To Learn or To Change: Optimal R&D Investments under Uncertainties in the Case of Climate Change

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Abstract

When studying R&D investments that mitigate climate change's damage (termed as "research to change" or RTC), current literature overlooks purchased learning (termed as "research to learn" or RTL) about climate change. We investigate interactions between investments in RTC and RTL under uncertainties about climate change and research outcomes. Results show that (1) if an RTL success does not contribute to an RTC breakthrough then it is almost never optimal to invest simultaneously in RTL and RTC; (2) if an RTL success does contribute to an RTC breakthrough, then simultaneously investing in both RTL and RTC may be optimal when the probability of harmful climate change is either moderate or very high; and (3) whenever RTL and RTC are conducted simultaneously then they are substitutes. We solve the model to identify parameters under which the precautionary principle and the learn-then-act principle should be followed regarding R&D investments.

Key words: climate change; R&D investments; uncertainties

JEL codes: Q54, Q55, D83

1 Introduction

Two types of uncertainties inevitably arise when faced with emerging problems whose possible impacts on human welfare are not conclusively understood. These concern: (1) the magnitude of the problems' human welfare impact, and (2) the future date when this unknown impact becomes clear. Responding to these two dimensions of uncertainty is likely to require two distinct lines of research. Take climate change as an example. To mitigate possible negative impact on humans, much research has been devoted to greenhouse gas (GHG) emission abatement technologies, energy efficient technologies, renewables, and adaptation technologies. We term this research category as"research to change" (RTC). On the other hand, research is also devoted to studying the uncertain impact of climate change itself. Will it be a manageable $2°C$ or a $4°C$ change? Or is climate change primarily caused by greenhouse gas accumulation or by something else, such as solar activity (Svensmark and Calder, 2008)? We term this category as "research to learn" (RTL). RTL can accelerate the resolution of the uncertain impact so as to improve decisions on extent of resources to be put into RTC. If in the future climate change is proved to have only a mild effect, some of the RTC investment will have turned out to have been wasted. However, if climate change proves disastrous, we would have wanted more RTC. If it turns out that the preponderant reason for climate change is other than GHG accumulation, then the tremendous effort to reduce GHG emission would be mis-targeted. RTL decreases the probability of making these mistakes. Since RTL is costly as well, optimal decisions on RTC investment should take into account interactions between RTL and RTC.

Although our analysis will focus on climate change, the message in this study can be applied to many other cases. For instance, while lacking firm evidence, some scientists fear that humans might contact Crohn's disease from the produce of cattle with Johne's

disease (Uzoigwe *et al.*, 2007). In this case, RTC includes research in preventing or treating Johne's disease or in technologies that can cut off channels through which Johne's disease affects humans. RTL may include research to identify the true relationship between these two diseases. The interaction between RTC and RTL should be considered when allocating resources to research regarding Johne's disease. If Johne's disease does cause Crohn's disease, then more RTC will be justified. But if it does not, then at least part of these RTC investments will have been wasted. Therefore, RTL is favored in the sense that it can prevent this kind of waste.

Our model can also be applied in the decision process at the firm level. When the concept of a new product that has potential to be profitable becomes available, a company in a related industry can either invest in new product market analysis to find the true state of its profitability (i.e., RTL) or invest into activities to study how to accommodate this new product into their existing production lines or even invest into building a new production line (i.e., RTC). In this scenario the study of interactions between RTL and RTC is of especial interest since the study can help firms make right decisions.

Research outcomes are also uncertain. RTC could either result in a failure or a breakthrough. By breakthrough we mean hereafter that the RTC reaches its goals and potential problems are solved. For instance, if we had a breakthrough in greenhouse emission abatement or alternative energy technologies like biofuels, wind or solar, the possible effects of additional climate change on welfare will be largely eliminated. RTL outcomes are also uncertain. In particular, the issue typically regards a neglected area of science and researchers have little initial guidance in their search for key relations.

Moreover, RTL and RTC are not necessarily independent of each other. The reason is that research is a complex activity and the outputs from one research project may have positive externality for other projects' success. Therefore, it is reasonable to as-

sume that the success of RTL can contribute to a breakthrough in RTC. For example, the investigation into stratospheric ozone depletion's causes directly contributed to solving the ozone-depletion problem: chlorofluorocarbon (CFC) phaseout. In the case of Johne's disease and Crohn's disease, during the process of RTL (i.e., research to find out the true relationship between these two diseases), some knowledge may be generated that can help the RTC (i.e., to prevent or treat Johne's disease). A breakthrough of RTC may accelerate success in RTL. For example, research in treating Johne's disease may help scientists better understand the true relationship between these two diseases.

This article explores the interaction between RTL and RTC as well as the optimal allocation of research resources to RTC and RTC in the face of uncertainties. This article lies in the strand of literature that studies the effect of learning on an irreversible decision. Research investments fit into the theoretical framework developed by this literature because the research cost cannot be redeemed later. Classic examples of early work in this literature includes Weisbrod (1964), Arrow and Fisher (1974), Henry (1974) and Hanemann (1989). Their studies show that the possibility of learning in the future that can mitigate uncertainties will encourage precautionary action upon irreversible development. Later work in this literature argued that if there are two opposing irreversibilities associated with the decision, the effect of uncertainty and learning on the irreversible decision is ambiguous. Examples are Olson (1990), Kolstad (1996), and Marwah and Zhao (2007). Under a Bayesian learning framework, Kelly and Kolstad (1999) show that there is a tradeoff between learning and GHG emission abatement. They argue that this is because GHG emission abatement can slow down learning about emissions' impacts on climate. Leach (2007) extends the results in Kelly and Kolstad (1999) to include learning on two types of uncertainties: a) GHG emissions' impact on climate and b) the persistence of temperature changes. By doing so, Leach (2007) concludes that the time to resolve these two types of

uncertainties may take hundreds or thousands of years considering the tradeoff between emission control and learning.

By applying the model in Hanemann (1989), Schimmelpfennig (1995) argues that in order to keep the option of using energy efficient technologies open, learning in the future will encourage decision makers to invest more in those technologies. By assuming linear-quadratic abatements costs and environmental damages, Karp and Zhang (2006) show that learning about GHG stock's damage magnitude decreases the optimal GHG emission abatement level. In a two-period model, Fisher and Narain (2003) show that capital sunkness decreases first-period abatement investment, but irreversible greenhouse gas accumulation increases such investment. Baker *et al.* (2006) study optimal R&D decisions under climate uncertainty. But in that study Baker *et al.* assume the outcome of R&D programs is certain. Studies on investments in risky R&D programs under climate uncertainties include Baker and Adu-Bonnah (2008), Baker and Shittu (2008), and Baker and Solak (2011, 2012)

The above literature implicitly assumes that uncertainties are only resolved by autonomous learning (i.e., the passage of time) or active learning (i.e., shock a system and learn about it) instead of by purchased learning (i.e., RTL).[1] In reality, however, purchased learning does consume a significant part of research resources.[2] To our best knowledge,

[1]Kolstad (1996) discussed three types of learning related to climate change: active learning, purchased learning, and autonomous learning.

[2]For example, the U.S. federal obligations for research in environmental sciences in FY 2006 was $3.4 billion (Data source: National Science Foundation (NSF)). Environmental sciences are defined by NSF as, "Environmental sciences (terrestrial and extraterrestrial) are, with the exception of oceanography, concerned with the gross nonbiological properties of the areas of the solar system that directly or indirectly affect human survival and

the only work that takes purchased learning into account when studying decision making under uncertainties is Hennessy and Moschini (2006). In that paper, the authors study the optimal level of scientific research (i.e., purchased learning) on the damage that a certain practice could cause when a social planner is considering whether to ban this practice. However, the model in Hennessy and Moschini (2006) is a binary choice model of a regulator's actions. In this article, we develop a continuous choice model that takes uncertainty about research outcomes into account. Utilizing this model we study the optimal RTL and RTC investment levels and explore interactions between RTC and RTL. We find that (1) if RTL's success does not contribute to an RTC breakthrough then it is almost never optimal to invest simultaneously in RTL and RTC, (2) if RTL's success contributes to an RTC breakthrough then simultaneously investing in both RTL and RTC may be optimal when the probability of harmful climate change is either moderate or very high, (3) whenever RTL and RTC are conducted simultaneously then an increase in RTL (or RTC) cost enhances the optimal RTC (or RTL) investments (i.e., RTL and RTC are substitutes), and (4) whether to follow the precautionary principle or the learn-then-act principle regarding R&D investments depends on investment costs and climate change damage's probability distribution. Parameters under which the two principles should be followed are identified. The precautionary principle favors taking immediate preventive measures to address potential hazards (Barrieu and Sinclair-Desgagné, 2006). The learn-then-act principle, on the contrary, favors not taking measures until we obtain more knowledge about the potential hazards. Opinions are divided among economists regarding which principle a society should follow under climate change uncertainties (e.g., Gollier *et al.*, 2000; Gollier and Treich, 2003; and Ulph and Ulph, 1997). Our study provides insight

welfare. " Available at: http://www.nsf.gov/statistics/nsf10303/tables/tab23.xls (accessed on 1/21/2012).

on unifying these two principles.

The rest of this article is structured as follows. Section 2 outlines the basic optimization model of a social planner. Section 3 studies the interaction between RTC and RTL. Then Section 4 conducts a comparative statics analysis of the optimal RTC and RTL decisions. Section 5 concludes with a discussion of possible extensions of this study.

2 Model Setup

A social planner seeks to minimize the expected negative welfare impact of climate change by investing in RTC and RTL. At time $t = 0$, the climate change's impact is uncertain. For simplicity we assume there are only two future states of nature: state T with probability $q \in [0,1]$ in which climate change imposes an constant instantaneous damage D and state F with probability $1-q$ in which it does not impose a damage.[3] Uncertainty about this welfare impact will be resolved at time $t = \tau_1$, which is an exponential random variable with density function $f(\tau_1) = le^{-l\tau_1}$, where $l \geq 0$ is determined by the social planner's investment in RTL.[4] The higher the investment is, the larger the value of l. If

[3]Of course the model could be written as damage D_T with probability q and damage $D_F \in [0, D_T)$ with probability $1-q$. Also, D_T and D_F can be written as functions of time to reflect climate change's increasing impacts when the greenhouse gas stock becomes larger over time. However, we do not do so in this article because our key interest here is to discover the salient points of optimal RTL and RTC investments and their interaction. Assuming a positive D_F, or letting D_T and D_F to be a function of time, will not add much more insight to the analysis.

[4]Autonomous learning can be viewed as a specific instance of our model when we fix the investment level in RTL.

there is no investment in RTL, then $l = 0$. Suppose that α amount of investment in RTL will increase l by one unit. At time $t = 0$ the social planner can also conduct RTC to respond to the potential damage of climate change. That is, the social planner can take a precautionary action to respond to possible damage from climate change. We assume that the success in RTC will happen at time $t = \tau_2$, which is also an exponentially random variable but with density function $g(\tau_2) = ce^{-c\tau_2}$. By choosing c, the social planner can govern the expectation of success time, $1/c$. In order to increase c by one unit, β units of investment in RTC is needed. We assume that investments in RTL and RTC are stock investments. Figure 1 is a visual presentation of the social planner's decision problem. From Figure 1 we can see the social planner's decision problem consists of four sub-problems, A) to D), which are explained next.

Sub-problem A) arises when an RTL success occurs before breakthrough in RTC and the true state of the world is T. In this sub-problem, since the RTC has not succeeded when the true state is revealed, an acceleration of RTC breakthrough may be desirable. Therefore, at time τ_1 the social planner will choose $c' \geq 0$ to minimize the total costs (i.e., the damage of climate change plus research costs). Here c' is the added investment into RTC at time τ_1. Since success in RTL may contribute to a breakthrough in RTC, then at time τ_1 the breakthrough time for RTC has a new probability density function. For notational clarity we utilize τ_3 to denote the breakthrough time of RTC when it happens after τ_1. Then the probability density function of τ_3 is

$$g'(\tau_3) = (c + c' + \eta)e^{-(c+c'+\eta)(\tau_3-\tau_1)}, \tag{1}$$

where $\tau_3 > \tau_1$ and $\eta \geq 0$.[5] Here η measures the magnitude of the contribution of an RTL

[5]This density function can be motivated in the following way. Let u_1 denote the

success to an RTC breakthrough. The smaller the η is, the smaller the contribution. If $\eta = 0$, then there is no such contribution. We assume that $\eta \geq 0$ and η is only defined when $\tau_2 > \tau_1$. This means that only the output (the success of RTL), not the input, of RTL can affect the breakthrough time of RTC.

Mathematically, Sub-problem A) can be written as

$$V^A(c,l) = \beta c + \alpha l + \min_{c' \geq 0} \Big\{ \beta c' e^{-r\tau_1} + E_{\tau_3}[\int_0^{\tau_3} De^{-rt} dt] \Big\}, \tag{2}$$

where $V^A(c,l)$ denotes the minimized total cost in sub-problem A) given the value of c and l. Here r is the continuous time discount rate. Hereafter we assume that $\eta < \sqrt{D/\beta} - r$. This assumption eliminates the possibility that the contribution of RTL to RTC (measured by η) is so high that the social planner will find is optimal to not invest into RTC at time τ_1 even if the true state is proved as T and if RTC has not been successful by time τ_1. This assumption implies that $\sqrt{D/\beta} - r > 0$, which means that were climate change harmful for sure (i.e., $q = 1$) then the optimal c will be greater than 0. Justification for this assumption is provided in Item A of Supplemental Materials (SM hereafter). We formally state this assumption as

Assumption 1. *The contribution of RTL's success to RTC's breakthrough will never be*

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breakthrough time for RTC governed by the newly added investment into RTC, c'; and let u_2 denote the breakthrough time for RTC governed by the outcome of RTL. Their density functions are $h_1(u_1) = c'e^{-c'(u_1-\tau_1)}$ and $h_2(u_2) = \eta e^{-\eta(u_2-\tau_1)}$, respectively. Here we have $u_1 > \tau_1$ and $u_2 > \tau_1$. The density of τ_2 conditional on $\tau_2 > \tau_1$ is $f(\tau_2|\tau_2 > \tau_1) = ce^{-c(\tau_2-\tau_1)}$. We assume u_1, u_2, and τ_2 are independent. Then we define $\tau_3 \equiv \min\{(\tau_2|\tau_2 > \tau_1), u_1, u_2\}$. It is easy to check that τ_3 has density function $(c + c' + \eta)e^{-(c+c'+\eta)(\tau_3-\tau_1)}$, where $\tau_3 > \tau_1$.

so high that the social planner will find it optimal to not invest into RTC at time τ_1 even if the true state is proved as T and if the RTC has not been successful yet. That is $\eta < \sqrt{D/\beta} - r$.

Assuming an interior solution for c', we show that

$$V^A(c,l) = \beta c + \alpha l + \frac{D}{r} + (2\sqrt{\beta D} - \beta(c + r + \eta) - \frac{D}{r})e^{-r\tau_1}. \tag{3}$$

The algebra to obtain equation (3) is presented in Item B of SM.

Sub-problems B) and D) are straightforward. Let $V^B(c,l)$ and $V^D(c,l)$ denote the total cost in sub-problems B) and D), respectively, given the values of c and l. If at time τ_1 it is proved that climate change is not harmful (i.e., State F), then the social planner will not invest more into RTC. Therefore the total costs for the social planner in sub-problems B) and D) are

$$V^B(c,l) = V^D(c,l) = \beta c + \alpha l. \tag{4}$$

Sub-problem C) occurs when RTC breakthrough happens earlier than RTL success (i.e., $\tau_2 \leq \tau_1$) and when the true state is T (i.e., climate change is harmful). For simplicity we assume there is no switch cost when the social planner adopts the RTC outcome. For a study of how a switch cost affects the optimal decisions when facing uncertainty, we refer readers to Hennessy and Moschini (2006). Therefore, the social planner will adopt the RTC outcome immediately even though at time τ_2 the true state of the world has not been realized. In this sub-problem the total cost is

$$V^C(c,l) \quad = \quad \beta c + \alpha l + \int_0^{\tau_2} De^{-rt}dt. \tag{5}$$

Therefore, at time 0 the social planner's problem is to choose $c \geq 0$ and $l \geq 0$ to

minimize total cost $V(c,l)$, which is

$$\begin{aligned} V(c,l) &= \int_0^\infty \int_{\tau_1}^\infty [qV^A(c,l)+(1-q)V^B(c,l)]g(\tau_2)f(\tau_1)d\tau_2 d\tau_1 \\ &\quad + \int_0^\infty \int_0^{\tau_1} [qV^C(c,l)+(1-q)V^D(c,l)]g(\tau_2)f(\tau_1)d\tau_2 d\tau_1, \end{aligned} \tag{6}$$

where the first (second) term on the right-hand side of equation (6) is the expected cost when $\tau_2 > \tau_1$ ($\tau_2 \leq \tau_1$). After some algebra, which is shown in SM Item C, we can simplify problem (6) to

$$V(c,l) = \alpha l + \beta c + \frac{q}{r+l+c}\{D + [2\sqrt{D\beta} - \beta(c+r+\eta)]l\}. \tag{7}$$

The first two terms on the right-hand side of equation (7) (i.e., αl and βc) are the research investments into RTL and RTC, respectively. The third term is the expected damage due to climate change when investment into RTC and RTL are c and l, respectively. It is easy to check that if $q = 0$ then the damage would be 0. Suppose $c = l = 0$, then the third term becomes qD/r, which is the expected adverse welfare impact of climate change when the social planner does nothing and when autonomous learning will take an infinite amount of time to reveal the true state of climate change's impact. An observation is that the cost of RTL, α, has no effect on the third term given (c,l). This is because the decision on RTL only happens at time 0. Unlike the decision on RTC, once the decision on RTL is made then the social planner will no longer need to make further decisions on RTL. Therefore, given (c,l), changing α does not affect expected damage due to climate change. The social planner's problem can be written as

$$\min_{c,l \geq 0} V(c,l), \tag{8}$$

whose first order conditions (FOCs) are:

$$\frac{\partial V}{\partial c} = \beta - q\frac{(\sqrt{D}+\sqrt{\beta}l)^2 - \beta\eta l}{(r+l+c)^2} \geq 0, \tag{9}$$

$$\frac{\partial V}{\partial l} = \alpha - q\frac{(\sqrt{D}-\sqrt{\beta}(c+r))^2 + \beta\eta(r+c)}{(r+l+c)^2} \geq 0. \tag{10}$$

Algebra to obtain the FOCs are shown in SM Item D, in which we also show that $V(c,l)$ is convex under Assumption 1. Therefore, the values (c^*, l^*) that satisfy FOCs (9) and (10) are the optimal solutions to problem (6). In the next section we study the optimal solution and the FOCs in detail.

3 Model Analysis

According to whether or not c^* and l^* are strictly positive, there are four possible cases. They are Case 1, $c^* = 0$ and $l^* = 0$; Case 2, $c^* = 0$ and $l^* > 0$; Case 3, $c^* > 0$ and $l^* = 0$; and Case 4, $c^* > 0$ and $l^* > 0$. Optimal decisions in Cases 1 and 2 (where $c^* = 0$ and $l^* \geq 0$) can be viewed as actions following the learn-then-act principle (favors delaying the investment in RTC). Optimal decisions in Cases 3 and 4 (where $c^* > 0$), however, can be viewed as actions following the precautionary principle (favors investing in RTC as early and as much as possible). We discuss these four cases below.

3.1 Four Cases of Optimal Solutions

Case 1. $c^* = 0$ ***and*** $l^* = 0$***.***

This case is of interest because it reveals conditions under which the social planner do nothing. If $c^* = 0$ and $l^* = 0$, then from the FOCs in (9) and (10) we can obtain

$$q \leq \beta r^2/D \equiv q_c, \tag{11}$$

$$q \leq \alpha r^2/[(\sqrt{D} - \sqrt{\beta} r)^2 + \beta \eta r] \equiv q_l. \tag{12}$$

Since the objective function in problem (8) is convex (SM Item D), the necessary conditions for (c^*, l^*) to be optimal are sufficient conditions. This means that whenever $q \leq \min\{q_c, q_l\}$ then $(c^*, l^*) = (0,0)$. By Assumption 1 we can check that $q_c < 1$. Intuitively, q_c (or q_l) is the probability at which the marginal cost of RTC (or RTL) investment equals the marginal benefit (i.e., the deduction of expected negative impact of climate change) of RTC (or RTL) investment when evaluated at $(c^*, l^*) = (0,0)$.

It is readily checked that q_l is decreasing in η. This is because a larger η will increase the marginal benefit of RTL investment given the level of q. Therefore, if $q_c > q_l$ then an increase in η will shrink the range of q that supports Case 1. So if the success of RTL could contribute to RTC at a larger magnitude, then it is less likely for the social planner to do nothing. However, if $q_c < q_l$, then an increase in η will not affect the range of q that supports Case 1. The reason is as follows. From the expression for q_c we can see that η does not affect q_c, which implies that η does not affect the marginal benefit of RTC as $l^* = c^* = 0$. We know that η can affect the RTC breakthrough only if success in RTL occurs before RTC. If $l^* = 0$, then success in RTL is very unlikely. Therefore, increasing η will not affect a breakthrough in RTC and hence the marginal benefit of RTC.

By studying q_c and q_l we can see that increasing the cost of RTC, β, will increase both

q_c and q_l (i.e., $\partial q_c/\partial\beta > 0$ and $\partial q_l/\partial\beta > 0$). However, increasing the cost of RTL, α, enlarges q_l but does not affect q_c (i.e., $\partial q_l/\partial\alpha > 0$ and $\partial q_c/\partial\alpha = 0$). Here, that q_c (or q_l) is increasing with β (or α) is quite intuitive because the higher the cost of RTC (or RTL), the higher will be the probability thresholds for making an investment in RTC (or RTL). But why q_l is increasing with β and why q_c is not affected by α needs some explanation. We know that one benefit of investing into RTL is to accelerate the realization of the true state of the world, so that an accurate decision on RTC can be made sooner to reduce the expected negative impact of climate change. But an increase in β will decrease the incentive to invest into RTC, and hence decrease the incentive to invest into RTL. In an extreme case, if investment into RTC is impossible (say, β is extremely high), then there is no need to invest into RTL at all to get more information about the state of the world. This is why q_l is increasing with β. The cost of RTL, α, does not affect q_c in this case because, given $l^* = 0$, RTC's marginal benefit is not affected by RTL and hence the cost of RTL has no effect on the probability threshold of investing in RTC. In Case 2 below we see that given $l^* > 0$ the probability threshold of investing in RTC is affected by RTL cost.

We summarize the analysis in this case as follows.

Remark 1. *Whenever the probability of having harmful climate change is small enough (i.e., $q < \min\{q_l, q_c\}$), then the social planner invests in neither RTL nor RTC. Given $(c^*, l^*) = 0$, (i) the probability threshold of investing in RTL, q_l, is increasing in both RTC cost and RTL cost; and (ii) the probability threshold of investing in RTC, q_c, is increasing in RTC cost but is not affected by RTL cost.*

Case 2. $c^* = 0$ ***and*** $l^* > 0$*.*

If $c^* = 0$ and $l^* > 0$ then from the FOC in (10) we obtain $l^* = (\sqrt{q/q_l} - 1)r$. Since in this case $l^* > 0$, we must have $q > q_l$. By plugging l^* into the FOC in (9) we obtain $q \leq q_{lc}$, where

$$q_{lc} \equiv q_l \left\{ 1 + \frac{0.5\beta^{-1}\left[(\beta\eta - 2\sqrt{D\beta}) + \sqrt{(2\sqrt{D\beta} - \beta\eta)^2 - 4\beta(D - \beta r^2/q_l)}\right]}{r} \right\}^2. \quad (13)$$

Algebra to show (13) is presented in SM Item E, in which we also show that the existence of q_{lc} requires that $\alpha \leq \beta((\sqrt{D} - \sqrt{\beta}r)^2 + \beta\eta r)/D$. Here q_{lc} is the probability threshold at which given the investment pair $c^* = 0$ and $l^* > 0$, the marginal cost of adding one more unit investment in RTC is equal to the marginal expected benefit of doing so. Therefore, if $q \in (q_l, q_{lc}]$ then it is optimal to only invest in RTL. The intuition here is that if the belief about the welfare impact is "ambiguous," i.e., q is neither very high nor very low, then investing in RTL only is more favorable. We summarize the analysis in this case as follows:

Remark 2. *Whenever the probability of having harmful climate change is moderate (i.e., $q \in (q_l, q_{lc}]$) and the RTL cost is not too high (i.e., $\alpha < \beta((\sqrt{D} - \sqrt{\beta}r)^2 + \beta\eta r)/D$), then at time 0 the social planner invests only in RTL.*

Remark 2 identifies conditions under which the social planner does not invest in RTC at time 0. This does not mean that the social planner will never invest in RTC when these conditions are satisfied. If at time τ_1 it turns out that climate change is harmful, then the investment in RTC will be positive. Remarks 1 and 2 can explain why some scientists do not support the idea of reducing greenhouse gas emissions (Wall Street Journal, 2012). Among their arguments, these scientists stated that "if elected officials feel compelled to 'do something' about climate, we recommend supporting the excellent scientists who

are increasing our understanding of climate with well-designed instruments on satellites, in the oceans and on land, and in the analysis of observational data," which is basically investment in RTL.

Case 3. $c^* > 0$ ***and*** $l^* = 0$.

If $c^* > 0$ and $l^* = 0$ then from the FOC in (9) we obtain $c^* = (\sqrt{q/q_c} - 1)r$. Since in this case $c^* > 0$, we must have $q > q_c$. By plugging c^* into FOC in inequality (10), we obtain

$$q + (\sqrt{\frac{\beta}{D}}\eta - 2)\sqrt{q} + 1 \leq \frac{\alpha}{\beta}. \tag{14}$$

In order for Case 3 to occur, inequality (14) must be satisfied by some q. This requires that $(\eta\sqrt{\beta/D} - 2)^2 \geq 4(1 - \alpha/\beta)$, i.e., $\alpha \geq \beta(\eta\sqrt{\beta/D} - \beta\eta^2/4D)$, from which we can see that Case 3 occurs only when α (i.e., RTL cost) is sufficiently large. Suppose equality in expression (14) holds. Then it has two solutions, q_1 and q_2, where $q_1 \leq q_2$. Therefore, the range of q that supports Case 3 is $[\max\{q_c, q_1\}, \min\{1, q_2\}]$. We can show that whenever q_1 and q_2 exist then $q_2 > q_c$. The proof is in SM Item F.

To facilitate the discussion, we define three critical values of α. They are

$$\alpha_1 \equiv \beta(\eta\sqrt{\frac{\beta}{D}} - \frac{\beta\eta^2}{4D}), \quad \alpha_2 \equiv \beta\eta\sqrt{\frac{\beta}{D}}, \quad \text{and} \quad \alpha_3 \equiv \frac{\beta}{D}((\sqrt{D} - \sqrt{\beta}r)^2 + \beta\eta r). \tag{15}$$

By Assumption 1 we can readily check that $\alpha_3 \geq \alpha_2 \geq \alpha_1 \geq 0$. According to the values of q_1 and q_2, the interval $[\max\{q_c, q_1\}, \min\{1, q_2\}]$ can take one of the following three possibilities. These three possibilities are: $[q_1, q_2]$, $[q_1, 1]$, and $[q_c, 1]$.[6] In SM Item G we show that these three possibilities require $\alpha \in [\alpha_1, \alpha_2]$, $\alpha \in [\alpha_2, \alpha_3]$, and $\alpha > \alpha_3$,

[6] In SM Item G we show that if $q_2 < 1$ then $q_1 > q_c$. Therefore, we can rule out $[q_c, q_2]$ as a possible value of $[\max\{q_c, q_1\}, \min\{1, q_2\}]$.

respectively.

Case 4. $c^* > 0$ ***and*** $l^* > 0$**.**

If $c^* > 0$ and $l^* > 0$ then equalities hold in FOCs (9) and (10). Due to the complexity of the equations we cannot explicitly solve for (c^*, l^*). But since Cases 1 to 4 are mutually exclusive and form a partition of all possible outcomes for (c^*, l^*), then the range of q that supports Case 4 contains any $q \in [0, 1]$ that does not support Case 1 to 3. Therefore, after we identify the intervals of q that support Cases 1 to 3, we can obtain the interval of q that supports Case 4 by removing the intervals of q supporting the other cases from interval $[0, 1]$. However, if we set $\eta = 0$ (i.e., RTL success does not contribute to an RTC breakthrough), then the FOCs in Case 4 can be simplified and analytical solutions can be identified. By analyzing FOCs (9) and (10) while setting $\eta = 0$ we obtain:

Remark 3. *Suppose the success of RTL does not contribute to an RTC breakthrough* (*i.e.,* $\eta = 0$). *When* $c^* > 0$ *and* $l^* > 0$*, then we must have* $q > \max\{q_c, q_l\}$ *and* $q = q_{lc}$.

The proof of Remark 3 is shown in SM Item H. Remark 3 shows that Case 4 is a "knife-edge" situation which happens only when the probability of having harmful climate change is equal to q_{lc}. This means that when an RTL success does not contribute to an RTC breakthrough (i.e., $\eta = 0$), it is almost always not optimal to carry out RTC and RTL simultaneously. The intuition is as follows. When $\eta = 0$ then the only benefit from investing in RTL is to make better decisions on RTC. The expected negative impact of climate change will not decrease just because of success in RTL. Therefore, the social planner will either conduct RTL before RTC or conduct RTC without any RTL. When success in RTL can contribute to a breakthrough in RTC (i.e., $\eta > 0$), then there will be an interval of q in which the social planner carries out RTC and RTL simultaneously, which will be discussed next.

3.2 Scenario Analysis

In this sub-section we analyze the possibilities for Cases 1 to 4 according to the values of RTL cost, α. We do so because when RTL cost varies then the q-intervals supporting the four cases change. Since the quantitative relationships between q_c, q_l, q_{lc}, q_1, and q_2 are important to the analysis that follows, here we summarize the relationships in a remark whose proof is shown in SM Item I.

Remark 4. *Whenever* (i) $\alpha \in (0, \alpha_1)$*, then* $q_l < q_c < q_{lc}$*;* (ii) $\alpha \in [\alpha_1, \alpha_2)$*, then* $q_l \leq q_c \leq q_{lc} \leq q_1 \leq q_2 \leq 1$*;* (iii) $\alpha = \alpha_1$*, then* $q_1 = q_2$*;* (iv) $\alpha \in [\alpha_2, \alpha_3)$*, then* $q_l \leq q_c \leq q_{lc} \leq q_1 \leq 1 \leq q_2$*;* (v) $\alpha = \alpha_2$*, then* $q_2 = 1$*;* (vi) $\alpha \geq \alpha_3$*, then* $q_1 \leq q_c \leq q_l \leq 1 \leq q_2$*; and* (vii) $\alpha = \alpha_3$*, then* $q_l = q_c = q_{lc} = q_1 < 1 < q_2$*.*

According to the values of α, we have four scenarios.

Scenario 1. $\alpha \in (0, \alpha_1)$**.** In this scenario Case 3 does not occur because no q satisfies inequality (14). By Remark 4 we know that $q_l < q_c < q_{lc}$ whenever $\alpha \in (0, \alpha_1)$. So Case 1 occurs when $q \in [0, q_l]$; Case 2 occurs when $q \in (q_l, q_{lc}]$; Case 4 occurs when $q \in (q_{lc}, 1]$. Therefore, we can see that if (1) the cost of RTL is low enough, and (2) an RTL success can contribute to an RTC breakthrough, then it is never optimal to only invest into RTC. It is easy to check that when $\eta = 0$ then $\alpha_1 = 0$. Therefore, Scenario 1 does not occur whenever $\eta = 0$. That is, when $\eta = 0$ then Case 3 always occur under some q. Figure 2 includes a visual presentation of Scenario 1. The upper-left panel of Figure 3 presents a numerical example of this scenario. For the parameter values of the numerical example in Figure 3, please see Section 4 for details.

Scenario 2. $\alpha \in [\alpha_1, \alpha_2)$**.** By Remark 4 we know that when $\alpha \in [\alpha_1, \alpha_2]$ then $q_l \leq q_c \leq q_{lc} \leq q_1 \leq q_2 \leq 1$. Under this scenario every case is possible. One interesting observation is that Case 4 occurs on disconnected intervals $[q_{lc}, q_1]$ and $[q_2, 1]$. An

explanation is that on the q-interval that supports Cases 3 and 4, the marginal benefit of RTL investment is first decreasing and then increasing in q. The reason is as follows. The benefit of RTL investment is two-fold. The first benefit is that by RTL investment the social planner can expect to identify the true state of the world sooner so that an accurate decision on RTC can be made. The second benefit is that the success in RTL can accelerate an RTC breakthrough. On one side, an increase in q makes the social planner more willing to invest in RTL given that the RTL success can accelerate an RTC breakthrough. On the other side, as RTC investment increases due to an increase in q, the benefit of RTL investment is reduced. This is because the larger the RTC investment is, the larger the probability that an RTC breakthrough will occur before the true state of the world is revealed. When q is moderate then the RTL benefit decrease caused by an RTC increase dominates. When q is large then the two-fold benefit of RTL dominates. Figure 2 includes a visual presentation of Scenario 2. The upper-right panel of Figure 3 presents a numerical example of this scenario.

Scenario 3. $\alpha \in [\alpha_2, \alpha_3)$**.** By Remark 4 we know that when $\alpha \in [\alpha_2, \alpha_3]$ then $q_l \leq q_c \leq q_{lc} \leq q_1 \leq 1 \leq q_2$. Under this scenario every case is possible as well. Figure 2 includes a visual presentation of Scenario 3. The lower-left panel of Figure 3 presents a numerical example of this scenario. We can see that Case 3 occurs whenever $q \in [q_1, 1]$; and Case 4 occurs whenever $q \in (q_{lc}, q_1)$.

Scenario 4. $\alpha \geq \alpha_3$**.** By Remark 4 we know that when $\alpha \geq \alpha_3$ then $q_l \geq q_c \geq q_1$. Under this scenario Case 2 does not occur because q_{lc} does not exist. Case 1 occurs when $q \in [0, q_c]$ and Case 3 occurs when $q \in (q_c, 1]$. Since the union of intervals of q that support Case 1 and Case 3 is $[0, 1]$, Case 4 does not occur either under this scenario. Therefore, we conclude that if $\alpha > \alpha_3$ then $l^* > 0$ will never be optimal. When the probability of having harmful climate change is lower than q_c, the social planner needs to do nothing.

When the probability is higher than q_c, the social planner will only conduct RTC. Figure 2 includes a visual presentation of Scenario 4. The lower-right panel of Figure 3 presents a numerical example of this scenario.

We summarize the analysis in this sub-section as Result 1.

Result 1. *Suppose an RTL success can accelerate an RTC breakthrough (i.e., $\eta > 0$). (i) When the cost of RTL is low enough $(i.e., \alpha \in (0, \alpha_1))$, then it is never optimal to only invest in RTC. (ii) When the cost of RTL is moderate $(i.e., \alpha \in [\alpha_1, \alpha_2))$ and when the probability of harmful climate change is either moderate or very high $(i.e., q \in [q_{lc}, q_1] \cup [q_2, 1]$), then the social planner invests in both RTC and RTL. (iii) When $\alpha \geq \alpha_3$ then $a)$ there is no RTL investment and $b)$ RTC investment occurs when q is large enough $($i.e., $q > q_c)$.*

From Result 1 we can see that whether to follow the precautionary principle or the learn-then-act principle regarding investment into new technology R&D depends on the costs of research activities and the probability distribution of climate change's damage. Therefore, the model provides an explicit resolution to the debate between the advocates for these two rules. We are also interested in how changes in exogenous parameters affect optimal RTC and RTL investment as well as q-intervals supporting precautionary principle and learn-then-act principle, which is the content of the next section.

4 Comparative Static Analysis

A marginal change of any exogenous parameters (i.e., α, β, η, q, r, and D) will not affect c^* in Cases 1 and 2 (or l^* in Cases 1 and 3) because c^* (or l^*) is zero in Cases 1 and 2 (or Cases 1 and 3). The effects of exogenous parameters on l^* in Case 2, c^* in Case 3, and

probability thresholds q_c, q_l, q_1, and q_2 can be readily identified after some algebra (see SM Item J).[7] The results are shown in Table 1.

However, the exogenous parameters' effects on c^* and l^* in Cases 4 and probability threshold q_{lc} are challenging to identify. Therefore, in this section we resort to numerical analysis to identify these effects. During the numerical analysis, the initial value of parameters are set as follows. We normalize the instantaneous damage rate, D, to 1. The continuous time discount rate, r, is assumed to be equal to 0.05. We further assume that η, the parameter that measures the magnitude of an RTL success' positive externality to an RTC breakthrough, is equal to 0.04. Suppose time unit is one year in our analysis. Then $\eta = 0.04$ signifies that if the social planner conducts a very small investment in RTC then an RTC breakthrough will occur about 25 years after the RTL success due to the RTL externality.

Given the values of D, r, and η, by Assumption 1 we know that $\beta < D/(r+\eta)^2 \approx 123.46$. In order to check the robustness of the numerical comparative static analysis, we select multiple values for β within the range $(0, 123.46)$.[8] For each value of β, together with the fixed values of D, r, and η, we calculate the three thresholds of RTL cost (i.e., α_i, $i \in \{1,2,3\}$). Since Case 4 and q_{lc} do not exist when $\alpha > \alpha_3$, we conduct numerical comparative static analysis for c^* and l^* in Case 4 and q_{lc} only when $\alpha \leq \alpha_3$. The results are summarized in Table 1 as well. We discuss these effects next.

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[7]We know that q_1 is relevant to our analysis only when $\alpha \leq \alpha_3$ because whenever $\alpha > \alpha_3$ then the q-interval supporting Case 3 is $[q_c, 1]$. Therefore, we are only interested in signs of $\partial q_1/\partial j$ when $\alpha \leq \alpha_3$. Here j stands for exogenous parameters.

[8]For the numerical example shown in Figure 3, we set $\beta = 100$ for better illustration. When β is small then the values of q_l and q_c could be very small and hence are difficult to depict in a figure.

4.1 Effects on c^* and l^*

We first study the effects of RTL cost, α, and RTC cost, β, on c^* and l^*. It is intuitive that an increase in RTL cost (or RTC cost) decreases the optimal level of investment in RTL (or RTC). We are more interested in the effect of RTL (or RTC) cost on RTC (or RTL) investment. From Table 1 we can see that in Case 2 an increase in RTC cost, β, will decrease the optimal RTL investment. In Case 3, however, an increase in RTL cost, α, has no impact on the optimal RTC investment. That is, in Cases 2 and 3, we have $\partial l^*/\partial\beta \leq 0$ and $\partial c^*/\partial\alpha = 0$, which means that in Case 2 RTL complements RTC and that in Case 3 RTL and RTC are neither complements or substitutes. The intuition is as follows. We know that Case 2 (where $c^* = 0$ and $l^* > 0$) occurs only when the probability of harmful climate change is moderate and when the RTL cost is not very high (i.e., $\alpha < \alpha_3$). In Case 2 the purpose of an RTL investment is to accelerate the realization of the true state of climate change, so that an accurate decision on RTC can be made later on to reduce the expected negative impact of climate change. But an increase in RTC cost, β, will decrease the incentive to invest in RTC, and hence decrease the incentive to invest in RTL. In Case 3 (where $l^* = 0$) RTL will never succeed and hence RTL has no effect on RTC. Therefore, changing the cost of RTL will not affect the optimal level of RTC.

In Case 4, however, an increase in RTL (or RTC) cost enhances the optimal RTC (or RTL) investment. That is, $\partial l^*/\partial\beta \geq 0$ and $\partial c^*/\partial\alpha \geq 0$, which means that in Case 4 RTL and RTC are substitutes. The intuition is as follows. Recall that RTL success generates a positive externality regarding an RTC breakthrough. When there is more RTL investment, then less RTC investment will be needed to achieve the same RTC outcome. Whenever RTC becomes more expensive, then the social planner will conduct more RTL expecting that the externality of RTL success will accelerate an RTC breakthrough. In reality RTL is more likely a public sector activity and RTC is more likely a private sector activity.

An implication of RTL and RTC being substitutes in Case 4 is that policies which reduce RTC cost in the private sector, such as subsidies or tax credits, will decrease the optimal public sector RTL investment. Similarly, policies which reduce public sector RTL cost, such as reducing bureaucracy, will decrease optimal RTC investment in the private sector.

Now let us study the effect of η, the magnitude of positive externality from an RTL success to an RTC breakthrough, on c^* and l^*. From Table 1 we can see that optimal RTL increases in η but optimal RTC decreases, at least weakly, in η. This is intuitive because a larger η means that the positive externality from RTL to RTC is bigger so that RTL is more valuable. Given that the RTL cost does not change, the social planner will invest more in RTL and less in RTC. If we view RTL as basic research and RTC as applied research, then the effects of η can explain why basic research that has larger potential to be used in applied research (i.e., larger η) is preferred by governments. For example, the broader impacts criterion was established by the U.S. National Science Foundation (NSF) to screen research proposals. To meet the broader impacts criterion, research proposals have to answer questions such as "how well does the activity advance discovery and understanding while promoting teaching, training and learning?" and "will the results be disseminated broadly to enhance scientific and technological understanding?" (NSF, 2007).

It is intuitive that an increase in q will always increase (at least weakly) RTC investment. However, the impact of q on the optimal RTL investment requires scrutiny. In Case 2 the optimal RTL investment, l^*, is increasing in q (Table 1). This is because when $c^* = 0$, then an increase in the probability of damage leads the social planner to put more resources into RTL in order to accelerate success in RTC at the second stage in case the true state of climate change is harmful. In Case 4 the impact of q on l^* is complicated. Table 1 shows that, under Scenarios 1 and 2, l^* in Case 4 is first decreasing and then

increasing with q. An explanation is that when the likelihood of the damage is small and when the cost of RTL is low enough (i.e., $\alpha \leq \alpha_2$), then the increasing c^* substitutes l^* out as q is increasing. However, if the likelihood of the damage becomes large and the cost of RTL is still low enough, then the social planner is willing to put more resources into both RTC and RTL in the expectation that the positive externality from RTL would promote an RTC breakthrough.

We summarize some key findings from above analysis as Result 2.

Result 2. (i) *In Case 2* $($*i.e.,* $c^* = 0$ *and* $l^* > 0)$*, RTL and RTC are complements. In Case 4* $($*i.e.,* $c^* > 0$ *and* $l^* > 0)$*, RTL and RTC are substitutes. In Case 3* $($*i.e.,* $c^* > 0$ *and* $l^* = 0)$*, RTL and RTC are neither complements nor substitutes.* (ii) *In Case 4,* l^* *is first decreasing and then increasing in q under Scenarios 1 and 2.*

4.2 Effects on q-intervals Supporting Precautionary Principle and Learn-then-act Principle

Our analysis has shown that whether or not to follow precautionary principle or the learn-then-act principle depends on the relationship between probability q and the probability thresholds (i.e., q_c, q_l, q_{lc}, q_1, and q_2). In this subsection we study the exogenous parameters' effects on these probability thresholds. Doing so allows us to see how the changes of exogenous parameters affect the q-interval that supports the two principles.

From Figure 2 we can see that under Scenarios 1 to 3, (1) actions following the learn-then-act principle (i.e., Cases 1 and 2) occurs whenever $q \leq q_{lc}$, where q_{lc} is a probability threshold at which given the investment pair $c^* = 0$ and $l^* > 0$, the marginal cost of RTC investment is equal to the marginal expected benefit of RTC investment; and (2) actions following the precautionary principle (i.e., Cases 3 and 4) occurs whenever $q >$

q_{lc}. Therefore, under Scenarios 1 to 3 we only need to focus on q_{lc}. From Table 1 we can see that q_{lc} is decreasing in α and D. So whenever RTL cost or climate change damage rate increases then the q-interval that supports the learn-then-act principle shrinks and the q-interval that supports the precautionary principle expands. This could explain why the precautionary principle was established in the Rio Declaration on Environment and Development (United Nations, 1992) considering climate change might be potentially catastrophic. From Table 1 we also see that q_{lc} is increasing in β, η, and r. So whenever RTC cost, positive externality of RTL success to RTC breakthrough, or discount rate increases, then the q-interval that supports the learn-then-act principle expands and the q-interval that supports the precautionary principle shrinks. The reason is that when RTC is more costly (or the positive externality of RTL success to RTC breakthrough is larger, or future generations welfare becomes less important), then the social planner will more likely follow the learn-then-act rule (Bartle and Vass, 2007; Gollier, 2001). For Scenario 4, a similar analysis applies.

We summarize the analysis in this sub-section as Result 3:

Result 3. *Whether or not to follow the precautionary rule or the learn-then-act rule depends on the relationship between the probability of harmful climate change, q, and the probability threshold, q_{lc}. Whenever $q \leq q_{lc}$ the social planner follows the learn-then-act principle. Otherwise she follows the precautionary principle. The q-intervals that supports the learn-then-act principle shrinks as α or D increases, but expands as β, η, or r increases. The opposite is true for the q-intervals that supports the precautionary principle.*

5 Conclusions and Future Research

How to face the challenge of climate change will be the focus of international policies before the world clearly understands the magnitude of climate change's welfare impacts, or before the world is confident that the technologies available could handle any possible effects of climate change. In this article we studied the optimal investments in two lines of research activities. One is research to learn the true welfare impacts of climate change (termed as "research to learn" or RTL). The other is research to develop new technologies that can mitigate climate change's negative impacts (termed as "research to change" or RTC). The results show that if RTL's success does not contribute to an RTC breakthrough then it is almost never optimal to invest simultaneously in RTL and RTC. If RTL's success contributes to an RTC breakthrough, however, then simultaneously investing in both RTL and RTC may be optimal when the probability of a harmful climate change is either moderate or very high. Whenever RTL and RTC are conducted simultaneously then an increase in RTL (or RTC) cost enhances the optimal RTC (or RTL) investment (i.e., RTL and RTC are substitutes). Factors that influence optimal investments in RTC and RTL are studied as well. We also show that whether to follow the precautionary principle or the learn-then-act principle regarding investment into R&D about new technologies depends on the costs of research activities and the probability distribution of damage due to climate change. We solved the model and identified parameters under which the two principles should be followed. Therefore, the article provides an explicit resolution to the debate between the advocates for these two principles.

There are several possible directions along which one could extend this research. One is to generalize the analysis into a formal Bayesian decision framework. The current analysis is a special case of a general Bayesian decision framework. But we expect that the generalization would cause challenging technical problems. The second is to calibrate

the current model and simulate what the optimal RTL and RTC should be, which could provide policy implications on optimally allocating scarce research resources. As Baker and Solak (2011) mentioned, every research breakthrough is unique and historical data on research breakthroughs have limited predictive power on future research breakthroughs. Therefore, expert elicitations may provide a reasonable estimation on success probabilities of future research regarding climate change. Third, in this study the social planner is modeled to minimize social costs, and hence risk aversion is absent from the model. Another possible extension to this study is to model the social planner as a utility maximizer so that risk aversion or even ambiguity aversion can be included. By doing so one can analyze how risk aversion or ambiguity aversion affects the interaction and resource allocation between RTL and RTC.

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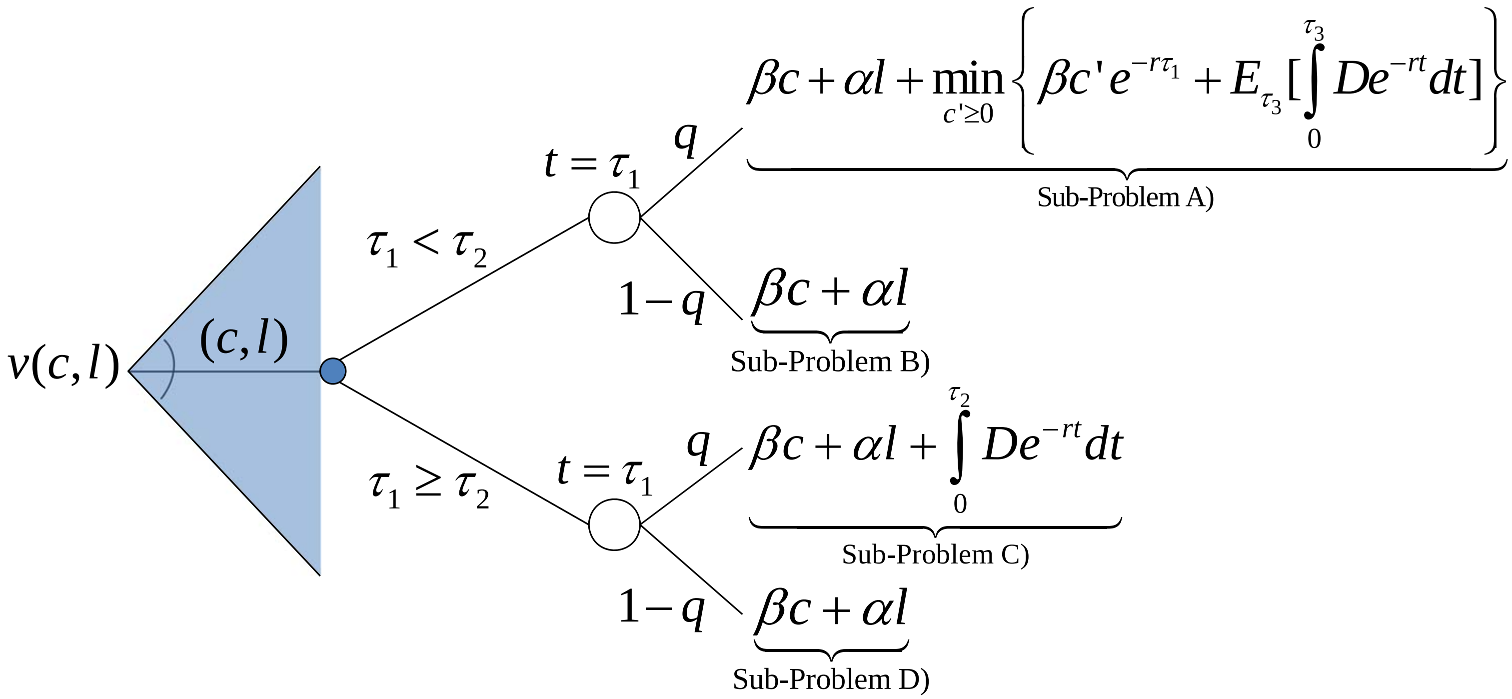
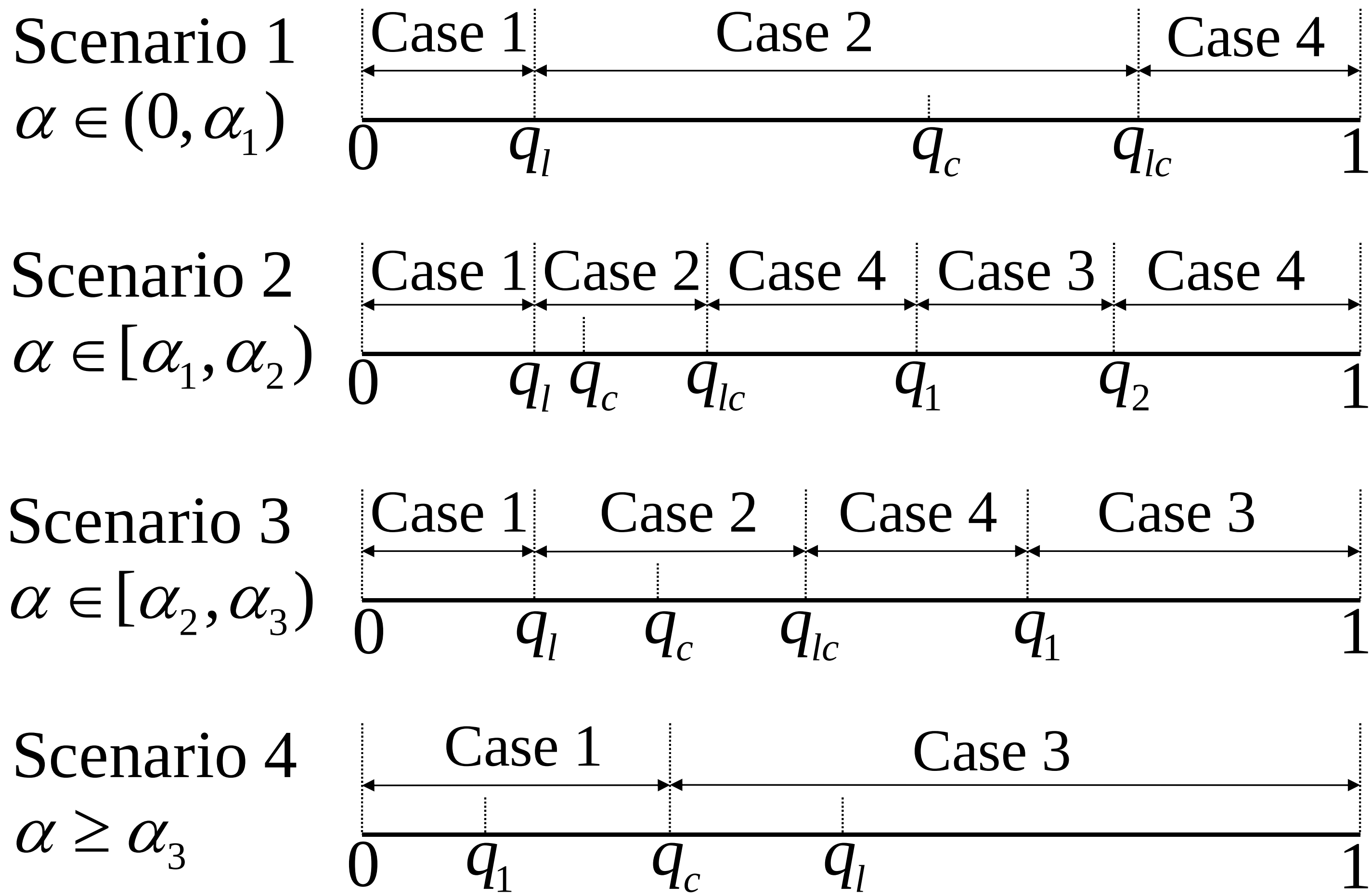


Fig. 1. The Social Planner's Decision Tree.



Case 1: $c^* = 0$, $l^* = 0$; Case 2: $c^* = 0$, $l^* > 0$;
Case 3: $c^* > 0$, $l^* = 0$; Case 4: $c^* > 0$, $l^* > 0$.

Fig. 2. Possible Cases under Scenarios 1 to 4.

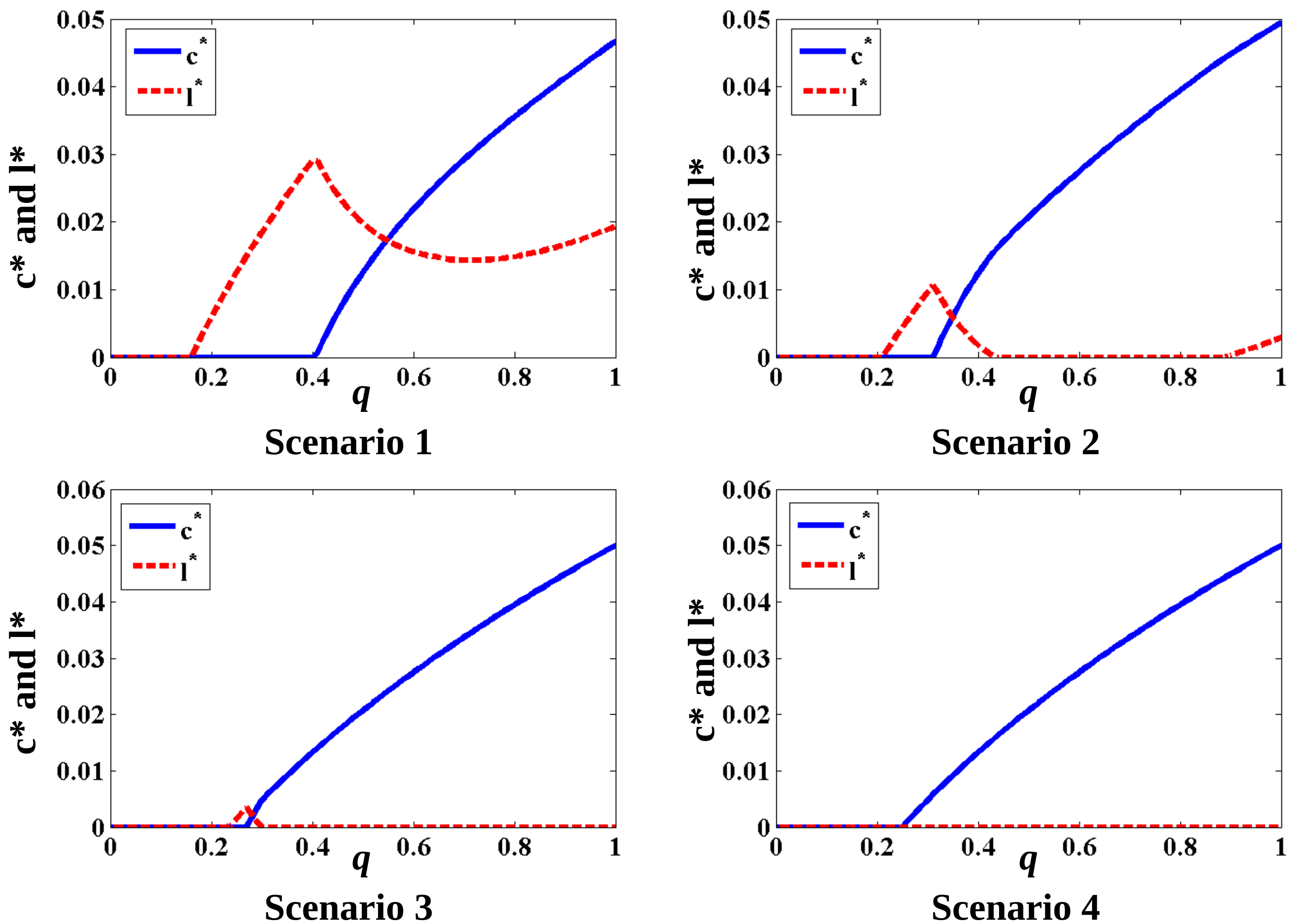


Fig. 3. Values of c^* and l^* under Various Scenarios: A Numerical Example.

Supplemental Materials

Item A

In this Item we present the justification for Assumption 1, i.e., $\eta < \sqrt{D/\beta} - r$.

If at time 0 there is no investment in RTC and at time τ_1 the true state is proved to be T, then at time τ_1 the social planner's problem is to choose an investment level, $c' \geq 0$, on RTC so that the sum of damage from climate change and research costs is minimized. That is,

$$\min_{c' \geq 0} \quad \beta c' e^{-r\tau_1} + \int_0^{\tau_1} De^{-rt}dt + E_{\tau_3}\left[\int_{\tau_1}^{\tau_3} De^{-rt}dt\right]. \tag{SM-1}$$

When $c = 0$ then the density function for τ_3 is $g'(\tau_3) = (c' + \eta)e^{-(c'+\eta)(\tau_3 - \tau_1)}$, where $\tau_3 > \tau_1$. We have

$$\begin{aligned}
E_{\tau_3}[\int_{\tau_1}^{\tau_3} De^{-rt}dt] &= \int_{\tau_1}^{\infty} \frac{D}{r}(e^{-r\tau_1} - e^{-r\tau_3})(c' + \eta)e^{-(c'+\eta)(\tau_3 - \tau_1)}d\tau_3 \\
&= \frac{D}{r}\left[e^{-r\tau_1} - \int_{\tau_1}^{\infty} e^{-r\tau_3}(c' + \eta)e^{-(c'+\eta)(\tau_3 - \tau_1)}\right] \\
&= \frac{D}{r}\left(1 - \frac{c' + \eta}{r + c' + \eta}\right)e^{-r\tau_1}.
\end{aligned}$$

Then the social planner's problem becomes

$$\min_{c' \geq 0} \quad \beta c' e^{-r\tau_1} + \int_0^{\tau_1} De^{-rt}dt + \frac{D}{r}\left(1 - \frac{c' + \eta}{r + c' + \eta}\right)e^{-r\tau_1}, \tag{SM-2}$$

which is equivalent to

$$\min_{c' \geq 0} \quad \beta c' e^{-r\tau_1} + \frac{D}{r}\left(1 - e^{-r\tau_1}\frac{c' + \eta}{r + c' + \eta}\right). \tag{SM-3}$$

The first order condition for an interior solution is

$$\beta - \frac{D}{(c'^{*} + r + \eta)^2} = 0. \tag{SM-4}$$

So $c'^{*} = \sqrt{D/\beta} - r - \eta$. An interior solution requires $\sqrt{D/\beta} - r - \eta > 0$. Naturally $\eta < \sqrt{D/\beta} - r$ follows.

Item B

In this item we show how to obtain equation (3). If at time $t = \tau_1$ it is proved that climate change is harmful and the RTC has not been successful yet, then the social planner will add more investment, $c' \geq 0$, to this research to accelerate the breakthrough time. Therefore, at time $t = \tau_1$ the social planner's problem is:

$$V^A(c,l) = \min_{c' \geq 0} \quad \beta c' e^{-r\tau_1} + E_{\tau_3}\left[\int_0^{\tau_3} De^{-rt} dt\right]. \tag{SM-5}$$

We know that τ_3 is greater than τ_1 and has probability density function $g'(\tau_3) = (c + c' + \eta)e^{-(c+c'+\eta)(\tau_3 - \tau_1)}$, where $\tau_3 \geq \tau_1$. Therefore,

$$\begin{aligned}
E_{\tau_3}[\int_0^{\tau_3} De^{-rt} dt] &= E_{\tau_3}[\frac{D}{r}(1 - e^{-r\tau_3})] \\
&= \int_{\tau_1}^{\infty} \frac{D}{r}(1 - e^{-r\tau_3})(c + c' + \eta)e^{-(c+c'+\eta)(\tau_3 - \tau_1)} d\tau_3 \\
&= \frac{D}{r}[1 - e^{-r\tau_1} \int_{\tau_1}^{\infty} (c + c' + \eta)e^{-(r+c+c'+\eta)(\tau_3 - \tau_1)} d\tau_3] \\
&= \frac{D}{r}(1 - \frac{c + c' + \eta}{r + c + c' + \eta} e^{-r\tau_1}).
\end{aligned}$$

Then at time τ_1 the social planner's problem becomes

$$\min_{c' \geq 0} \beta c' e^{-r\tau_1} + \frac{D}{r}\left(1 - \frac{c+c'+\eta}{r+c+c'+\eta} e^{-r\tau_1}\right). \tag{SM-6}$$

The first order condition is

$$\beta - \frac{D}{(r+c+c'^*+\eta)^2} \geq 0, \tag{SM-7}$$

so the interior solution is

$$c'^* = \sqrt{D/\beta} - c - r - \eta.$$

The interior solution requires $\sqrt{D/\beta} - c - r - \eta > 0$ and hence $\eta < \sqrt{D/\beta} - r - c \leq \sqrt{D/\beta} - r$. Upon Plugging the interior solution into the objective function (SM-6), we get

$$\begin{aligned} V^A(c,l) &= \beta(\sqrt{D/\beta} - c - r - \eta)e^{-r\tau_1} + \frac{D}{r}(1 - \frac{\sqrt{D/\beta} - r}{\sqrt{D/\beta}} e^{-r\tau_1}) \\ &= \frac{D}{r} + [\sqrt{\beta D} - \beta(c+r+\eta) - \frac{D}{r}\frac{\sqrt{D/\beta} - r}{\sqrt{D/\beta}}]e^{-r\tau_1} \\ &= \frac{D}{r} + (2\sqrt{\beta D} - \beta(c+r+\eta) - \frac{D}{r})e^{-r\tau_1}. \end{aligned}$$

Item C

In this Item we show how to obtain equation (7).

Plugging equations (3), (4), and (5) into equation (6), we get

$$
\begin{aligned}
V(c,l) &= \int_0^\infty \int_{\tau_1}^\infty \{q\left[\frac{D}{r} + \left(2\sqrt{\beta D} - \beta(c+r+\eta) - \frac{D}{r}\right)e^{-r\tau_1}\right] \\
&\quad +(1-q)(\beta c + \alpha l)\} g(\tau_2) f(\tau_1) d\tau_2 d\tau_1 \\
&\quad + \int_0^\infty \int_0^{\tau_1} \{q[\frac{D}{r}(1-e^{-r\tau_2})] + (1-q)(\beta c + \alpha l)\} g(\tau_2) f(\tau_1) d\tau_2 d\tau_1 \\
&= \beta c + \alpha l + q \int_0^\infty \int_{\tau_1}^\infty [\frac{D}{r} + (2\sqrt{\beta D} - \beta(c+r+\eta) - \frac{D}{r})e^{-r\tau_1}] g(\tau_2) f(\tau_1) d\tau_2 d\tau_1 \\
&\quad +q \int_0^\infty \int_0^{\tau_1} [\frac{D}{r}(1-e^{-r\tau_2})] g(\tau_2) f(\tau_1) d\tau_2 d\tau_1,
\end{aligned}
$$

where $f(\tau_1) = le^{-l\tau_1}$, $g(\tau_2) = ce^{-c\tau_2}$.

Since

$$
\begin{aligned}
& \int_0^\infty \int_{\tau_1}^\infty \left[\frac{D}{r} + (2\sqrt{\beta D} - \beta(c+r+\eta) - \frac{D}{r})e^{-r\tau_1}\right] g(\tau_2) f(\tau_1) d\tau_2 d\tau_1 \\
= & \int_0^\infty e^{-c\tau_1} [\frac{D}{r} + (2\sqrt{\beta D} - \beta(c+r+\eta) - \frac{D}{r})e^{-r\tau_1}] le^{-l\tau_1} d\tau_1 \\
= & \frac{D}{r} \int_0^\infty e^{-c\tau_1} le^{-l\tau_1} d\tau_1 + \int_0^\infty [2\sqrt{\beta D} - \beta(c+r+\eta) - \frac{D}{r}] e^{-r\tau_1} le^{-l\tau_1} d\tau_1 \\
= & \frac{Dl}{r(l+c)} + [2\sqrt{\beta D} - \beta(c+r+\eta) - \frac{D}{r}] \frac{l}{r+l+c},
\end{aligned}
$$

and

$$
\begin{aligned}
& \int_0^\infty \int_0^{\tau_1} [\frac{D}{r}(1-e^{-r\tau_2})]g(\tau_2)f(\tau_1)d\tau_2 d\tau_1 \\
= \;& \int_0^\infty \int_0^{\tau_1} [\frac{D}{r}(1-e^{-r\tau_2})]ce^{-c\tau_2}le^{-l\tau_1}d\tau_2 d\tau_1 \\
= \;& \frac{D}{r}\left[\int_0^\infty \int_0^{\tau_1} ce^{-c\tau_2}le^{-l\tau_1}d\tau_2 d\tau_1 - \int_0^\infty \int_0^{\tau_1} cle^{-l\tau_1}e^{-(r+c)\tau_2}d\tau_2 d\tau_1\right] \\
= \;& \frac{D}{r}\left[\frac{c}{l+c} - \frac{cl}{r+c}\int_0^\infty e^{-l\tau_1}(1-e^{-(r+c)\tau_1})d\tau_1\right] \\
= \;& \frac{D}{r}\left[\frac{c}{l+c} - \frac{cl}{r+c}[\int_0^\infty e^{-l\tau_1}d\tau_1 - \int_0^\infty e^{-(r+l+c)\tau_1}d\tau_1]\right] \\
= \;& \frac{D}{r}\left[\frac{c}{l+c} - \frac{cl}{r+c}[\frac{1}{l} - \frac{1}{r+l+c}]\right] \\
= \;& \frac{D}{r}\left[\frac{c}{l+c} - \frac{c}{r+c} + \frac{cl}{(r+c)(r+l+c)}\right] \\
= \;& \frac{D}{r}\left[\frac{c}{l+c} + \frac{cl-c(r+l+c)}{(r+c)(r+l+c)}\right] \\
= \;& \frac{D}{r}\left[\frac{c}{l+c} - \frac{c}{r+l+c}\right] \\
= \;& \frac{Dc}{(l+c)(r+l+c)},
\end{aligned}
$$

we have

$$\begin{aligned}
V(c,l) &= \beta c+\alpha l+q\left\{\frac{Dl}{r(l+c)}+[2\sqrt{\beta D}-\beta(c+r+\eta)-\frac{D}{r}]\frac{l}{r+l+c}\right. \\
&\quad \left.+\frac{Dc}{(l+c)(r+l+c)}\right\} \\
&= \beta c+\alpha l+q\left\{\frac{Dl}{r}(\frac{1}{l+c}-\frac{1}{r+l+c})\right. \\
&\quad \left.+[2\sqrt{\beta D}-\beta(c+r+\eta)]\frac{l}{r+l+c}+\frac{Dc}{(l+c)(r+l+c)}\right\} \\
&= \beta c+\alpha l+q\left\{\frac{Dl+Dc}{(l+c)(r+l+c)}+[2\sqrt{\beta D}-\beta(c+r+\eta)]\frac{l}{r+l+c}\right\} \\
&= \beta c+\alpha l+q\left\{\frac{D}{r+l+c}+[2\sqrt{\beta D}-\beta(c+r+\eta)]\frac{l}{r+l+c}\right\} \\
&= \beta c+\alpha l+\frac{q}{r+l+c}\left\{D+[2\sqrt{\beta D}-\beta(c+r+\eta)]l\right\},
\end{aligned}$$

which is equation (7).

Item D

In this Item we derive the FOCs of problem (7) and show the function $V(c,l)$ is convex under Assumption 1. FOCs are:

$$\begin{aligned}
\frac{\partial V(c,l)}{\partial c} &= \beta-\frac{q\beta l(r+l+c)+q\{D+[2\sqrt{D\beta}-\beta(c+r+\eta)]l\}}{(r+l+c)^2} \\
&= \beta-q\frac{\beta l(r+l+c)+D+2\sqrt{D\beta}l-\beta(c+r+\eta)l}{(r+l+c)^2} \\
&= \beta-q\frac{\beta l^2+2\sqrt{D\beta}l+D-\beta\eta l}{(r+l+c)^2} \\
&= \beta-q\frac{(\sqrt{D}+\sqrt{\beta}l)^2-\beta\eta l}{(r+l+c)^2}\geq 0,
\end{aligned}$$

and

$$
\begin{aligned}
\frac{\partial V(c,l)}{\partial l} &= \alpha + q\frac{[2\sqrt{D\beta}-\beta(c+r)](r+l+c)-\left\{D+[2\sqrt{D\beta}-\beta(c+r+\eta)]l\right\}}{(r+l+c)^2} \\
&= \alpha + q\frac{[2\sqrt{D\beta}-\beta(c+r+\eta)](r+c)-D}{(r+l+c)^2} \\
&= \alpha - q\frac{\beta(r+c)^2-2\sqrt{D\beta}(r+c)+D+\beta\eta(c+r)}{(r+l+c)^2} \\
&= \alpha - q\frac{(\sqrt{D}-\sqrt{\beta}(c+r))^2+\beta\eta(r+c)}{(r+l+c)^2} \geq 0.
\end{aligned}
$$

Next we show $V(c,l)$ is convex when $0 \leq \eta \leq \sqrt{D/\beta}-r$. To do this we need to show that if $0 \leq \eta \leq \sqrt{D/\beta}-r$, then $\partial^2 V(c,l)/\partial c^2 \geq 0$, $\partial^2 V(c,l)/\partial l^2 \geq 0$, and $(\partial^2 V(c,l)/\partial c^2)(\partial^2 V(c,l)/\partial l^2)-(\partial^2 V(c,l)/\partial c\partial l)^2 \geq 0$. We have

$$
\begin{aligned}
\frac{\partial^2 V(c,l)}{\partial c^2} &= 2q\frac{(r+l+c)[(\sqrt{D}+\sqrt{\beta}l)^2-\beta\eta l]}{(r+l+c)^4} \\
&= 2q\frac{(\sqrt{D}+\sqrt{\beta}l)^2-\beta\eta l}{(r+l+c)^3}, \\
\frac{\partial^2 V(c,l)}{\partial l^2} &= 2q\frac{(\sqrt{D}-\sqrt{\beta}(r+c))^2+\beta\eta(r+c)}{(r+l+c)^3}, \\
\frac{\partial^2 V(c,l)}{\partial c\partial l} &= -q\frac{(r+l+c)[-2\sqrt{\beta}(\sqrt{D}-\sqrt{\beta}(r+c))+\beta\eta]-2[(\sqrt{D}-\sqrt{\beta}(r+c))^2+\beta\eta(r+c)]}{(r+l+c)^3} \\
&= -q\frac{-2[\sqrt{D}-\sqrt{\beta}(r+c)][\sqrt{D}+\sqrt{\beta}l]+\beta\eta[l-(r+c)]}{(r+l+c)^3} \\
&= q\frac{2[\sqrt{D}-\sqrt{\beta}(r+c)][\sqrt{D}+\sqrt{\beta}l]+\beta\eta[(r+c)-l]}{(r+l+c)^3}.
\end{aligned}
$$

Define $\phi \equiv q^2/(r+l+c)^6$. Then we have

$$
\begin{aligned}
& \frac{\partial^2 V(c,l)}{\partial c^2}\frac{\partial^2 V(c,l)}{\partial l^2} - (\frac{\partial^2 V(c,l)}{\partial c \partial l})^2 \\
= \quad & \phi\Bigg\{4[(\sqrt{D}+\sqrt{\beta}l)^2 - \beta\eta l][(\sqrt{D}-\sqrt{\beta}(r+c))^2 + \beta\eta(r+c)] \\
& -\big\{2[\sqrt{D}-\sqrt{\beta}(r+c)][\sqrt{D}+\sqrt{\beta}l] + \beta\eta[(r+c)-l]\big\}^2\Bigg\} \\
= \quad & \phi\beta\eta\Bigg\{4(r+c)(\sqrt{D}+\sqrt{\beta}l)^2 - 4l(\sqrt{D}-\sqrt{\beta}(r+c))^2 \\
& -4[(r+c)-l][\sqrt{D}+\sqrt{\beta}l][\sqrt{D}-\sqrt{\beta}(r+c)] \\
& -\beta\eta[4l(r+c)+((r+c)-l)^2]\Bigg\} \\
= \quad & \phi\beta\eta\Bigg\{4(r+c)(\sqrt{D}+\sqrt{\beta}l)[(\sqrt{D}+\sqrt{\beta}l)-(\sqrt{D}-\sqrt{\beta}(r+c))] \\
& +4l(\sqrt{D}-\sqrt{\beta}(r+c))[(\sqrt{D}+\sqrt{\beta}l)-(\sqrt{D}-\sqrt{\beta}(r+c))] \\
& -\beta\eta(r+l+c)^2\Bigg\} \\
= \quad & \phi\beta\eta\Bigg\{4(r+c)(\sqrt{D}+\sqrt{\beta}l)[\sqrt{\beta}(r+l+c)] \\
& +4l(\sqrt{D}-\sqrt{\beta}(r+c))[\sqrt{\beta}(r+l+c)] \\
& -\beta\eta(r+l+c)^2\Bigg\} \\
= \quad & \phi\beta\eta\Bigg\{4[\sqrt{\beta}(r+l+c)]\Big[(r+c)(\sqrt{D}+\sqrt{\beta}l)+l(\sqrt{D}-\sqrt{\beta}(r+c))\Big] \\
& -\beta\eta(r+l+c)^2\Bigg\} \\
= \quad & \phi\beta\eta\Bigg\{4\sqrt{D\beta}(r+l+c)^2 - \beta\eta(r+l+c)^2\Bigg\} \\
= \quad & \phi\beta\eta\Bigg\{4\sqrt{D\beta} - \beta\eta\Bigg\}(r+l+c)^2 \\
= \quad & \phi\beta^2\eta\Bigg\{4\sqrt{\frac{D}{\beta}} - \eta\Bigg\}(r+l+c)^2 \\
\geq \quad & 0 \qquad \text{by Assumption 1.}
\end{aligned}
$$

It is obvious that $\partial^2 V(c,l)/\partial l^2 \geq 0$. Now we only need to check that $\partial^2 V(c,l)/\partial c^2 \geq 0$ when $0 \leq \eta \leq \sqrt{D/\beta} - r$. We have

$$\begin{aligned}
\frac{\partial^2 V(c,l)}{\partial c^2} &= 2q\frac{(\sqrt{D}+\sqrt{\beta}l)^2 - \beta\eta l}{(r+l+c)^3} \\
&\geq 2q\frac{(\sqrt{D}+\sqrt{\beta}l)^2 - \beta(\sqrt{D/\beta}-r)l}{(r+l+c)^3} \\
&= 2q\frac{D+2\sqrt{D\beta}l+\beta l^2 - \sqrt{D\beta}l + r\beta l}{(r+l+c)^3} \\
&= 2q\frac{D+(\sqrt{D\beta}+r\beta)l+\beta l^2}{(r+l+c)^3} \\
&\geq 0.
\end{aligned}$$

Hence we have shown that $V(c,l)$ is convex whenever $0 \leq \eta \leq \sqrt{D/\beta} - r$.

Item E

In this Item we show how to obtain q_{lc} expressed in equation (13). Plugging $l^* = (\sqrt{q/q_l} - 1)r$ and $c^* = 0$ into inequality (10) we obtain

$$\begin{aligned}
& \beta r^2 \geq q_l[D + (2\sqrt{D\beta} - \beta\eta)l^* + \beta l^{*2}] \\
\Rightarrow \quad & \beta(\sqrt{\frac{q}{q_l}}r - r)^2 + (2\sqrt{D\beta} - \beta\eta)(\sqrt{\frac{q}{q_l}}r - r) + D - \frac{\beta r^2}{q_l} \leq 0 \\
\Rightarrow \quad & \beta y^2 + (2\sqrt{D\beta} - \beta\eta)y + D - \frac{\beta r^2}{q_l} \leq 0, \qquad \text{(SM-8)}
\end{aligned}$$

where $y \equiv (\sqrt{q/q_l} - 1)r$. Then the solutions of equation

$$\beta y^2 + (2\sqrt{D\beta} - \beta\eta)yD - \frac{\beta r^2}{q_l} = 0$$

are

$$y_1 = \frac{-(2\sqrt{D\beta}-\beta\eta) - \sqrt{(2\sqrt{D\beta}-\beta\eta)^2 - 4\beta(D-\beta r^2/q_l)}}{2\beta},$$

and

$$y_2 = \frac{-(2\sqrt{D\beta}-\beta\eta) + \sqrt{(2\sqrt{D\beta}-\beta\eta)^2 - 4\beta(D-\beta r^2/q_l)}}{2\beta}.$$

Existence of the solutions requires that

$$(2\sqrt{D\beta}-\beta\eta)^2 - 4\beta(D-\beta r^2/q_l) \geq 0,$$

from which we can get

$$\begin{aligned}
& (2\sqrt{D\beta}-\beta\eta)^2 - 4\beta(D-\beta r^2/q_l) \geq 0 \\
\Rightarrow \quad & (2\sqrt{D\beta}-\beta\eta)^2 - 4\beta D + 4\frac{\beta^2}{\alpha}((\sqrt{D}-\sqrt{\beta}r)^2 + \beta\eta r) \geq 0 \\
\Rightarrow \quad & 4\frac{\beta^2}{\alpha}[(\sqrt{D}-\sqrt{\beta}r)^2 + \beta\eta r] \geq 4\beta\eta\sqrt{D\beta} - \beta^2\eta^2 \\
\Rightarrow \quad & \alpha \leq \frac{4\beta^2[(\sqrt{D}-\sqrt{\beta}r)^2 + \beta\eta r]}{4\beta\eta\sqrt{D\beta} - \beta^2\eta^2} \\
\Rightarrow \quad & \alpha \leq \frac{\beta[(\sqrt{D}-\sqrt{\beta}r)^2 + \beta\eta r]}{D(\eta\sqrt{\beta/D} - \frac{1}{4}\eta^2\beta/D)} \\
\Rightarrow \quad & \alpha \leq \frac{\beta[(\sqrt{D}-\sqrt{\beta}r)^2 + \beta\eta r]}{D(\frac{\eta}{\sqrt{D/\beta}})(1-\frac{1}{4}\frac{\eta}{\sqrt{D/\beta}})} \equiv \alpha_l.
\end{aligned}$$

Inequality (SM-8) requires $y \in [y_1, y_2]$. By Assumption 1 we have $-(2\sqrt{D\beta}-\beta\eta)/2\beta < 0$. Due to $q > q_l$, we must have $y_1 > 0$ or $y_2 > 0$ or both. It is easy to check $y_1 < 0$. Therefore, y_1 is not the solution we want. To guarantee $y_2 > 0$, we must have $4\beta(D - \beta r^2/q_l) < 0$, that is $\alpha < \beta((\sqrt{D}-\sqrt{\beta}r)^2 + \beta\eta r)/D \equiv \alpha_3$. By Assumption 1 we have

$\alpha_3 < \alpha_l$. Hence the existence of a positive y_1 requires $\alpha \leq \alpha_3$.

$$\begin{aligned}
& y \leq y_1 \\
\Rightarrow \quad & \sqrt{\frac{q}{q_l}}r - r \leq y_1 \\
\Rightarrow \quad & \sqrt{q} \leq \frac{\sqrt{q_l}(r+y_1)}{r} \\
\Rightarrow \quad & q \leq q_l(\frac{r+y_1}{r})^2 \\
\Rightarrow \quad & q \leq q_l \frac{\left\{r+(2\beta)^{-1}\left[(\beta\eta - 2\sqrt{D\beta}) + \sqrt{(2\sqrt{D\beta}-\beta\eta)^2 - 4\beta(D-\beta r^2/q_l)}\right]\right\}^2}{r^2} \equiv q_{lc}.
\end{aligned}$$

Item F

In this Item we show that when q_1 and q_2 exist, then we always have $q_2 > q_c$. By construction we know $q_1 \leq q_2$. If the equality in expression (14) holds then we have

$$q_2 = \left\{\frac{(2-\eta\sqrt{\beta/D}) + \sqrt{(2-\eta\sqrt{\beta/D})^2 - 4(1-\alpha/\beta)}}{2}\right\}^2.$$

Then

$$
\begin{aligned}
\sqrt{q_2} &\geq \frac{2-\eta\sqrt{\beta/D}}{2} \\
&= 1-\frac{1}{2}\eta\sqrt{\frac{\beta}{D}} \\
&\geq 1-\frac{1}{2}\sqrt{\frac{\beta}{D}}\left(\sqrt{\frac{D}{\beta}}-r\right) \qquad \text{by Assumption 1} \\
&= 1-\frac{1}{2}\left(1-r\sqrt{\frac{\beta}{D}}\right) \\
&= \frac{1}{2}+\frac{1}{2}r\sqrt{\frac{\beta}{D}} \\
&\geq r\sqrt{\frac{\beta}{D}} \qquad \text{by } r\sqrt{\beta/D}<1 \text{ from Assumption 1} \\
&= \sqrt{q_c}.
\end{aligned}
$$

Therefore $q_2 > q_c$.

Item G

In this item we show that the three possibilities for q-intervals that support Case 3, $[q_1, q_2]$, $[q_1, 1]$, and $[q_c, 1]$, require $\alpha \in [\alpha_1, \alpha_2]$, $\alpha \in [\alpha_2, \alpha_3]$, and $\alpha > \alpha_3$, respectively.

First we show that if $q_2 \leq 1$ then $q_1 \geq q_c$. Since

$$
q_1 = \left\{ \frac{(2-\eta\sqrt{\beta/D}) - \sqrt{(2-\eta\sqrt{\beta/D})^2 - 4(1-\alpha/\beta)}}{2} \right\}^2,
$$

and

$$
q_2 = \left\{ \frac{(2-\eta\sqrt{\beta/D}) + \sqrt{(2-\eta\sqrt{\beta/D})^2 - 4(1-\alpha/\beta)}}{2} \right\}^2,
$$

$q_2 \leq 1$ implies that

$$
\begin{aligned}
& \frac{(2-\eta\sqrt{\beta/D})+\sqrt{(2-\eta\sqrt{\beta/D})^2-4(1-\alpha/\beta)}}{2} \leq 1 \\
\Rightarrow \quad & -\eta\sqrt{\beta/D}+\sqrt{(2-\eta\sqrt{\beta/D})^2-4(1-\alpha/\beta)} \leq 0 \\
\Rightarrow \quad & \sqrt{(2-\eta\sqrt{\beta/D})^2-4(1-\alpha/\beta)} \leq \eta\sqrt{\beta/D}.
\end{aligned}
$$

Then

$$
\begin{aligned}
\sqrt{q_1} &= \frac{(2-\eta\sqrt{\beta/D})-\sqrt{(2-\eta\sqrt{\beta/D})^2-4(1-\alpha/\beta)}}{2} \\
&\geq \frac{(2-\eta\sqrt{\beta/D})-\eta\sqrt{\beta/D}}{2} \\
&= 1-\eta\sqrt{\frac{\beta}{D}} \\
&= \sqrt{\frac{\beta}{D}}(\sqrt{\frac{\beta}{D}}-\eta) \\
&\geq \sqrt{\frac{\beta}{D}}r \quad \text{by Assumption 1} \\
&= \sqrt{q_c}.
\end{aligned}
$$

Second we show that $q_2 \leq 1$ requires $\beta(\eta\sqrt{\beta/D}-\beta\eta^2/4D) \leq \alpha \leq \beta\eta\sqrt{\beta/D}$. Since the existence of q_2 requires $\alpha \geq \beta(\eta\sqrt{\beta/D}-\beta\eta^2/4D)$, we only need to show that $q_2 \leq 1$

requires $\alpha \leq \beta\eta\sqrt{\beta/D}$. Calculations are

$$
\begin{aligned}
& q_2 \leq 1 \\
\Rightarrow\ & \frac{(2-\eta\sqrt{\beta/D}) + \sqrt{(2-\eta\sqrt{\beta/D})^2 - 4(1-\alpha/\beta)}}{2} \leq 1 \\
\Rightarrow\ & -\eta\sqrt{\beta/D} + \sqrt{(2-\eta\sqrt{\beta/D})^2 - 4(1-\alpha/\beta)} \leq 0 \\
\Rightarrow\ & \sqrt{(2-\eta\sqrt{\beta/D})^2 - 4(1-\alpha/\beta)} \leq \eta\sqrt{\beta/D} \\
\Rightarrow\ & (2-\eta\sqrt{\beta/D})^2 - 4(1-\alpha/\beta) \leq \frac{\beta}{D}\eta^2 \\
\Rightarrow\ & 4 - 4\eta\sqrt{\beta/D} + \frac{\beta}{D}\eta^2 - 4 + 4\frac{\alpha}{\beta} \leq \frac{\beta}{D}\eta^2 \\
\Rightarrow\ & \frac{\alpha}{\beta} \leq \eta\sqrt{\beta/D} \\
\Rightarrow\ & \alpha \leq \beta\eta\sqrt{\beta/D}.
\end{aligned}
$$

Now we show that conditions $q_1 \geq q_c$ and $q_2 > 1$ require that $\beta\eta\sqrt{\beta/D} \leq \alpha \leq \beta[(\sqrt{D} - \sqrt{\beta}r)^2 + r\eta\beta]/D$. Since we have shown that $q_2 > 1$ implies $\alpha \geq \beta\eta\sqrt{\beta/D}$, we only need to show $q_1 \geq q_c$ implies $\alpha \leq \beta[(\sqrt{D} - \sqrt{\beta}r)^2 + r\eta\beta]/D$.

$$
\begin{aligned}
& q_1 \geq q_c \\
\Rightarrow\ & \frac{(2-\eta\sqrt{\beta/D}) - \sqrt{(2-\eta\sqrt{\beta/D})^2 - 4(1-\alpha/\beta)}}{2} \geq r\sqrt{\beta/D} \\
\Rightarrow\ & \sqrt{(2-\eta\sqrt{\beta/D})^2 - 4(1-\alpha/\beta)} \leq (2-\eta\sqrt{\beta/D}) - 2r\sqrt{\beta/D} \\
\Rightarrow\ & (2-\eta\sqrt{\beta/D})^2 - 4(1-\alpha/\beta) \leq (2-\eta\sqrt{\beta/D})^2 - 4r\sqrt{\beta/D}(2-\eta\sqrt{\beta/D}) + 4r^2\frac{\beta}{D} \\
\Rightarrow\ & -4 + 4\alpha/\beta \leq -4r\sqrt{\beta/D}(2-\eta\sqrt{\beta/D}) + 4r^2\frac{\beta}{D} \\
\Rightarrow\ & \frac{\alpha}{\beta} \leq 1 - 2r\sqrt{\beta/D} + r^2\beta/D + \beta r\eta/D \\
\Rightarrow\ & \alpha \leq \frac{\beta}{D}[(\sqrt{D} - \sqrt{\beta}r)^2 + r\eta\beta].
\end{aligned}
$$

Item H

In this Item we prove Remark 3.

Proof. Suppose $\eta = 0$. From FOC (9) we have

$$\begin{aligned} q &= \beta\left\{\frac{r+c^*+l^*}{\sqrt{D}+\sqrt{\beta}(l^*)}\right\}^2 \\ &> \beta\left\{\frac{r+l^*}{\sqrt{D}+\sqrt{\beta}(l^*)}\right\}^2 \qquad \text{by } c^*>0 \\ &= \left\{\frac{\sqrt{D}+\sqrt{\beta}(l^*)-(\sqrt{D}-\sqrt{\beta}r)}{\sqrt{D}+\sqrt{\beta}(l^*)}\right\}^2 \\ &= \left\{1-\frac{\sqrt{D}-\sqrt{\beta}r}{\sqrt{D}+\sqrt{\beta}(l^*)}\right\}^2 \\ &> \left\{1-\frac{\sqrt{D}-\sqrt{\beta}r}{\sqrt{D}}\right\}^2 \qquad \text{by } l^*>0 \\ &= q_c. \end{aligned}$$

From FOC (10) we have

$$\begin{aligned} q &= \alpha\left\{\frac{r+c^*+l^*}{\sqrt{D}-\sqrt{\beta}(r+c^*)}\right\}^2 \\ &> \alpha\left\{\frac{r+c^*}{\sqrt{D}-\sqrt{\beta}(r+c^*)}\right\}^2 \qquad \text{by } l^*>0 \\ &> \alpha\left\{\frac{r}{\sqrt{D}-\sqrt{\beta}r}\right\}^2 \qquad \text{by } c^*>0 \text{ and } \sqrt{D}-\sqrt{\beta}(r+c^*)>0 \\ &= q_l. \end{aligned}$$

Therefore we have $q > \max\{q_c, q_l\}$. Next we show that the existence of a solution (c^*, l^*) such that FOCs for Case 4 requires $q = q_{lc}$. It is readily checked that when $\eta = 0$ then $q_{lc} = (1-\sqrt{\alpha/\beta})^2$.

Define $X \equiv l^*$ and $Y \equiv r + c^*$. Then the FOCs for Case 4 become

$$\sqrt{\frac{\beta}{q}} = \frac{\sqrt{D}+\sqrt{\beta}X}{X+Y}, \tag{SM-9}$$

$$\sqrt{\frac{\alpha}{q}} = \frac{\sqrt{D}-\sqrt{\beta}Y}{X+Y}, \tag{SM-10}$$

where $X > 0$ and $Y > r$. From equation (SM-9) and (SM-10) we have

$$Y = \sqrt{\frac{qD}{\beta}} - (1-\sqrt{q})X \tag{SM-11}$$

$$Y = \frac{\sqrt{D}}{\sqrt{\alpha/q}+\sqrt{\beta}} - \frac{\sqrt{\alpha/q}}{\sqrt{\alpha/q}+\sqrt{\beta}}X, \tag{SM-12}$$

where $X > 0$ and $Y > r$. When $q = q_{lc}$, then

$$\begin{aligned} & \sqrt{q} = 1-\sqrt{\alpha/\beta} \\ \Rightarrow \quad & \sqrt{q}+\sqrt{\alpha/\beta} = 1 \\ \Rightarrow \quad & \sqrt{q\beta}+\sqrt{\alpha} = \sqrt{\beta} \\ \Rightarrow \quad & \sqrt{\beta}+\sqrt{\alpha/q} = \sqrt{\beta/q}. \end{aligned}$$

Therefore, equation (SM-12) becomes

$$\begin{aligned} Y &= \frac{\sqrt{D}}{\sqrt{\beta/q}} - \frac{\alpha/q}{\sqrt{\beta/q}}X \\ &= \sqrt{\frac{qD}{\beta}} - \sqrt{\frac{\alpha}{\beta}}X \\ &= \sqrt{\frac{qD}{\beta}} - (1-\sqrt{q})X \qquad \text{by } \sqrt{q} = 1-\sqrt{\alpha/\beta}. \end{aligned}$$

We can see that whenever $q = q_{lc}$ then equations (SM-9) and (SM-10) are identical. So any solution (c^*, l^*) such that equation (SM-9) must be such that equation (SM-10). Next we are going to show that whenever $q \neq q_{lc}$ then there is no solution (c^*, l^*) satisfying equations (SM-9) and (SM-10).

First, we consider the situation in which $q > q_{lc}$. If $q > q_{lc}$, then $\sqrt{\beta} + \sqrt{\alpha/q} > \sqrt{\beta/q}$. This is because when $\eta = 0$ then $q_{lc} = (1 - \sqrt{\alpha/\beta})^2$. Therefore, $\sqrt{D}/(\sqrt{\alpha/q} + \sqrt{\beta}) < \sqrt{qD/\beta}$. This means that the Y-intercept of function (SM-11) is smaller than the Y-intercept of function (SM-12). If we can show that the absolute value of the slope of the curve of function (SM-11) is greater than the slope of the curve of function (SM-12) then we can conclude that the two curves have no intersection point in the first quadrant. The following algebra shows this to be true.

$$
\begin{aligned}
& \frac{\sqrt{\alpha/q}}{\sqrt{\alpha/q} + \sqrt{\beta}} > 1 - \sqrt{q} \\
\Leftrightarrow \quad & \sqrt{\alpha/q} > (1 - \sqrt{q})(\sqrt{\frac{\alpha}{q}} + \sqrt{\beta}) \\
\Leftrightarrow \quad & \sqrt{\alpha/q} > \sqrt{\frac{\alpha}{q}} + \sqrt{\beta} - \sqrt{\alpha} - \sqrt{q\beta} \\
\Leftrightarrow \quad & 0 > \sqrt{\beta} - \sqrt{\alpha} - \sqrt{q\beta} \\
\Leftrightarrow \quad & 0 > 1 - \sqrt{\alpha/\beta} - \sqrt{q} \\
\Leftrightarrow \quad & \sqrt{q} > 1 - \sqrt{\alpha/\beta} \\
\Leftrightarrow \quad & q > q_{lc}.
\end{aligned}
$$

By the same procedure we can show that if $q > q_{lc}$ then there is no solution (c^*, l^*) satisfying equations (SM-9) and (SM-10) either. This concludes the proof. $\square$

Item I

In this Item we prove Remark 4.

Item (i): if $\alpha \in (0, \alpha_1)$ then $q_l < q_c < q_{lc}$.

Proof. Once we show that whenever $\alpha \in (0, \alpha_3)$ then $q_l < q_c < q_{lc}$, the conclusion in item (i) follows naturally. We first show that whenever $\alpha < \alpha_3$ then $q_l < q_c$. That is,

$$
\begin{aligned}
& q_l < q_c \\
\Leftrightarrow \quad & \frac{\alpha r^2}{(\sqrt{D} - \sqrt{\beta} r)^2 + \beta \eta r} < \frac{\beta r^2}{D} \\
\Leftrightarrow \quad & \alpha < \frac{\beta}{D}[(\sqrt{D} - \sqrt{\beta} r)^2 + \beta \eta r] = \alpha_3.
\end{aligned}
$$

It is easy to check that equality holds when $\alpha = \alpha_3$.

Then we show that if $\alpha < \alpha_3$ then $q_c < q_{lc}$. To show this we first show that whenever $\alpha = \alpha_3$ then $q_c = q_{lc}$. Then we are going to show that if $\alpha < \alpha_3$ then $\partial q_{lc}/\partial \alpha < 0$. Since $\partial q_c / \partial \alpha = 0$ then the result follows.

We have shown that if $\alpha = \alpha_3$, then $q_l = q_c$. Therefore, when $\alpha = \alpha_3$ then we have

$$
\begin{aligned}
q_{lc} &= q_c \frac{\left\{ r + (2\beta)^{-1} \left[(\beta\eta - 2\sqrt{D\beta}) + \sqrt{(2\sqrt{D\beta} - \beta\eta)^2 - 4\beta(D - \beta r^2/q_c)} \right] \right\}^2}{r^2} \\
&= q_c \frac{\left\{ r + (2\beta)^{-1} \left[(\beta\eta - 2\sqrt{D\beta}) + \sqrt{(2\sqrt{D\beta} - \beta\eta)^2 - 4\beta(D - \frac{\beta r^2}{\beta r^2/D})} \right] \right\}^2}{r^2} \\
&= q_c.
\end{aligned}
$$

Plugging $q_l\alpha r^2/[(\sqrt{D}-\sqrt{\beta}r)^2+\beta\eta r]$ into equation (13) we have

$$
\begin{aligned}
& q_{lc} \\
= \;& \frac{\alpha}{(\sqrt{D}-\sqrt{\beta}r)^2+\beta\eta r}\Bigg\{r+ \\
& \frac{(\beta\eta-2\sqrt{D\beta})+\sqrt{\beta^2\eta^2-4\beta\eta\sqrt{D\beta}+\frac{4\beta^2}{\alpha}((\sqrt{D}-\sqrt{\beta}r)^2+\beta\eta r)}}{2\beta}\Bigg\}^2 \\
= \;& \frac{1}{(\sqrt{D}-\sqrt{\beta}r)^2+\beta\eta r}\Bigg\{(r+\frac{\eta}{2}-\sqrt{\frac{D}{\beta}})\sqrt{\alpha}+\sqrt{\alpha}\sqrt{\frac{\eta^2}{4}-\eta\sqrt{\frac{D}{\beta}}+\frac{4}{\alpha}((\sqrt{D}-\sqrt{\beta}r)^2+\beta\eta r)}\Bigg\}^2 \\
= \;& \frac{1}{(\sqrt{D}-\sqrt{\beta}r)^2+\beta\eta r}\Bigg\{(r+\frac{\eta}{2}-\sqrt{\frac{D}{\beta}})\sqrt{\alpha}+\sqrt{(\frac{\eta^2}{4}-\eta\sqrt{\frac{D}{\beta}})\alpha+4((\sqrt{D}-\sqrt{\beta}r)^2+\beta\eta r)}\Bigg\}^2.
\end{aligned}
$$

Define $\Phi \equiv (r+\frac{\eta}{2}-\sqrt{\frac{D}{\beta}})\sqrt{\alpha}+\sqrt{(\frac{\eta^2}{4}-\eta\sqrt{\frac{D}{\beta}})\alpha+4((\sqrt{D}-\sqrt{\beta}r)^2+\beta\eta r)}$. Therefore, we have

$$\frac{\partial q_{lc}}{\partial\alpha}=\frac{2\Phi\partial\Phi/\partial\alpha}{(\sqrt{D}-\sqrt{\beta}r)^2+\beta\eta r}.$$

We can check that $\Phi\geq 0$ when $\alpha\leq\alpha_3$. Therefore, $sign(\partial q_{lc}/\partial\alpha)=sign(\partial\Phi/\partial\alpha)$. Next we need to show $\partial\Phi/\partial\alpha<0$.

$$\frac{\partial\Phi}{\partial\alpha} \;=\; (r+\frac{\eta}{2}-\sqrt{\frac{D}{\beta}})\frac{1}{2\sqrt{\alpha}}+\frac{\frac{\eta^2}{4}-\eta\sqrt{\frac{D}{\beta}}}{2\sqrt{(\frac{\eta^2}{4}-\eta\sqrt{\frac{D}{\beta}})\alpha+4((\sqrt{D}-\sqrt{\beta}r)^2+\beta\eta r)}}.$$

By Assumption 1 we have $r+\eta/2-\sqrt{D/\beta}<0$ and $\eta^2/4-\eta\sqrt{D/\beta}<0$. Therefore $\partial\Phi/\partial\alpha<0$ when $\alpha<\alpha_3$. This finishes proving item (*i*) of Remark 4. □

Item (*ii*): if $\alpha\in[\alpha_1,\alpha_2)$, then $q_l\leq q_c\leq q_{lc}\leq q_1\leq q_2\leq 1$.

Proof. We first show that whenever $\alpha \in [\alpha_1, \alpha_3]$ then $q_1 \geq q_{lc}$. Recall that q_1 exists only when $\alpha \geq \alpha_1$ and q_{lc} exists only when $\alpha \leq \alpha_3$. In Item G we show that $q_1 \geq q_c$ is equivalent to $\alpha \leq \alpha_3$ and equality holds when $\alpha = \alpha_3$. From the proof of item (*i*) in Remark 4 we know that when $\alpha \leq \alpha_3$ then $q_c \geq q_l$. Therefore, we know that when $\alpha \leq \alpha_3$ then $q_1 \geq q_l$. From Item G we know that when $\alpha \in [\alpha_1, \alpha_3]$ then the lower bound of the q-interval that supports Case 3 is q_1. We know that the upper bound of the q-interval that supports Case 2 is q_{lc}. Were $q_{lc} > q_1$ when $\alpha \in [\alpha_1, \alpha_3]$, then there must be a q-interval that supports both Case 2 and Case 3, i.e., there must be a q-interval such that for any q in this interval we have $c^* > 0$ and $c^* = 0$ given all other parameters. This is impossible. Therefore, whenever $\alpha \in [\alpha_1, \alpha_3]$ then $q_1 \geq q_{lc}$.

In SM Item G we have shown that when $\alpha \leq \alpha_2$ then $q_2 \leq 1$. By construction we have $q_1 \leq q_2$. In the proof of item (*i*) in Remark 4 we have shown that whenever $\alpha \in (0, \alpha_3)$ then $q_l < q_c < q_{lc}$. Therefore, item (*ii*) in Remark 4 is proved. □

Once items (*i*) and (*ii*) are proved, it is straightforward to show that the remainder of Remark 4 is true.

Item J

In this item we show how to obtain the results in Table 1. The derivation of signs of $\partial l^*/\partial j$ (Case 2), $\partial c^*/\partial j$ (Case 3), $\partial q_c/\partial j$, and $\partial q_l/\partial j$ is trivial, where j stands for parameters. Here we only show the signs of $\partial q_1/\partial j$ and $\partial q_2/\partial j$. We identify signs of $\partial q_1/\partial j$ and $\partial q_2/\partial j$ by studying signs of $\partial \sqrt{q_1}/\partial j$ and $\partial \sqrt{q_2}/\partial j$. We know that

$$\sqrt{q_1} = \frac{(2 - \eta\sqrt{\beta/D}) - \sqrt{(2 - \eta\sqrt{\beta/D})^2 - 4(1 - \alpha/\beta)}}{2},$$

and

$$\sqrt{q_2} = \frac{(2-\eta\sqrt{\beta/D}) + \sqrt{(2-\eta\sqrt{\beta/D})^2 - 4(1-\alpha/\beta)}}{2}.$$

Here we only study the signs of $\partial\sqrt{q_1}/\partial j$. A similar procedure applies when studying the signs of $\partial\sqrt{q_2}/\partial j$. We define $X \equiv 2-\eta\sqrt{\beta/D}$. Then $\sqrt{q_1}$ becomes

$$\sqrt{q_1} = \frac{X - \sqrt{X^2 - 4(1-\frac{\alpha}{\beta})}}{2}.$$

Therefore,

$$\frac{\partial\sqrt{q_1}}{\partial D} = \frac{1 - X(X^2-4(1-\frac{\alpha}{\beta}))^{-\frac{1}{2}}}{2}\frac{\partial X}{\partial D},$$

and

$$\frac{\partial\sqrt{q_1}}{\partial \eta} = \frac{1 - X(X^2-4(1-\frac{\alpha}{\beta}))^{-\frac{1}{2}}}{2}\frac{\partial X}{\partial \eta}.$$

When $\alpha < \alpha_3$ then $X(X^2-4(1-\frac{\alpha}{\beta}))^{-\frac{1}{2}} > 1$. This is because when $\alpha < \alpha_3$ then $\alpha < \beta$. It is readily checked that $\partial X/\partial\eta < 0$ and $\partial X/\partial D > 0$. Therefore, we have $\partial\sqrt{q_1}/\partial\eta > 0$ and $\partial\sqrt{q_1}/\partial D < 0$.

Regarding $\partial\sqrt{q_1}/\partial\beta$, we have

$$\begin{aligned}
\frac{\partial\sqrt{q_1}}{\partial\beta} &= \frac{1}{2}\left[-\frac{\eta}{2\sqrt{D\beta}} - \frac{-\eta(2-\eta\sqrt{\beta/D})/\sqrt{D\beta} - 4\alpha/\beta^2}{2\sqrt{(2-\eta\sqrt{\beta/D})^2 - 4(1-\alpha/\beta)}}\right] \\
&= -\frac{1}{4}\left[\frac{\eta}{\sqrt{D\beta}} - \frac{-\eta(2-\eta\sqrt{\beta/D})/\sqrt{D\beta} - 4\alpha/\beta^2}{\sqrt{(2-\eta\sqrt{\beta/D})^2 - 4(1-\alpha/\beta)}}\right] \\
&= -\frac{\eta}{4\sqrt{D\beta}}\left[1 - \frac{(2-\eta\sqrt{\beta/D}) + 4\alpha\sqrt{D\beta}/(\eta\beta^2)}{\sqrt{(2-\eta\sqrt{\beta/D})^2 - 4(1-\alpha/\beta)}}\right].
\end{aligned}$$

Since $\alpha < \alpha_3 < \beta$, we have

$$1 - \frac{(2 - \eta\sqrt{\beta/D}) + 4\alpha\sqrt{D\beta}/(\eta\beta^2)}{\sqrt{(2 - \eta\sqrt{\beta/D})^2 - 4(1 - \alpha/\beta)}} < 0.$$

Therefore, $\partial(\sqrt{q_1})/\partial\beta > 0$ whenever $\alpha < \alpha_3$.