

# Technology, Trade Costs, and the Pattern of Trade with Multi-Stage Production

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## Abstract

Comparative advantage and trade costs shape the geography of cross-border supply chains and trade flows. To quantify these forces, we build a model of trade with sequential, multi-stage production that features technology differences both across and within individual production stages. We estimate technology and trade costs in the model via simulated method of moments, matching bilateral shipments of final and intermediate goods for sixteen countries. We then apply the estimated model in a series of counterfactual experiments. For example, we show that changes in the level of trade costs generate concentration in gross relative to value-added trade, and that technological improvements in one country induce changes in regional supply chains. Surprisingly however, multi-stage production does not substantially inflate the gravity distance elasticity of trade.

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In a global supply chain, sequential production stages are ‘sliced up’ and allocated across countries to minimize total production costs. Comparative advantage and trade costs govern the allocation of stages to countries. First, countries differ in the cost with which they perform individual production stages. Some countries have comparative advantage in downstream production stages (e.g., manufacturing assembly China), while other have comparative advantage upstream stages (e.g., production of disk drives in Japan). Second, as inputs are shipped from country to country through the chain, producers incur trade costs. Often these costs are paid multiple times as goods travel back and forth across borders. Further, the burden of these trade costs is large: ad valorem costs are paid on the gross value of goods shipped, but cost savings from moving marginal production stages only apply to a fraction of that gross value.

In this paper, we build a quantitative model of trade with cross-border supply chains to study the role of comparative advantage and trade costs in shaping production fragmentation and trade patterns. As in Yi (2003, 2010), the production of each good requires a discrete number of stages, which must be performed in sequence.<sup>1</sup> These stages are allocated across countries to minimize production costs, given both bilateral trade frictions and differences in technologies across countries and stages. In contrast to workhorse Ricardian models, such as Eaton and Kortum (2002), that emphasize comparative advantage across goods, the multi-stage model features comparative advantage across and within individual production stages. Further, the discrete multi-stage production process allows supply chains to amplify the role of trade costs in shaping trade flows, an effect that is shut down by assumption in standard Ricardian models.

To quantify the role of technology and trade costs, we develop a new procedure to match the multi-stage model to data on cross-border input-output linkages. Specifically, we build a model-based global input-output table that tracks final and intermediate shipments across stages and countries. We then estimate technology and trade costs via simulated method of moments by minimizing deviations between final and intermediate trade shares in the model and data.<sup>2</sup> Because we ask the model to match observed trade flows, as is standard in the trade literature, we are able to estimate both trade costs and productivity as free parameters. This allows a tighter mapping between theory and data than previous calibration

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<sup>1</sup>Related models with a discrete number of stages include Markusen and Venables (2007) and Baldwin and Venables (2010). Costinot, Vogel, and Wang (forthcoming) develop a model with a continuum of production stages, building on Dixit and Grossman (1982) and Sanyal (1983). Arkolakis and Ramanarayanan (2009) and Bridgman (2008, 2012) also work with models that feature vertical specialization.

<sup>2</sup>In using a simulated method of moments procedure, we face the technical challenge that simulated moments in Ricardian models are not continuous in model parameters. This typically makes estimation for a large number of countries infeasible. We overcome this problem by borrowing smoothing techniques from the discrete choice literature, drawing on McFadden (1989).

procedures.<sup>3</sup> Not only does this facilitate comparison between the multi-stage model and competing alternatives, but it also paves the way for use of the multi-stage model in future applications.

We implement this estimation procedure in a two-sector, two-stage version of the model using data for 15 industrial and emerging market countries, plus a composite rest-of-the-world region, in 2004.<sup>4</sup> Our estimates suggest that there are large differences in technology levels across countries and stages. Though technology levels covary strongly across stages, countries exhibit pronounced differences in relative technology across stages, which induce countries to specialize across stages. For example, we find that China has comparative advantage in downstream (stage 2) production, while Australia has comparative advantage in upstream (stage 1) production. Comparative advantage in stage 2 production is not correlated with country income. However, it is negatively correlated with the share of commodities in exports, reflecting the upstream position of commodities in the production chain.

We also find that ad valorem trade costs are large in our framework, on the order of 250% for a typical country pair. Thus, our multi-stage framework returns estimates of average trade costs that coincide with levels obtained in standard Armington or Ricardian gravity-style models. As we discuss further below, this observation has important implications for interpreting the response of our model economy in counterfactual experiments. Finally, using the model-based input-output table, we can measure simulated trade on a value-added basis, comparable to data-based measures of trade in value added. We show the model matches key stylized facts about value added trade. For example, the model matches both the share of foreign value added in final goods produced in each country and the positive correlation of bilateral ratios of value added to gross trade with distance.

We apply the estimated model in three quantitative exercises that advance our understanding of how trade costs shape cross-border fragmentation and bilateral trade flows. First, we examine the role of multi-stage production in explaining the large negative elasticity of bilateral trade to distance. Yi (2003) first pointed out that multi-stage production inflates the effect of tariffs on trade, while Yi (2010) argues that it also increases the influence of

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<sup>3</sup>Yi (2010) calibrates a similar multi-stage model for the US and two Canadian regions using a mixture of data (on production, labor allocations, income, etc.) and ad hoc parameter restrictions. For example, Yi assumes that productivity is equal across stages and sectors in several regions, but not in others. We do not need to impose these restrictions on technology differences. Further, we are also able to estimate trade costs, unlike Yi (2010) who measures trade costs based on auxiliary data.

<sup>4</sup>It is straightforward to extend our procedure to allow for more sectors and/or stages, or to vary the number of countries. Because the estimation procedure is computationally costly, one needs to trade off these dimensions in practice. We have opted here for a relatively few stages and a high level of aggregation to maximize country coverage, which allows us to conduct gravity-style analysis of trade and allow richer cross-country analysis of counterfactual experiments. We intend to examine other permutations of the basic model in future iterations of this paper, as well as future work.

country borders on trade. By analogy, one would expect that multi-stage production also inflates the distance elasticity of trade.

Surprisingly, given our model estimates, we conclude that multi-stage production does not play an important role in explaining the elasticity of trade to distance. The reason is that the distance elasticity of trade is a function of the level of trade costs in the multi-stage model. Like standard models, the multi-stage model requires large trade costs to explain observed home bias and concentration in trade. These high trade costs dampen the extent to which stages are separated across borders in the model, and therefore limit the extent to which multi-stage production changes the mapping from trade costs to trade flows. To demonstrate the role of trade costs, we show that the distance elasticity does increase (in absolute value) as we lower trade costs uniformly across trade partners. However, sizable inflation effects require much lower trade costs, on the order of 100-200 percentage points than we estimate.

Second, we examine the response of trade flows and wages to a uniform 10% reduction in international trade costs across all countries and partners.<sup>5</sup> In this exercise, we distinguish ‘short run’ versus ‘long run’ responses to this change, where the ‘short run’ scenario holds the location from which stage 2 producers source stage 1 inputs fixed, while the ‘long run’ scenario allows producers to reoptimize their sourcing choices. Comparing these scenarios shed light on the extent to which reallocation of stages to countries is important in explaining responses to trade.

We find that holding suppliers fixed significantly dampens the response of gross trade and metrics of vertical specialization (e.g., value added to export ratios) in response to changes in trade costs. In the long run, we find that gross trade becomes more concentrated relative to value added trade following the decline in trade costs. That said, we do not find that the multi-stage model yields larger changes in aggregate trade than more conventional Ricardian frameworks.<sup>6</sup> Similar to our conclusion regarding the distance elasticity, the reason is that the elasticity of aggregate trade to uniform changes in trade costs is also a function of the initial level of trade costs. Thus, the multi-stage model yields macro-level responses comparable to more standard models, despite important differences in micro-level adjustment mechanisms.

Third, we examine the response of production chains and trade patterns to an increase in productivity in China. We show that changes in vertical specialization, as measured both by value added to export ratios and the share of foreign value added in final goods production,

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<sup>5</sup>This liberalization exercise is only one of many possibly interesting scenarios that could be analyzed in our estimated model. We will be adding additional exercises to future drafts.

<sup>6</sup>Specifically, we benchmark our results against a similar liberalization exercise in a two-sector version of the Eaton-Kortum model with input-output linkages developed by Caliendo and Parro (2012), where we parameterize that model using our simulated data.

are concentrated among China's Asian neighbors. Changes in vertical specialization are very small for most other countries outside the region. This emphasizes the local character of production chains in the model.

The rest of the paper proceeds as follows. Section 1 lays out the many-country, multi-stage model, presents a solution procedure, and discusses some key features of the model. Section 2 discusses how we assemble the model-based input-output framework. Section 3 describes how we calibrate elements of the model and estimate technology and trade costs, as well as how we construct alternative measures value added trade. Section 4 presents our estimates and statistics regarding model fit, while Section 5 presents our counter-factual analysis. Section 6 concludes.

# 1 Framework

We start this section by laying out the basic elements of the framework. The model draws heavily on models developed by Yi (2003, 2010), with the exposition here adapting the model to a many country, multi-sector setting. Because the model does not admit analytical solution, we then discuss how to solve the model numerically.

## 1.1 Production

Consider a world economy with many countries, indexed by  $i, j, k \in \{1, \dots, C\}$ . Within each country, we divide economic activity into two sectors  $s \in 1, 2$ , standing for goods and services. Within each sector, there is a unit continuum of goods, indexed by  $z$ , and each good requires two stages to produce. Production in both stages is perfectly competitive.

By way of notation, we generally put country labels in the superscript and stage labels in the subscript. We put good and sector subscripts in parentheses, so that  $(z, s)$  denotes good  $z$  in sector  $s$ .

Production in stage 1 uses labor and a composite input, and we assume the production function for good  $z$  in sector  $s$  is:

$$q_1^i(z, s) = T_1^i(z, s)\Theta(s)M^i(z, s)^{\theta(s)}l_1^i(z, s)^{1-\theta(s)} \quad (1)$$

where  $T_1^i(z, s)$  is the good-sector specific productivity of country  $i$  in stage 1,  $l_1^i(z, s)$  and  $M^i(z, s)$  are the quantities of labor and the composite input used in production,  $\theta(s)$  is the sector-specific share of the composite input in production, and  $\Theta(s) = (1 - \theta)^{1-\theta} \theta^\theta$  is a constant normalization. The output of the first stage is an input that is used in stage 2 production of good  $z$  in sector  $s$ .

Production in stage 2 combines the first stage input and labor, with the production function given by:

$$q_2^i(z, s) = T_2^i(z, s)\Theta(s)x_1^i(z, s)^{\theta(s)}l_2^i(z, s)^{1-\theta(s)}, \quad (2)$$

where  $T_2^i(z, s)$  is productivity in stage 2,  $x_1^i(z, s)$  is the quantity of the stage 1 input used,  $l_2^i(z, s)$  is labor used,  $\theta(s)$  is again the cost share attached to the stage 1 input, and  $\Theta(s)$  is the same normalization as above.<sup>7</sup>

Output in each stage may be produced in any location, but every time output is shipped it incurs an bilateral sector-specific iceberg transportation cost  $\tau^{ij}(s)$ .<sup>8</sup>

## 1.2 Aggregation

Stage 2 goods are aggregated to form a non-traded composite good in each sector, and these composite goods are sold to final consumers and used to form the composite input used in stage 1.<sup>9</sup> The composite goods, denoted  $Q^i(s)$ , are Cobb-Douglas combinations of stage 2 goods:

$$Q^i(s) = \exp \left( \int_0^1 \log(\tilde{q}^i(z, s)) dz \right), \quad (3)$$

where  $\tilde{q}^i(z, s)$  is the quantity of stage 2 good  $z$  in sector  $s$  purchased (from home or abroad) by country  $i$ .

These sector-level composite goods are combined to form an aggregate final good and the composite input used by stage 1 producers. We assume that the aggregate final good is given by:  $F^i = AF^i(1)^\alpha F^i(2)^{1-\alpha}$ , where  $F^i(s)$  denotes the amount of the composite good in sector  $s$  that is sold to final consumers and  $A = (1 - \alpha)^{1-\alpha} \alpha^\alpha$ . Similarly, the composite input is given by:  $M^i = BM^i(1)^\beta M^i(2)^{1-\beta}$ , with  $M^i = \sum_s \left[ \int_0^1 M^i(z, s) dz \right]$  in equilibrium

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<sup>7</sup>In contrast to Yi (2003), we do not explicitly include capital as a produced factor in the model. This implies that differences in capital are impounded in to the productivity term in our estimation. In computing counterfactuals, we implicitly hold all factors fixed. Including endogenous capital stocks would be a straightforward extension.

<sup>8</sup>Two points are worth noting here. First, we do not assume that the cost is stage-specific. Extensions in which trade costs for final and intermediate goods differ would allow one to consider the effects of input-tariff liberalization. Second, we assume that trade costs are ad valorem. Extensions with per unit trade costs would give rise to differences in trade costs across stages because the gross value per unit shipped differs across stages.

<sup>9</sup>One can think of aggregation step as a third production stage, with zero value added. Because there is zero value added, one can alternatively write down the model with aggregation explicitly incorporated into preferences and production functions.

and  $B = (1 - \beta)^{1-\beta} \beta^\beta$ .<sup>10</sup> Finally, adding up requires that:  $Q^i(s) = F^i(s) + M^i(s)$ .

### 1.3 Households

Consumers supply labor inelastically to firms and consume the composite final good  $F_i$ . In effect, they therefore have Cobb-Douglas preferences over stage 2 goods. The consumer budget constraint is:  $w^i L^i = P_F^i F^i + T B^i$ , where  $w^i$  is the wage,  $L^i$  is the labor endowment,  $P_F^i$  is the price of the final composite, and  $T B^i$  is the nominal trade balance. The trade balance appears here in the budget constraint, since we treat it as an exogenous nominal transfer necessary to equate income and expenditure for each country.

### 1.4 Solving the Model

To estimate and simulate the model, we solve a discrete approximation of the continuum model described above. We assume that there are a large number ( $R$ ) of goods within each sector, and let  $r = \{1, \dots, R\}$  index discrete products.

We describe the solution to the model by walking through a three step numerical procedure here. First, given wages  $w^i$ , we determine prices and the assignment of stages to countries for each good. Second, given prices and this assignment, we find equilibrium quantities produced of each good. Third, given prices and quantities, we compute labor demand and check whether this matches labor supply in each country.

#### 1.4.1 Prices and Assignment of Stages to Countries

Given wages, we can solve for the assignment of stages to countries for production of all goods delivered to each destination.<sup>11</sup> To do so, we construct prices for all possible assignments of stages to countries for delivery of a given good to each destination, and pick the assignment that minimizes costs.

This takes the form of a nested minimization problem:

$$\begin{aligned} \tilde{p}_2^k(r, s) &= \min_j \tau^{jk}(s) p_2^j(r, s), \quad \text{with} \quad p_2^j(r, s) = \frac{(w^j)^{1-\theta(s)} (\tilde{p}_1^j(r, s))^{\theta(s)}}{T_2^j(r, s)} \\ \text{and} \quad \tilde{p}_1^j(r, s) &= \min_i \tau^{ij}(s) p_1^i(r, s), \quad \text{with} \quad p_1^i(r, s) = \frac{(w^i)^{1-\theta(s)} (P_M^i)^{\theta(s)}}{T_1^i(r, s)}, \end{aligned}$$

<sup>10</sup>Note that we assume the composite input is not sector-specific.

<sup>11</sup>The allocation of stages to countries for each good depends on the destination at which that good is consumed.

where the price of the composite intermediate input is given by  $P_M^i = P^i(1)^\beta P^i(2)^{1-\beta}$ , where  $P^i(s)$  denotes the price of the composite good in each sector. This composite price is itself a function of the prices of stage 2 output delivered to country  $i$ :  $\log(P^i(s)) = \int_0^1 \log(\tilde{p}_2^k(z, s)) dz \approx \frac{1}{R} \sum_r \log(\tilde{p}_2^k(r, s))$ .

To be clear,  $\tilde{p}_2^k(r, s)$  is the realized price of stage 2 output of good  $r$  in sector  $s$  (i.e., the price at which  $k$  actually purchases good  $(r, s)$ ). It is equal the minimum over  $\tau^{jk}(s)p_2^j(r, s)$ , the possible prices at which each country  $j$  could deliver the stage 2 good if  $j$  chooses the minimum cost source for stage 1. These prices are in turn a function of the cost of stage 1 inputs in each source  $j$ , where the low cost supplier of stage 1 goods delivers inputs to country  $j$  at price  $\tilde{p}_1^j(r, s)$ . This input supply price is the minimum over delivered prices from alternative source countries ( $i$ ):  $\tau^{ij}(s)p_1^i(r, s)$ . Finally, those input supply prices depend on the composite input price in country  $i$ , which itself is a function of the realized prices of stage 2 output in country  $i$ .

Starting with a guess for the composite input prices  $P_M^i$ , we can solve the minimization problem for prices  $\{\tilde{p}_1^j(r, s), \tilde{p}_2^k(r, s)\}$ . We then use these prices to update the value of  $P_M^i$ , and solve for an updated set of stage 1 and stage 2 prices. We iterate on this fixed point problem to convergence. Having converged on a value for  $P_M^i$ , we can easily compute the solution for equilibrium stage 1 and stage 2 prices, as well as the allocation of stages to countries. We denote the set of countries to which country  $i$  is the low cost supplier for a particular stage of each good as:  $\{\Omega_1^i(r, s), \Omega_2^i(r, s)\}$ .

### 1.4.2 Quantities Supplied and Demanded

Given the prices obtained in the previous step, we can compute production of goods at each stage in each country by working backwards from final demand. Total demand for the sector-level composite goods is given by:

$$P^k(1)Q^k(1) = \alpha P_F^k F^k + \beta P_M^k M^k \quad (4)$$

$$P^k(2)Q^k(2) = (1 - \alpha)P_F^k F^k + (1 - \beta)P_M^k M^k, \quad (5)$$

where  $P^k(s)$  is the price index for the sector composite. Since we have taken the wage as given and observe the trade balance, we know  $P_F^k F^k = w^k L^k - TB^i$ . However, we do not directly observe expenditure on the composite input  $P_M^k M^k$ . We therefore need to solve for this value.

Given  $P_F^k F^k$  and a guess for  $M^k$ , we can compute  $P^k(s)Q^k(s)$ . These then imply demand



for individual stage 2 goods in destination  $k$  given by:

$$\tilde{q}^k(r, s) = \frac{\frac{1}{R}P^k(s)Q^k(s)}{\tilde{p}_2^k(r, s)}, \quad (6)$$

where again  $\tilde{p}_2^k(r, s)$  is the delivered price from the actual source that supplies market  $k$ .

Tracing these demands back to the countries that supply those goods, we can compute the quantity of stage 2 goods produced in each source  $j$  as:

$$q_2^j(r, s) = \sum_{k \in \Omega_2^j(r, s)} \tau^{jk} \tilde{q}^k(r, s). \quad (7)$$

Then, given this stage 2 production in country  $j$ , demand for stage 1 inputs in country  $j$  is:

$$x_1^j(r, s) = \frac{\theta(s)p_2^j(r, s)q_2^j(r, s)}{\tilde{p}_1^j(r, s)}. \quad (8)$$

These input demands allow us to then solve for the quantity of each stage 1 good supplied by country  $i$  as:

$$q_1^i(r, s) = \sum_{j \in \Omega_1^i(r, s)} \tau^{ij} x_1^j(r, s). \quad (9)$$

Finally, given this stage 1 production, we can compute demand for the composite input:

$$M^i = \frac{1}{P_M^i} \sum_s \sum_r \theta(s) p_1^i(r, s) q_1^i(r, s). \quad (10)$$

This gives us an updated value for purchases of the composite input  $M^i$ , and hence updated values for the total amount of the composite goods supplied in each sector  $P^k(s)Q^k(s)$ . We iterate on this fixed point problem to convergence.

### 1.4.3 Labor Market Clearing

The candidate solution above involved a guess for wages, so we need to check whether this guess clears the labor market. We can calculate total labor demand from both stages as:

$$\begin{aligned} l_1^i(r, s) &= (1 - \theta(s)) \frac{p_1^i(r, s) q_1^i(r, s)}{w^i} \\ l_2^i(r, s) &= (1 - \theta(s)) \frac{p_2^i(r, s) q_2^i(r, s)}{w^i}. \end{aligned}$$

Then total labor demand is:  $L_D^i(\mathbf{w}) = \sum_s \sum_r l_1^i(r, s) + l_2^i(r, s)$ , where we have made total labor demand explicitly a function of the wage vector. The equilibrium wage vector then

sets labor demand equal to labor supply:  $L_D^i = L^i$  for  $i = 2, \dots, N$  (where market 1 is dropped appealing to Walras' law).

## 1.5 Discussion

### 1.5.1 Snakes and Spiders

The model mixes sequential, multi-stage production with a roundabout input loop. We illustrate the basic set-up in a closed economy in Figure 1. Borrowing terminology from Baldwin and Venables (2010), we can think of this aggregate model as one with recursive ‘snakes’ and ‘spiders.’

To isolate the ‘snake’ part of the model, suppose that we were to set the share of the composite input in stage 1 to zero. In that case, we would re-write the first stage production function as  $q_1^i(z, s) = T_1^i(z, s)l_1^i(z, s)$ . And the stage 2 output would be used only to satisfy final demand. This would turn the full model above into one with a continuum of two-stage ‘snakes’ – an economy of the sort analyzed in Yi (2003). If (as above) we classify both stages of each good in the same sector, then there would be no inter-sectoral linkages in this simplified model, which clearly is at odds with the data. This then leads us to the full model, which includes both intra-sectoral and inter-sectoral input linkages.

In the full model, a ‘spider’ production process links the two ends of the ‘snakes.’ Output from the second stage is aggregated within and between sectors to form a composite input that is fed back into the first stage of the production process. Because the composite input links ends of the sequential production process, it converts the two-stage process into a multi-stage process with an effectively infinite number of production stages, where some fraction of output is drawn out at each stage to satisfy final demand.

Not only does the spider link the ends of the sequential production process, it also links the production process across sectors. Manufactures uses non-manufactures in production (and vice versa) because the composite input is made from all goods. Put differently, inputs flow across sectors through spiders, while inputs flow within each sector both via snakes and spiders.

### 1.5.2 Elasticity of Trade to Trade Costs

In the model with multi-stage production above, the elasticity of trade flows to trade costs depends on the level of trade costs. We discuss this relationship at length in simulations of the model below, but pause here to develop some intuition for this result. This intuition draws on arguments in Yi (2003, 2010), reworked here to emphasize points relevant to our

empirical analysis. There are two observations that underpin our interpretation of the trade elasticity.

First, as trade costs rise in the discrete multi-stage model, it becomes increasingly costly to split up discrete stages of the production process across countries. Therefore, as trade costs increase, the model behaves more like a standard multi-sector Ricardian model – an analog to the multi-sector extension of the Eaton and Kortum (2003) model by Caliendo and Parro (2012). With the standard assumption that productivities are drawn from the Fréchet distribution, then the elasticity of trade to changes in trade costs is equal to the Fréchet shape parameter. This elasticity anchors the trade elasticity in the multi-stage model for high values of trade costs.

Second, as trade costs fall in the multi-stage model, it becomes increasingly attractive to split up discrete production stages across borders to take advantage of cost differences. The ability to substitute over the location of individual stages of the production process, rather than simply over entire goods themselves, tends to amplify the sensitivity of trade to changes in trade costs.

One force for amplification arises because trade costs are incurred multiple times when inputs are shipped abroad and then embodied in imported final goods. For example, if a good that is exported uses an imported input, then one pays *ad valorem* costs on the input twice – once when it is imported, and again when it is exported embodied in final good. Yi (2010) refers this as the ‘multiple border crossing’ force.

A second force for amplification arises because agents evaluate the burden of trade costs relative to the cost savings on shifting the location of a single stage of the production process. The benefits of moving the location of a single stage depend on that stage’s share in total value added (equivalently, the value of the final good), and benefits are lower when the share in total value added is also low. The trade costs incurred in shifting a production stage are thus perceived to be more burdensome when total value added in the marginal stage is low. Yi (2010) refers to this as the ‘effective rate of protection’ force.

At intermediate levels of for the trade cost, the model economy features both standard Ricardian trade, where consumers substitute across entire goods, and trade through multi-stage production chains in which agents substitute over production locations for each stage. Therefore, the aggregate model elasticity of trade to trade costs depends on the mix of Ricardian vs. multi-stage multistage trade. As trade costs fall, the share of trade via multi-stage production chains rises, and the elasticity of trade to trade costs does as well.

## 2 A Multi-Stage Input-Output Framework

In the model, there are input-output linkages across production stages, sectors, and countries. These linkages can be represented in the form of a model-based global input-output (IO) table. This model based table is helpful for explaining the structure of trade in final goods and intermediate inputs in the model, and in describing the link between model and data. Further, we can apply the model based input-output table to construct measures of value-added trade in the model. We present the input-output framework here and then describe this application in the next section.

To build the model-based IO table, we start with market clearing conditions. These market clearing conditions hold good-by-good and stage-by-stage. In forming the model-based IO table, we aggregate across goods within each sector to re-write these market clearing conditions so that they hold stage-by-stage and sector-by-sector. We then assemble these market clearing conditions into standard IO table formatting.

To start, we need to define notation for values of bilateral shipments and gross output. Using the letter  $y$  to denote prices times delivered quantities, then we can write bilateral shipments as:

$$y_1^{ij}(r, s) \equiv \mathbb{I}(j \in \Omega_1^i(r, s)) \tilde{p}_1^j(r, s) x_1^j(r, s), \quad (11)$$

$$y_2^{ij}(r, s) \equiv \mathbb{I}(j \in \Omega_2^i(r, s)) \tilde{p}_2^j(r, s) \tilde{q}^j(r, s), \quad (12)$$

where the indicator functions  $\mathbb{I}(\cdot)$  take the value 1 when country  $i$  is the low cost supplier of good  $(r, s)$  to country  $j$  in a given stage. Using this notation, we can then aggregate across goods and destinations as necessary. Denoting stages by  $k$ , we can define total production by good  $y_k^i(r, s)$ , total bilateral shipments by sector  $y_k^{ij}(s)$ , and total production by sector  $y_k^i(s)$  as:

$$y_k^i(r, s) \equiv \sum_j y_k^{ij}(r, s) \quad (13)$$

$$y_k^{ij}(s) \equiv \sum_r y_k^{ij}(r, s) \quad (14)$$

$$y_k^i(s) \equiv \sum_j \sum_r y_k^{ij}(r, s). \quad (15)$$

We now turn to the market clearing condition for stage 1 output. Note that bilateral

shipments of stage 1 inputs for sector  $s$  can be written as:

$$\begin{aligned} y_1^{ij}(s) &= \sum_r \mathbb{I}(j \in \Omega_1^i(r, s)) \tilde{p}_1^j(r, s) x_1^j(r, s) \\ &= \sum_r \mathbb{I}(j \in \Omega_1^i(r, s)) \theta(s) y_2^j(r, s), \end{aligned} \quad (16)$$

where the second line uses the fact that  $\tilde{p}_1^j(r, s) x_1^j(r, s) = \theta(s) y_2^j(r, s)$ . Then the market clearing condition for stage 1 output can be written as:

$$\begin{aligned} y_1^i(s) &= \sum_j y_1^{ij}(s) \\ &= \sum_j \left[ \frac{y_1^{ij}(s)}{y_2^j(s)} \right] y_2^j(s) \\ &= \sum_j \left[ \theta(s) \sum_r \mathbb{I}(j \in \Omega_1^i(r, s)) \left( \frac{y_2^j(r, s)}{y_2^j(s)} \right) \right] y_2^j(s). \end{aligned} \quad (17)$$

In the second line, the ratio  $\frac{y_1^{ij}(s)}{y_2^j(s)}$  records the share of stage 1 inputs from country  $i$  used by country  $j$  in sector  $s$  as a share of stage 2 output of sector  $s$  in country  $j$ . The third line says that this ratio is equal to the Cobb-Douglas input cost share times the weighted count of goods in which country  $i$  is the low cost supplier of stage 1 inputs to country  $j$ , where the weights equal the share of country  $j$ 's stage 2 production of each good in total stage 2 production in  $j$ .

Turning to stage 2 output, we need to divide this output across uses since it is both absorbed as a final good and used to form the composite input. We can break down output for sector  $s$  as follows:

$$\begin{aligned} y_2^{ij}(r, s) &= \mathbb{I}(j \in \Omega_2^i(r, s)) \tilde{p}_2^j(r, s) \tilde{q}^j(r, s) \\ &= \mathbb{I}(j \in \Omega_2^i(r, s)) \frac{P^j(s) Q^j(s)}{R} \\ &= \frac{\mathbb{I}(j \in \Omega_2^i(r, s))}{R} [\alpha(s) P_F^j F^j + \beta(s) P_M^j M^j] \\ &= \frac{\mathbb{I}(j \in \Omega_2^i(r, s))}{R} \left[ \alpha(s) P_F^j F^j + \beta(s) \sum_{s'} \theta(s') y_1^j(s') \right], \end{aligned} \quad (18)$$

where the last line uses the fact that  $P_M^j M^j = \sum_{s'} \sum_r \theta(s') y_1^j(r, s')$  and we introduce the notation  $\{\alpha(s), \beta(s)\}$  here to denote the Cobb-Douglas shares attached to sector  $s$ .<sup>12</sup> The

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<sup>12</sup>To be clear,  $\alpha(1) = \alpha$ ,  $\beta(1) = \beta$ ,  $\alpha(2) = 1 - \alpha$ , and  $\beta(2) = 1 - \beta$ .

last line breaks down stage 2 shipments into final use and intermediate use by sector.

Then the full sector-level market clearing conditions for stage 2 output are given by:

$$\begin{aligned}
y_2^i(s) &= \sum_j y_2^{ij}(s) \\
&= \sum_j \sum_r y_2^{ij}(r, s) \\
&= \sum_j \left[ \frac{\sum_r \mathbb{I}(j \in \Omega_2^i(r, s))}{R} \right] \left[ \alpha(s) P_F^j F^j + \beta(s) \sum_{s'} \theta(s') y_1^j(s') \right].
\end{aligned} \tag{19}$$

The ratio  $\left[ \frac{\sum_r \mathbb{I}(j \in \Omega_2^i(r, s))}{R} \right]$  is the fraction of stage 2 goods for which  $i$  is the low cost supplier to country  $j$  in sector  $s$ . For shorthand, we define  $R^{ij}(s) \equiv \sum_r \mathbb{I}(j \in \Omega_2^i(r, 1))$ , so then this fraction is given by:  $\frac{R^{ij}(s)}{R}$ .

With these market clearing conditions, we can set up the input-output table. The component pieces are bilateral input use matrices  $A^{ij}$  and bilateral final goods shipments, which we will denote  $f^{ij}$ . The input use matrices have four rows/columns, corresponding to stages and sectors, and take the form:

$$\begin{aligned}
A^{ij} &= \begin{bmatrix} 0 & \frac{y_1^{ij}(1)}{y_2^j(1)} & 0 & 0 \\ \frac{R_2^{ij}(1)}{R} \beta(1) \theta(1) & 0 & \frac{R_2^{ij}(1)}{R} \beta(1) \theta(2) & 0 \\ 0 & 0 & 0 & \frac{y_1^{ij}(2)}{y_2^j(2)} \\ \frac{R_2^{ij}(2)}{R} \beta(2) \theta(1) & 0 & \frac{R_2^{ij}(2)}{R} \beta(2) \theta(2) & 0 \end{bmatrix}, \\
\text{with } \frac{y_1^{ij}(s)}{y_2^j(s)} &= \theta(s) \sum_r \mathbb{I}(j \in \Omega_1^i(r, s)) \left( \frac{y_2^j(r, s)}{y_2^j(s)} \right).
\end{aligned} \tag{20}$$

The ordering of rows/columns is (sector 1, stage 1), (sector 1, stage 2), (sector 2, stage 1), and (sector 2, stage 2).

These bilateral matrices can be arrayed to form the  $4N \times 4N$  dimensional global input-output matrix:

$$A \equiv \begin{pmatrix} A^{11} & A^{12} & \dots & A^{1N} \\ A^{21} & A^{22} & \dots & A^{2N} \\ \vdots & \vdots & \ddots & \vdots \\ A^{N1} & A^{N2} & \dots & A^{NN} \end{pmatrix} \tag{21}$$

Then we can organize the use of stage 2 goods as final goods in vector form as:

$$f^{ij} = \begin{bmatrix} 0 \\ \frac{R_2^{ij}(1)}{R} \alpha(1) P_F^j F^j \\ 0 \\ \frac{R_2^{ij}(1)}{R} \alpha(2) P_F^j F^j \end{bmatrix}. \quad (22)$$

And let  $F$  be the  $4N \times N$  matrix of all  $f^{ij}$  vectors, where destinations  $j$  are arrayed along columns and source countries  $i$  are stacked vertically. And defining  $\iota$  as a  $N \times 1$  column of ones, note that  $F\iota$  is the vector of final goods produced by source  $i$ .

Finally, let us assemble output by stage and sector into vectors:

$$y^i = \begin{bmatrix} y_1^i(1) \\ y_2^i(1) \\ y_1^i(2) \\ y_2^i(2) \end{bmatrix}. \quad (23)$$

And let us stack these vertically to form an  $4N \times 1$  dimensional vector  $Y$ .

Given this set-up, the standard input-output accounting identity holds:  $Y = AY + F\iota$ . We will use this input-output system to compute different model-based measures of trade in value added.

### 3 Mapping the Model to Data

In this section, we discuss how we fit the model presented in Sections 1 and 2 to data. In this, we now assign identities to the two sectors in the model and refer to sector 1 as the ‘goods’ sector and sector 2 as the ‘services’ sector. We begin by presenting our data source and discuss how we assemble the global input-output framework from that data. We then discuss how we calibrate a subset of the parameters of the model, and estimate the remainder via a simulated method of moments procedure. We then conclude this section by describing how we use the global input-output framework to compute several measures of trade in value added.

### 3.1 Global Input-Output Data

Our data source is the GTAP 7.1 Data Base assembled by the Global Trade Analysis Project at Purdue University, which includes trade, production, and input-output data for 2004.<sup>13</sup> While the underlying data includes more than 90 countries, we cannot use this fine country detail due to computational constraints. Therefore, we retain 15 major countries – United States, China, Japan, Germany, Italy, India, Great Britain, France, Canada, Spain, Brazil, Australia, Russia, Mexico, and South Korea – and aggregate the remaining countries to form a composite rest-of-the-world region.<sup>14</sup> Further, there are 57 sectors in the underlying data.

In the data, we have information on 6 objects for each country:

1.  $y^i$  is a  $57 \times 1$  vector of total gross production.
2.  $f^{Di}$  is a  $57 \times 1$  vector of domestic final expenditure, which includes consumption, investment, and government purchases.
3.  $f^{Ii}$  is a  $57 \times 1$  vector of domestic final import expenditure.
4.  $A^{ii}$  is a  $57 \times 57$  domestic input-output matrix.
5.  $A^{Ii}$  is a  $57 \times 57$  import input-output matrix.
6.  $\{x^{ij}\}$  is a collection of  $57 \times 1$  bilateral export vectors for exports from  $i$  to  $j$ .

To form data-based bilateral input-output matrices  $\bar{A}^{ji}$  and final expenditure vectors  $\bar{f}^{ji}$ , we apply a proportionality assumption. Within each of the 57 sectors, we assume that imports from each source country are split between final and intermediate in proportion to the overall split of imports between final and intermediate use in the destination. And conditional on being allocated to intermediate use, we assume that imported intermediates from each source are split across purchasing sectors in proportion to overall imported intermediate use in the destination. Mathematically, we compute  $\bar{A}^{ji}$  and  $\bar{f}^{ji}$  as:

$$\bar{A}^{ji}(s, s') = Mshare^{ji} A^{Ii} \quad \text{and} \quad \bar{f}^{ji}(s) = Mshare^{ji} f^{Ii},$$

where  $Mshare^{ji}$  is a  $57 \times 57$  matrix with elements  $\frac{x^{ji}(s)}{\sum_j x^{ji}(s)}$  along the diagonal.

Stacking the production data to form an  $57N \times 1$  vector of gross output  $\bar{Y}$ , assembling the input output matrices to form a  $57N \times 57N$  global input-output matrix  $\bar{A}$ , and arranging

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<sup>13</sup>See the GTAP website at <http://www.gtap.agecon.purdue.edu> for documentation of the source data. This is the same dataset used in Johnson and Noguera (2012a).

<sup>14</sup>Because we have input-output data for the countries that comprise the rest-of-the-world region, we can aggregate them in a way that preserves basic input-output identities for the world as a whole.



the final expenditure vectors  $\bar{f}^{ij}$  into  $57N \times N$  matrix  $\bar{F}$ , we write the input-output identity in the data as:  $\bar{Y} = \bar{A}\bar{Y} + \bar{F}\iota$ .

We use this system to compute value added in trade, as described in the next section. We also use the final and intermediate shipments in this data in estimation of the model. Specifically, we aggregate bilateral final and intermediate shipments to the two-sector level, focusing on goods versus services shipments.<sup>15</sup> Due to the proportionality assumptions used above, variation in aggregate final and intermediate goods trade shares across trade partners for a given destination arises solely due to differences in the composition of imports across partners.<sup>16</sup>

## 3.2 Fitting the Model

There are a number of free parameters, including technology levels  $\{T_1^i(r, s), T_1^i(r, s)\}$ , trade costs  $\tau^{ij}(s)$ , and several share parameters in production functions and preferences  $\{\theta(s), \alpha, \beta\}$ . We mix calibration and estimation in pinning down these parameters.

### 3.2.1 Calibrated Parameters

We calibrate  $\{\theta(s), \alpha, \beta\}$  to match ratios for ‘typical’ countries in the data. The parameter  $\theta(s)$  governs the value added to output ratio in each sector.<sup>17</sup> We therefore set  $\theta(s)$  to match the median value added to output ratio across countries in each sector. These median values are:  $\theta(1) = 0.62$  and  $\theta(2) = 0.41$ .<sup>18</sup> These imply that the value added to output ratio is lower by roughly 0.2 in the goods sector relative to the services sector.

The parameter  $\alpha$  is also straightforward to calibrate, since it is the share of goods in final expenditure. We also set this value equal to the median across countries, given by  $\alpha = 0.25$ .<sup>19</sup> Finally,  $\beta$  governs extent to which goods versus services are used in forming the composite input. As we describe in Appendix A, we choose  $\beta$  to match inter-sectoral flows for the world economy as a whole. This yields a value  $\beta = .6$ . Note that  $\beta > \alpha$ , so goods

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<sup>15</sup>Goods covers sectors 1-42 in the GTAP data, which includes agriculture, natural resources, and manufacturing.

<sup>16</sup>This likely understates true variation in final and intermediate input shares across partners. An alternative approach to measuring final goods trade flows across countries would be to use trade data classified according the Broad Economic Categories (BEC) system, which categories goods based on final versus intermediate use. We intend to check our results using this alternative approach.

<sup>17</sup>To see this, note that value added is equal to the wage bill for each good  $r$ :  $va^i(r, s) = w^i l_1^i(r, s) + w^i l_2^i(r, s) = (1 - \theta(s))[y_1^i(r, s) + y_2^i(r, s)]$ . Adding up across goods yields:  $1 - \theta(s) = \frac{va^i(s)}{[y_1^i(s) + y_2^i(s)]}$ .

<sup>18</sup>The input share for goods varies from roughly 0.55 to 0.73 across countries, with most countries between 0.6 and 0.7. The input share for services varies from 0.31 to 0.53.

<sup>19</sup>Expenditure shares on manufactures tend to be higher than this baseline value emerging markets (up to 0.5), and near this benchmark in industrial countries.

receive a larger weight in the composite input than in final demand.

### 3.2.2 Estimation via Simulated Method of Moments

The remaining unknown parameters are technology levels and trade costs.

For technology levels, we assume that countries draw productivity from country, stage, and sector specific Fréchet distributions, where draws are assumed to be independent across countries/stages/sectors.<sup>20</sup> We parameterize these distributions with a common shape parameter  $\kappa$ , and location parameters  $\{T_1^i(s), T_2^i(s)\}$  for sector  $s$  in country  $i$ . We set  $\kappa = 4.12$ , guided by Simonovska and Waugh (2011).<sup>21</sup> [To simplify computation, we restrict  $T_k^i(1) = T_k^i(2)$  in this draft, so that productivity in goods and services are equal within countries. We plan to relax this assumption in future drafts.]

We also parameterize trade costs by assuming that bilateral trade costs are a power function in distance. Specifically, we estimate a function of the form:  $\tau^{ij}(s) = \tau(s) (d^{ij})^\rho$ , where  $d^{ij}$  is the distance between country  $i$  and country  $j$ ,  $\tau(s)$  is a level parameter, and  $\rho$  is the elasticity of trade costs to distance. We set trade costs on domestic shipments to one in all countries ( $\tau_{ii}(s) = 1$ ).

We denote the set of parameters to be estimated as  $\Theta = \{T_1^i(s), T_2^i(s), \rho\}$ , and without loss of generality set  $T_1^1(s) = T_2^1(s) = 1$  so that technology levels are measured relative to country 1. We estimate these parameters by matching the simulated data to measured shipments of final and intermediate goods across countries. In effect, we choose parameters to ensure that the model-based input-output table mimics data on global input-output linkages. In this estimation, we generate simulated data taking final expenditure in each market  $P_F^i F^i$  and relative wages  $w^i$  as given and set equal to values in the data (with  $w^1 = 1$  as a normalization).<sup>22</sup>

**Moments** In this draft, we estimate the model using moments based on trade in goods only, and therefore assume that services are non-traded (as if  $\tau(s) \rightarrow \infty$ ). Specifically, we estimate the model under the restrictions that stage 1 inputs in the services sector must be source domestically, and that the stage 2 output of services must either be used to satisfy

<sup>20</sup>We have experimented with allowing draws to be correlated across stages for individual goods. The parameter governing this correlation is weakly identified by the data and introducing this correlation does not materially affect the results.

<sup>21</sup>Though we set the shape parameter to this value based on Simonovska and Waugh, we recognize that this parameter is estimated in a model that is different than ours. Obtaining appropriate estimates for this parameter in the context of our model is a topic for future work.

<sup>22</sup>We compute total expenditure in each market by dividing observed final expenditure on goods value by  $\alpha$ . We also observe the trade surplus for goods  $TB^i$  in the data. Since the consumer budget constraint is given by  $w^i L^i = P_F^i F^i + TB^i$ , then we back out wages as:  $w^i = (P_F^i F^i + S^i) / L^i$ . We use 2004 population data from the Penn World Table 7.1 to proxy for labor endowment  $L^i$ .

domestic final demand for services or shipped to domestic stage 1 producers to form the composite input. We discuss the model-based global input-output table in this special case further in Appendix C. [We restrict services trade here to reduce computation time, and plan to estimate the full model in future drafts.]

Given calibrated parameters  $\{\kappa, \theta(s), \alpha, \beta\}$ , relative wages, and final expenditure, we choose values of  $\Theta$  and draw productivities for each good and sector, which we denote  $T_1^j(r, s)$  and  $T_2^j(r, s)$  with  $r = 1, \dots, R$  indexing the sequence of the draw. Given these parameters, we solve the model following the procedure in Section 1.4.<sup>23</sup> Using the simulated data from the model, we then compute a vector of moments  $M(\Theta)$ , which we match to analogous moments  $M$  in the data.

We form the first set of moments using trade shares for final goods. From Equation (22), final goods shipments from  $i$  to  $j$  in sector 1 are  $\left(\frac{R_2^{ij}(1)}{R}\right) \alpha(1) P_F^j F^j$ , where  $\alpha(1) P_F^j F^j$  is total expenditure on goods in  $j$ . So the share of final goods from source  $i$  in country  $j$  expenditure is:  $Fshare^{ij} = \frac{R_2^{ij}(1)}{R}$ .<sup>24</sup>

The second set of moments consists of trade shares for inputs. Input shipments from country  $i$  to  $j$  include both stage 1 goods and stage 2 goods destined for the composite input. Define input shipments from  $i$  to  $j$  of sector 1 goods as:  $In^{ij}(1) = \sum_r \mathbb{I}(j \in \Omega_1^i(r, 1)) [\theta(s) y_2^j(r, 1)] + \frac{1}{R} \sum_r \mathbb{I}(j \in \Omega_2^i(r, 1)) [\beta(1) P_M^j M^j]$ . Define total input purchases of sector 1 goods as:  $In^j(1) = \theta(1) y_2^j(1) + \beta(1) P_M^j M^j$ . Then the share of inputs from source  $i$  in country  $j$ 's total purchases of inputs from the goods sector is:  $Inshare^{ij} = \frac{In^{ij}(1)}{In^j(1)}$ . And

Since  $\sum_i \pi_n^{ij} = 1$ , we only use off diagonal trade shares, in total  $2(N^2 - N)$  moments. We denote the log difference between actual and simulated moments  $\mu^{ij}(\Theta) = \ln m^{ij} - \ln \hat{m}^{ij}(\Theta)$ , and stack all  $\mu^{ij}$ 's in a column vector  $M(\Theta)$ .

**Estimation Procedure** Our estimation procedure is based on the moment condition  $E[M(\Theta_0)] = 0$ , where  $\Theta_0$  is the true value of  $\Theta$ . Hence, we estimate a  $\hat{\Theta}$  that satisfies:

$$\arg \min_{\Theta} \{M(\Theta)' M(\Theta)\} \quad (24)$$

Since we have  $2N$  variables and  $2(N^2 - N)$  moments, the model is over identified. [Though we do not compute standard errors in this draft, we will report them in future drafts.]

We note here that this minimization problem is not straightforward to solve numerically, since the simulated moments are not continuous in the underlying parameters. To circumvent

<sup>23</sup>Since we take wages as given in the estimation, we drop the final stage of the procedure in Section 1.4 in which the wage is determined via labor market clearing.

<sup>24</sup>Note that in the model, these trade shares are identical to the trade shares for all stage 2 output, including output dedicated for intermediate use. So they can alternatively be computed using total stage 2 trade flows.

this problem, we borrow a technique to smooth the objective function from the discrete choice literature. With this smoothing, we can turn to standard numerical routines to solve the minimization problem. We discuss this technical issue further in Appendix B.

### 3.3 Value Added in Trade

Using the input-output frameworks described above, we can compute the value added content of final goods shipments in both model and data. We focus on two alternative metrics. First, we compute the amount of value added from each source country embodied in final goods produced by a given country, which we refer to as ‘value-added inputs.’ Second, we compute the amount of value added from each source country consumed in each destination, which we refer to as ‘value-added exports’ as in Johnson and Noguera (2012a). The common element in both calculations is the observation that multiplying the Leontief inverse of the global input-output matrix by a vector of final goods returns the amount of gross output (by country and sector) needed to produce those final goods. These gross output requirements can then easily be converted to value added requirements, by multiplying by value added to output ratios.

Starting with the data, we compute value-added inputs for final output from the goods sector as follows. We construct total final goods shipped from each country as  $\bar{F}\iota$ , and then reshape the resulting vector into corresponding  $57 \times 1$  vectors of final goods shipped from each country, which we write  $\bar{f}^i$ . Zeroing out elements of these vectors corresponding to services sectors, we get modified vectors  $\bar{f}^{i,goods}$ . Then we arrange the collection of  $\bar{f}^{i,goods}$  for all countries to form a  $57N \times N$  block diagonal matrix  $\bar{F}^{VAI}$ , and compute foreign value added in final output in the goods sector as:

$$\overline{VAInputs} \equiv \bar{R}(I - \bar{A})^{-1}\bar{F}^{VAI}, \quad (25)$$

where  $\bar{R}$  is a  $N \times 57N$  block diagonal matrix with row vectors of value added to output ratios for each country along the diagonal.

To explain this calculation, note that  $(I - \bar{A})^{-1}\bar{F}_{VAI}$  returns a  $57N \times N$  matrix where column  $j$  is the vector of output needed to produce final goods shipped from  $j$  to all destinations. To compute value added embodied in those goods, we multiply by sector-level value added to output ratios and sum across sectors, where both operations are accomplished simultaneously via pre-multiplication by  $\bar{R}$ . The  $ij$  elements of the resulting matrix are the amount of value added from country  $i$  embodied in final goods produced in country  $j$ . For example, it measures the amount of Mexican value added in final goods produced in the United States. We construct value added inputs in the model in an identical way, but slide

in model-based definitions for the input-output matrix, final goods production, and value added to output ratios.<sup>25</sup> We denote the resulting values  $VAI_{inputs}$ .

We can also compute value added exports in the model and data. The procedure is similar to that above, except that final goods are distinguished according to the destination in which they are consumed.<sup>26</sup> We express value added exports for the goods sector in matrix form as:

$$\overline{VAExports} = \bar{R}^{VAX}(I - \bar{A})^{-1}\bar{F}, \quad (26)$$

where here  $\bar{R}^{VAX}$  takes the same form as  $\bar{R}$ , but replaces all value added to output ratios for services with zeros. The  $ij$  elements of  $\overline{VAExports}$  record the amount of value added from the goods sector in country  $i$  that is absorbed in destination  $j$ , embodied in the final goods that  $j$  consumes. As above, we can construct the model-equivalent measures using an identical formula with values from the model-based input-output framework substituted for values from data, denoting resulting values  $VAExports$ .

## 4 Estimation Results

### 4.1 Technology and Trade Costs

**Technology** We present estimates for technology levels by stage for the 14 countries and the composite region in Table 1. All values are expressed relative to the United States. The first two columns present estimates of the parameters  $\{T_1^i(1), T_1^i(1)\}$ , while the third and fourth columns convert these estimates into average productivity levels, describing the geometric means of  $T_n^i(z, 1)$  in each country.<sup>27</sup> The final column computes the productivity of stage 2 relative to stage 1 production in each country.

The estimates indicate that most countries have technology/productivity levels lower than the U.S. level. We plot the aggregate productivity level for each country again income per capita in Figure 2, computed as an unweighted geometric mean of stage 1 and stage 2 productivities. As expected, average productivity is highly correlated with income per capita.

Based on examination of Table 1, it is evident that technology and productivity levels are correlated across stages, in that countries with high absolute productivity in stage 1 tend

<sup>25</sup>One point to note is that value added to output ratios in the model are pinned down by parameters, equal to  $1 - \theta(s)$  in each sector and common to all countries by assumption.

<sup>26</sup>See Johnson and Noguera (2012a,b) for further discussion of value-added exports.

<sup>27</sup>Since  $T_n^i(z, 1)$  is drawn from a Fréchet, the geometric mean is given by  $T_n^i(1)^{1/\kappa} \exp(\gamma/\kappa)$ , where  $\gamma$  here is the Euler-Mascheroni constant.

to also have high absolute productivity in stage 2. Despite this correlation, there are sizable differences in relative stage productivities across countries. To be clear about interpretation, the final column Table 1 measures relative productivity in each country relative to relative productivity in the U.S. So numbers greater than one indicate that a country has greater comparative advantage in stage 2 production than does the U.S. Scanning the table, China, Germany, India, Japan, and South Korea all have comparative advantage in stage 2 production, while all other countries tend to have comparative advantage in stage 1 production. In Figure 3, we plot the log difference between exports in the second stage versus the first stage against relative productivity across stages. As is evident, comparative advantage manifests itself strongly in export composition in the model.

Two points regarding patterns of comparative advantage across stages are worth further comment. First, relative productivities across stages are uncorrelated with income per capita. As is evident in Figure 4, there is significant variation in relative productivities at both high and low income levels, but virtually zero correlation between relative productivity and income.

Second, the relative productivity of stage 2 production is negatively correlated with a country's commodity share of exports. We plot relative productivities against commodity export shares in Figure 5. This strong correlation makes sense from the perspective of the model. Commodities are heavily used as intermediate inputs, so countries that export commodities tend to account for a higher share of intermediate goods imports than final goods imports. The model rationalizes this fact by assigning those countries relatively high productivity in supplying stage 1 inputs. In this sense, the model describes comparative advantage across goods based on how goods are used by importers.

**Trade Costs** Turning to estimated trade costs, we assumed that the trade cost function took the form:  $\tau^{ij} = \tau (d^{ij})^\rho$ . Our estimate of the elasticity of trade costs to distance is  $\rho = 0.28$ , and the level parameter is  $\tau = .31$ .<sup>28</sup> For the country pair separated by the median distance in our data (8500km), these estimates imply that international trade costs are roughly 3.76 times (276% higher than) domestic trade costs:  $\tau^{median} = \exp(\ln \kappa + \rho \ln(8500)) = 3.76$ . These costs are large, but in line with standard estimates from gravity-style models.<sup>29</sup> We examine what our estimates imply for gravity-style analyses of trade further below.

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<sup>28</sup>This magnitude of this level parameter is not directly interpretable since it depends on the units in which we measure distance. Therefore, we focus on total implied trade costs, which are interpretable as costs of international relative to domestic trade.

<sup>29</sup>For example, Eaton and Kortum (2002) return estimated distance costs of roughly 300% for country pairs in the 3000 to 6000 mile distance range. Anderson and van Wincoop argue trade costs are equivalent to an ad-valorem tax of 170% for a representative rich country, which would translate presumably into higher estimates for poorer countries.

**Model Fit** Before turning to detailed analysis of the model, we quickly summarize how the model fits various moments in the data. We start by examining how the model fits the moments that we have targeted in estimation – intermediate and final goods trade shares – in Figure 6, the true trade shares are on the x-axis and simulated trade shares are on the y-axis.<sup>30</sup> The model generally fits these trade shares well.<sup>31</sup>

Turning to untargeted moments, we are able to reproduce variation in several key metrics of global production sharing. In the left panel of Figure 7, we plot value added input shares – the share of value added from each bilateral source country in production of final goods for a given exporter. In the right panel of Figure 7, we plot bilateral value added to gross shipments ratios, which equal value added to export ratios for cross-border trade.<sup>32</sup>

The model does an excellent job at replicating bilateral sourcing of value added. The model also generates a positive correlation between value added to export ratios in actual and simulated data, though the overall fit here is not as tight. The dimension on which the model misses is in generating value added to export ratios near/above one, which are observed in the actual data but not the simulated data. Consistent with these bilateral data, we show in Figure 8 that the model fits aggregate value adding sourcing patterns better than aggregate VAX ratios. Though the model gets the level of the VAX ratio right for most countries, it misses in explaining the cross-country variation of those levels.

Finally, we report several reduced form correlations between bilateral trade and distance that are helpful for interpreting counterfactuals below. To do this, we estimate a simple gravity regression of the form:  $\log y^{ij} = \chi^i + \chi^j + \delta \ln \text{Distance}^{ij} + e^{ij}$ , where  $y^{ij}$  is either actual or simulated total bilateral exports ( $Exports^{ij}$ ) or VAX ratios ( $VAX^{ij} = VAXExports^{ij}/Exports^{ij}$ ), and  $\chi^i$  and  $\chi^j$  are exporter and importer fixed effects.<sup>33</sup> We present the results in Table 2. Not surprisingly, the model is able to reproduce the well known dampening effect of distance on trade, producing a distance coefficient of  $-1.07$ , slightly larger than the  $-1$  in the actual data. Further, the model reproduces the positive

<sup>30</sup>The cluster of points in the upper right corner is the share of each country’s purchases from itself. Not surprisingly, these own shares are uniformly large.

<sup>31</sup>Examining the more detailed data, the model struggles to fit import shares for each country vis-a-vis the rest of the world. This is not surprising, since we force the model to assign a single productivity estimate to the rest of the world, even though individual countries trade with different countries within that composite. The model also struggles in fitting trade shares between the U.S. and its NAFTA partners.

<sup>32</sup>In both figures, we include domestic as well as cross-border transactions. Specifically, the cluster of points in the upper right of the left panel is the share of domestic value added in final goods production for each country, while the cluster of points in the lower left of the right panel is the value added to gross shipment ratio for domestic shipments (as in, ‘exports’ to one-self). We include these domestic transactions because these are important moments for the model to replicate in order to generate the correct degree of aggregate openness for each country.

<sup>33</sup>In estimating this regression, we include exports to/from the rest of the world. Results are virtually identical if we exclude these flows.

correlation between value added to export ratios and distance, though the magnitude is somewhat smaller in the simulated than actual data.<sup>34</sup>

## 4.2 Interpreting the Distance Coefficient in Gravity Regressions

In trade models that yield CES import demand equations, the elasticity of bilateral trade to distance – the ‘distance coefficient’ in a gravity regression – is equal to the (constant) elasticity of imports times the elasticity of trade costs to distance. If the elasticity of trade costs to distance is constant, then the gravity distance coefficient is also constant and therefore independent of the level of trade costs. Further, if one knows the elasticity of import demand (e.g., the Fréchet shape parameter in the Eaton-Kortum model) then one can infer the elasticity of trade costs to distance by dividing the gravity distance coefficient by that parameter.

In contrast to this CES benchmark, the elasticity of bilateral trade to distance depends on the level of trade costs in the multi-stage model. Therefore, the gravity distance coefficient is not necessarily directly informative regarding the true elasticity of trade costs to distance. The extent to which the CES logic above would lead one to mis-estimate the elasticity of trade costs to distance is an empirical matter. So, we turn to our estimated model to evaluate this issue.

Our baseline estimates suggest that the standard procedure does provide a reasonable guide for interpreting the data. In our data, as in most data sets, the elasticity of trade to distance is close to -1 (see Table 2). Given this estimate and an import elasticity near 4, one would infer that the elasticity of trade costs to distance is near 0.25. This is very close to the actual distance elasticity we estimate (0.28), allowing for multi-stage production. Thus, one would reach similar conclusions about the elasticity of trade costs to distance in our multi-stage model and standard CES models.

This conclusion is surprising. Yi (2010) argues that multi-stage production is important in inflating estimated border effects relative to true border frictions. By implication, Yi’s work suggests that one should see inflation in other reduced form gravity coefficients as well. Yet, we do not find significant inflation, at least relative to the CES benchmark.

The reason is that our model returns high estimates for the level of international trade costs. As noted above, the elasticity of bilateral trade to distance is a function of the level of trade costs in our model. In particular, the elasticity of trade to distance increases (in absolute value) as the level of trade costs falls. Therefore, if trade costs were lower, then our estimated model would generate significant inflation in gravity coefficients.

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<sup>34</sup>Johnson and Noguera (2012b) present and discuss this correlation of VAX ratios and distance at length.



To illustrate this point, we estimate gravity regressions (with fixed effects) in simulated data from our model for alternative levels of international trade costs. In these experiments, we lower trade costs uniformly across all partners, leaving relative distances and hence relative trade costs unchanged across bilateral partners.<sup>35</sup> Starting from our baseline simulated data, we reduce the level of trade costs by 200 percentage points in 10 increments. This takes the mean bilateral trade cost in the model from near 250% to 50%. We then resimulate bilateral trade flows at each of these new levels of trade costs, and estimate a standard gravity regression in each simulated data set.

We plot the resulting distance coefficients against the mean bilateral trade cost in alternative scenarios in Figure 9. The distance coefficient in our baseline simulation is the furthest point in the upper right corner, where average iceberg costs are near 250%. As trade costs fall, the absolute value of the elasticity increases in the multistage case. As we lower trade costs, this increase is initially gradual, but starts to increase sharply as average trade costs fall to around 150%. With ad valorem trade costs at 50%, then the multistage elasticity is inflated by roughly 13% relative to the baseline elasticity. In other words, a lower *level* of trade costs make trade more sensitive to distance in a world with multistage production.

Our intuition for this result is as follows. For high levels of trade costs, breaking up production stages across borders is costly and occurs less frequently. Hence, most substitution is across goods, rather than over the location of production stages. And substitution across goods is governed by the Fréchet elasticity, as in the Eaton-Kortum model. As we lower the level of trade costs, fragmentation becomes more common and so the Yi-type amplification effects kick in. Seen in this light, Yi apparently finds larger inflation in the influence of borders and tariffs on trade because he analyzes cases in which trade costs are low. For example, Yi (2010) focuses on US-Canada trade where ad valorem trade costs are in the range of 10-40%, while Yi (2003) analyzes tariff liberalization starting with initial tariffs near 15%.<sup>36</sup> This level of trade costs is far too low to rationalize the observed home bias and

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<sup>35</sup>In lowering the level of trade costs by large amounts, we run into a subtle technical constraint in analyzing the resulting data. As the level of trade costs falls, country pairs with initially low trade costs hit the lower bound of  $\tau^{ij} = 1$ . Once at this bound, we cannot lower trade costs further for these pairs. As a result, we start to distort relative distances in the data when we proceed to lower trade costs among all other pairs (i.e., pairs not at the lower bound). To estimate gravity coefficients using a constant set of relative distances and trade costs at different absolute levels of trade costs, we drop these country pairs for which the lower bound is attained from all regressions (both the high and low trade cost regressions). Because we drop some countries here, the reduced form gravity coefficient is somewhat higher than in our full dataset, which is reflected in a larger initial absolute value at the highest level of trade costs in Figure 9.

<sup>36</sup>There are of course many other differences between our model, calibration, and estimation procedure and the quantitative models in Yi (2003, 2010), but we believe this to be the key difference between models driving our results. Given this, it is natural to ask how the low level of trade costs assume in Yi's analysis yield realistic levels of trade. In Yi (2003), one reason is that the model is calibrated to two symmetric countries. For example, suppose that we examine a symmetric two-country Ricardian model, and let us assume for simplicity that relative wages are equal to one and constant (e.g., set in an outside, freely traded

concentration in trade flows observed in the data.<sup>37</sup>

## 5 Technology and Trade Cost Counterfactuals

We now turn to analysis of several counterfactuals in the model. We start with a scenario in which we lower trade costs across all partners. We then examine cross-country spillovers and trade effects from an increase in productivity in China.

### 5.1 Reduction in Trade Costs

Starting from the baseline estimated equilibrium, we now lower trade costs by 10% for all country pairs, so that  $\tau_{new}^{ij} = 0.9\tau^{ij}$ ,  $i \neq j$ .<sup>38</sup> We examine three main outcomes: changes in real wages, changes in gross trade, and changes in vertical specialization and value added trade. In analyzing trade and vertical specialization, we describe ‘short run’ and ‘long run’ effects of the change in trade costs separately. By short run response, we mean changes the model equilibrium holding the source from which each country purchases stage 1 inputs fixed.<sup>39</sup> The long run responses then allow producers to re-optimize stage 1 sourcing decisions. In dubbing these short run versus long run, we implicitly have in mind a more complicated model in which producers are tied to suppliers in the short run due fixed costs or time lags associated with forming new supplier relationships.<sup>40</sup>

In Figure 10, we plot changes in consumer prices ( $P_F^i$ ), wages ( $w^i$ ), and real wages ( $w^i/P_F^i$ ) for each of the countries in our sample. The median increase in real wages is 1 percent, but there is considerable heterogeneity across countries. Overall, real wages increase more in markets with higher initial import shares (e.g. France and Spain), since imported goods have a larger share of the price index in these countries.

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sector). With Fréchet productivity distributions, then the share of imports in GDP is given by:  $\frac{\tau^{-n}}{1+\tau^{-n}}$ . If  $n \in (3, 6)$ , then one needs trade costs between 20-44% to achieve an import share of 0.25. In our model and data, even very small countries exhibit extreme home bias, which requires much higher trade costs.

<sup>37</sup>At the same time, it might be possible to detect amplification effects at current levels of trade costs if we were to use more disaggregated data, featuring sectors with lower implied trade costs. This remains a topic for further work.

<sup>38</sup>In computing the new equilibrium, we hold trade imbalances constant at their initial levels.

<sup>39</sup>In response to the decline in trade costs, a given country may start to produce stage 2 goods that it did not produce in the initial equilibrium. In these cases, we assume that the country sources stage 1 inputs from the country that would have been the low cost supplier in the initial equilibrium. We could alternatively allow new stage 2 producers to choose sources based on the new configuration of trade costs and prices in the new equilibrium.

<sup>40</sup>One could of course introduce these frictions directly into the model, at the cost of increasing the complexity of the optimization problem. We take this ad hoc approach to simply illustrate the functioning of the model.

The response of exports and imports is plotted in Figure 11. Overall, the median increase in exports (imports) is 39% (30%) percent, and world trade overall rises by 32% in response to this 10% change in trade costs. The short run effect is denoted by the black segment, while the white segment is the additional long run effect. For most countries the short run effect dominates the overall response of trade, but there are large differences across countries in the relative importance of short run and long run adjustment.

Differences between short and long run changes in exports and imports are also naturally linked to one another across countries. Countries that are upstream in the production process (i.e., specialize in stage 1 goods) tend to see larger long run relative to short run responses in their exports. On the flip side, countries that are downstream (i.e., specialize in stage 2 production) tend to see larger long run than short run responses in their imports. Further, trade partners also tend to see similar symmetric adjustments. For example, the United States sees a relatively small change in imports in the short run, but a much larger change in the long run. This is matched by relatively large long run adjustment in exports for Canada and Mexico, which makes sense since much of the increased foreign sourcing in the U.S. will be concentrated on these markets.

Turning to trade measured in value added terms, we plot responses for two measures – the share of foreign value added in final goods production and the value-added to export ratio – in Figure 12. Lower trade costs lead to more fragmented production, as measured by both metrics. For the median country, the foreign value added input share increases by 3.8 percentage points, while the VAX ratio decreases by 2.9 percentage points. Further, for most countries, the bulk of the adjustment in these metrics comes from long run re-optimization of stage 1 sourcing decisions.

In looking at bilateral changes in value added to export ratios, one finds that adjustments tend to be larger among nearby partners. To illustrate this, we plot changes in VAX ratios at the bilateral level in Figure 13. Overall, VAX ratios tend to fall among most (though not all) bilateral pairs. And these changes are largest on average in the lower left portion of the figure, where distances are smallest. This differential change by distance implies that a uniform reduction in trade costs makes gross exports more concentrated relative to value-added exports in the multi-stage model. This pattern of suggests that a declining level of trade costs may be able to explain the increasing concentration in gross relative to value-added exports documented in Johnson and Noguera (2012b, 2012c).

As in the evaluation of our estimates of trade costs above, it is natural to ask how the response of trade and wages differs in the multi-stage model relative to more conventional Ricardian models. Because the multi-stage model does not strictly speaking nest the standard Ricardian model, this question is not straightforward to answer. However, we can broadly

compare the multi-stage results to results from the a two-sector version of the Ricardian model used by Caliendo and Parro (2012).

To do this, we construct a simulated data set from our estimated model, including bilateral trade shares, income, sector-level production and expenditure, sector-level input cost shares, and the trade surplus. Along with an assumed Fréchet shape parameter, these are all the parameters needed to compute counterfactuals in the Caliendo-Parro model, following techniques from Dekle, Eaton, and Kortum (2008).<sup>41</sup> Parameterizing the Caliendo-Parro model using our simulated data ensures that we start both the counterfactuals in the multi-stage model and the Ricardian model at an observationally identical equilibrium.<sup>42</sup> We then feed the same change in trade costs used above ( $\tau_{new}^{ij} = 0.9\tau^{ij}$ ,  $i \neq j$ ) through the Caliendo-Parro model. For reference, we describe the exact calibration and solution procedure in Appendix D.

The responses of exports, imports, and real wages in the Caliendo-Parro benchmark are reported in Figures 14 and 15. Overall, the response is similar to the multi-stage model. Real wages rise by 1 percent for the median country, while exports (imports) increase by 47% (38%). Global trade rises by nearly 40%. Like the gravity results above, the fact that the Caliendo-Parro model generates changes in trade as large as the multi-stage model also surprising. Yi (2003) finds that a comparably sized liberalization generates a strong non-linear response of trade in a multi-stage model, far in excess of what would be predicted by a generic Ricardian model. Our intuition for why we do not find these large effects is largely the same: at the high trade costs needed to match observed trade shares, the amplification effects of vertical cross-border production are weak. That said, the mechanics of adjustment and shuffling of the allocation of stages to countries in the multi-stage model are of course different than substitution across goods in the benchmark Ricardian model. So the same macro-behavior hides different underlying micro-behavior.

## 5.2 Changes in Technology

We now turn to investigating how improvements in local technology in one country induce changes in global production chains. We focus on one experiment here: an increase in technology in China. Starting with our estimated model, we increase both  $T_1^{CHN}$  and  $T_2^{CHN}$  by two log points, which brings Chinese technology roughly to the level of Mexico.

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<sup>41</sup>Specifically, we do not need information about the trade cost function and technology parameters to solve for counterfactuals in the model. See Caliendo and Parro (2012) and Dekle, Eaton, and Kortum (2008) for details.

<sup>42</sup>To be clear, our simulated data naturally includes information about shipments between production stages. We discard this information in calibrating the Caliendo-Parro model, since the meaning of a production stage is undefined in that model.

Figure 16 illustrates the changes in value added to export ratios and the share of foreign value added in final goods production following the change in Chinese technology. The largest adjustment occurs within China itself, where the VAX ratio rises and the foreign value added sourcing share falls. Both adjustments reflect the fact that China sources a larger fraction of inputs from itself in the new equilibrium.

Nearby countries – such as Japan, Australia and South Korea – experience the opposite adjustment. For those countries, VAX ratios fall and foreign value-added input shares rise, as China supplies more intermediate inputs into production in the Asian region. For European countries and the U.S., the adjustments are generally very small. In Figure 17, we plot changes in foreign value-added input shares by country on the vertical axis against distance from China on the horizontal axis. The takeaway from both figures is that adjustments in the production chain are mostly confined to proximate trading partners. As in the case of the decline in trade costs above, this again reflects the predominantly local scope of production chains.

## 6 Conclusion

[To Be Added]

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Table 1: Estimated Technology

|                | Technology |         | Average Productivity |         |                         |
|----------------|------------|---------|----------------------|---------|-------------------------|
|                | Stage 1    | Stage 2 | Stage 1              | Stage 2 | $\frac{Stage2}{Stage1}$ |
| United States  | 1          | 1       | 1                    | 1       | 1                       |
| China          | 0.004      | 0.022   | 0.27                 | 0.40    | 1.47                    |
| Japan          | 0.719      | 0.929   | 0.92                 | 0.98    | 1.06                    |
| Germany        | 0.496      | 1.701   | 0.84                 | 1.14    | 1.35                    |
| Italy          | 0.619      | 0.535   | 0.89                 | 0.86    | 0.97                    |
| India          | 0.001      | 0.005   | 0.18                 | 0.28    | 1.57                    |
| United Kingdom | 0.888      | 0.381   | 0.97                 | 0.79    | 0.81                    |
| France         | 0.562      | 0.378   | 0.87                 | 0.79    | 0.91                    |
| Canada         | 0.724      | 0.181   | 0.92                 | 0.66    | 0.71                    |
| Spain          | 0.247      | 0.224   | 0.71                 | 0.70    | 0.98                    |
| Brazil         | 0.026      | 0.011   | 0.41                 | 0.34    | 0.82                    |
| Australia      | 1.814      | 0.125   | 1.16                 | 0.60    | 0.52                    |
| Russia         | 0.039      | 0.007   | 0.46                 | 0.30    | 0.66                    |
| Mexico         | 0.080      | 0.037   | 0.54                 | 0.45    | 0.83                    |
| South Korea    | 0.122      | 0.234   | 0.60                 | 0.70    | 1.17                    |
| Rest of World  | 0.710      | 0.003   | 0.92                 | 0.23    | 0.25                    |

Note: Average productivity is the geometric mean of the Fréchet distribution with estimated technology parameters given in columns 1 and 2:  $\exp(\gamma/\kappa)T_n^{1/\kappa}$ , where  $\gamma$  is the Euler-Mascheroni constant and  $\kappa = 4.12$  as in our simulated model. Technology and average productivity levels are normalized to one in the U.S. in both stages. Relative productivities therefore measure comparative advantage across stages relative to U.S. comparative advantage.

Table 2: Distance and Trade in Data and Model

|              | $\log(Exports^{ij})$ |                      | $\log(VAX^{ij})$    |                     |
|--------------|----------------------|----------------------|---------------------|---------------------|
|              | Data                 | Model                | Data                | Model               |
| Log Distance | -0.996***<br>(0.074) | -1.075***<br>(0.012) | 0.230***<br>(0.020) | 0.100***<br>(0.006) |
| $R^2$        | 0.92                 | 0.98                 | 0.62                | 0.81                |
| N            | 240                  | 240                  | 240                 | 240                 |

All regressions include exporter and importer fixed effects. Robust standard errors in parentheses.



Figure 1: Diagram of Production Process in a Closed Economy

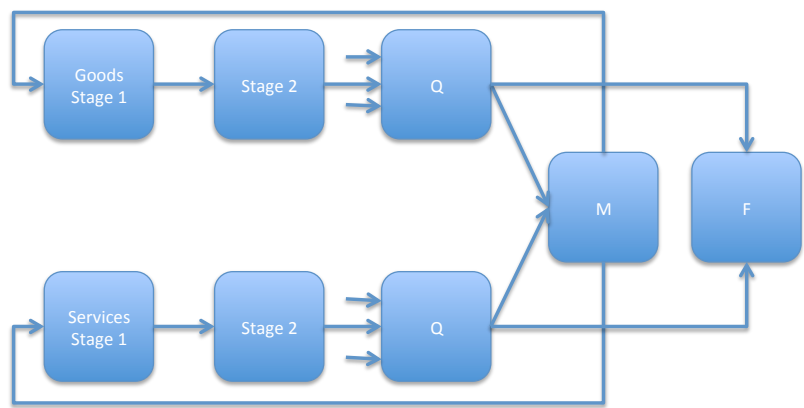
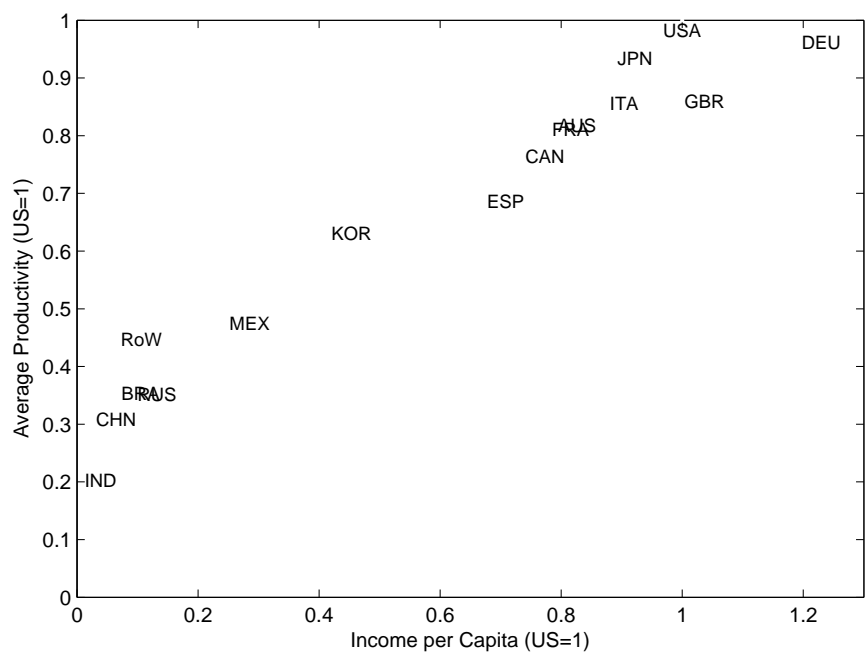
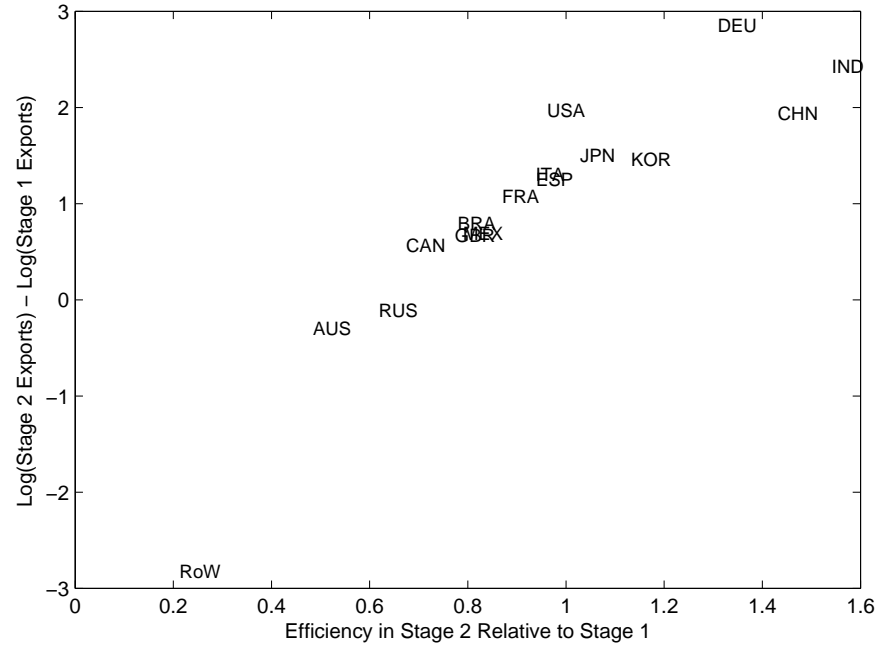


Figure 2: Average Productivity and Income per Capita



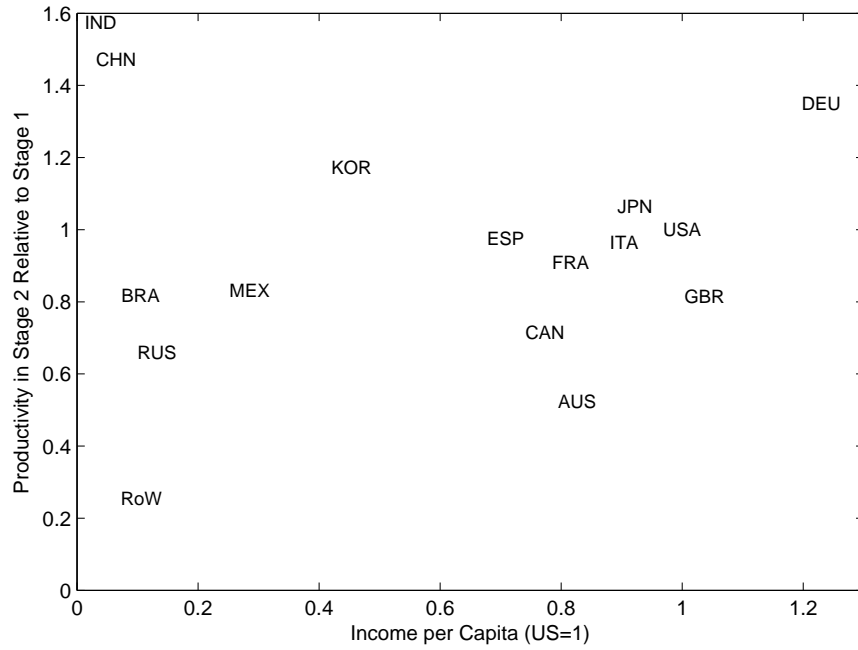
Average productivity is an unweighted geometric mean of Stage 1 and Stage 2 productivity estimates, both measured relative to the United States. Income per capita is nominal expenditure at market exchange rates divided by population.

Figure 3: Relative Productivity and Export Composition by Stage



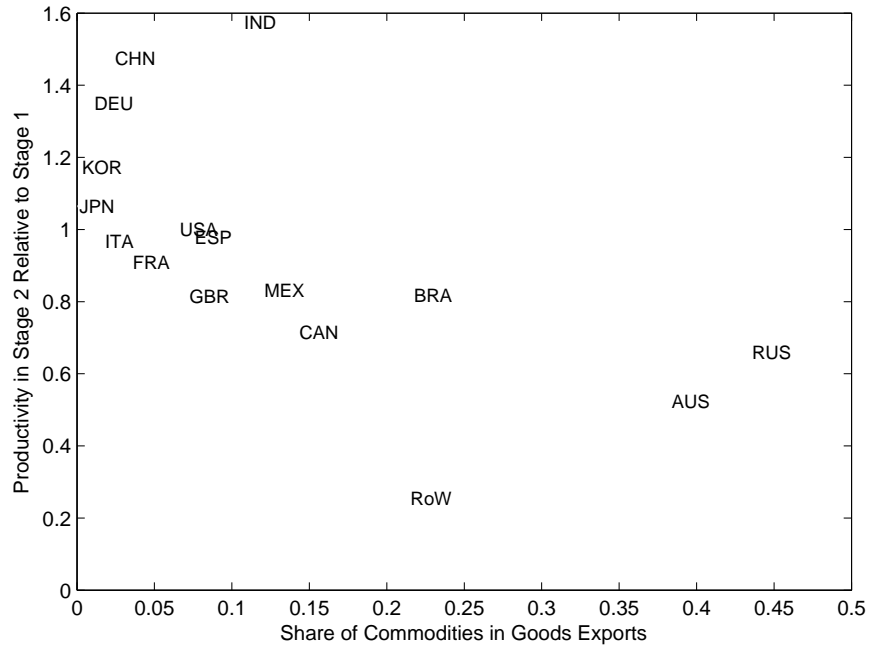
Relative productivity is Stage 2 productivity divided by Stage 1 productivity, both measured relative to the United States

Figure 4: Relative Productivity and Income per Capita



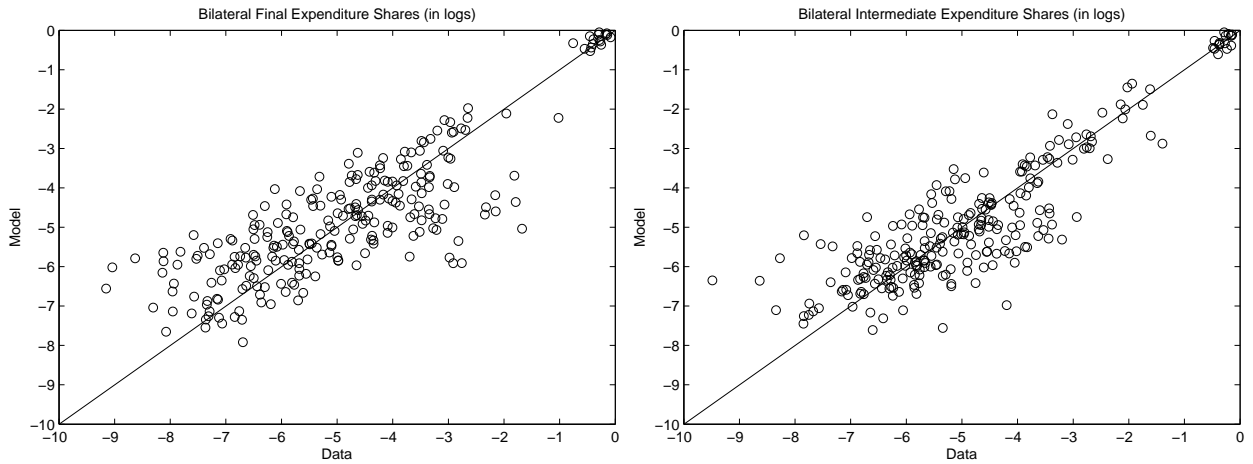
Relative productivity is Stage 2 productivity divided by Stage 1 productivity, both measured relative to the United States. Income per capita is GDP at market exchange rates divided by population.

Figure 5: Relative Productivity and Commodity Share of Exports



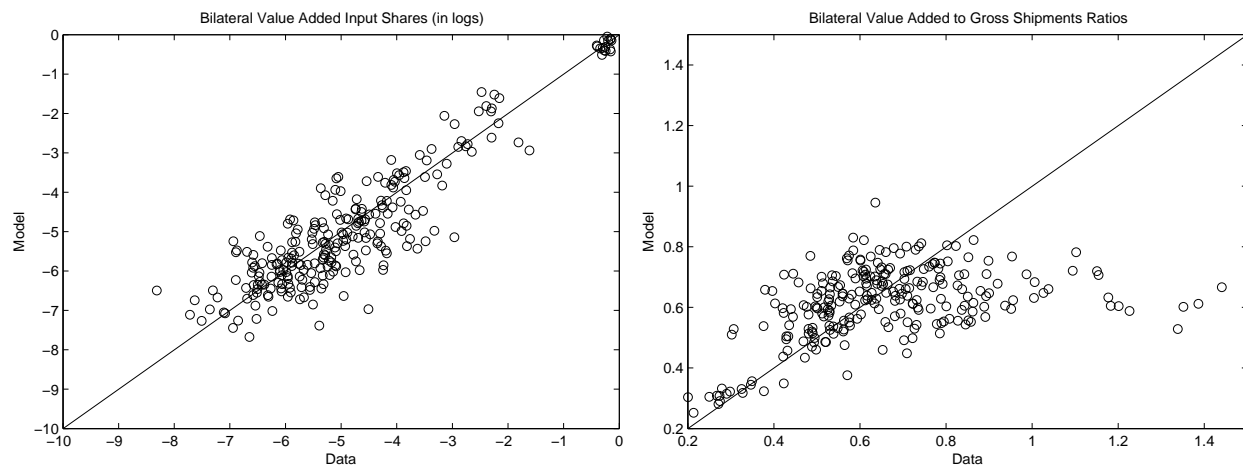
Relative productivity is Stage 2 productivity divided by Stage 1 productivity, both measured relative to the United States. The commodity share of exports is computed from the GTAP 7.1 Database and equals agriculture plus natural resources exports, divided by total goods exports.

Figure 6: Trade Shares in Data and Model



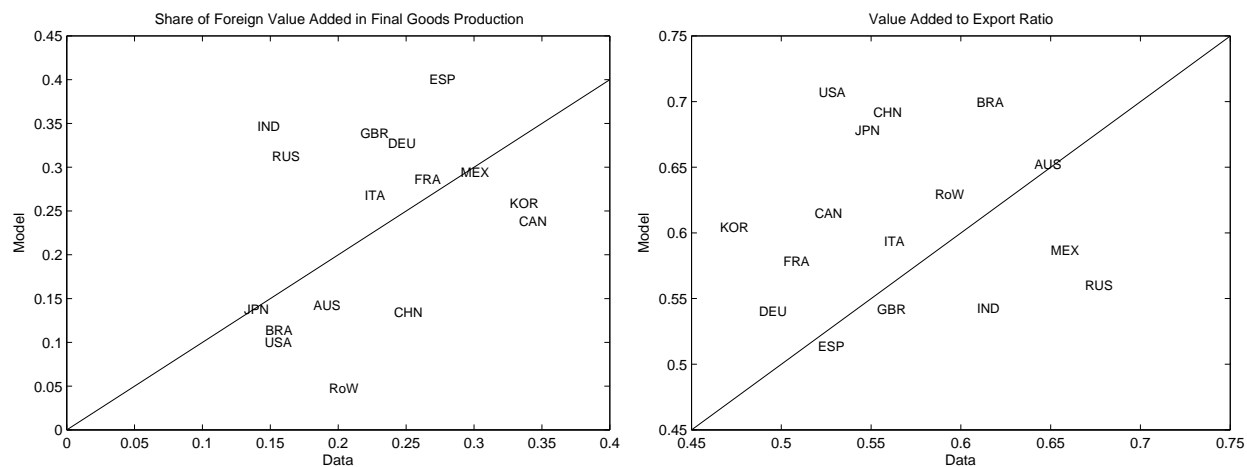
Note: The solid line in both figures is the 45-degree line.

Figure 7: Bilateral Value Added Trade in Data and Model



Note: The solid line in both figures is the 45-degree line.

Figure 8: Aggregate Value Added Trade in Data and Model



Note: The solid line in both figures is the 45-degree line.

Figure 9: The Gravity Distance Coefficient and the Level of Trade Costs in the Estimated Model

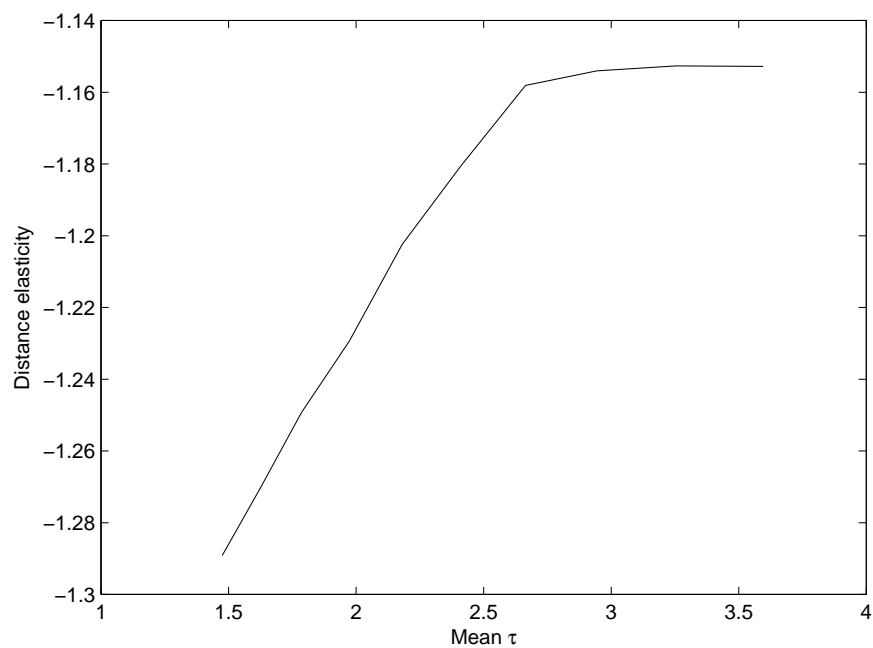


Figure 10: Price Responses to 10% Reduction in Trade Costs in Multi-Stage Model

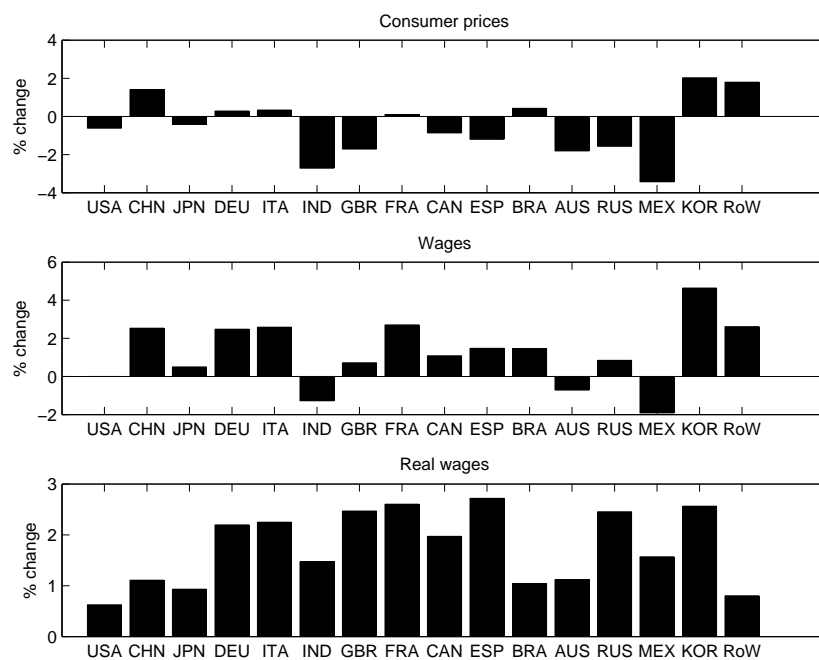


Figure 11: Trade Responses to 10% Reduction in Trade Costs in Multi-Stage Model

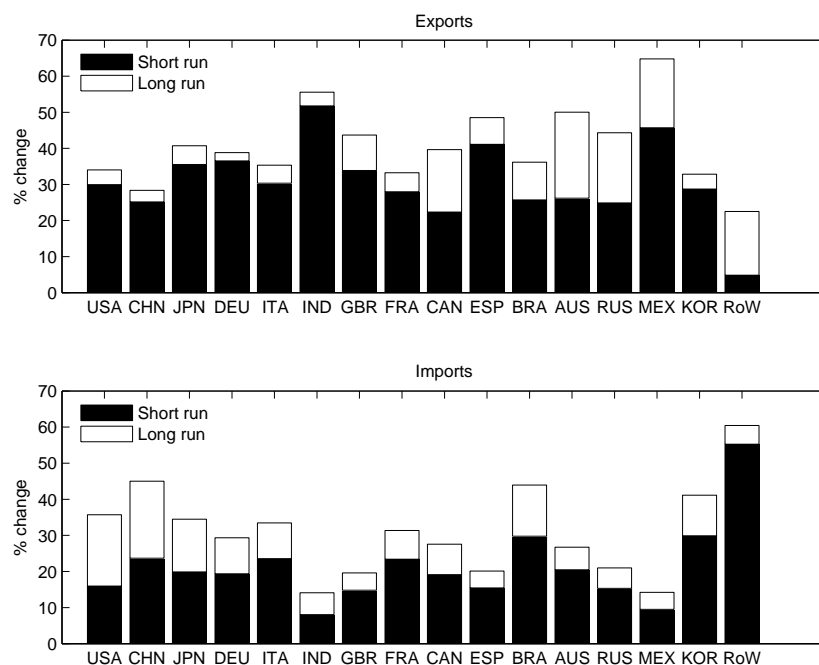


Figure 12: Value Added Trade Responses to 10% Reduction in Trade Costs in Multi-Stage Model

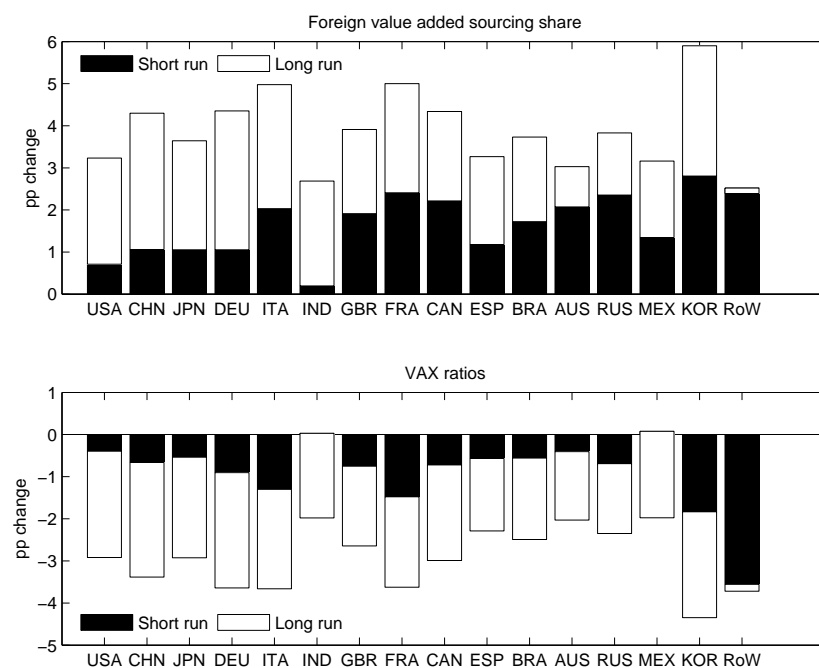


Figure 13: Value Added to Export Ratio Changes by Distance in Response to 10% Reduction in Trade Costs in Multi-Stage Model

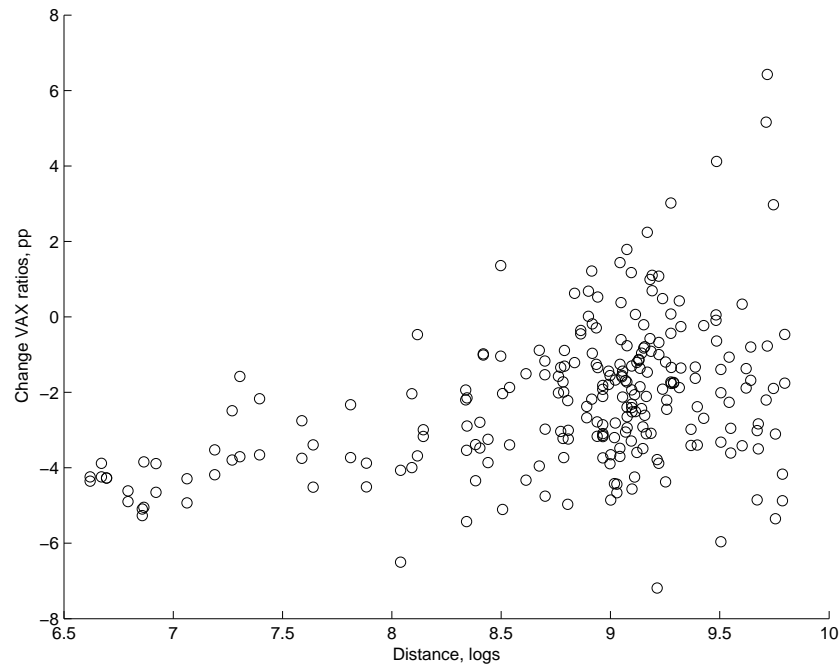


Figure 14: Price Responses to 10% Reduction in Trade Costs in Two-Sector Ricardian Model

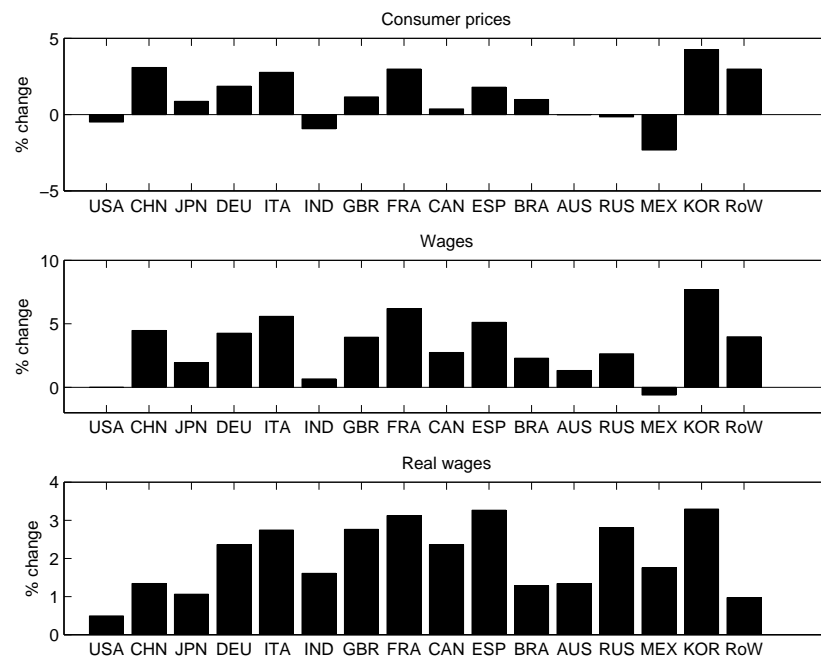


Figure 15: Trade Responses to 10% Reduction in Trade Costs in Two-Sector Ricardian Model

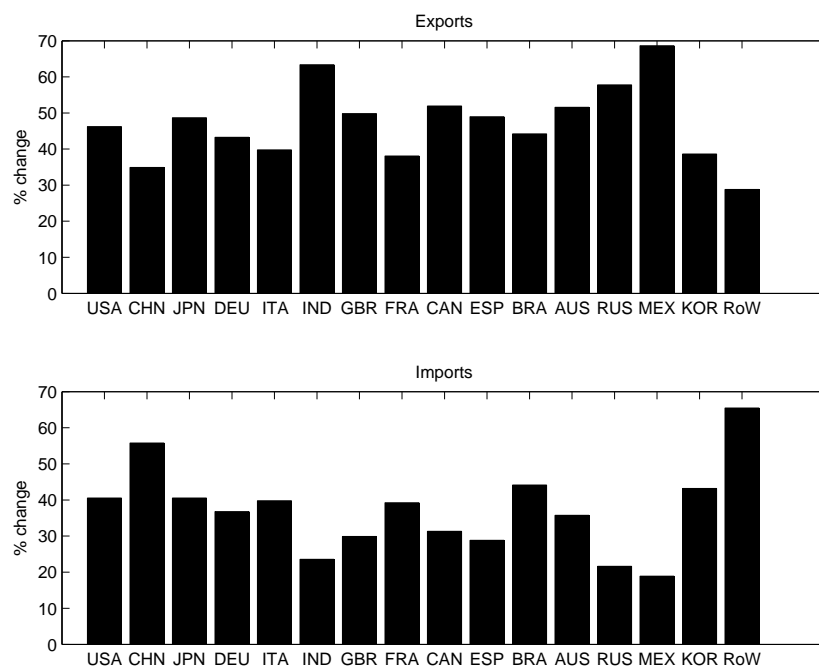


Figure 16: Value Added Trade Responses to Increase in Chinese Technology in Multi-Stage Model

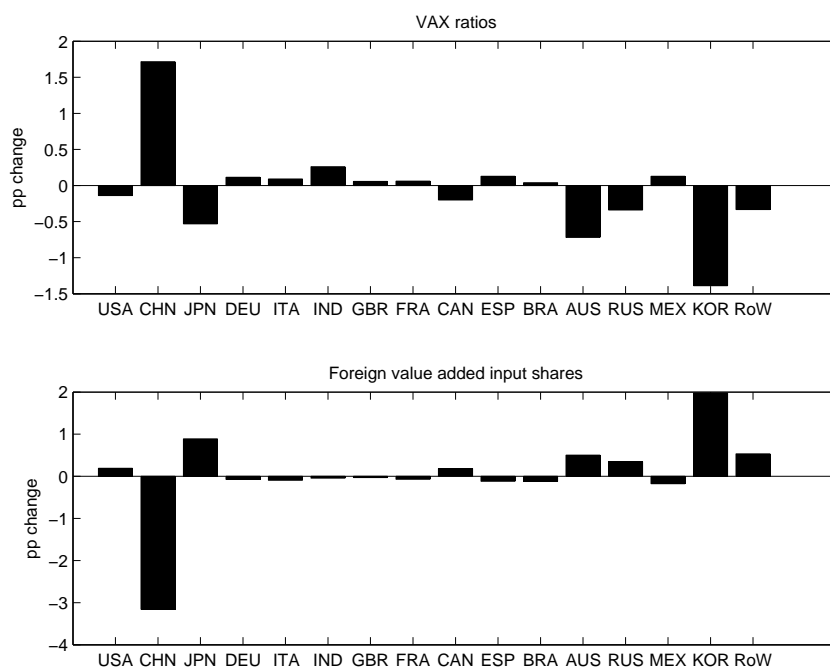
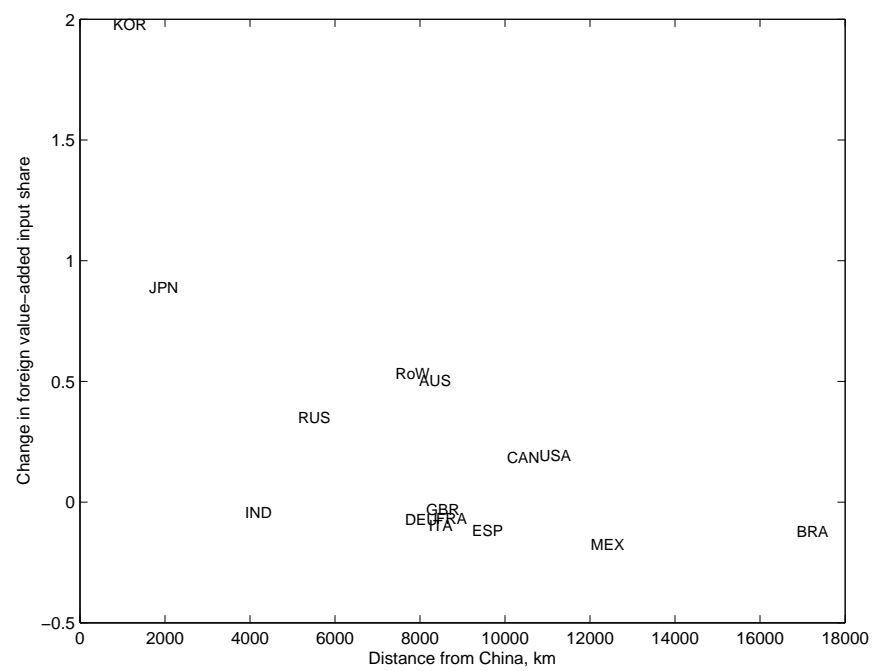




Figure 17: Changes in Foreign Sourcing of Value Added in Response to Increase in Chinese Technology in Multi-Stage Model



## A Calibration of the Composite Input Aggregator

This appendix discusses our basis for calibrating the weight of manufacturing and non-manufacturing goods in the composite intermediate input, i.e., the parameter  $\beta$ . To pick a value for this parameter, we lay out an approach to choosing a value for  $\beta$  that is appropriate in a closed economy. We then use data for the world economy as a whole to calibrate  $\beta$ , which is by definition closed. Because we focus on a closed economy here, we suppress the country superscript on variables below.

Using notation similar to Section 2, we can write total gross output in each sector as:  $y(s) = y_1(s) + y_2(s)$ . In the closed economy, all stage 2 goods are produced and use domestic stage 1 goods as inputs. Therefore,  $y_1(s) = \theta(s)y_2(s)$ . Using this fact, we re-write gross output as:  $y(s) = (1 + \theta(s))y_2(s)$ . This links stage two output to observable sector level output  $y(s)$  and a parameter  $\theta(s)$  that can be measured from data. This is the first useful accounting identity.

The second useful accounting identity is the market clearing for stage 2 goods from sector 1:  $y_2(1) = \alpha P_F F + \beta P_M M$  in the closed economy. We then recall that total purchases of the composite intermediate inputs are given by:  $P_M M = \theta(1)y_1(1) + \theta(2)y_1(2)$ . Combining these yields:

$$y_2(1) = \alpha P_F F + \beta \theta(1)y_1(1) + \beta \theta(2)y_1(2). \quad (27)$$

Recall that in our data, final purchases are observed at the sector level, so  $\alpha P_F F$  is data. Then we can also link  $y_2(1)$ ,  $y_1(1)$ , and  $y_1(2)$  to data on gross output at the sector level, as in the previous paragraph. This leaves us with one equation in one unknown  $\beta$ :

$$(1 + \theta(1))^{-1}y(1) = \alpha P_F F + \beta \theta(1) \left[ \frac{\theta(1)}{1 + \theta(1)} \right] y(1) + \beta \theta(2) \left[ \frac{\theta(2)}{1 + \theta(2)} \right] y(2). \quad (28)$$

We pick  $\beta$  guided by this equation. To implement the calibration for the world, we aggregate all countries in our data to form the composite input-output table for the world, which records sector-to-sector sales of inputs, gross output, and final demand by sector. Using this data, we compute the sector-level input shares (i.e.,  $\{\theta(1), \theta(2)\}$ ) that are consistent with this world-level data, which happen to be nearly identical to cross-country median input shares. Plugging in these values along with values for final demand and gross output into the equation and solving yields a value of  $\beta \approx .6$ .

## B Smoothing the Objective Function

There are some technical details related to the discrete approximation of the model with a continuum of goods that are worth mentioning here. In practice, we search for  $\hat{\Theta}$  using a standard gradient based optimization algorithm (fmincon in Matlab).<sup>43</sup> To do this effectively, we need the objective function to be relatively smooth in the underlying parameters. In the

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<sup>43</sup>Gradient techniques are helpful to us, since the parameter space has relatively high dimensionality. We attempted to use non-gradient methods initially, but they generally performed poorly (i.e., were both slow and had difficulty finding the minimum).

discrete approximation, the trade shares are discontinuous in the underlying parameters.<sup>44</sup>

Our approach to simulating the trade shares borrows from the discrete choice literature, building on the observation that the trade shares are mathematically equivalent to choice probabilities. We use the logit-smoothed AR simulator to compute the trade shares (see McFadden (1989) and Train (2009)), which replaces the indicator functions with a logit function.

The first step in performing this transformation is to note that our indicator functions above can be re-written as statements about supply prices from alternative sources. Country  $j$  buys stage  $n$  output from country  $i$  if  $i$  is the low cost supplier, which means that:  $\mathbb{I}(j \in \Omega_n^i(r, s)) = \mathbb{I}(p_n^{ij}(r, s) < p_n^{kj}(r, s), \forall k \neq i)$ . For example, the final trade shares can be written as:  $Fshare^{ij} = \frac{1}{R} \sum_r \mathbb{I}(p_2^{ij}(r, 1) < p_2^{kj}(r, 1), \forall k \neq i)$ . The second step then approximates the indicator function with logit function, as in:

$$Fshare^{ij} = \frac{1}{R} \sum_r \frac{e^{-p_2^{ij}(r, 1)/\lambda}}{\sum_k e^{-p_2^{kj}(r, 1)/\lambda}}, \quad (29)$$

where  $\lambda$  is a scale factor.

Similarly, we can approximate the input trade shares as:

$$\begin{aligned} Inshare^{ij} &= \sum_r \mathbb{I}(j \in \Omega_1^i(r, 1)) \left[ \frac{\theta(s)y_2^j(r, 1)}{In^j(1)} \right] + \frac{1}{R} \sum_r \mathbb{I}(j \in \Omega_2^i(r, 1)) \left[ \frac{\beta(1)P_M^j M^j}{In^j(1)} \right] \\ &= \sum_r \frac{e^{-p_1^{ij}(r, 1)/\lambda}}{\sum_k e^{-p_1^{kj}(r, 1)/\lambda}} \left[ \frac{\theta(s)y_2^j(r, 1)}{In^j(1)} \right] + \frac{e^{-p_2^{ij}(r, 1)/\lambda}}{\sum_k e^{-p_2^{kj}(r, 1)/\lambda}} \left[ \frac{\beta(1)P_M^j M^j}{In^j(1)} \right]. \end{aligned} \quad (30)$$

The scale factor  $\lambda$  determines the degree of smoothing. As  $\lambda \rightarrow 0$ , the logit function converges to the indicator function and the smoothed trade shares approach the exact trade shares (in the discrete model). There is little guidance on the appropriate level of  $\lambda$  in general. By trial and error, we find that  $\lambda = 0.02$  yields a very good approximation to the exact trade shares. Finally, we also need to choose  $R$  which yields an acceptable trade-off between simulation accuracy and computing time. In Monte Carlo simulations, we have found that the empirical model is able to recover the true parameters of the model when  $R = 20,000$ , so we use this value.

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<sup>44</sup>For example, trade shares for final goods are given by:  $\frac{R^{ij}(s)}{R} = \frac{\sum_r \mathbb{I}(j \in \Omega_2^i(r, 1))}{R}$ . This is essentially an accept-reject (AR) simulator for the trade shares in the true model with a continuum of goods, where we determine whether the indicator functions are zero or one for each discrete good, and then take the average.

## C Input-Output Framework for Restricted Model

In this appendix, we re-write the input-output framework in the restricted version of the model in which services are non-traded. The bilateral input-output matrices take the form:

$$\begin{aligned} \text{For } i \neq j : \quad A^{ij} &= \begin{bmatrix} 0 & \frac{y_1^{ij}(1)}{y_2^j(1)} & 0 & 0 \\ \frac{R_2^{ij}(1)}{R} \beta(1) \theta(1) & 0 & \frac{R_2^{ij}(1)}{R} \beta(1) \theta(2) & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ \text{For } i = j : \quad A^{ii} &= \begin{bmatrix} 0 & \frac{y_1^{ii}(1)}{y_2^i(1)} & 0 & 0 \\ \frac{R_2^{ii}(1)}{R} \beta(1) \theta(1) & 0 & \frac{R_2^{ii}(1)}{R} \beta(1) \theta(2) & 0 \\ 0 & 0 & 0 & \frac{y_1^{ii}(2)}{y_2^i(2)} \\ \frac{R_2^{ii}(2)}{R} \beta(2) \theta(1) & 0 & \frac{R_2^{ii}(2)}{R} \beta(2) \theta(2) & 0 \end{bmatrix}. \end{aligned}$$

The vectors of bilateral final goods shipments take the form:

$$\begin{aligned} \text{For } i \neq j : \quad f^{ij} &= \begin{bmatrix} 0 \\ \frac{R_2^{ij}(1)}{R} \alpha(1) P_F^j F^j \\ 0 \\ 0 \end{bmatrix}, \\ \text{For } i = j : \quad f^{ii} &= \begin{bmatrix} 0 \\ \frac{R_2^{ii}(1)}{R} \alpha(1) P_F^i F^i \\ 0 \\ \alpha(2) P_F^i F^i \end{bmatrix}. \end{aligned}$$

These components can be collected to form the full global input-output table and manipulated as in the main text.

## D Benchmark Ricardian Trade Model with Input-Output Linkages

To benchmark our model, we use a two-sector version of the model in Caliendo and Parro (2012). In this appendix, we write down the key equilibrium conditions from that model, translated into our notation and making minor modifications.

[To Be Added]