximize the discounted sum of present and future profits. The linkage between present and future profits occurs through the evolution of deposits. Because consumers have adjustment costs, if deposits are high today they will want to hold more deposits tomorrow, all else equal. These dynamics will affect the value of deposits to the bank. If the bank gives consumers a high interest rate today (by lowering δ_t^x), then it will induce consumers to purchase more deposits and will raise demand for deposits tomorrow. In the future, the bank can exploit this demand by lowering the interest rate. We make this connection explicit by solving for the bank's Euler equation. Writing the argument of $g(\cdot)$ as

$$\tilde{\pi}_t = d_B + \delta_t^x x_t(\delta_t^x, s_{t-1}, x_{t-1}, \delta_t^s) - x_{t-1} - \gamma x_t(\delta_t^x, s_{t-1}, x_{t-1}, \delta_t^s)$$

we can write the bank's first order condition as

$$\begin{aligned} 0 \quad = \quad & g'(\tilde{\pi}_t)\left(x_t(\delta_t^x, s_{t-1}, x_{t-1}, \delta_t^s) + (\delta_t^x - \gamma)x_{\delta^x}(\delta_t^x, s_{t-1}, x_{t-1}, \delta_t^s)\right) + \\ & \beta E_{\delta_{t+1}^s|\delta_t^s} \frac{\partial \pi_t(x_t(\delta_t^x, s_{t-1}, x_{t-1}, \delta_t^s), \delta_{t+1}^s)}{\partial x_t} x_\delta(\delta_t^x, s_{t-1}, x_{t-1}, \delta_t^s). \end{aligned} \tag{6}$$

consumers own a security that pays a dividend which reflects bank profits across all markets. In this model the bank's stochastic discount factor will be derived from the consumer problem. Since we model consumers as risk-averse, the bank will also behave in a risk-averse way. A difficulty that arises is that when computing the bank's optimization problem we have to track both the state of the bank's market, which determines what the bank's profits will be next period, and the aggregate state of all markets, which will determine how the dividend paid to consumers will evolve over time. In the current specification, we only need to track the individual bank's market state.

It is also useful to derive the two envelope conditions associated with the x_{t-1} and s_{t-1} state variables, which we lay out below (suppressing the arguments of π, x, and s to keep the notation tractable):

$$\begin{aligned}
\frac{\partial \pi_t}{\partial x_{t-1}} &= g'(\tilde{\pi}_t)\left((\delta_t^x - \gamma)\frac{\partial x_t}{\partial x_{t-1}} - 1\right) + \beta E_{\delta_{t+1}^s|\delta_t^s}\left[\frac{\partial \pi_{t+1}}{\partial s_t}\frac{\partial s_t}{\partial x_{t-1}} + \frac{\partial \pi_{t+1}}{\partial x_t}\frac{\partial x_t}{\partial x_{t-1}}\right]. \\
\frac{\partial \pi_t}{\partial s_{t-1}} &= g'(\tilde{\pi}_t)\left((\delta_t^x - \gamma)\frac{\partial x_t}{\partial s_{t-1}}\right) + \beta E_{\delta_{t+1}^s|\delta_t^s}\left[\frac{\partial \pi_{t+1}}{\partial s_t}\frac{\partial s_t}{\partial s_{t-1}} + \frac{\partial \pi_{t+1}}{\partial x_t}\frac{\partial x_t}{\partial s_{t-1}}\right].
\end{aligned}$$

These envelope conditions can be manipulated and combined with the first order condition in (6) to derive an intuitive formula for the bank's pricing rule:

$$\delta_t^x = \gamma + \mu + \delta^F + \beta E_{\delta_{t+1}^s|\delta_t^s} X(\delta_{t+1}^s). \tag{7}$$

The discount rate δ^x is equal to the marginal cost, γ, plus a monopoly markup $\mu = -\frac{x_t}{\frac{\partial x_t}{\partial \delta_t^x}}$, plus a risk-free rate, $\delta^F = \beta E_{\delta_{t+1}^s|\delta_t^s}\left[\frac{g'(\tilde{\pi}_{t+1})}{g'(\tilde{\pi}_t)}\right]$, plus a complicated function of the future value function and its derivatives,

$$X(\delta_{t+1}^s) = \frac{g'(\tilde{\pi}_{t+1})}{g'(\tilde{\pi}_t)} \frac{\frac{\partial s_{t+1}}{\partial x_t}\frac{\partial x_{t+1}}{\partial s_t} - \frac{\partial s_{t+1}}{\partial s_t}\frac{\partial x_{t+1}}{\partial x_t}}{\frac{\partial x_{t+1}}{\partial \delta_{t+1}^x}\frac{\partial s_{t+1}}{\partial s_t} - \frac{\partial s_{t+1}}{\partial \delta_{t+1}^x}\frac{\partial x_{t+1}}{\partial s_t}} x_{t+1} + \frac{1}{g'(\tilde{\pi}_t)}\left[\frac{\frac{\partial x_{t+1}}{\partial \delta_{t+1}^x}\frac{\partial s_{t+1}}{\partial x_t} - \frac{\partial s_{t+1}}{\partial \delta_{t+1}^x}\frac{\partial x_{t+1}}{\partial x_t}}{\frac{\partial x_{t+1}}{\partial \delta_{t+1}^x}\frac{\partial s_{t+1}}{\partial s_t} - \frac{\partial s_{t+1}}{\partial \delta_{t+1}^x}\frac{\partial x_{t+1}}{\partial s_t}} + \frac{\frac{\partial s_t}{\partial \delta_t^x}}{\frac{\partial x_t}{\partial \delta_t^x}}\right]\frac{\partial \pi_{t+1}}{\partial s_t}.$$

Note that μ is positive - x is decreasing in δ^x because as δ^x rises interest rates fall and fewer consumers will choose deposits from the bank. Intuitively, higher markups mean higher δ^x's and hence lower interest rates. This equation also can be used to compute the gross output of the bank, which is $\gamma + \mu$. If one observes the deposit rate, δ_t^x, the bank's gross output will be the deposit rate minus the sum of the risk-free rate and the term which accounts

for the impact of changing deposits today on future profits. The term $X(\delta^s_{t+1})$ is a function of the derivatives of future state variables and future profits with respect to today's state variables and δ^x, and it captures the impact of future profits on the firm's current price. We would expect $X(\delta^s_{t+1})$ to be positive if adjustment costs exist and the bank has market power, because adjustment costs make deposits more valuable to the bank. We have not been able to prove this conjecture theoretically, but we find it to be true in our calibration exercise.

5 Numerical Method

We will numerically solve and simulate the model described in Section 4 in order to examine the properties of our measure of bank output. To give a short summary of how we solve the model, we first begin by solving the period T problem. In period T, we solve the consumer's problem within the firm's optimization problem: when the firm chooses a candidate δ^x_t, we numerically solve the Euler equations in (4). Once we have solved for the optimal policy in period T, we solve the problem in period $T-1$, taking the period T policy functions for the consumers and the bank as given.

In each period, we solve the problem on a discretization of the state space points. There are 3 state variables we must discretize: last period's savings s_{t-1}, deposits x_{t-1}, and the discount rate in the savings market δ^s_t. We choose a finite grid of $k = 1, ..., K$ points in each dimension, and index a state grid point as $(s^{k_1}, x^{k_2}, \delta^{s,k_3}), k_j = 1, ..., K, j \in \{1, 2, 3\}$. The problem will have K^3 total grid points. Note that when we solve the problem in period $T-1$, we know exactly what the consumer's policy in period T will be, so we get an exact solution for the firm and the consumer's problem. We cannot get an exact solution in periods prior to $T-1$, however; for example, in period $T-2$, we will know the consumer and firm policies on the grid points, but we will need to estimate it off the grid points to compute

the expectations of the policy and value functions in equations (4) and (5). To address this, we interpolate the policy and value functions using the multilinear interpolation method of Weiser and Zarantonello (1988).[9]

We solve the problem using Matlab, and for the consumer problem we generally use the fsolve procedure from the Optimization Toolbox for solving nonlinear systems to solve the Euler equations. When solving the consumer problem, we optimize over deposits x_t and total savings, $x_t + s_t$ - we have found that our solution tends to be more stable if we optimize over total savings rather than just s_t. In rare situations fsolve fails to find a zero, in which case we solve the system by repeatedly using a one dimensional solver: given a starting value for savings, we find the value of x_t that minimizes the squared error in the Euler equations, then we solve for savings conditional on the x_t, and we repeat until a solution is found. The optimal deposit rate δ_t^x is found using a one dimensional optimizer fminbnd.

A complication arises when solving the consumer and firm problems due to the fact that there are bounds on the values of s_t, x_t, and δ_t^x. The consumer's total savings, $s_t + x_t$, can never get so negative that the consumer could never pay it off. In period $T-1$, this means that

$$x_{T-1} + s_{T-1} > -d_C,$$

and in period $T-\tau$ it must be the case that

$$x_{T-\tau} + s_{T-\tau} > -\sum_{t=0}^{\tau-1} (\underline{\delta}^s)^t d_C,$$

where $\underline{\delta}^s$ is the smallest possible value of δ_t^s. There are also bounds on the x_t implied by the firm's choice of δ_t^x. Because we model the firm as being risk averse, in a given period t the firm cannot choose x_t low enough that its profits would be negative:

[9] An algorithm for the interpolation is supplied in Weiser and Zarantonello (1988), and we implement the algorithm in C.

$$d_B + \delta_t^x x_t - x_{t-1} \geq 0.$$

If, instead of choosing δ_t^x conditional on demand for deposits x_t, we think of the firm choosing x_t against an inverse demand $\delta_t^x(x_t, s_{t-1}, x_{t-1}, \delta_t^s)$, then we can write the lower bound as

$$\underline{x}_t = \min\{x_t : d_B + \delta_t^x(x_t, s_{t-1}, x_{t-1}, \delta_t^s)x_t - x_{t-1} \geq 0\}.$$

On the other hand, x_t can never be so high that the firm would end up having negative profits at some point in the future. For example, in order to keep profits in period T positive we need

$$x_{T-1} \leq L + d_B.$$

In period t, we need to keep period $t+1$ profits positive, so it must be the case that

$$\overline{x}_t = \max\{x_t : \max_{x_{t+1}}[d_B + \delta_t^x(x_{t+1}, s_t, x_t, \delta_t^s)x_{t+1} - x_t] \geq 0\}.$$

When solving the problem, it is difficult to tell if a state space point is out of bounds, and solving for the above $\overline{x}_t$ is hard. Operationally, what we do is in period $t+1$ is if we find that we cannot find an x_{t+1} where profits are positive at the grid point x^k, we flag that grid point. When solving the period t optimization problem, we penalize choices of x_t which lead to grid points that are flagged. This generates a somewhat more conservative upper bound on x_t than $\overline{x}_t$, but as the interpolation grid gets finer we will approach the correct bound.

6 Results

Table 1 shows the parameter values used for the calibration exercise. We interpret a period in our calibration exercise as a quarter. The discount factor β is therefore set to be consistent

with an annual discount factor of 0.96. We set the parameters ϕ and ψ to values that generate an average deposit discount rate of about 1. We choose a discrete Markov distribution for δ^s, and calibrate the distribution to approximate the quarterly rates of 3 month Treasury bonds.[10] Table 2 shows the four values of δ^s and the probabilities of transitioning between them. Consumers and owners of bank equity are each given outside income of $d_C = 0.5$ and $d_B = 0.5$. We interpolate x_{t-1} over 14 points between 0.005 and 1 (we put more grid points near the lower bound than the upper bound because we found the policy functions became flat as one approaches larger values of x_{t-1}) and $x_{t-1} + s_{t-1}$ on an 10 dimensional grid of equally spaced points between -2 and 3.

Table 1: Parameter Values for Calibration

Parameter	Value
β	0.9898
r^L	0.5
η	0.1
ϕ	0.006
ψ	0.5
γ	0

We solve for the consumer and firm policy and value functions for $T = 30$ periods. We run our simulation for 530 periods, keeping periods 100 to 500. We have found that when solving for the policy functions, they rapidly stabilize for periods prior to around the $T-10$th period, such that there are no numerically perceptible changes between policy functions in preceding periods. Hence, we assume that the policy function for period 1 obtained from the $T = 30$ solution approximates well any policy functions for earlier periods, and we use that policy function in every period of our simulation exercise. When we simulate

[10] We take the empirical distribution of interest rates and fit a histogram with 4 equally spaced bins, assuming that the realized interest rate is the average within a bin. The probability of switching between bins is calculated from the data.

Table 2: Markov Process for δ^s

Value	Transition Probability			
	0.9725	0.9799	0.9877	0.9958
0.9725	0.5	0.5	0	0
0.9799	0.04	0.87	0.09	0
0.9877	0	0.02	0.93	0.06
0.9958	0	0	0.05	0.95

The first column and row shows the four possible values of δ^s. The probabilities show the probability of transitioning from a value of δ^s in a given row to a value in a given column.

the model, we remove the first one hundred periods to remove the impact of the starting points. Average simulated moments are shown in Table 3. Our simulation currently produces slightly negative average deposit rates; deposit rates are positive about 40% of the time. As we would expect, the deposit rate δ^x is positively correlated with the discount rate on savings δ^s. Our simulation currently produces an average level of consumer deposits of 0.26 and an average level of savings of -1.9, implying that overall consumers are borrowing to finance consumption. Our estimated risk-free rate is about 1%, while the estimated reference rate is about 3%. Recall that our model's measure of bank output is the difference between the reference rate and the deposit rate.

Our results show that the estimated service margin with adjustment costs and bank market power is much higher than the standard formula, which takes the difference between the risk free rate and the deposit rate. This can be seen in the sixth and seventh rows of Table 3, where the service margin benchmarked to the risk-free rate is 0.014, while the service margin benchmarked to the reference rate is almost 3 times higher at 0.035. Additionally, the service margin that uses our reference rate is less volatile than the one that uses the risk-free rate, as can be seen in the last two rows. This can also be seen in Figure 2, which shows the simulated service margins (the spread between the deposit rate and some other rate) using

the implied reference rate and the risk free rate, as well as the level of the interest rate on savings, s. When the risk-free rate is used to construct the service margin, the implied value of bank output is quite low. In contrast, when one uses the reference rate implied by our model, a higher service margin is obtained.

Table 3: Simulated Moments

Simulated Moment	Result
Avg Deposit Rate	-0.0035
Avg x	0.26
Avg s	-1.92
Avg Risk-Free Rate	0.0105
Avg Reference Rate	0.0317
Avg Service Margin (Risk-Free)	0.0140
Avg Service Margin (Reference)	0.0352
S.D. Service Margin (Risk-Free)	0.0081
S.D. Service Margin (Reference)	0.0059

Simulated results for 400 periods.

7 Discussion

In this paper, we have proposed a model of banking services in which banks have market power, banks and consumers are forward-looking, and consumers face adjustment costs when altering the level of deposits that they consume. This model is used to inform measurement of output of financial services furnished by banks. Theoretical results show that the appropriate reference rate for measuring the implicitly priced services that banks offer though deposits is one which combines the risk-free rate with a second term that represents the effect of the bank's actions today on its future profits. Through simulation of this model, we are able to show that the second term may be quite significant, with a service margin that is about three times the service margin computed from the risk-free rate alone. In addition, bank output is estimated to be 27% less volatile than when computed using the risk-free rate alone.

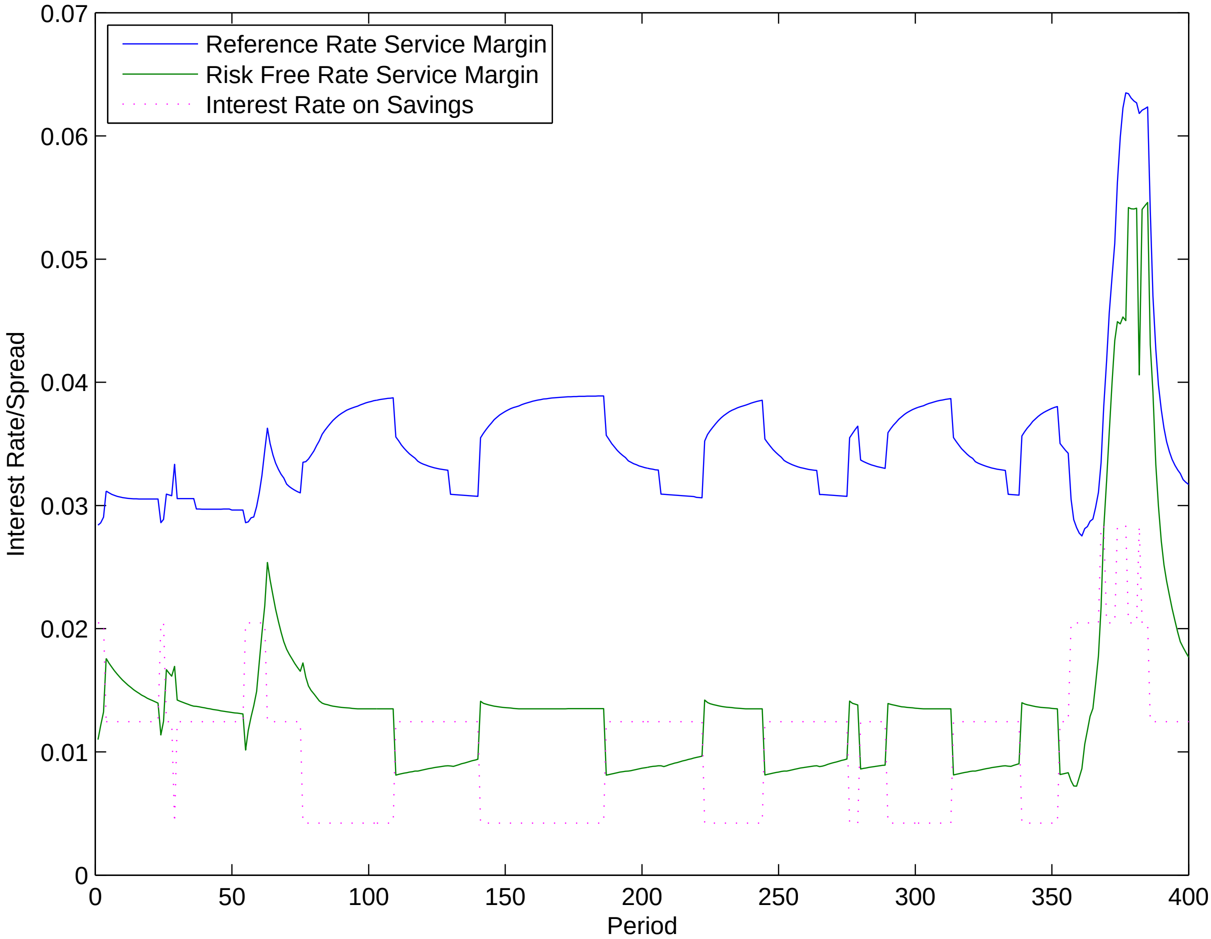

Figure 2: Simulated Service Margins Using Reference Rate and Risk-Free Rate
Notes: Simulation results for 400 periods, with 100 initial burn-in periods removed. Y Axis shows the interest rate (dotted line), or spread implied by estimated service margin.

In addition to addressing the question of how to appropriately measure depositor services, our model serves as a first attempt to capture the phenomenon of sticky deposit demand and consumer-level adjustment costs. While the economic literature on this topic is rather lean, there is evidence that such a phenomenon is present. Our model is able to make predictions about the effect of this phenomenon in a variety of contexts.

Further extensions we are planning include closing the model of investment supply and demand, for example, by allowing the banks to invest in other borrowings, by explicitly modeling loan demand, and by allowing consumers to own shares in the banks themselves.

In addition, we wish to consider alternate assumptions for bank competition and market structure.

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