FACTOR AUGMENTED AUTOREGRESSIVE DISTRIBUTED LAG MODELS

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Preliminary, Comments Welcome

Abstract

This paper proposes a factor augmented autoregressive distributed lag (FADL) framework for analyzing the dynamic effects of common and idiosyncratic shocks. We first estimate the common shocks from a large panel of data with a strong factor structure. Impulse responses are then obtained from an autoregression, augmented with a distributed lag of the estimated common shocks. The approach has three distinctive features. First, identification restrictions, especially those based on recursive or block recursive ordering, are very easy to impose. Second, the dynamic response to the common shocks can be constructed for variables not necessarily in the panel. Third, the restrictions imposed by the factor model can be tested. The relation to other identification schemes used in the FAVAR literature is discussed. The methodology is used to study the effects of monetary policy and news shocks.

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1 Introduction

This paper proposes a new approach for analyzing the dynamic effects of q common shocks such as due to monetary policy and technology on q or more observables. We assume that a large panel of data $X^{ALL} = (X, X^{OTH})$ is available and use the sub-panel X that is likely to have a strong factor structure to estimate the common shocks. Identification is based on restrictions on a q dimensional subset of X. The impulse response coefficients are obtained from an autoregression in each variable of interest augmented with current and lagged values of the identified common shocks. Observed factors can coexist with latent factors. We refer to this approach as Factor Augmented Autoregressive Distributed Lag (FADL).

An important feature of the FADL is that it estimates the impulse responses using minimal restrictions from the factor model. The approach has several advantages. First, while X is large in dimension, identification is based on a subset of variables whose dimension is the number of common shocks. This reduces the impact of invalid restrictions on variables that are not of direct interest. Second, the impulse responses are the coefficients estimated from a regression with common shocks as predictors. Restrictions are easy to impose, and for many problems the impulse responses can be estimated on an equation by equation basis. Third, the analysis only requires a strong factor structure to hold in X and is less likely to be affected by the possibility of weak factors in X^{OTH}.

The proposed FADL methodology lets the data speak whenever possible and is in the spirit of vector-autoregressions (VAR) proposed by Sims (1980). The FADL also shares some similarities with the Factor Augmented Vector Autoregressions (FAVAR) considered in Bernanke and Boivin (2003). Their FAVAR expands the econometrician's information set without significantly increasing the dimension of the system. Our FADL further simplifies the analysis by imposing restrictions only on the variables of interest. Recursive and non-recursive restrictions can be easily implemented.

The FADL is derived from a structural dynamic factor model which has a restricted FAVAR as its reduced form. A factor model imposes specific assumptions on the covariance structure of the data. Even though many variables are available for analysis, a factor structure may not be appropriate for every series. As noted in Boivin and Ng (2006), more data may not be beneficial for factor analysis if the additional data are noisy and/or do not satisfy the restrictions of the factor model. We treat X like a training sample. Using it to estimate the common shocks enables us to validate the factor structure in X^{OTH}, the series not in X.

The FADL approach stands in contrast to structural FAVARs that impose all restrictions of a dynamic factor model in estimation, as Forni, Giannone, Lippi, and Reichlin (2009). The FADL estimates will necessarily be less efficient if the restrictions are correct, but are more robust when the restrictions do not hold universally. As in Stock and Watson (2005), our FADL also permits

implications of the factor model to be tested. However, we go one-step further by letting the data determine the Wold representation instead of inverting a large FAVAR.

The paper proceeds as follows. Section 2 first sets up the problem of identifying the effects of common shocks from the perspective of a dynamic factor model. It then presents the FADL framework without observed factors. Estimation and identification of a FADL is discussed in Section 3. Relation of FADL to alternative structural dynamic factor analysis is discussed in Section 4, and FADL is extended to allow for observed factors. Simulations are presented in Section 5. Section 6 considers the identification of monetary and news shocks. Both examples highlight the two main features of FADL:- the ability to perform impulse responses analysis and to test the validity of the factor structure of variables not used in estimation or identification of the common shocks.

2 Dynamic Factor Models and the FADL Framework

Let N be the number of cross-section units and T be the number of time series observations where N and T are both large. We observe data $X^{ALL} = (X,\ X^{OTH})$ which are stationary or have been transformed to be covariance stationary. It is assumed that $X_t = (X_{1t}, \dots, X_{Nt})'$ has a (strong) factor representation and can be decomposed into a common and an idiosyncratic component:

$$X_t = \lambda(L) f_t + u_{Xt} \tag{1}$$

where $f_t = (f_{1t}, \dots, f_{qt})'$ is a vector of q common factors and $\lambda(L) = \lambda_0 + \lambda_0 L + \dots \lambda_s L^s$ is a polynomial matrix of factor loadings in which the $N \times q$ matrix $\lambda_j = (\lambda_{j1} \dots, \lambda_{jN})'$ quantifies the effect of the common factors at lag j on X_t. The series-specific errors $u_{Xt} = (u_{X1t}, \dots, u_{XNt})'$ are mutually uncorrelated but can be serially correlated. We assume

$$(I_N - D(L)L) u_{Xt} = v_{Xt} \tag{2}$$

where v_{Xt} is a vector white noise process. The q latent dynamic factors are assumed to be a vector autoregressive process of order h. Without loss of generality, we assume $h = 1$ and thus

$$f_t = \Gamma_1 f_{t-1} + \Gamma_0 v_{ft} \tag{3}$$

where the characteristic roots of Γ_1 are strictly less than one. The $q \times 1$ vector v_{ft} consists of structural common shocks (such as monetary policy or technology). These structural shocks can affect several dynamic factors simultaneously. Hence, the $q \times q$ matrix Γ_0 need not be an identity. By assumption, $E(v_{Xit} v_{Xjt}) = 0$ and $E(v_{Xit} v_{fkt}) = 0$ for all $i \neq j$ and for all $i = 1, \dots N$ and $k = 1, \dots q$.

Assuming that $I - D(L)L$ is invertible, the vector-moving average representation of X_t in terms of the structural common and idiosyncratic shocks is

$$X_t = \Psi^f(L)v_{ft} + \Psi^X(L)v_{Xt}.$$

The structural impulse response coefficients Ψ_j^X and Ψ_j^f are defined from

$$\begin{aligned}\Psi^X(L) &= \sum_{j=0}^{\infty} \Psi_j^X L^j = (I - D(L)L)^{-1} \\ \Psi^f(L) &= \sum_{j=0}^{\infty} \Psi_j^f L^j = (I - D(L)L)^{-1}\lambda(L)(I - \Gamma_1 L)^{-1}\Gamma_0.\end{aligned}$$

For each $j \geq 0$, Ψ_j^f is a $N \times q$ matrix summarizing the effect of a unit increase in v_{ft} after j periods. We use $\Psi^f_{j,i_1:i_2,k_1:k2}$ to denote the submatrix in the i_1 to i_2 rows and k_1 to k_2 columns of Ψ_j^f. When $i_1 = i_2 = i$ and $k_1 = k_2 = k$, we use $\psi^f_{j,i,k}$ to denote the effect of shock k in period t on series i in period $t + j$.

The objective of the exercise is to uncover the dynamic effects (or the impulse response) of the structural common shocks v_{ft} on variables of interest. By using $X_{1t}, \ldots, X_{Nt}$ for factor analysis, the econometrician's information set is of dimension N. Forni, Giannone, Lippi, and Reichlin (2009) argue that non-fundamentalness is generic of small scale models but cannot arise in a large dimensional dynamic factor model. The reason is that $\Psi^f(z)$ is a rectangular rather than a square matrix and its rank is less than q for some z only if all $q \times q$ sub-matrices of $\Psi^f(z)$ are singular, which is highly unlikely. Assuming that N is large ensures that the common shocks are fundamental for X.

However, even if N is large, nothing distinguishes one common shock from another. In a VAR analysis with q endogenous variables and q shocks, $q(q-1)/2$ restrictions will be necessary. A popular approach is to impose contemporaneous exclusion restrictions such that a rank condition is satisfied, see, eg. Deistler (1976), Rubio-Ramírez, Waggoner, and Zha (2010). If the identification restrictions imply a recursive ordering, then the parameters can be identified sequentially and estimation can proceed on an equation by equation basis.

While $\Psi_0^X = I_N$ in a dynamic factor model, the contemporaneous response of X_t to common shocks v_{ft} is given by

$$\Psi_0^f = \Lambda_0\Gamma_0 = \begin{pmatrix} \lambda_{0,1,1} & \lambda_{0,1,2} & \ldots & \lambda_{0,1,q} \\ \vdots & & & \\ & \vdots & & \\ \lambda_{0,q,1} & \lambda_{0,q,2} & \ldots & \lambda_{0,q,q} \\ \vdots & & \vdots & \\ \lambda_{0,N,1} & \lambda_{0,N,2} & \ldots & \lambda_{0,N,q} \end{pmatrix} \begin{pmatrix} \Gamma_{0,1,1} & \ldots & \Gamma_{0,1,q} \\ \vdots & & \\ \Gamma_{0,q,1} & \ldots & \Gamma_{0,q,q} \end{pmatrix}.$$

The (i,k) entry of Λ_0 is the contemporaneous effect of factor k on series i, and the (k,j) entry of Γ_0 is the effect of the j-th common shock on factor k. In general, Ψ_0^f will not be an identity matrix.

Two additional problems make the identification problem non-standard. First, while having more total shocks than endogenous variables should facilitate identification, the common shocks also restrict the co-movements across series. Imposing constraints on an isolated number of series is actually quite difficult within the factor framework. Zero restrictions on the entries of Λ_0 or Γ_0 alone are not usually enough to ensure that a particular entry of Ψ_0^f takes on the desired value (often zero). Second, the dynamic factors are themselves latent. Thus, not only do we need to identify the effects of v_f, we also need to identify v_f.

Our analysis is based on the following assumptions.

Assumption 1: $E(v_{ft}) = 0, \quad E(v_{ft}v'_{ft}) = I_q.$

Assumption 2 $D(L)$ is a diagonal matrix with $\delta_i(L)$ in the i-th diagonal, ie

$$D(L) = \begin{pmatrix} \delta_1(L) & 0 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & \delta_N(L) \end{pmatrix},$$

Assumption 3: For some j, a $q \times q$ matrix of Ψ_j^f is full rank.

Assumption 1 is a normalization restriction as we cannot separate the size of the common shocks from their impact effects. Assumption 2 is a form of exclusion restriction. We assume univariate autoregressive dynamics idiosyncratic errors:

$$u_{Xit} = \delta_i(L)u_{Xit-1} + v_{Xit}.$$

This implies that dynamic correlations between any two series are due entirely to the common factors, which is the defining feature of a dynamic factor model. Diagonality of $D(L)$ in turn allows X_{it} to be characterized by an autoregressive distributed lag model

$$X_{it} = \delta_i(L)X_{it-1} + (1 - \delta_i(L))\lambda_i(L)f_t + v_{Xit} \tag{4}$$

where $\lambda_i(L) = \lambda_{0i} + \lambda_{1i}L + \dots \lambda_{si}L^s$ is the i-th row of $\lambda(L)$. A representation that is more useful for impulse response analysis is an autoregressive distributed lag in the primitive shocks v_{ft}:

$$X_{it} = \delta_i(L)X_{it-1} + \psi_i^f(L)v_{ft} + v_{Xit} \tag{5}$$

where

$$\psi_i^f(L) = (1 - \delta_i(L))\lambda_i(L)(I - \Gamma_1 L)^{-1}\Gamma_0.$$

We will henceforth refer to (5) as the FADL representation of X_{it}. Note that $\psi_i^f(L) = \sum_{j=0}^{\infty} \psi_{j,i,1:q}^f L^j$ is precisely the i-th row of $\Psi^f(L)$, with

$$\psi_{0,i,1:q}^f = \lambda_{0i}\Gamma_0 = \begin{pmatrix}\lambda_{0,i,1} & \lambda_{0,i,2} & \dots & \lambda_{0,i,q}\end{pmatrix} \begin{pmatrix}\Gamma_{0,1,1} & \dots & \Gamma_{0,1,q} \\ \vdots & & \\ \Gamma_{0,q,1} & \dots & \Gamma_{0,q,q}\end{pmatrix}.$$

The dynamic effects of the common shocks v_{ft} on X_{it} are defined by the coefficients $\psi_i^f(L)$. If v_f were observed and $N = q$, equation (5) defines a dynamic simultaneous equations system in which identification can be achieved by excluding some v_f or its lags from certain equations. For example, contemporaneous restrictions can be imposed so that the $q \times q$ matrix Ψ_0^f has rank q. As our system is tall with $N \geq q$, Assumption 3 is modified to require that a $q \times q$ submatrix of Ψ_j^f is full rank. If all restrictions are imposed on Ψ_0^f, Assumption 3 will hold if the top $q \times q$ submatrix of Ψ_0^f has rank q. However, long run and sign restrictions are also permitted.

Assumptions 1 to 3 are fairly standard. But our factors are also latent and we can only identify the space spanned by the factors and not the factors themselves. To make the procedure operational, we need to replace v_{ft} by estimates $\widehat{v}_{ft}$ which have the same properties as Assumption 1. These identification conditions will be further developed below.

3 Estimation and Identification

If there are q common shocks, we will need at least q series for identification. Without loss of generality, let Y_t be the first q series in X_t. Since each $y_t \subset Y_t$ admits a dynamic factor structure, it holds that

$$y_t \quad = \quad \alpha_{yy}(L)y_{t-1} + \alpha_{yf}(L)v_{ft} + v_{yt}. \tag{6}$$

Estimation of (6) is not possible because we do not observe v_{ft}. Our impulse response analysis is based on least squares estimation of the FADL

$$y_t \quad = \quad \alpha_{yy}(L)y_{t-1} + \alpha_{yf}(L)\widehat{v}_{ft} + v_{yt} \tag{7}$$

where a prior restrictions are be imposed on $\alpha_{yf}(L)$ for identification. We now explain how v_{ft} is estimated and how restrictions are imposed on the FADL.

Let Λ be the $N \times r$ matrix of loadings, F_t be a $r = q(s+1) \times 1$ vector of static factors, where

$$\Lambda = \begin{pmatrix}\Lambda_1 \\ \Lambda_2 \\ \vdots \\ \Lambda_N\end{pmatrix}, \qquad F_t = \begin{pmatrix} f_t \\ f_{t-1} \\ \vdots \\ f_{t-max(h,s)}\end{pmatrix}, \qquad \Phi_F = \begin{pmatrix}\Gamma_1 & \Gamma_2 & . & . & . & \Gamma_s \\ I_q & 0 & . & 0 & . & 0 \\ 0 & I_q & & 0 & . & 0 \\ . & 0 & & . & . & 0 \\ 0 & 0 & . & I_q & . & 0\end{pmatrix} \qquad \Lambda_i = \begin{pmatrix}\lambda_{i0} & \lambda_{i1} & \dots \lambda_{is}\end{pmatrix}.$$

The starting point is the static factor representation of the pre-whitened data, $x_{it} = (1-\delta_i(L)L)X_{it}$:

$$\begin{aligned} x_{it} &= \Lambda_i F_t + v_{Xit} && (8) \\ F_t &= \Phi_F F_{t-1} + \varepsilon_{Ft} && (9) \\ \varepsilon_{Ft} &= G\varepsilon_{ft}. && (10) \end{aligned}$$

The ε_{Ft} are the reduced form errors of F_t and are themselves linear combinations of the structural shocks v_{ft} and $\varepsilon_{ft} = \Gamma_0 v_{ft}$ is the vector of reduced form common shocks, see (3),. The $r \times q$ matrix G maps the structural dynamic shocks to the reduced form static shocks. Since X_t is assumed to have a strong factor structure, $\Lambda'\Lambda/N \to \Lambda > 0$ as $N \to \infty$, and the $N \times N$ matrix $\frac{1}{T}\sum_{t=1}^T x_t x_t'$ has r eigenvalues that diverge as $N, T \to \infty$ while the largest eigenvalue of the $N \times N$ covariance matrix of v_{Xt} is bounded.

From $v_{Xit} = x_{it} - \Lambda_i F_t = x_{it} - \Lambda_i(\Phi_F F_{t-1} + \varepsilon_{Ft})$, define

$$\begin{aligned} \varepsilon_{Xit} &= x_{it} - \Lambda_i \Phi_F F_{t-1} \\ &= \Lambda_i \varepsilon_{Ft} + v_{Xit}. && (11) \end{aligned}$$

As noted in Stock and Watson (2005), the rank of the $r \times 1$ vector ε_{Ft} is only q, since F_t is generated by q common shocks.[1] In other words, ε_{Xit} itself has a factor structure with common factors ε_{ft}. But ε_{ft} are themselves linear combinations of v_{ft}. Let

$$v_{ft} = H\varepsilon_{ft}.$$

The $q \times q$ matrix H maps the reduced form dynamic shocks to the structural dynamic shocks. The objective is to identify v_{ft} and to trace out its effects on the variables of interest. If there are q common shocks, $q(q-1)/2$ restrictions are necessary to identify v_{ft} via H.

Estimation proceeds in five steps.

Step E1: Estimate F_t from the full panel of data X by iterative principal components (IPC).

i Initialize $\delta_i^X(L)$ using estimates from a univariate AR(q) regression in X_{it}. Let $D(L)$ be a diagonal matrix with $\delta_i^X(L)L$ on the i-th diagonal.

ii Iterate until convergence

$$\min_{D(L),\Lambda,F} SSR = \sum_{t=1}^T \Big((I - D(L)L)X_t - \Lambda F_t\Big)'\Big((I - D(L)L)X_t - \Lambda F_t\Big).$$

[1] Bai and Ng (2007) thus suggest using the number of diverging eigenvalues in the covariance of ε_{Ft} to estimate q.

a Let $\widehat{F}_t$ be the first k principal components of xx' using the normalization that $F'F/T = I_k$, where k is the assumed number of static factors.

b Estimate $D(L)$ and Λ by regressing X_{it} on $\widehat{F}_t$ and lags of X_{it}.

The method of principal components (PC) estimates k factors as the eigenvectors corresponding to the k largest eigenvalues of $XX'/(NT)$. Under the assumption of strong factors, Bai and Ng (2006) show that the estimates are consistent for the space spanned by the true factors in the sense that $\frac{1}{T}\sum_{t=1}^{T}\left\|\widehat{F}_t - HF_t\right\|^2 = O_p(\min(N,T))$, where H is a $k \times r$ matrix of rank r. However, the idiosyncratic errors may not be white noise. Stock and Watson (2005) suggest using IPC to iteratively update $\delta_i^X(L)$, which is then used to define x_{it}. The static factors form the common component of x_{it}.

Step E2: Estimate a VAR in $\widehat{F}_t$ to obtain $\widehat{\Phi}_F$ and $\widehat{\varepsilon}_{Ft}$ and let $\widehat{\varepsilon}_{Xit} = x_{it} - \widehat{\Lambda}_i'\widehat{\Phi}_F\widehat{F}_{t-1}$, where $\widehat{\Lambda}$ and $\widehat{F}_{t-1}$, $\widehat{\Phi}_F$ are obtained from Step (E1). Amengual and Watson (2007) show that the q principal components of $\widehat{\varepsilon}_{Xt}$ can precisely estimate the space spanned by ε_{ft}.

Step E3: Identification of v_{ft}: The common shocks $\widehat{\varepsilon}_{ft}$ are unorthogonalized and, in general, are mutually correlated. We seek a matrix H such that

$$\widehat{v}_{ft} = H\,\widehat{\varepsilon}_{ft}, \tag{12}$$

and $\widehat{v}_{ft}$ is a vector of mutually uncorrelated structural common shocks. We consider two approaches. The first condition (abbreviated as RO) is lower triangularity of a $q \times q$ sub-matrix so that the shocks can be identified recursively from q equations. The second condition (abbreviated as BO) requires organizing the data into blocks using *a priori* information so that the factors estimated from each block can be given meaningful interpretation.

Assumption Recursive Ordering (RO) Method (a) is based on an assumed causal structure. Just like a VAR, this would require knowledge of which of the q variables to order first. For $j = 1:q$ consider estimating the regression:

$$y_{tj} = \alpha_{yy,j}(L)y_{t-1,j} + \sum_{k=1}^{q} a_{yf,j,k}(L)\widehat{\varepsilon}_{fkt} + v_{yt,j}$$

where $\widehat{\varepsilon}_{ft}$ are the q principal components of the N residuals $\widehat{e}_{Xt}$.

i Let $\widehat{A}_{f0}$ be the estimated contemporaneous response to the q unorthogonalized shocks $\widehat{\varepsilon}_{ft}$:

$$\widehat{A}_{f0} = \begin{pmatrix} \widehat{a}_{yf,0,1,1} & \widehat{a}_{yf,0,1,2} & \cdots & \widehat{a}_{yf,0,1,q} \\ \vdots & \vdots & \vdots & \\ \widehat{a}_{yf,0,q,1} & \widehat{a}_{yf,0,q,2} & \cdots & \widehat{a}_{yf,0,q,q} \end{pmatrix}.$$

ii Define the $q \times q$ matrix $H = [\text{chol}(\widehat{A}_{f0}\widehat{A}'_{f0})]^{-1}\widehat{A}_{f0}$. Now let

$$\begin{aligned}\widehat{v}_{ft} &= H\widehat{\varepsilon}_{ft} \\ \widehat{\alpha}_{yf,j} &= \widehat{\alpha}_{yf,j}(L)H^{-1}.\end{aligned}$$

By construction, $\widehat{v}_{ft}$ is orthonormal. The method achieves exact identification by using the causal ordering of the q variables selected for analysis.

Imposing a causal structure through the ordering of variables is the most common way to achieve identification of FAVAR. Stock and Watson (2005) also use Assumption RO to identify the primitive shocks. Their implementation differs from ours in that we apply Choleski decomposition to the FADL estimates of $\alpha_{yf}(0)$ and hence we do not impose all the restrictions of the factor model. In contrast, Stock and Watson (2005) impose restrictions implied by the FAVAR in X_t and F_t. The results are likely to be more sensitive to the choice of X_t.

Assumption Block Ordering (BO) Method (b) is useful when the data can be organized into blocks. Let $X = (X^1, X^2, \ldots X^q)$ be data organized into q blocks. To see how data blocks facilitate identification, observe that the factor estimates $\widehat{\varepsilon}^0_{ft}$ are linear combinations of $\widehat{\varepsilon}_{Xt}$. Let $\widehat{\varepsilon}^0_f = \widehat{\varepsilon}_{X,:,:}\zeta^0$ be the $T \times q$ matrix of factor estimates where for each t,

$$\widehat{\varepsilon}^0_{ft} = \begin{pmatrix} \zeta^0_{11} & \zeta^0_{12} & \cdots & \cdots & \zeta^0_{1N} \\ \zeta^0_{21} & \zeta^0_{22} & \cdots & \cdots & \zeta^0_{2N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \zeta^0_{q1} & \zeta^0_{q2} & \cdots & \cdots & \zeta^0_{qN} \end{pmatrix} \begin{pmatrix} \widehat{\varepsilon}_{X1t} \\ \widehat{\varepsilon}_{X2t} \\ \vdots \\ \widehat{\varepsilon}_{XNt} \end{pmatrix}. \tag{13}$$

Identification requires a priori information on the ζ.

i. For $b = 1, \ldots q$, let $\widehat{\varepsilon}^b_f$ be the matrix of eigenvector corresponding to the largest eigenvalues of the $n_b \times n_b$ matrix $\widehat{\varepsilon}^{b\prime}_X \widehat{\varepsilon}^b_X$.

ii. Let H be the Choleski decomposition of the $q \times q$ sample covariance of $\widehat{\varepsilon}_{ft}$. Then $\widehat{v}_{ft} = H\widehat{\varepsilon}_{ft}$.

The identification strategy can be understood as follows. From (11), we see that $\varepsilon_{Xt} = \begin{pmatrix}\varepsilon^{1\prime}_{Xt} & \varepsilon^{2\prime}_{Xt} & \ldots & \varepsilon^{q\prime}_{Xt}\end{pmatrix}'$ have ε_{ft} as common factors. Since the factors are pervasive by definition, the factors are also common to all ε^b_{Xt} for arbitrary b. Thus for each $b = 1, \ldots q$, consider a factor model for $\varepsilon^b_{Xit} = \Lambda^b_i \varepsilon^b_{ft} + v^b_{Xit}$. If ε^b_{Xit} were observed, the factors for block b can be estimated by principal components which are linear combinations of series in ε^b_{Xt}. We do not observe ε^b_{Xt}, but we have $\widehat{\varepsilon}_{Xt} = x_t - \widehat{\Lambda}\widehat{\Phi}_F\widehat{F}_{t-1}$ from Step (E2). For example, if X^1 is a $T \times N_1$ panel of employment

data, the first principal component of $\widehat{\varepsilon}_X^{1\prime}\widehat{\varepsilon}_X^1$ is a labor market factor $\widehat{\varepsilon}_{f1t}$, and if X^2 is a panel of price data, $\widehat{\varepsilon}_{f2t}$ is a price factor. Collecting the factors estimating from all blocks into $\widehat{\varepsilon}_{ft}$, we have

$$\widehat{\varepsilon}_{ft} = \begin{pmatrix} \zeta^1_{1,1:N_1} & 0 & 0 & \dots & 0 & 0 \\ 0 & \zeta^2_{1,1:N_2} & 0 & \dots & 0 & 0 \\ \vdots & 0 & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & \zeta^q_{1,1:N_q} \end{pmatrix} \begin{pmatrix} \widehat{\varepsilon}^1_{Xt} \\ \widehat{\varepsilon}^2_{Xt} \\ \vdots \\ \widehat{\varepsilon}^q_{Xt} \end{pmatrix} \tag{14}$$

Obviously, the factors are defined by assuming a structured covariance relation in the observables. The appeal is that we can now associate the q factors with the block of variables from which they are estimated. However, these factors can still be correlated across blocks. To orthogonalize them, step (ii) performs q regressions beginning with $\widehat{v}_{f1} = \widehat{\varepsilon}^1_f$. For $m = 2, \dots q$, $\widehat{v}_{fb} = M_b \widehat{\varepsilon}^b_f$ are the residuals from projecting $\widehat{\varepsilon}^b_f$ onto the space orthogonal to $\widehat{v}_{f1}, \dots, \widehat{v}_{f,b-1}$, and M_b is the corresponding projection matrix.

Bernanke, Boivin, and Eliasz (2005) treat the interest rate as an observed factor, organize the macro variables into a fast and a slow block, and estimate the one factor from the slow variables. Their identification is based on a Choleski decomposition of the residuals in the slow variables and the observed factor. Their implementation is specific to the question under investigation while our methodology is general. Our identification algorithm is generic, provided blocks of variables with meaningful interpretation can be defined.[2]

In conventional VAR models, the structural impulse responses are obtained by rotating the reduced form impulse response matrix by a matrix, say, H. The primitive shocks are then obtained by rotating the reduced form errors with the inverse of the same matrix. In our setup, identification of structural common shocks precedes estimation of the impulse responses. This allows us to impose economic restrictions on the impulse response functions without simultaneously affecting the structural shocks. As presented, H is a lower triangular matrix. However, sign, long run and other structural restrictions can be imposed.

Step E4: Construct Impulse Response Function: Estimate a q dimensional FADL by OLS with restrictions on $\alpha_{Yf}(L)$:

$$Y_t = \alpha_{YY}(L)Y_{t-1} + \alpha_{Yf}(L)\widehat{v}_{ft} + v_{yt} \tag{15}$$

where $\alpha_{YY}(L)$ is a diagonal polynomial in the L of order p^y, and α_{Yf} is of order p^f. Given interpretation of $\widehat{v}_f$ identified from Step E3, short and long-run economic restrictions on the impulse

[2] Moench and Ng (2011) construct regional factors from data organized geographically. Ludvigson and Ng (2009) study the relative importance of the factor loadings and find that factor one loads heavily on real activity series, factor two on money and credit variables, while factor three loads on price variables.

responses can be directly imposed on α_{yf}. The estimated responses of y_t to a unit increase in the common shocks $\widehat{v}_{ft}$ and idiosyncratic shocks v_{yt} are defined by

$$\widehat{\psi}_y^f(L) = \frac{\widehat{\alpha}_{yf}(L)}{1-\widehat{\alpha}_{yy}(L)L} \qquad \widehat{\psi}_y^y(L) = \frac{1}{1-\widehat{\alpha}_{yy}(L)L}.$$

Since $\widehat{\alpha}_{yy}(L)$ is a scalar rational polynomial, the impulse responses are easy to compute using the FILTER command in MATLAB. Note that by Assumption 1, the standard deviation of all common shocks are normalized to unity. The response to a unit shock is thus the same as the response to a standard deviation shock.

Step E5: Model Validation Our maintained assumptions are that F_t are pervasive amongst X_t rather than (X_t, X_t^{OTH}) and by assumption, X_t have a strong factor structure. We refer to X as a 'training sample'. This is useful because once the estimated common shocks $\widehat{v}_{ft}$ are available, they can be treated as regressors in a FADL model for z_t (scalar) not necessarily in X_t. This is because if (X_t, X_t^{OTH}) have a factor structure, the shocks v_{ft} common to X_t are also common to variables in X_t^{OTH}. If the common factors are important for $z_t \subset X_t^{OTH}$, then FADL coefficients on v_{ft} and its lags should be statistically significant.

4 Relation to the Other Methods and Allowing for Observed Factors

An important difference between our approach and existing structural FAVAR analysis is that we estimate the impulse responses directly rather than inverting a VAR. Chang and Sakata (2007). estimates the shocks as residuals from long vector autoregressions in observed variables. The authors show that their estimated impulse responses are asymptotically equivalent to the local projections method proposed by Jorda (2005). Our analysis has the additional complication that the factors are latent. Thus, we first estimate the space spanned by common factors, then estimate the space spanned by the common shocks, before finally estimating the impulse response functions.

It is useful to relate our estimate of $\Psi^f(L)$ with the conventional FAVAR approach which starts with the representation

$$\begin{pmatrix} F_t \\ X_t \end{pmatrix} = \begin{pmatrix} \Phi & 0 \\ \Lambda\Phi & D(L) \end{pmatrix} \begin{pmatrix} F_{t-1} \\ X_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{Ft} \\ \Lambda\varepsilon_{Ft} + v_{Xt} \end{pmatrix}$$

from which it follows that

$$\begin{aligned} x_t &= \Lambda\Phi L(I-\Phi L)^{-1}\varepsilon_{Ft} + \Lambda\varepsilon_{Ft} + v_{Xt} \\ &= \Big(\Lambda\Phi L(I-\Phi L)^{-1} + \Lambda\Big)\varepsilon_{Ft} + v_{Xt}. \end{aligned}$$

The dynamic effects of shocks ε_{Ft} to the static factors on (prewhitened) data are determined by

$$\Lambda\Big(\Phi L(I-\Phi L)^{-1}+I\Big)=\Lambda\sum_{j=0}^{\infty}\Phi^{i+1}L^{i+1}. \tag{16}$$

At lag j, the $N\times N$ response matrix

$$\Lambda\Phi^j=\begin{pmatrix}\Lambda_i' & \Lambda_2' & \vdots & \Lambda_N'\end{pmatrix}'\Phi^j.$$

Intuitively, the total effect of ε_{Ft} depends on X_t through F_t and hence depends on the dynamics of F_t and the importance of the factor loadings on X_t. Assuming that the reduced form shocks are related to the structural shocks via $\varepsilon_{Ft}=A_0 v_{ft}$, the response to the structural shocks estimated by a FAVAR is

$$\widehat{\Psi}^f=\widehat{\Lambda}\widehat{\Phi}^j A_0^{-1}$$

which is a product of three terms: two that are the same for all i, and one $(\widehat{\Lambda})$ that is specific to unit i's. Since $\widehat{\Lambda}_i$ is only available for any $x_{it}\in X_t$, Ψ^f can be constructed only for N series. This is a consequence of the fact that the FAVAR estimates the impulses without directly estimating v_{ft}. Since we estimate v_{ft}, we can construct impulse responses for series not in X.

In contrast, our estimator of Ψ^f is $\widehat{\Lambda_i'\Phi^j A}_0$, which may not equal $\widehat{\Lambda}_i'\widehat{\Phi^j}A_0$, because we do not fully impose restrictions of the dynamic factor model on the static factor representation. Instead of a large FAVAR system, we estimate the FADL one variable at a time. Cross parameter restrictions between $\alpha_{yf}(L)$ and $\alpha_{yy}(L)$ are also not imposed. As is usually the case, system estimation is more efficient if the restrictions are true. However, misspecification in one equation can adversely affect the estimates of all equations. This possibility increases with N. The single equation FADL estimates are more robust to misspecification than those that rely on a large number of overidentifying restrictions which are often imposed on variables that are not of primary interest, or whose factor structure may not be strong.

Finally, restrictions on Γ_0 and Λ_0 alone may not be enough for identification. Consequently, it is not always easy to directly define A_0. FAVARs typically require several auxiliary regressions to determine A_0. In addition to incurring sampling variations at each step, the identification procedure requires tricks that are problem specific. In a FADL setting, the restrictions are directly imposed when the FADL is estimated. It is more straightforward, as will be illustrated in Sections 6 and 7.

4.1 Extension to m Observed Factors

Some economic analysis involves identification of shocks to observed variables in the presence of latent shocks. For example, Bernanke, Boivin, and Eliasz (2005), Stock and Watson (2005) and Forni and Gambetti (2010) consider identification of monetary policy shocks in the presence of other

shocks, using the information that some variables have instantaneous, while others have delayed response to shocks to the observed factor, being the Fed Funds Rate. These studies, summarized in Appendix A, impose restrictions of the factor models on all series. Our proposed FADL approach imposes significantly fewer restrictions on the factor model.

To extend the dynamic factor model to allow for m observed common factors W_t, let

$$\begin{aligned} X_t &= \lambda^f(L)f_t + \lambda^w(L)w_t + u_{Xt} \\ u_{Xt} &= D(L)u_{Xt-1} + v_{Xt} \\ \begin{pmatrix} f_t \\ w_t \end{pmatrix} &= \begin{pmatrix} \Gamma_{1,ff} & \Gamma_{1,fw} \\ \Gamma_{1,wf} & \Gamma_{1,ww} \end{pmatrix} \begin{pmatrix} f_{t-1} \\ w_{t-1} \end{pmatrix} + \begin{pmatrix} \Gamma_{0,ff} & \Gamma_{0,fw} \\ \Gamma_{0,wf} & \Gamma_{0,ww} \end{pmatrix} \begin{pmatrix} v_{ft} \\ v_{wt} \end{pmatrix} \end{aligned}$$

with $\Gamma_{1,fw} \neq 0$ and $\Gamma_{0,wf} \neq 0$. Without these assumptions, w_t is weakly exogenous and can be excluded from the analysis. Let W_t be a vector consisting of w_t and its lags. Assume that its dynamics can be represented by a VAR(1):

$$W_t = \Phi_W W_{t-1} + \varepsilon_{Wt}. \tag{17}$$

The reduced form model is

$$\begin{aligned} X_t &= (I - D(L)L)^{-1} \begin{pmatrix} \lambda^f(L) & \lambda^w(L) \end{pmatrix} (I - \Gamma_1)^{-1}\Gamma_0 \begin{pmatrix} v_{ft} \\ v_{wt} \end{pmatrix} + (I - D(L)L)^{-1})v_{Xt} \\ &= \Psi^f(L)v_{ft} + \Psi^w(L)v_{wt} + \Psi^X(L)v_{Xt}. \end{aligned}$$

The static factors are estimated from prewhitened data that also nets out the effects of the observed factors, and the construction of the structural shocks v_{ft} must take into account that the reduced form innovations to the static factors can be correlated with the innovations to the reduced form representation of the observed factors. Let $x_{it} = X_{it} - \delta_i X_{it-1}$ and define

$$x_{it} = \Lambda'_{iF}F_t + \lambda'_{Wi}W_t + \varepsilon_{it}$$

where $W_t = (w'_t \; w'_{t-1}, \; \dots w_{t-p})'$. The steps can be summarized as follows.

Step W1: Estimate F_t conditional on W_t by iterating until convergence

$$\min_{D(L),\Lambda,F} = \sum_{t=1}^{T} \Big((I - D(L)L)X_t - \Lambda_F F_t - \Lambda_F W_t \Big)' \Big((I - D(L)L)X_t - \Lambda_F F_t - \Lambda_W W_t \Big).$$

i Let $\widehat{F}_t$ be the k principal components of xx' using the normalization that $F'F/T = I_k$.

ii Estimate $D(L)$, Λ_F and Λ_W by regressing X_{it} on $\widehat{F}_t$ and W_t.

Step W2: Estimate Φ_F and Φ_W from a VAR in $\widehat{F}_t$ and W_t, respectively. Also let $\widehat{\varepsilon}_{Wt}$ be the residuals from estimation of (17), the VAR in W_t.

Step W3: Estimate v_{ft}:

i. Let $\widehat{\varepsilon}_{Xit} = x_{it} - \widehat{\Lambda}'_{iF}\widehat{\Phi}_F\widehat{F}_{t-1} - \widehat{\Lambda}'_{iW}\widehat{\Phi}_W W_{t-1}$, where $\widehat{F}_t$ are the iterative principal components of the full panel.

ii. Let $X = (X^1, X^2, \dots X^q)$ be data organized into q blocks. For $b = 1, \dots q$, let $\widehat{\varepsilon}_{fb}$ be the eigenvector corresponding to largest eigenvalue of the $n_b \times n_b$ matrix $\widehat{\varepsilon}'_{X^b}\widehat{\varepsilon}_{X^b}$.

iii. Orthogonalize $\widehat{\varepsilon}_t = (\widehat{\varepsilon}'_{ft}\ \widehat{\varepsilon}'_{wt})'$ using the causal or block ordering of the variables.

Step W4: Construct the impulse response: Estimate by OLS with restrictions on $\alpha_{yf}(L)$ and $\alpha_{yw}(L)$:

$$y_t = \alpha_{yy}y_{t-1} + \alpha_{yf}(L)\widehat{v}_{ft} + \alpha_{yw}(L)v_{wt} + v_{yt}. \tag{18}$$

Then $\widehat{\psi}^f(L) = \frac{\widehat{\alpha}_{yf}(L)}{(1-\widehat{\alpha}_{yy}(L))}$ gives the response of y_t to v_{ft} holding W_t fixed.

5 Simulations

We use simulations to evaluate the finite sample properties of the identified impulse responses. Data are simulated from equations (1)-(3) with $\lambda(L)$ being a polynomial of degree $s = 1$. The persistence parameter δ_i is uniformly distributed over (.2,.5). The errors v_{Xit}, v_{ft} and the non-zero factor loadings are normally distributed with variances $\sigma_X^2, 1, \sigma_\lambda^2$ respectively. We set $T = 200$ and $N = 120$ to mimic the macroeconomic panels used in empirical work.

The structural moving-average representation is

$$X_{it} = 1 - \delta_i L)^{-1}\begin{pmatrix}\lambda_{0i} & \lambda_{1i}L\end{pmatrix}\begin{pmatrix}I - \Gamma_1 L\end{pmatrix}^{-1}\Gamma_0\begin{pmatrix}v_{f1t}\\ v_{f2t}\end{pmatrix} + v_{Xit}.$$

This implies that the impact response of X_{it} to the shocks is summarized by

$$X_{it} = (\lambda_{0,i,1} \quad \lambda_{0,i,2})\begin{pmatrix}\gamma_{0,11} & \gamma_{0,12}\\ \gamma_{0,21} & \gamma_{0,22}\end{pmatrix}\begin{pmatrix}v_{f1t}\\ v_{f2t}\end{pmatrix} + v_{Xit}. \tag{19}$$

DGP 1: $q = 2$ factors, $\Gamma_1 = \begin{pmatrix}0.75 & 0\\ 0 & 0.7\end{pmatrix}$, $\sigma_{\lambda,1k} = 1$.

case a: $\Gamma_0 = I$, **case b:** $\Gamma_0 = \begin{pmatrix}1 & 0\\ 0.5 & 1\end{pmatrix}$, $\sigma_{\lambda,2k} = 0.8$.

The N variables are ordered such that the first $N/2$ variables respond contemporaneously to both shocks and are labeled ‘fast’. The last $N/2$ do not respond contemporaneously to shock 2 and are labeled slow’. By design, X_{1t} is a fast variable and X_{Nt} is a slow variable. This structure is achieved by specifying

$$(\lambda_{0,i,1} \quad \lambda_{0,i,2}),\ \ i = 1, \dots, N/2 \qquad \text{and} \qquad (\lambda_{0,i,1} \quad 0)\ \ i = N/2+1, \dots, N.$$

Let $Y_t = (X_{1t}, X_{N,t})$ be the two variables whose impulse responses are of interest. Since there are no observed factors, estimation begins with E1 and E2. We consider both identification strategies and estimate two FADL regressions, one for each variable in Y_t. As a benchmark, we also estimate the (infeasible) FADL regressions using the true common shocks, v_{ft}.

The results are summarized in Table 1. The top panel of Table 1 shows that for DGP 1a, the correlation between v_{fjt} and $\widehat{v}_{fjt}$ are well above 0.90 for both identification strategies. For DGP 1b, Method (a) is more precise than (b) but the latter is still quite precise. The correlation between v_{fjt} and v_{fkt} are statistically different from zero, but are numerically small. Panel B of Table 1 reports the RMSE of the estimated impulse responses when the shocks are observed. Given that there are two shocks, there are two impulse responses to consider for each of the two variables. We use $v_{fj} \to X_k$ to denote the response of X_k to shock j, where $k = 1$ is the fast variable, and $k = N$ is the slow variable. Panel C reports results when the common shocks have to be estimated. The $\widehat{\psi}$ are practically identical to the analytical ones given by (19). Furthermore, the impact response of slow variable to second shock is not statistically different from zero.

When the FADL models are estimated on $\widehat{v}_f$ instead of v_f, we observe that (i) $corr(v_{ft}, \widehat{v}_{ft}) \approx I$; In case 2, off-diagonal elements (ii) $\widehat{\psi}(L)$ are very close to true impulse response coefficients (iii) the non-zero coefficients have statistically significant estimates.

DGP 2: $q = 2$ latent and $m = 1$ observed factors Let $\sigma_{\lambda,jk} = 1$, $\sigma_{\lambda,1k} = 1$, $\sigma_{\lambda,2k} = 0.8$, and $\sigma_{\lambda,3k} = 0.7$ and $\Gamma_1 = \begin{pmatrix} 0.75 & 0 & 0 \\ 0 & 0.7 & 0 \\ 0 & 0 & 0.65 \end{pmatrix}$.

case a: $\Gamma_0 = I$, **case b:** $\Gamma_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0.4 & 1 & 0 \\ 0.3 & 0.2 & 1 \end{pmatrix}$.

The ordering of structural shocks is $v_{ft} = (v_{ft}^{slow}, \quad v_{ft}^{mp}, \quad v_{ft}^{fast})$. The goal is (partial) identification of the effects of v_{ft}^{mp}. Again, the N variables are divided between fast and slow: slow variables do not respond on impact to second and third shocks, and at least one variable does not respond immediately only to the third shock, such that the causal ordering holds.

After the common shocks are estimated and identified according to Methods RO and BO, two FADL regressions are estimated for the two components in Y_t: one fast and one slow variable. As we are interested in partial identification of the second shock, we only report results on the approximation of v_{ft}^{mp}, and impulse responses of two variables to this shock. As in the previous exercise, FADL regressions with true shocks produce impulse response coefficients practically identical to the analytical ones. The estimated second shock is very close to the true one and $\widehat{\psi}(L)$ very close to true impulse response coefficients.

6 Two Examples

In this section, we use FADL to analyze two problems:- measuring the effects of monetary policy in the presence of other common shocks, and news shocks.

6.1 Example 1: Effects of a Monetary Policy Shock

As in Bernanke, Boivin, and Eliasz (2005), the monetary authority observes N_{slow} variables (such as measures of real activity and prices) collected into X_t^{slow} when setting the interest rate R_t but does not observe N_{fast} variables (such as financial data) collected into X_t^{fast}. In this exercise, R_t is an observed factor. Let $v_{ft} = (v_{ft}^{slow}, \quad v_{ft}^{mp}, \quad v_{ft}^{fast})$ be the vector of q common shocks, where v_{ft}^{mp} is the monetary policy shock, v_{ft}^{fast} is a vector q_1 shocks, specific to X^{fast}, and v_{ft}^{slow} is a vector of q_2 shocks, specific to X_t^{slow} respectively, with $q = q_1 + q_2 + 1$. The issue of interest is (partial) identification of the effects of monetary policy shock, meaning that the effects due to v_{ft}^{slow} and v_{ft}^{fast} are not of interest.

Bernanke, Boivin, and Eliasz (2005) identify the monetary policy shock by assuming that Ψ_0^f is a block lower triangular structure. This involves restrictions o on $N_{slow} > q_2$ variables. In a data rich environment, some of these restrictions could well be invalid. We consider two alternative identification strategies, both using fewer restrictions. The first is based on Assumption RO which can be achieved by choosing the first q variables to compose of q_1 (slow) indicators of real activity and prices, followed by the monetary policy instrument.[3] The second is based on Assumption BO which identifies the shocks at the block level. The data are ordered as $Y_t = (X_t^{slow\prime}, \quad R_t, \quad X_t^{fast\prime})'$. After estimating v_{ft}^{slow} from X_t^{slow} and and v_{ft}^{fast} from X_t^{fast}, the monetary policy shocks are the residuals from a regression of R_t on current and lag values of $\widehat{v}_{ft}^{slow}$. By construction, the estimated structural shocks are mutually uncorrelated under both RO and RO assumptions. A FADL in all the shocks is then estimated for each variable of interest.

In terms of matrix Ψ_0^f, Bernanke, Boivin, and Eliasz (2005) assumes:

$$\Psi_0^f = \begin{pmatrix} \underbrace{\psi_{0,1,1}}_{N_{slow}\times q_1} & \underbrace{0}_{N_{slow}\times 1} & \underbrace{0}_{N_{slow}\times q_2} \\ \underbrace{\psi_{0,2,1}}_{1\times q_1} & \underbrace{\psi_{0,2,2}}_{1\times 1} & \underbrace{0}_{1\times q_2} \\ \underbrace{\psi_{0,3,1}}_{N_{fast}\times q_1} & \underbrace{\psi_{0,3,2}}_{N_{fast}\times 1} & \underbrace{\psi_{0,3,3}}_{N_{fast}\times q_2} . \end{pmatrix}$$

[3] One may also add q_2 financial indicators at the end of the recursion, but Bernanke, Boivin, and Eliasz (2005) found that there is little informational content in the fast moving factors that is not already accounted for by the federal funds rate.

Assumptions RO and BO both assume that the top $q \times q$ block of Ψ_0^f is lower triangular:

$$\Psi^f_{0,1:q,1:q} = \begin{pmatrix} \underbrace{\psi_{0,1:q,1}}_{q_1 \times q_1} & \underbrace{0}_{q_1 \times 1} & \underbrace{0}_{q_1 \times q_2} \\ \underbrace{\psi_{0,2:q,1}}_{1 \times q_1} & \underbrace{\psi_{0,2:q,2}}_{1 \times 1} & \underbrace{0}_{1 \times q_2} \\ \underbrace{\psi_{0,3:q,1}}_{q_2 \times q_1} & \underbrace{\psi_{0,3:q,2}}_{q_2 \times 1} & \underbrace{\psi_{0,3:q,3}}_{q_2 \times q_2} . \end{pmatrix}$$

However, the Ψ_0^f matrix and $\widehat{v}_{ft}$ identified by RO will be different from those identified by BO. Under Assumption RO, all N series are used to estimate the q vector ε_{ft}. Thus any q series in the training sample can be used to identify primitive shocks v. Under Assumption BO, ε^j_{ft} is estimated from block j of X_t. Thus, the j shock in v_{ft} is identified from one of the N_j series in block j of X_t. Assumption BO also allows *a priori* economic restrictions to be imposed on some or all variables within the blocks. For example, we can restrict all N_{slow} series not to react on impact to a monetary policy shock, while the response of fast moving variables is unrestricted. Since these restrictions are imposed on equation by equation basis, they do not affect the estimation nor the identification of structural shocks.[4]

6.1.1 Data and Results

The training sample used to estimate the factors consists of 107 quarterly aggregate macroeconomic and financial indicators over the extended sample 1959Q1- 2009 Q1. This data set consists of fast and slow moving variables. The Federal funds rate (FFR) is treated as an observed factor. All data are assumed stationary or transformed to be covariance stationary. The complete list of variables is given in the Appendix.

Our estimation differs from Bernanke, Boivin, and Eliasz (2005) in two ways. First, we use quarterly data. Second, we estimate the factors by IPC to take care of autocorrelation in residuals. According to information criteria in Amengual and Watson (2007) and Bai and Ng (2007), there are $q = 3$ latent dynamic factors in the training sample. Identification is achieved by imposing a causal ordering. We order commodity price inflation first, followed by GDP deflator inflation, unemployment rate, and then FFR. Hence monetary policy is the last variable in this causal ordering, which implies zero contemporaneous response to monetary policy by the slow moving variables. We only impose restrictions on q series (one from each block) while Bernanke, Boivin, and Eliasz (2005) impose restrictions on all series belonging to the slow moving block.

Compared to Stock and Watson (2005), we impose the same minimal number of restrictions to identify the structural shocks, but our approach differs in estimating the impulse response functions.

[4]The restrictions can vary across series in the block. For example, one series could be restricted to respond only 2 periods after the shock, the sign of another variables could be fixed, the shape of the impulse response function could be constrained for a third variables, and so on.

Instead of constructing impulse response coefficients of X_t as $(I - \widehat{D}(L))\widehat{\Lambda}(I - \widehat{\Gamma}_1(L))^{-1}\widehat{\Gamma}_0$, we rather estimate the product, $\psi_i^f(L)$, equation by equation for any element of X_t and X_t^{OTH}.

The 12 period impulse responses are presented in Figure 1. As in Bernanke, Boivin, and Eliasz (2005), controlling for the presence of common shocks resolves anomalies found in the literature. After a monetary policy shock, the fast moving variables such as Treasury bills increase immediately, while stock prices, housing starts, and consumer expectations fall. Furthermore, many measures of the slow variables including real activity and prices decline as a result of the shock without evidence of a price puzzle. The exchange rate appreciates fully on impact, with no evidence of overshooting. The results for the variables of interest are in line with Christiano, Eichenbaum, and Evans (2000) who use recursive and non-recursive identification schemes to study the effects of monetary policy, using small VARs. However, once the common shocks are estimated, the effects of monetary policy can be studied for many variables, not just the q variables used in identification. The scope of the analysis is much larger than a small VAR.

To check the validity of the factor structure in series not in the training sample, we consider X_t^{OTH} consisting of 107 disaggregated series. Amongst these are (i) 3 sectoral CPI, 55 PCE, and 3 PPI measures of inflation, (ii) 10 disaggregated employment series, (iii) 18 investment measures, and (iv) 18 consumption series. For each of these additional variables, the Wald test is used to test the null hypothesis that all coefficients in $\alpha_{yf}(L)$ are jointly zero. The null hypothesis cannot be rejected at the five percent level for many series including one sectoral CPI, 15 PCE, one employment, one investment and two consumption series. For these series, the data does not support the presence of a factor structure.

We then proceed to analyze the effects of monetary policy on variables in X_t^{OTH}. Interestingly, the impulse responses of variables not affected by v_{ft} display price-puzzle like features. As seen in the top panel of Figure 2 for some of these variables, an increase in the Fed Funds rate increases rather than lowers prices. The bottom panel displays results for four series with significant $\widehat{\alpha}_{yf}(L)$. For these latter set of variables, the impulse responses are similar to those reported for the variables in the training sample, namely, that an increase in the Fed funds rate lowers prices.

The impulse responses of all sectoral variables are presented in Figure 3. The responses of many disaggregated series are in line with theory: a decline of real activity and price indicators across several sectors after an adverse monetary policy shock. In case of employment variables, only mining and government sector series diverge from others during the first year after the shock, while the price indicators of some nondurable goods sectors present the price puzzle behavior.

6.2 Example 2: Effects of a News Shock

Beaudry and Portier (2006) consider technology shock and news shocks, $v_{ft} = (v_t^{TFP}\ v_t^{NS})'$, inter-

preted as an announcement of future change in productivity. They are interested in the effects of these two shocks on productivity X_{1t}. Consider identification by the short run restrictions. Suppose that the first N_1 variables $X_t^1 \subset X_t$ do not respond immediately to v_t^{NS}, but their response to v_t^{TFP} is unrestricted. Then Ψ_0^f is lower block triangular, viz:

$$\Psi_0^f = \begin{pmatrix} \psi_{0,1,1}^f & 0 \\ \vdots & \vdots \\ \psi_{0,N_1+1,1}^f & \psi_{0,N_1+1,2}^f \\ \vdots & \vdots \\ \psi_{0,N,1}^f & \psi_{0,N,2}^f \end{pmatrix} \equiv \begin{pmatrix} \Psi_{0,1:N_1,1}^f & 0_{1:N_1,1} \\ \cdots & \cdots \\ \Psi_{0,N_1+1:N,1}^f & \Psi_{0,N_1+1:N,2}^f \end{pmatrix}.$$

This structure can be achieved if Λ_0 and Γ_0 are both *lower* block triangular, ie.

$$\underbrace{\Lambda_0}_{N\times 2} = \begin{pmatrix} \lambda_{0,1,1} & 0 \\ \vdots & \vdots \\ \lambda_{0,N_1+1,1} & \lambda_{0,N_1+1,2} \\ \vdots & \vdots \\ \lambda_{0,N,1} & \lambda_{0,N,2} \end{pmatrix} = \begin{pmatrix} \Lambda_{0,1:N_1,1} & \vdots & 0_{N_1\times 1} \\ \Lambda_{0,N_1+1:N,1} & \vdots & \Lambda_{0,N_1+1:N,2} \end{pmatrix} \qquad \text{and} \qquad \underbrace{\Gamma_0}_{2\times 2} = \begin{pmatrix} \Gamma_{0,11} & 0 \\ \Gamma_{0,21} & \Gamma_{0,22}. \end{pmatrix}$$

The zero restriction should hold for all series in the first block. But since there are only two shocks, any two series permit exact identification provided one is from X_t^1, one from X_t^2, and one restriction is imposed on Ψ_0^f. Beaudry and Portier (2006) only uses two variables (X_{1t}, X_{Nt}) for analysis where X_{1t} is a measure of TFP and X_{Nt} is stock price. We allow for $N > 2$ variables. But unlike standard VARs which require restrictions of order N^2 to identify N shocks, we use q series to exactly identify $q = 2$ shocks. As discussed earlier, instead of putting restrictions on Γ_0 or Λ_0 separately, our restrictions are imposed on the relevant row(s) of $\Psi_0^f = \Gamma_0\Lambda_0$. The bivariate system has the property that

$$\begin{pmatrix} X_{1t} \\ X_{Nt} \end{pmatrix} = \begin{pmatrix} \psi_{0,11}^f & 0 \\ \psi_{0,21}^f & \psi_{0,22}^f \end{pmatrix} \begin{pmatrix} v_t^{TFP} \\ v_t^{NS} \end{pmatrix} + \sum_{j=1}^{\infty} \begin{pmatrix} \psi_{j,11}^f & \psi_{j,12}^f \\ \psi_{j,21}^f & \psi_{j,22}^f \end{pmatrix} \begin{pmatrix} v_{t-j}^{TFP} \\ v_{t-j}^{NS} \end{pmatrix}.$$

The number of identifying restrictions used in the FADL is of order q^2 irrespective of N. This also contrasts with standard FAVARs which impose many overidentifying restrictions. In our setup, a large N is desirable for FADL because it improves estimation of v_{ft}. Long run restrictions can similarly be imposed so that $\Psi^f(1)$ is block lower triangular. A FADL leads to exact identification using the salient features of the factor model.

6.2.1 Data and Results

Our data consists of $X_t = (X_t^{TFP}, X_t^{SP}, X_t^{OTH})$, where X_t^{TFP} contains six TFP measures from FRB San Francisco, X_t^{SP} is a vector of eight S&P and Dow Jones aggregate stock price indicators,

and X_t^{OTH} is a vector of 104 macroeconomic time series used in the previous example but with the stock prices removed[5]. Beaudry and Portier (2006) only use one series of the six series in X_t^{TFP} and one series in X_t^{SP} at the time. Forni, Gambetti, and Sala (2011) use the same TFP series and some of our stock price measures.

Two identification strategies are considered:

i (Causal Ordering) estimate two common shocks from $X_t = (X_t^{TFP}, X_t^{SP})$. Two series, one from X_t^{TFP} and one from X_t^{SP} are selected. By ordering the TFP series ordered first, the H that identifies the technology and the news shock.

ii (Block Ordering) ε_t^{TFP} is estimated exclusively from X_t^{TFP} and ε_t^{SP} is estimated from X_t^{SP}. The identification is based on the structure

$$\begin{pmatrix} \varepsilon_t^{TFP} \\ \varepsilon_t^{SP} \end{pmatrix} = \begin{pmatrix} a_{11} & 0 \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} v_t^{TFP} \\ v_t^{NS} \end{pmatrix}.$$

Effectively, $\widehat{v}_t^{TFP} = \varepsilon_t^{TFP}$ and $\widehat{v}_t^{NS}$ are the residuals from a projection of $\widehat{\varepsilon}_t^{SP}$ onto $\widehat{v}_t^{TFP}$. Note that under both identification strategies the estimated shocks are mutually uncorrelated.

Once $\widehat{v}_t^{TFP}$ and $\widehat{v}_t^{NS}$ are available, variable by variable FADL equations are estimated for all series in X_t. The zero impact restrictions are imposed for all TFP measures, while all other FADL regressions are left unrestricted. The results for the two identification strategies and for both technology and news shocks (v_t^{TFP} and v_t^{NS} respectively) are given in Figures 4-7. We report results for differenced data.

The Table 3 contains p-values for Wald test for the null hypothesis of no factor structure in X_t^{TFP}, X_t^{SP} and X_t^{OTH} variables. The abbreviations 'RO', 'BO' stand for Assumption RO and BO respectively. The null hypothesis is strongly rejected for many series. Turning to the impulse responses, the effects of technology shocks are in line with Christiano, Eichenbaum, and Vigfusson (2003) who suggest that technology improvements are pro-cyclical for real activity and hours measure, but contrary to Basu, Fernald, and Kimball (2006) and Gali (1999).

Of special interest here are the responses to a positive news shock. The forward looking variables such as stock prices, housing starts, new orders and consumer expectations increase on impact. Consumption reacts positively. The wealth effect does not seem important enough such that the worked hours also increase on impact.

Our results are in line with Beaudry and Portier (2006) for the pro-cyclical response of worked hours. However, Barsky and Sims (2011) also estimate positive response of consumption and find an immediate decrease of hours. Forni, Gambetti, and Sala (2011) find that both consumption and

[5]The complete list of additional variables used in news shock application is available in Appendix

hours respond negatively on impact. These differences can be due to the choice of variables used to identify the shocks and to the variables selected for analysis. In particular, these studies used a small set of worked hours measures. We check the sensitivity of our results to a much broader set of available indicators.

To this end, we assess the sensitivity of our results (under the assumption of a block structure) to additional variables as follows:

a Estimate ε_t^{OTH} from the macro data X_t^{OTH}. Identification is now based on

$$\begin{pmatrix} \varepsilon_t^{TFP} \\ \varepsilon_t^{SP} \\ \varepsilon_t^{OTH} \end{pmatrix} = \begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} v_t^{TFP} \\ v_t^{NS} \\ v_t^{OTH} \end{pmatrix}.$$

b change the ordering to ε_t^{TFP}, ε_t^{OTH} with ε_t^{SP} ordered last in view of the forward looking nature of stock prices.

These results are denoted Block 2 and Block 3 respectively. In a VAR setup, there would be 104 VARs to consider when there are 104 macro variables that might not be econometrically exogenous to TFP and stock prices. In the factor setup, we only need to estimate one set of macro shocks from 104 macro series. As shown in Figure 5, the effects of news shocks are smaller when the macro shocks are present. In other words, omitted variables from the VAR could have biased the estimated effects of news shocks. However, for an assumed q, the identified impulse responses are robust to the ordering of the variables.

As is well known, VARs involving hours worked are sensitive to whether the hours series is in level or in difference, see for example, Féve and Guay (2009). We use the specification labeled Block 3 to further understand the dynamic response the level (Figure 8) and growth (Figure 9) of average weekly hours (AWH) level to news shock. The dynamic responses of AWH and total hours indices are plotted in Figure 9. Regardless of the data transformation, the hours variables are pro-cyclical after the news technology shock. This exercise illustrates the FADL can be used to check the robustness of the results to many other measures without affecting the identification of structural shocks.

7 Conclusion

In this paper, we have proposed a new approach to analyze the dynamic effect of common shocks in a data-rich environment. After estimating the common shocks from a large panel of data and imposing a minimal set of identification restrictions, the impulse responses are obtained from an autoregression in each variable of interest, augmented with a distributed lag of structural shocks.

The FADL framework presents several advantages. The method is more robust to a fully structural factor model when the identifying factor restrictions do not hold universally. Since the impulse responses are obtained from a set of regressions, the restrictions are easy to impose, and implications of the factor model can be tested. The estimation of common shocks is less likely to be affected by the presence of weak factors. The FADL methodology is used to measure the effects of monetary policy shocks, and to news and technology shocks. The approach allows us to go beyond existing structural FAVAR, and to validate restrictions of the factor model.

Appendix: Relation to Other Methods with Observable Factors

The estimated common shocks are treated as regressors of a FADL. As such, *a priori* restrictions on the impulse response functions can be directly imposed in estimation of the FADL by least squares. The approach is simpler and more transparent than existing implementations of structural FAVARs.

Consider the identification of monetary policy shocks in the presence of other shocks as in Bernanke, Boivin, and Eliasz (2005). Their point of departure is a static factor model with latent and observed factors:

$$X_t = \Lambda^F F_t + \Lambda^R R_t + u_t \tag{20}$$

$$\begin{bmatrix} F_t \\ R_t \end{bmatrix} = \Phi \begin{bmatrix} F_{t-1} \\ R_{t-1} \end{bmatrix} + \eta_t \tag{21}$$

where F_t is vector of r latent factors and R_t is the observed factor (usually Federal Funds Rate or 3-month Treasury Bill). The authors organize the $N = 120$ data vector X_t into a block of slow-moving' variables that are largely predetermined, and another consisting of 'fast moving' variables that are sensitive to contemporaneous news. The idiosyncratic errors are assumed to be serially uncorrelated.

BBE Identification

1 Estimate F_t.

  i Let $\widehat{C}(F_t, R_t)$ be the K principal components of X_t.

  ii Let X_t^S be N_S 'slow' moving variables that do not respond immediately to a monetary policy shock. Let the K principal components of X_t^S be $C^\star(F_t)$.

  iii Define $\widehat{F}_t = \widehat{C}(F_t, R_t) - \widehat{b}_R R_t$ where $\widehat{b}_R$ is obtained by least squares estimation of the regression

$$\widehat{C}(F_t, R_t) = b_C C^\star(F_t) + b_R R_t + e_t.$$

2 Estimate the loadings by regressing X_t on $\widehat{F}_t$ and R_t: $\widehat{\Lambda^F}$ and $\widehat{\Lambda^R}$.

3 Estimate the FAVAR given by (21) and let $\widehat{\eta}_t$ be the residuals. From the triangular decomposition of the covariance of $\widehat{\eta}_t$, let A_0 be a lower triangular matrix with ones on the main diagonal Then $\widehat{\eta}_t = \widehat{A}_0 \widehat{\varepsilon}_t$ are the monetary policy shocks.

4 Obtain IRFs for $\widehat{F}_t$ and R_t by inverting (21) and using $\widehat{A}_0$

5 Multiplying the IRFs in (3) by $\widehat{\Lambda^F}$ and $\widehat{\Lambda^R}$ to obtain the IRFs for X_t.

The novelty of the BBE analysis is that Step (1) accommodates the observed factor R_t when $\widehat{F}_t$ is being estimated. By construction, $\widehat{C}(F_t, R_t)$ spans the space spanned by F_t and R_t while $C^*(F_t)$ spans the space of common variations in variables that do not respond contemporaneously to monetary policy. Since R_t is observed, the regression then constructs the component of $\widehat{C}_t$ that is orthogonal to R_t. Once $\widehat{F}_t$ is available, Step (2) is straightforward. Under the BBE scheme, the common shocks are identified in Step (3) when a FAVAR in $(\widehat{F}_t, R_t)$ is estimated. Because $\widehat{F}_t$ may be correlated contemporaneously with R_t, the monetary policy shocks are identified by ordering R_t after $\widehat{F}_t$ in (21).[6]

The lower triangular of A_0 is not enough to identify the structural shocks as the response depends on the product $\begin{pmatrix}\Lambda^F & \Lambda^R\end{pmatrix} A_0$.[7] Thus, BBE impose additional restrictions. In particular, the K slow moving variables are ordered first in X_t. Furthermore, the $K \times K$ block of Λ^F is identity, and the first element in Λ^R is zero. As a result, the first $K+1 \times K+1$ part of the product $\begin{pmatrix}\Lambda^F & \Lambda^R\end{pmatrix} A_0$ is lower triangular. For $K = 2$,

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \lambda_{31} & \lambda_{32} & \lambda_{33} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ a_{21} & 1 & 0 \\ a_{31} & a_{32} & 1 \end{pmatrix}$$

The structural model is just-identified.

Stock and Watson (2005) The SW approach treats monetary policy as a dynamic factor. The identification assumptions are that (i) the monetary policy shock does not affect the slow-moving variables contemporaneously; and (ii) the slow-moving shock and monetary policy affects the Fed Funds rate contemporaneously. Thus, as in Bernanke, Boivin, and Eliasz (2005), the slow-moving variables first, followed by the Fed funds rate, and then the fast-moving variables. The point of departure is that $\varepsilon_{Xt} = \Lambda \varepsilon_{ft} + v_{Xt}$ is assumed to have a factor structure and $\varepsilon_{ft} = G\eta_t = GHv_{ft}$. Letting $C = GH$, the errors are related by

$$\varepsilon_{Xt} = \Lambda C v_{ft} + v_{Xt}$$

where v_{ft} is of dimension q. The steps are as follows:

[6] Boivin, Giannoni, and Stevanović (2009) suggests an alternative way to estimate F_t that does not rely on organizing the variables into fast and slow.

1 Initialize $\widehat{F}_t$ to be the K first principal components of X_t.

2 (i) Regress X_t on $\widehat{F}_t$ and R_t, to obtain $\widehat{\Lambda}_t^{F,j}$ and $\widehat{\Lambda}_t^{R,j}$. (ii) Compute $\widetilde{X}_t^j = X_t - \widehat{\Lambda}_t^{R,0} R_t$ (iii) Update $\widehat{F}_t$ as the first K principal components of $\widetilde{X}_t$

By construction, $\widehat{F}_t$ is contemporaneously uncorrelated with R_t This is possible because the step that estimates the latent factors controlling for the presence of the observed factors is separated from identification of structural shock. In BBE, η_t depends on the choice of variables used in the first stage to estimate F_t.

[7] In BBE application, Step 1 estimates the loadings of slow moving variables to R_t close to zero.

1 Let $\widehat{\varepsilon}_{Xt}$ into ($\widehat{\varepsilon}^{slow}_{X,t}$, $\widehat{\varepsilon}^{fed}_{X,t}$ and $\widehat{\varepsilon}^{fast}_{X,t}$) corresponding to the three types of variables.

2 Let $\widehat{u}_{Ft}$, the residuals from a VAR in the static factors constructed from the full panel, X.

3 Let the factor component of $\varepsilon^{fed}_{X,t}$ be the fit from a reduced rank regression of $\widehat{\varepsilon}^{slow}_{X,t}$ and $\widehat{u}_{Ft}$.

4 Take the monetary shocks to be the residuals from a projection of of $\widehat{\varepsilon}^{fed}_{X,t}$ onto $\widehat{v}^{slow}_{X,t}$.

If there are q^{slow} and q^{fast} factors in $\widehat{\varepsilon}^{slow}_{Xt}$ and $\widehat{\varepsilon}^{fast}_{Xt}$ respectively, then $q = q^{slow} + q^{fast} + 1$. The identification scheme makes use of the fact that $v^{slow}_{X,t}$ spans the space of $\varepsilon^{slow}_{X,t}$ and can thus be identified from a projection of $\widehat{\varepsilon}^{slow}_{X,t}$ on $\widehat{u}_{Ft}$. An additional step is needed to estimate the common variations between $\widehat{u}_{Ft}$ and $\widehat{\varepsilon}^{slow}_{X,t}$. This procedure sequentially estimates the rotation matrix H[8]. Note that the identification restrictions are imposed directly on the impact coefficients matrix of the structural moving average representation of X_t, and the structural model is overidentified. The method is not easily generalizable to other models in which the shocks do not have a block recursive structure implicit in the model.

FGLR: Forni, Giannone, Lippi, and Reichlin (2009) provides a framework for structural FAVAR analysis. The method is applied to identify monetary policy in Forni and Gambetti (2010).

1 let $\widehat{\Lambda}$ be a $N \times r$ matrix of estimated loadings and $\widehat{F}_t$ be the static principal components. Estimate a VAR in $\widehat{F}_t$ to get $\widehat{\Gamma}(L)$ and the residuals $\widehat{u}_{Ft}$.

2 Perform a spectral decomposition of the covariance matrix of $\widehat{u}_{Ft}$. Let M be a diagonal matrix consisting of the largest eigenvalue of $\widehat{u}_F\widehat{u}_F'$ and let $\mathcal{K}$ be the $r \times q$ matrix of eigenvectors.

3 Let $S = \mathcal{K}M$. The non-orthogonalized impulse responses are given by

$$\ddot{\Psi}^{\eta}(L) = \widehat{\Lambda}(I - \widehat{\Gamma}(L))^{-1}S.$$

Step (2) is a consequence of the fact that the VAR in $\widehat{F}_t$ is singular. Step (3) rotates $\ddot{\Psi}^{\eta}$ by a $q \times q$ matrix of restrictions. Unlike the partial identification analysis of Stock and Watson (2005), this method estimates the impulse responses for the system as a whole. Mis-specification in a sub-system can affect the entire analysis, but the estimates are more efficient if every aspect of the factor model is correctly specified.

[8] Boivin, Giannoni, and Stevanović (2009b) find that the rotation of principal components by $\widehat{H}$ gives interpretable factors.

Table 1: Simulations DGP 1

	$corr(v_{ft}, \widehat{v}_{ft})$							
	a. Recursive Ordering				b. Block Ordering			
DGP 1a	$\begin{pmatrix} 0.9844 & 0.0308 \\ 0.0636 & 0.9790 \end{pmatrix}$				$\begin{pmatrix} 0.9825 & 0.0562 \\ 0.0953 & 0.9030 \end{pmatrix}$			
DGP 1b	$\begin{pmatrix} 0.9843 & 0.0313 \\ 0.0638 & 0.9777 \end{pmatrix}$				$\begin{pmatrix} 0.9805 & 0.0874 \\ 0.1188 & 0.8706 \end{pmatrix}$			
	RMSE of Impulse Responses: v_{ft} observed							
	$v_{f1} \to X_1$	$v_{f2} \to X_1$	$v_{f1} \to X_N$	$v_{f2} \to X_N$				
DGP 1a	0.0415	0.0412	0.0426	0.0414				
DGP 1b	0.0409	0.0394	0.0422	0.0425				
	RMSE of Impulse Responses Using $\widehat{v}_{ft}$							
	a. Recursive Ordering				b. Block Ordering			
	$v_{f1} \to X_1$	$v_{f2} \to X_1$	$v_{f1} \to X_N$	$v_{f2} \to X_N$	$v_{f1} \to X_1$	$v_{f2} \to X_1$	$v_{f1} \to X_N$	$v_{f2} \to X_N$
DGP 1a	0.2452	0.2228	0.2299	0.1963	0.2558	0.2642	0.2366	0.2032
DGP 1b	0.2994	0.2453	0.2702	0.1953	0.2954	0.3556	0.2556	0.2578

Table 2: Simulations DGP 2

	$corr(v_{ft}^{mp}, \widehat{v}_{ft}^{mp})$			
	a. Recursive Ordering		b. Block Ordering	
DGP 2a	0.9747		0.9629	
DGP 2b	0.9774		0.9620	
	RMSE of Impulse Responses: v_f Observed			
	$v_{f2} \to X_1$	$v_{f2} \to X_N$		
DGP 2a	0.0406	0.0417		
DGP 2b	0.0399	0.0419		
	RMSE of Impulse Responses: v_f Estimated			
	a. Recursive Ordering		b. Block Ordering	
	$v_{f2} \to X_1$	$v_{f2} \to X_N$	$v_{f2} \to X_1$	$v_{f2} \to X_N$
DGP 2a	0.2743	0.2285	0.2760	0.1996
DGP 2b	0.3155	0.2781	0.3391	0.2466

Table 3: News Shock, p-values from Wald test for $H_0 : \alpha_{yf}(L) = 0$

Variables in (X_t^{TFP}, X_t^{SP})	RO	BO	BO2	BO3
TFP	0	0	0	0
TFP-util	0	0	0	0
TFP-I	0	0	0	0
TFP-C	0	0	0	0
TFP-I-util	0	0	0	0
TFP-C-util	0	0	0	0
S&P: composite	0	0	0	0
S&P: industrial	0	0	0	0
S&P: dividend	0	0	0	0
S&P: price/earning	0	0	0	0
DJ: industrial	0	0	0	0
DJ: composite	0	0	0	0
DJ: transportation	0	0	0	0
DJ: utilities	0	0	0	0
Variables in X_t^{OTH}				
GDP	0	0	0	0
IP	0	0	0	0
RONSUMPTION	0	0	0	0
INVESTMENT	0	0	0	0
HOURS	0	0	0	0
HOURS: Overtime	0	0	0	0
EMPLOYEES: NONFARM	0,0262	0,0399	0	0
CLF: EMPLOYED	0	0	0	0
HELP-WANTED ADV	0	0	0	0
AVG HOURLY EARNINGS	0	0	0	0
CAPACITY UTILIZATION	0	0	0	0
RONSUMER EXPECTATIONS	0	0	0	0
UR	0	0	0	0
EMPLOYEES ROMPENSATION	0	0	0	0
RONSUMPTION: NONDUR	0	0	0	0
RONSUMPTION: DURAB	0	0	0	0
RONSUMER CREDIT	0,2422	0,2772	0	0
HOUSING STARTS	0,0062	0,0027	0	0
NEW ORDERS	0	0	0	0
INVENTORIES	0,0718	0,1742	0	0
FFR	0,3513	0,2612	0	0
ROMMODITY PRICES	0,0299	0,1384	0	0
CPI	0,3527	0,5556	0	0
GDP DEFLATOR	0,0438	0,0729	0	0

RO and BO refer to identification by causal and block ordering, respectively. The two blocks of data are X_t^{TFP} and X_t^{SP}. Data from the macro block X_t^M are also used in BO2 and BO3. BO2 uses the ordering TFP, SP, Macro. BO3 orders the macro variables before the stock prices.

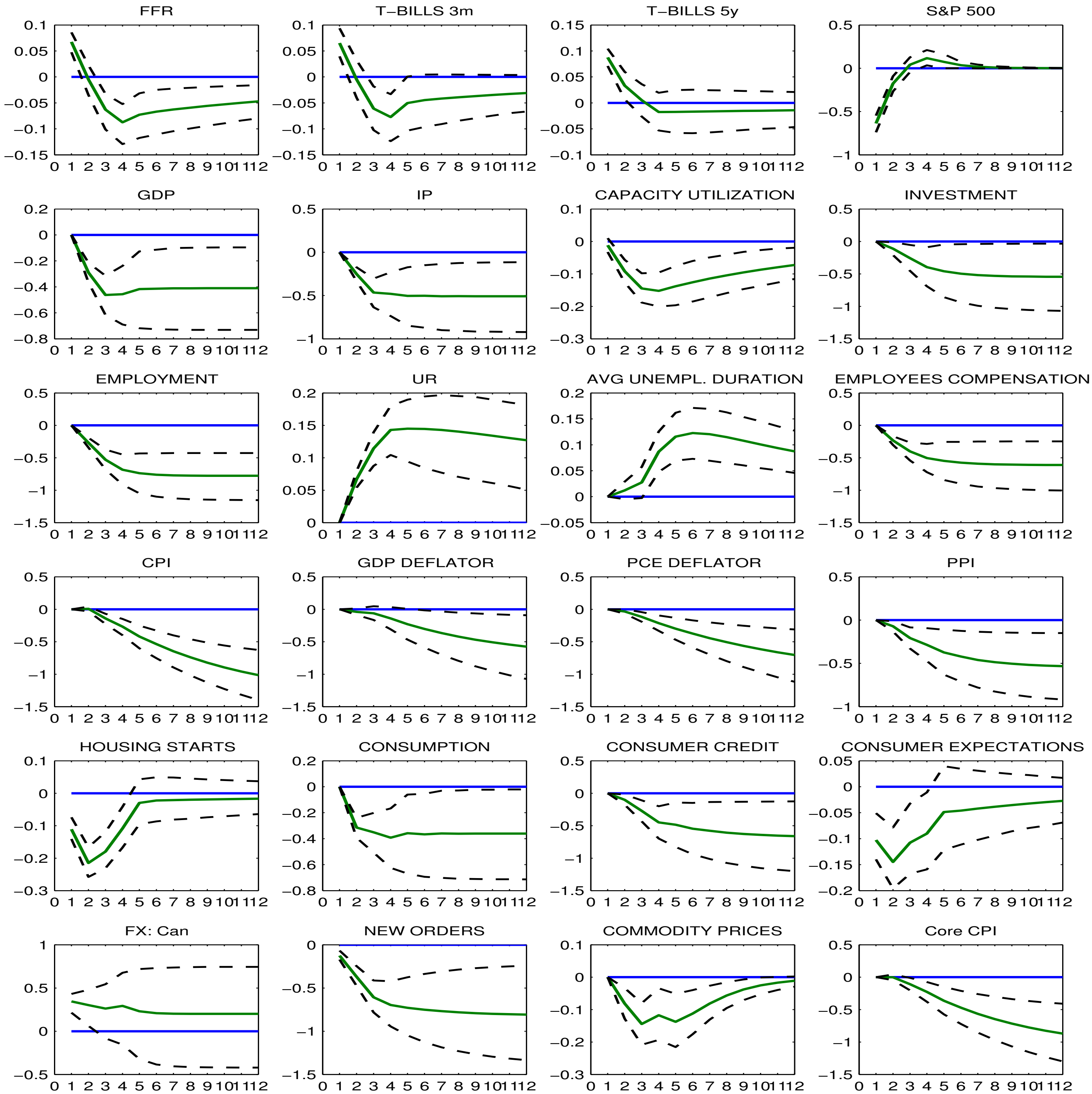

Figure 1: Example 1: dynamic responses to a contractionary monetary policy shock

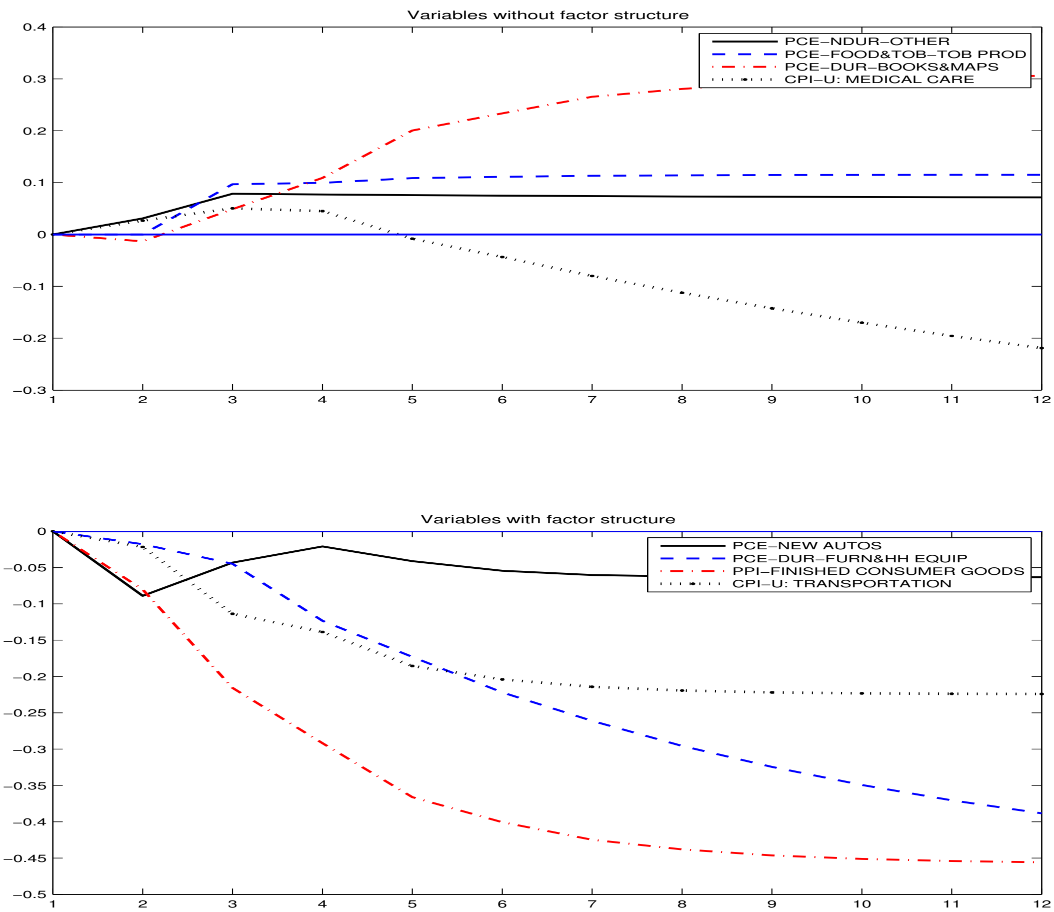

Figure 2: Example 1: dynamic responses to a monetary policy shock (aggregate data)

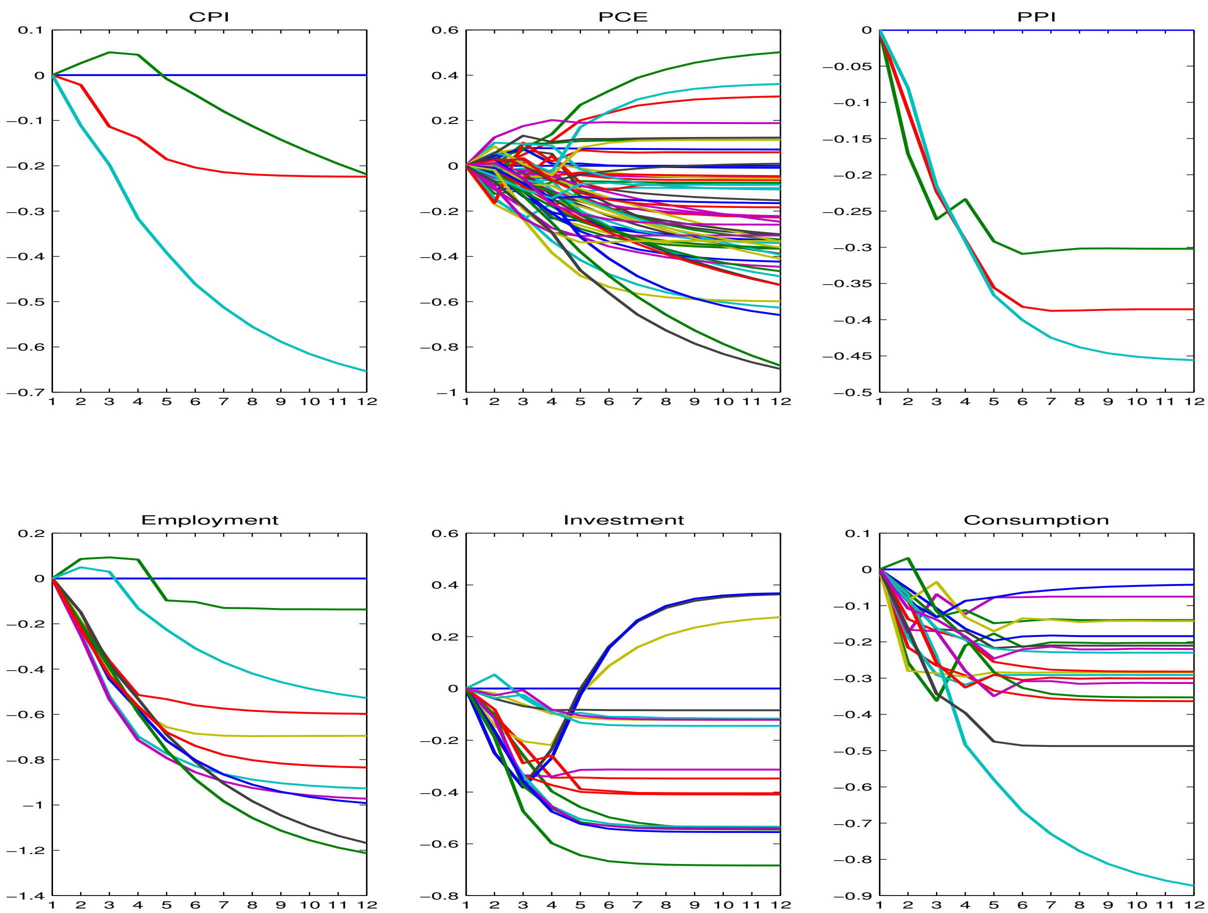

Figure 3: Example 1: dynamic responses to a monetary policy shock (disaggregate data)

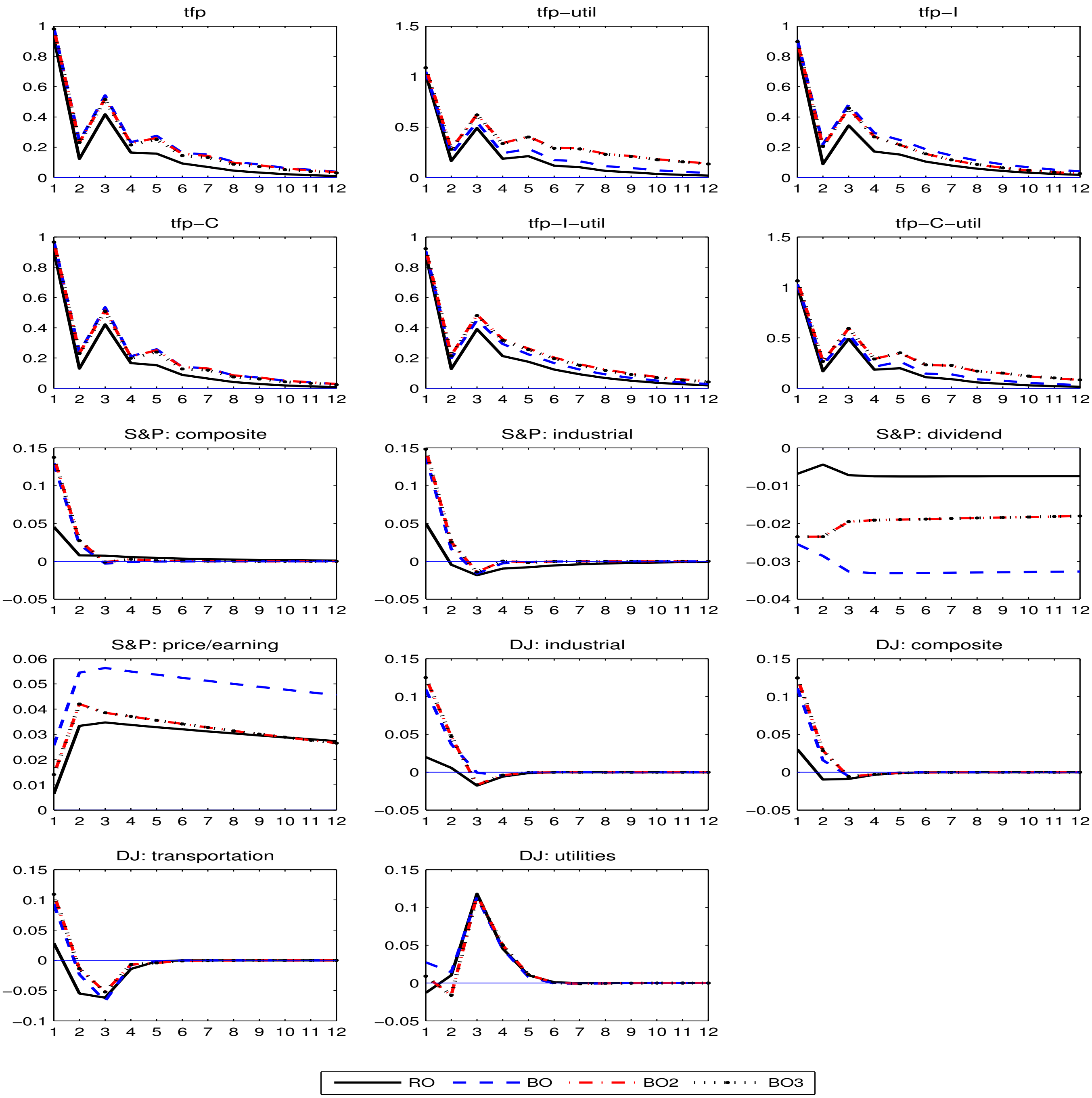

Figure 4: Example 2, dynamic responses of X_t^{TFP} and X_t^{SP} to a positive technology shock

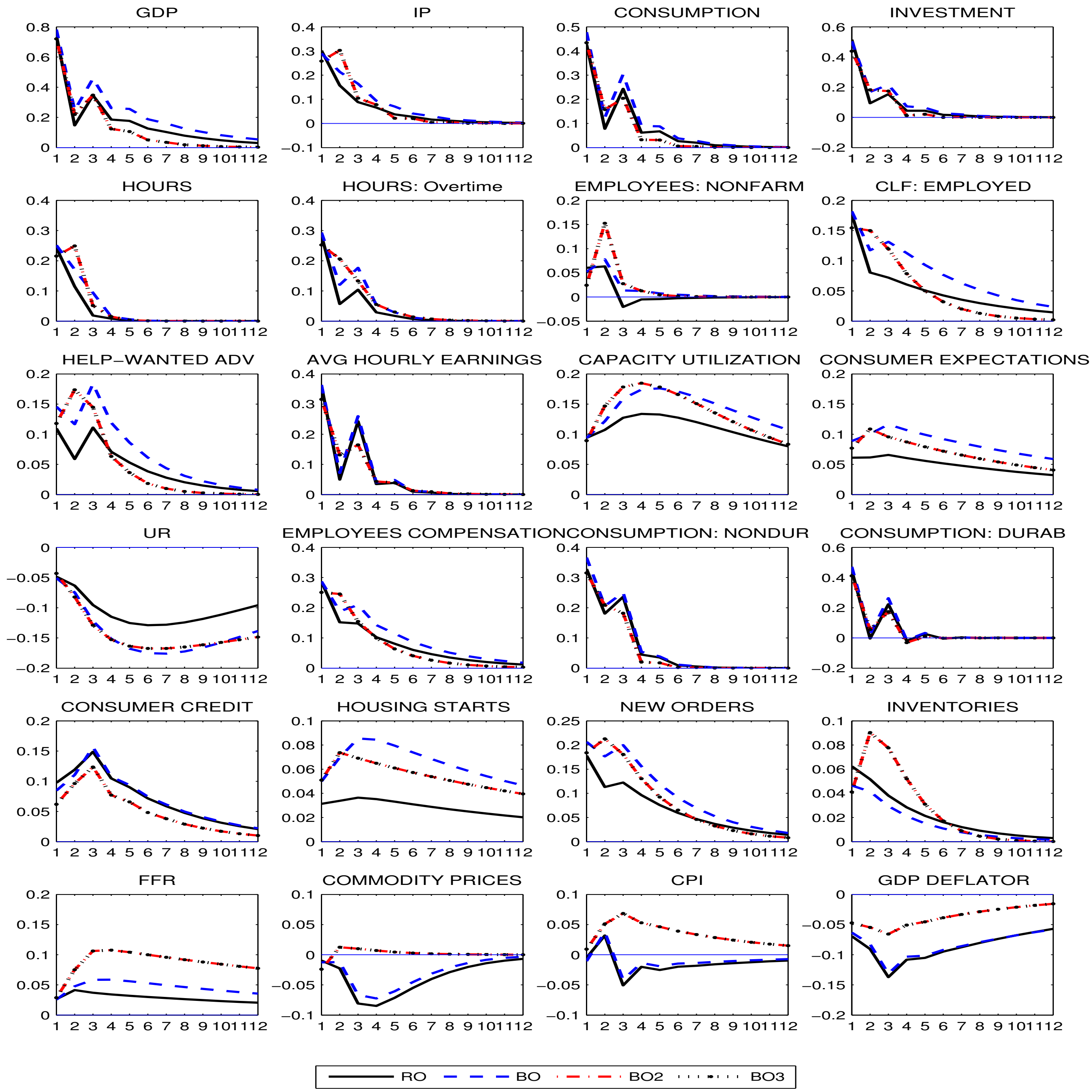

Figure 5: Example 2, dynamic responses of some series in X_t^{OTH} to a positive technology shock

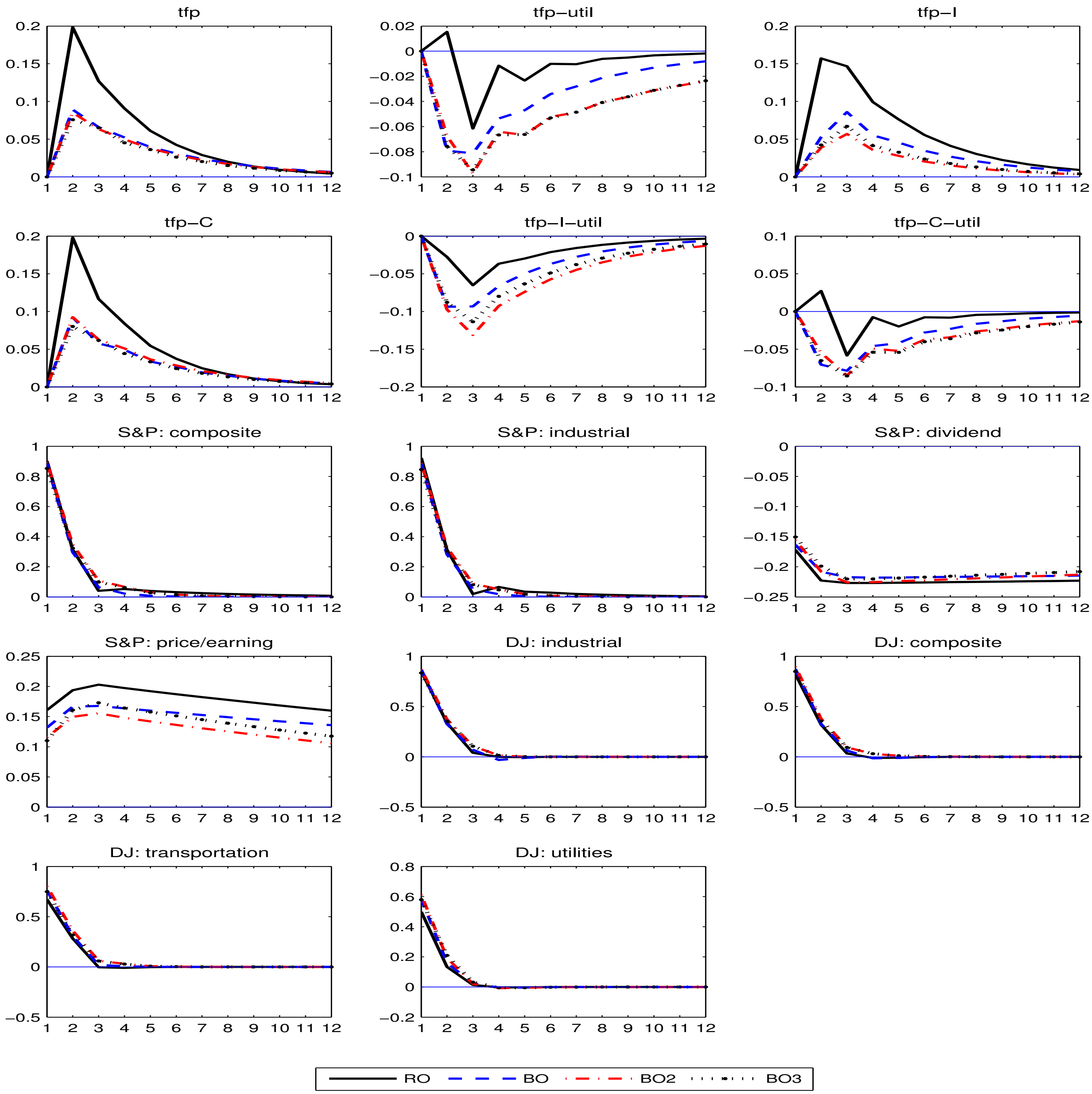

Figure 6: Example 2, dynamic responses of X_t^{TFP} and X_t^{SP} to a positive news shock

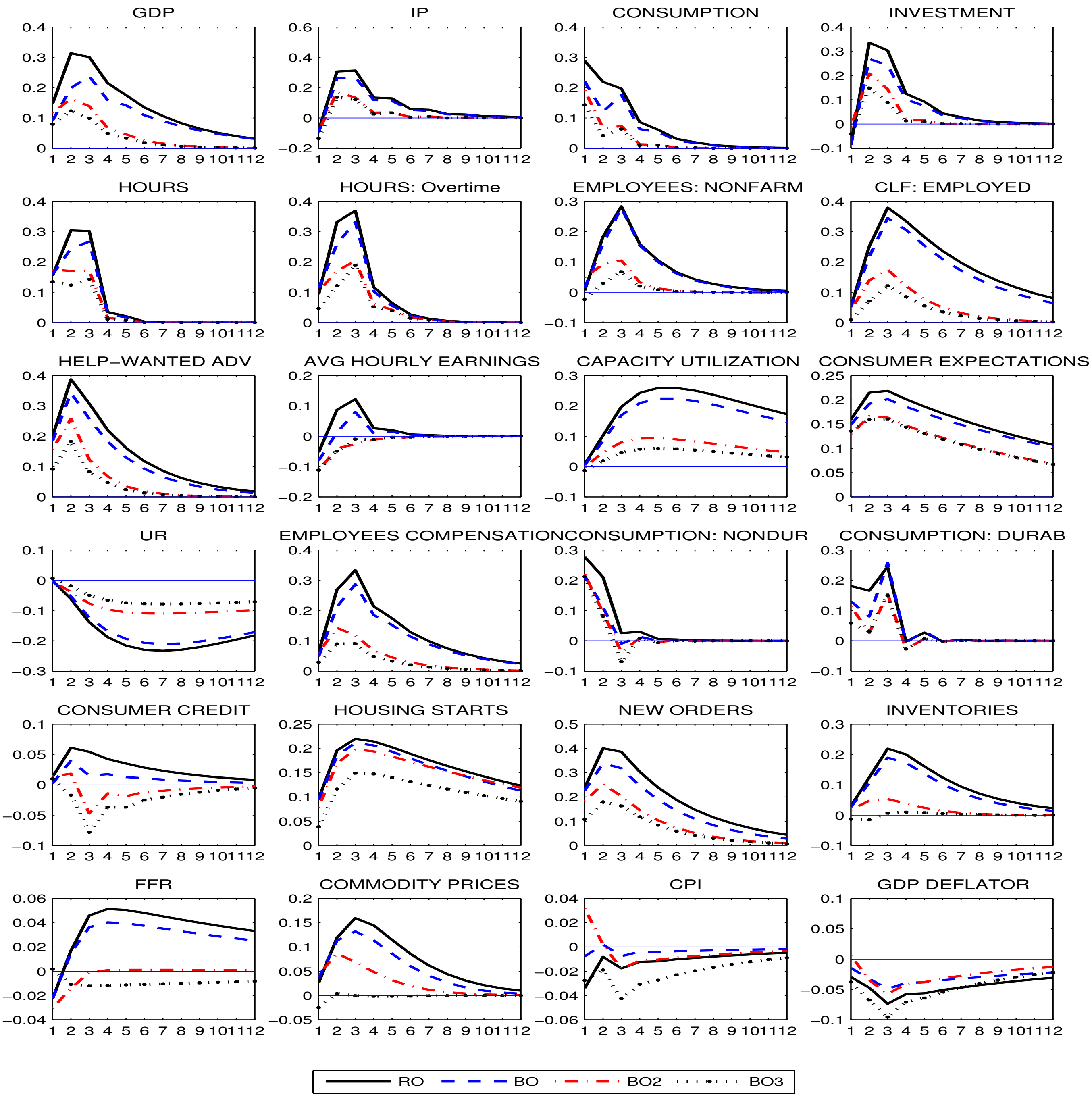

Figure 7: Example 2, dynamic responses of selected series in X_t^{OTH} to a positive news shock

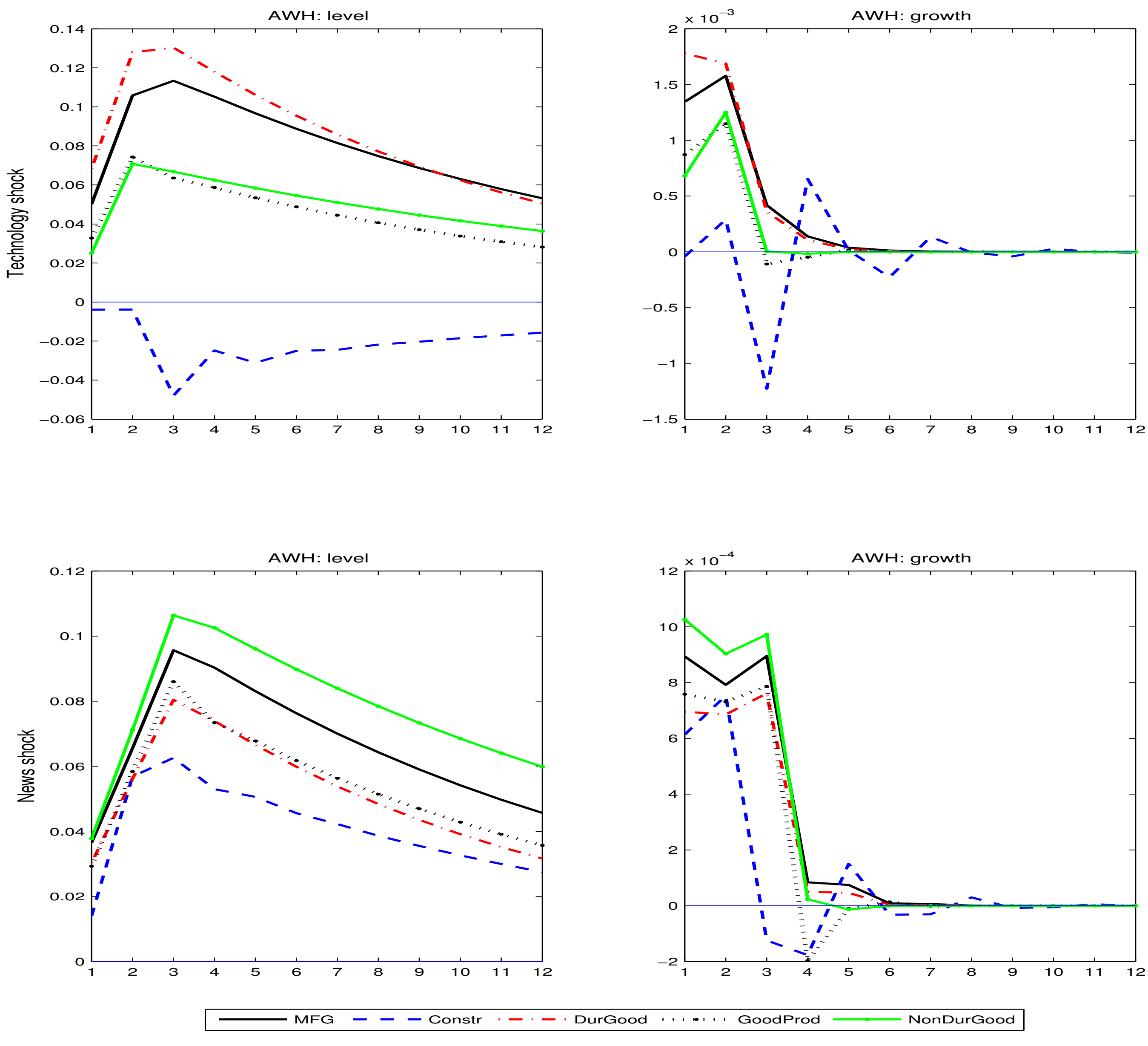

Figure 8: Example 2, dynamic responses of selected hours measures

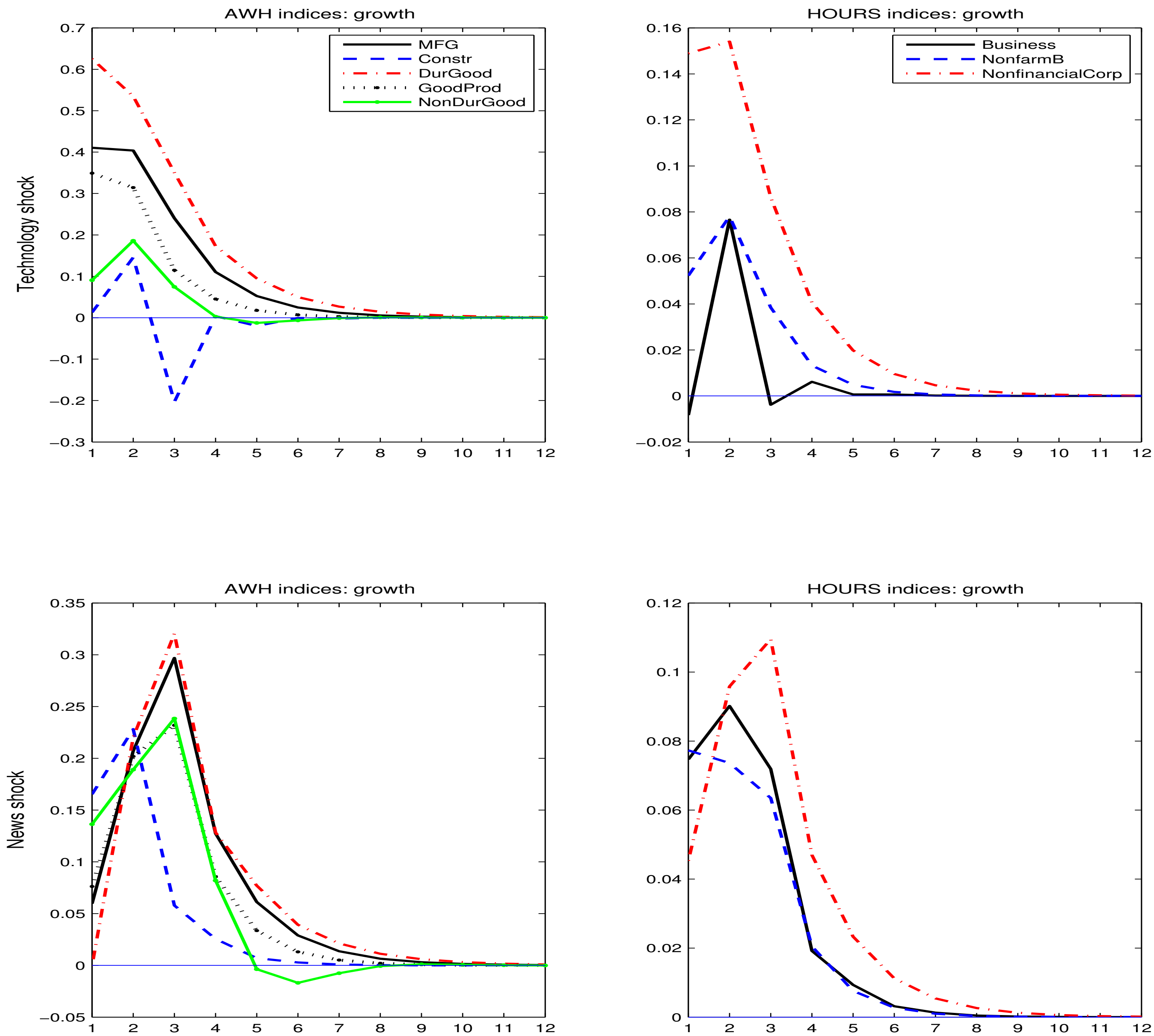

Figure 9: Example 2, dynamic responses of hours measured as indices

8 Appendix C: Data Sets

The transformation codes are: 1 no transformation; 2 first difference; 4 logarithm; 5 first difference of logarithm; 0 variable not used in the estimation (only used for transforming other variables). A * indicate a series that is deflated with the GDP deflator (series #89).

8.1 Data used in monetary policy shock example

No.	Series Code	T-Code	Series Description
1	DRIINTL:GDPRC@US.Q	5	NIA REAL GROSS DOMESTIC PRODUCT (CHAINED-2000), SA - U.S.
2	USCEN:GDPGDR.Q	5	REAL GDP-GDS,BILLIONS OF CH (2000) $,SAAR-US
3	USCEN:GDPSVR.Q	5	REAL GDP-SVC,BILLIONS OF CH (2000) $,SAAR-US
4	USCEN:GDPSR.Q	5	REAL GDP-STRUC,BILLIONS OF CH (2000) $,SAAR-US
5	BASIC:IPN11.M	5	INDUSTRIAL PRODUCTION INDEX - PRODUCTS, TOTAL
6	BASIC:IPN300.M	5	INDUSTRIAL PRODUCTION INDEX - FINAL PRODUCTS
7	BASIC:IPN12.M	5	INDUSTRIAL PRODUCTION INDEX - CONSUMER GOODS
8	BASIC:IPN13.M	5	INDUSTRIAL PRODUCTION INDEX - DURABLE CONSUMER GOODS
9	BASIC:IPN18.M	5	INDUSTRIAL PRODUCTION INDEX - NONDURABLE CONSUMER GOODS
10	BASIC:IPN25.M	5	INDUSTRIAL PRODUCTION INDEX - BUSINESS EQUIPMENT
11	BASIC:IPN32.M	5	INDUSTRIAL PRODUCTION INDEX - MATERIALS
12	BASIC:IPN34.M	5	INDUSTRIAL PRODUCTION INDEX - DURABLE GOODS MATERIALS
13	BASIC:IPN38.M	5	INDUSTRIAL PRODUCTION INDEX - NONDURABLE GOODS MATERIALS
14	BASIC:IPN10.M	5	INDUSTRIAL PRODUCTION INDEX - TOTAL INDEX
15	USCEN:UTLB00004.M	1	CAPACITY UTILIZ-MFG,SA-US
16	BASIC:PMI.M	1	PURCHASING MANAGERS' INDEX (SA)
17	BASIC:PMP.M	1	NAPM PRODUCTION INDEX (PERCENT)
18	DRIINTL:WS@US.Q	5*	NIA NOMINAL TOTAL COMPENSATION OF EMPLOYEES, SA - U.S.
19	USCEN:YPR.M	5	PERS INCOME CH 2000 $,SA-US
20	USCEN:YP@V00C.M	5	PERS INCOME LESS TRSF PMT CH 2000 $,SA-US
21	USCEN:AHPMF.M	5*	AHE,PROD WORKERS: MFG,SA-US
22	USCEN:AHPCON.M	5*	AHE,PROD WORKERS: CONSTR,SA-US
23	USCEN:HPMF.M	5	AWH,PROD WORKERS: MFG,SA-US
24	USCEN:HOPMD.M	5	AVG WEEKLY OT,PROD WORKERS: DUR,SA-US
25	BASIC:LHEL.M	5	INDEX OF HELP-WANTED ADVERTISING IN NEWSPAPERS (1967=100;SA)
26	BASIC:LHELX.M	1	EMPLOYMENT: RATIO; HELP-WANTED ADS:NO. UNEMPLOYED CLF
27	BASIC:LHEM.M	5	CIVILIAN LABOR FORCE: EMPLOYED, TOTAL (THOUS.,SA)
28	BASIC:LHNAG.M	5	CIVILIAN LABOR FORCE: EMPLOYED, NONAGRIC.INDUSTRIES (THOUS.,SA)
29	BASIC:LHUR.M	1	UNEMPLOYMENT RATE: ALL WORKERS, 16 YEARS & OVER (%,SA)
30	BASIC:LHU680.M	1	UNEMPLOY.BY DURATION: AVERAGE(MEAN)DURATION IN WEEKS (SA)
31	BASIC:LHU5.M	5	UNEMPLOY.BY DURATION: PERSONS UNEMPL.LESS THAN 5 WKS (THOUS.,SA)
32	BASIC:LHU14.M	5	UNEMPLOY.BY DURATION: PERSONS UNEMPL.5 TO 14 WKS (THOUS.,SA)
33	BASIC:LHU15.M	5	UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 WKS + (THOUS.,SA)
34	BASIC:LHU26.M	5	UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 TO 26 WKS (THOUS.,SA)
35	BASIC:CES001.M	5	EMPLOYEES, NONFARM - TOTAL NONFARM
36	BASIC:CES002.M	5	EMPLOYEES, NONFARM - TOTAL PRIVATE
37	BASIC:CES003.M	5	EMPLOYEES, NONFARM - GOODS-PRODUCING
38	USCEN:CR.Q	5	REAL PCE,BILLIONS OF CH (2000) $,SAAR-US
39	USCEN:JQCDR.Q	5	REAL PCE-DUR,QTY INDEX (2000=100),SA,SA-US
40	USCEN:JQCNR.Q	5	REAL PCE-NDUR,QTY INDEX (2000=100),SA,SA-US
41	USCEN:JQCSVR.Q	5	REAL PCE-SVC,QTY INDEX (2000=100),SA,SA-US
42	USCEN:JQCXFAER.Q	5	REAL PCE EX FOOD&ENERGY,QTY INDEX (2000=100),SAAR-US
43	DRIINTL:CGRCUS.Q	5	REAL GOVERNMENT CONS. EXPEND.& GROSS INVESTMENT (CHAINED-2000), SA - U.S.
44	USCEN:I.Q	5*	GROSS PRIV DOM INVEST,BILLIONS OF $,SAAR-US
45	USCEN:IF.Q	5*	GROSS PRIV DOM INVEST-FIXED,BILLIONS OF $,SAAR-US
46	USCEN:IFNRE.Q	5*	GROSS PRIV DOM INVEST-FIXED NONRES,BILLIONS OF $,SAAR-US
47	USCEN:IFRES.Q	5*	PRIV FIXED INVEST-RES-STRUC,BILLIONS OF $,SAAR-US
48	USCEN:IFRE.Q	5*	GROSS PRIV DOM INVEST-FIXED RES,BILLIONS OF $,SAAR-US
49	USCEN:II.Q	1	GROSS PRIV DOM INVEST-CH IN PRIV INVENT,BILLIONS OF $,SAAR-US
50	USCEN:IIF.Q	1	GROSS PRIV DOM INVEST-CH IN PRIV INVENT-FARM,BILLIONS OF $,SAAR-US
51	BASIC:HSFR.M	4	HOUSING STARTS:NONFARM(1947-58);TOTAL FARM&NONFARM(1959-)(THOUS.,SA
52	BASIC:HMOB.M	4	MOBILE HOMES: MANUFACTURERS' SHIPMENTS (THOUS.OF UNITS,SAAR)
53	BASIC:PMNV.M	1	NAPM INVENTORIES INDEX (PERCENT)
54	BASIC:PMNO.M	1	NAPM NEW ORDERS INDEX (PERCENT)
55	BASIC:PMDEL.M	1	NAPM VENDOR DELIVERIES INDEX (PERCENT)
56	BASIC:MOCMQ.M	5	NEW ORDERS (NET) - CONSUMER GOODS & MATERIALS, 1996 DOLLARS (BCI)
57	BASIC:MSONDQ.M	5	NEW ORDERS, NONDEFENSE CAPITAL GOODS, IN 1996 DOLLARS (BCI)
58	USCEN:M.Q	5	IMPORTS OF GDS&SVC,BILLIONS OF $,SAAR-US
59	USCEN:X.Q	5	EXPORTS OF GDS&SVC,BILLIONS OF $,SAAR-US
60	BASIC:FSPCOM.M	5	S&P'S COMMON STOCK PRICE INDEX: COMPOSITE (1941-43=10)
61	BASIC:FSPIN.M	5	S&P'S COMMON STOCK PRICE INDEX: INDUSTRIALS (1941-43=10)
62	BASIC:FSDXP.M	1	S&P'S COMPOSITE COMMON STOCK: DIVIDEND YIELD (% PER ANNUM)
63	BASIC:FSPXE.M	1	S&P'S COMPOSITE COMMON STOCK: PRICE-EARNINGS RATIO (%,NSA)
64	BASIC:EXRUK.M	5	FOREIGN EXCHANGE RATE: UNITED KINGDOM (CENTS PER POUND)
65	BASIC:EXRCAN.M	5	FOREIGN EXCHANGE RATE: CANADA (CANADIAN $ PER U.S.$)
66	BASIC:FYGM3.M	1	INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,3-MO.(% PER ANN,NSA)
67	BASIC:FYGM6.M	1	INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,6-MO.(% PER ANN,NSA)
68	BASIC:FYGT1.M	1	INTEREST RATE: U.S.TREASURY CONST MATURITIES,1-YR.(% PER ANN,NSA)
69	BASIC:FYGT5.M	1	INTEREST RATE: U.S.TREASURY CONST MATURITIES,5-YR.(% PER ANN,NSA)
70	BASIC:FYGT10.M	1	INTEREST RATE: U.S.TREASURY CONST MATURITIES,10-YR.(% PER ANN,NSA)
71	BASIC:FYAAAC.M	1	BOND YIELD: MOODY'S AAA CORPORATE (% PER ANNUM)
72	BASIC:FYBAAC.M	1	BOND YIELD: MOODY'S BAA CORPORATE (% PER ANNUM)

73	FYGM6-FYFF	1	
74	FYGM3-FYFF	1	
75	FYGT1-FYFF	1	
76	FYGT5-FYFF	1	
77	FYGT10-FYFF	1	
78	FYAAAC-FYFF	1	
79	FYBAAC-FYFF	1	
80	BASIC:FM1.M	5	MONEY STOCK: M1(CURR,TRAV.CKS,DEM DEP,OTHER CK'ABLE DEP)(BIL$,SA)
81	BASIC:FM2.M	5	MONEY STOCK:M2(M1+O'NITE RPS,EURO$,G/P&B/D MMMFS&SAV&SM TIME DEP(BIL$)
82	USCEN:MNY2@00.M	5	MONEY SUPPL-M2 IN 2000 $,SA-US
83	BASIC:FMFBA.M	5	MONETARY BASE, ADJ FOR RESERVE REQUIREMENT CHANGES(MIL$,SA)
84	BASIC:FMRRA.M	5	DEPOSITORY INST RESERVES:TOTAL,ADJ FOR RESERVE REQ CHGS(MIL$,SA)
85	BASIC:FMRNBA.M	2	DEPOSITORY INST RESERVES:NONBORROWED,ADJ RES REQ CHGS(MIL$,SA)
86	USCEN:ALCIBL00Z.M	5	COML&IND LOANS OUTST,SA-US
87	BASIC:FCLBMC.M	1	WKLY RP LG COM'L BANKS:NET CHANGE COM'L & INDUS LOANS(BIL$,SAAR)
88	BASIC:CCINRV.M	5	CONSUMER CREDIT OUTSTANDING - NONREVOLVING(G19)
89	DRIINTL:PGDP@US.Q	5	NIA PRICE DEFLATOR - GROSS DOMESTIC PRODUCT, SA - U.S.
90	DRIINTL:PCP@US.Q	5	NIA PRICE DEFLATOR - PRIVATE CONSUMPTION EXPENDITURE, SA - U.S.
91	USCEN:PDII.Q	5	GROSS PRIV DOM INVEST,PRICE DEFLATORS (2000=100),SA,SA-US
92	USCEN:JPCD.Q	5	PCE-DUR,PRICE INDEX (2000=100),SA,SA-US
93	USCEN:JPCN.Q	5	PCE-NDUR,PRICE INDEX (2000=100),SA,SA-US
94	USCEN:JPCSV.Q	5	PCE-SVC,PRICE INDEX (2000=100),SA,SA-US
95	BASIC:PUXM.M	5	CPI-U: ALL ITEMS LESS MEDICAL CARE (82-84=100,SA)
96	BASIC:PUXHS.M	5	CPI-U: ALL ITEMS LESS SHELTER (82-84=100,SA)
97	BASIC:PUXF.M	5	CPI-U: ALL ITEMS LESS FOOD (82-84=100,SA)
98	BASIC:PUS.M	5	CPI-U: SERVICES (82-84=100,SA)
99	BASIC:PUCD.M	5	CPI-U: DURABLES (82-84=100,SA)
100	BASIC:PUC.M	5	CPI-U: COMMODITIES (82-84=100,SA)
101	BASIC:PUNEW.M	5	CPI-U: ALL ITEMS (82-84=100,SA)
102	BASIC:PWFSA.M	5	PRODUCER PRICE INDEX: FINISHED GOODS (82=100,SA)
103	BASIC:PMCP.M	1	NAPM COMMODITY PRICES INDEX (PERCENT)
104	UOMO83	1	COMPONENT INDEX OF CONSUMER EXPECTATIONS, NSA, CONFBOARD AND U.MICH.
105	DRIINTL:JLEAD@US.Q	5	COMPOSITE CYCLICAL INDICATOR (1996) - LEADING, SA - U.S.
106	DRIINTL:JLAG@US.Q	5	COMPOSITE CYCLICAL INDICATOR (1996) - LAGGING, SA - U.S.
107	DRIINTL:JCOIN@US.Q	5	COMPOSITE CYCLICAL INDICATOR (1996) - COINCIDENT, SA - U.S.
108	USCEN:NC16&Z.M	0	CIVILIAN NONINSTITUTIONAL POP: 16 YEARS&OVER,SA-US
109	BASIC:FYFF.M	1	INTEREST RATE: FEDERAL FUNDS (EFFECTIVE) (% PER ANNUM,NSA)

8.2 Additional series used in news shock example

No.	Series Code	T-Code	Series Description
			TFP measures
1		1	FERNALDS'S BUSINESS SECTOR TFP
2		1	FERNALDS'S BUSINESS SECTOR UTILIZATION-ADJUSTED TFP
3		1	FERNALDS'S INVESTMENT SECTOR TFP
4		1	FERNALDS'S INVESTMENT SECTOR UTILIZATION-ADJUSTED TFP
5		1	FERNALDS'S CONSUMPTION SECTOR TFP
6		1	FERNALDS'S CONSUMPTION SECTOR UTILIZATION-ADJUSTED TFP
			SP measures
7	BASIC:FSPCOM.M	5	S&P'S COMMON STOCK PRICE INDEX: COMPOSITE (1941-43=10)
8	BASIC:FSPIN.M	5	S&P'S COMMON STOCK PRICE INDEX: INDUSTRIALS (1941-43=10)
9	BASIC:FSDXP.M	1	S&P'S COMPOSITE COMMON STOCK: DIVIDEND YIELD (% PER ANNUM)
10	BASIC:FSPXE.M	1	S&P'S COMPOSITE COMMON STOCK: PRICE-EARNINGS RATIO (%,NSA)
11		5	DOW JONES INDEX: INDUSTRIALS
12		5	DOW JONES INDEX: COMPOSITE
13		5	DOW JONES INDEX: TRANSPORTATION
14		5	DOW JONES INDEX: UTILITIES
			OTHER measures of hours worked
15			AVG WEEKLY HOURS: MANUFACTURING
16			AVG WEEKLY HOURS: CONSTRUCTION
17			AVG WEEKLY HOURS: DURABLE GOODS
18			AVG WEEKLY HOURS: GOODS PRODUCING
19			AVG WEEKLY HOURS: NONDURABLE GOODS
20			AWH INDEX: MANUFACTURING
21			AWH INDEX: CONSTRUCTION
22			AWH INDEX: DURABLE GOODS
23			AWH INDEX: GOODS PRODUCING
24			AWH INDEX: NONDURABLE GOODS
25			TOTAL HOURS INDEX: BUSINESS SECTOR
26			TOTAL HOURS INDEX: NONFARM BUSINESS SECTOR
27			TOTAL HOURS INDEX: NONFINANCIAL CORPORATION SECTOR

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