Optimal policy commitment: investment deterrence versus option value

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Abstract

The consistency and predictability of regulation is thought to be highly important for costly investment decisions. When the government lacks commitment power to maintain a promised policy, and firms are not perfectly competitive, investment levels are typically lower than when the government can commit. However, flexibility in policymaking is beneficial in that it allows for adjustment when conditions change or new information arrives. Shocks to growth or to individuals sectors of the economy, for example, can change the optimal level of taxation. In the environmental context, an important factor for policy decisions is the expected damage caused by a pollutant, which can change over time as new economic conditions and information arrive. What is the optimal frequency and intensity of changes in legislation when both option value and less-than-full commitment power are present? The solution requires balancing the harm of unpredictable legislation with the benefits of option value. This paper considers this tradeoff to derive the optimal policy commitment in the presence of both irreversible investment decisions and informational uncertainty about the damage done by a pollutant. I show that in many, but not all, cases the government will want to commit to lower-than-Pigouvian tax rates in the state of the world where the pollutant is damaging. However, equilibrium investment rates in this

*I thank Leonid Chindelevitch, Don Fullerton, Julian Reif and Mar Reguant Rido for helpful feedback.

partial commitment model are not always larger than when the government lacks the power to commit to a specific tax rate and are not always lower than when the government has full commitment power.

1 Introduction

The consistency and predictability of environmental regulation is thought to be highly important for costly investment decisions. When the government lacks credibility in its announcement of future policies, investment is costly, and firms have some market power or are risk averse, investment levels are typically lower than when the government can commit to a given policy. This problem can arise in many settings, from multi-million dollar companies planning a power plant to individuals deciding what fraction of their income to save for pensions and whether to put it into a traditional or Roth retirement account.[1] At the same time, commitment to a particular policy can be suboptimal if there is potential for learning about what the optimal policy is over time.

This paper considers this tradeoff to derive the optimal level of commitment to future policy in the presence of both costly investment decisions and informational uncertainty about the damage done by a pollutant. I build a simple model where firms may not invest at the socially optimal level, even once the potential for externalities is accounted for, because of expectations of higher taxes in the future. Specifically, I assume that N firms make and sell a consumption good that may have a harmful externality. Each firm faces a decision about how much to invest in lowering its future marginal cost of production. Firms' investment decisions are affected by their expectations of the future tax rate. After the investment decision is made, the uncertainty is resolved, and the government chooses the appropriate tax rate. A distinguishing characteristic of this paper is to assume that the government has some power to make it costly to deviate from a promised future tax rate and can credibly announce this commitment level prior to the investment decisions. I show that, under certain conditions, the government will commit to deviate from ex-post efficient

[1] Retirement savings are a costly investment because they typically cannot be withdrawn until the contributor is close enough to retirement age.

taxation in order to encourage investment. However, committing to no taxation is never optimal. I then analyze the effect on the optimal level of commitment of (a) the extent of certainty about the pollutant's harm, (b) the extent to which the pollutant is harmful, and (c) the relationship between future taxes and investment. I also compare this setting to one where the government can subsidize investment directly.

To ensure that costly investment is profitable for firms I assume that they have some market power. Otherwise, the firms cannot recoup the investment costs in the production stage. The presence of market power also means that the second-period tax when the government has no commitment power may differ from the Pigouvian tax. At the extreme, the government may actually want to *subsidize* production. I separate the effect of partial commitment power from the effect of firms' market power by comparing the realized tax to both the Pigouvian tax and the second period tax when the government has no commitment power.

Although I have not yet completed formal proofs, preliminary simulation results show that the tax rate in the state of the world where the pollution created by production is always weakly lower than the marginal damage of the pollutant. In addition, this tax rate increases with the extent of damage, but decreases weakly with the probability of damage. The relationship between investment when the government has partial commitment power, as in the model developed here, and when the government has no or full commitment power is ambiguous. Similarly, contrary to popular intuition, allowing the government to have some commitment power does not unambiguously lead to a lower tax rate than in the case where the government cannot commit.

I contribute to two strands of literature. First, a substantial literature examines optimal dynamic regulation of pollution under uncertainty (see for example Kolstad, 1996a,b; Heutel, 2011). Many of these papers assume, explicitly or implicitly, that the government can commit perfectly to its future course of action or that lack of commitment has no effect. The second strand examines the role of uncertainty in investment decisions (see for example Bernanke, 1983; Abel and Eberly, 1999; Abel et al., 1996; Dixit and Pindyck, 1994; Caballero, 1991; Dixit, 1995). Abel and Eberly (1999) show that uncertainty can either increase or decrease the long-run capital stock

when there is irreversibility. The ambiguity of uncertainty's impact on investment is due to two countervailing effects. The first effect of uncertainty is a larger user cost of capital, which tends to reduce the capital stock. The second is a "hangover effect", where the firm cannot reverse the investment decision, which tends to increase it. Bloom (2009) studies the impact of uncertainty shocks with a time-varying second moment and shows that an increase in uncertainty causes firms to temporarily pause investment and hiring. Bloom et al. (2007) show that combining uncertainty and irreversibility reduces the responsiveness of investment to demand shocks. I contribute to this literature by allowing the government to choose the uncertainty that firms face and ask how this changes investment decisions.

I am aware of two papers that examine the tradeoff between uncertainty and option value. In the area of macroeconomics, Athey et al. (2005) examine the problem of a time-inconsistent monetary authority and shows that the optimal policy can be achieved by legislating a cap on inflation, which reduces the amount of discretion the monetary authority has. Amador et al. (2003) examine the issue of commitment in the area of pension policy, where savers experience both temptation and taste shocks, which are unobservable to the policy maker. They show that the optimal policy is to mandate a minimum savings level.

The rest of the paper is organized as follows. In Section 2, I describe how the government can affect firms' expectations about future legislation. Section 3 describes the model and its implications for the optimal level of commitment. Section 4 presents simulation results and Section 5 concludes.

2 Institutions, rules, and policy expectations

Governments can affect firms' expectations about the extent of future legislation in numerous ways. At the most basic level, institutional characteristics can affect the probability of a particular policy being implemented. Allowing filibusters, for example, makes it more difficult to pass a given policy in the case of a diverse distribution of beliefs among the legislators, as is requiring a super-majority

of two-thirds. One permanent feature that may reduce the probability of changes in legislation is having two chambers of representatives instead of one (assuming each has to pass legislation with a majority vote). Another is giving the Executive Branch veto power. Some institutional features have an ambiguous effect on predictability, such as delegating certain policy-making to states or other sub-divisions of the country and having less frequent congressional elections.

How a policy is written may affect its probability of changing as well. Policies with sunset provisions, such as renewable energy credits and tax cuts for the wealthy, are more likely to change than policies with no expiration date. A policy may also be written with built-in adjustments that depend on the realization of the state of the world. Joining an international treaty or organization (e.g., WTO, Kyoto) and thus agreeing to abide by joint rules also affects expectations about future national policy, although whether this makes policy changes more or less frequent depends on the particular setting.

How salient a policy is also affects its likelihood of changing. More salient policies, such as Medicare and income taxes, may be harder to implement or alter than less salient ones, such as import quotas on obscure goods. In some cases, a government can make a policy more salient by bringing it to the public's attention, as Barack Obama has attempted with the American Jobs Act. Relatedly, expectations may differ about policy that is written by Congress, the President or Federal Agencies ("executive branch regulation"). Charging a government agency, such as the EPA, with writing the detailed rules for a bill also affects the expected policy. In short, one can map permanent and temporary features of policy-making into policy expectations in a plethora of ways.

The idea is that the government lacks full commitment power but can nevertheless commit itself to facing high costs of changing a tax in the future. This idea might sound implausible. None of the above settings ensure perfect commitment to a particular policy. Given the right circumstances, for example, a government may find it beneficial to alter the filibuster rules. However, government does not have to literally announce the cost of changing the future tax and have the actual cost be

exactly equal to the announcement. Similar to the revelation principle in mechanism design,[2] as long as a one-to-one mapping between what the government says and the probability of it being able to set a certain tax rate exists, one can think of this problem in terms of the government setting a future cost for implementing a tax. For example, if the government announces that it will eschew new taxes, one does not have to believe that the government will set no new taxes. As long as the announcement lowers the probability of a tax being implemented, it can be used as a partial commitment mechanism. While modeling the multiple layers of commitment may be informative, I abstract from the complicated structures of the real world and assume that government has a single commitment parameter that it can set.

3 Model

3.1 Setup

In this section, I outline the simplest model that captures the tradeoff between the harmful effects of policy uncertainty and the benefits of waiting for the resolution of environmental uncertainty.

I assume a production externality of the form $\theta\phi\left(Q\right)$, where Q is aggregate production of a good, and $\theta \in \{0,1\}$ is not known until the second time period ($t = 2$). If $\theta = 0$, production has no externality. The marginal cost of production is a constant $\kappa\left(I\right)$, where I is the amount of investment chosen by the firm in the first time period ($t = 1$). To avoid confusing notation, I denote the state of the world where the pollutant is damaging $(\theta = 1)$ by the subscript "D" and the state of the world where it is safe $(\theta = 0)$ by the subscript "S". Social surplus, net of production costs or damages, is given by $SS\left(Q\right)$. Specifically, if $P\left(q\right)$ is the inverse consumer demand function, social surplus is equal to $\int_0^Q P\left(q\right)dq - \theta\phi\left(Q\right) - \kappa\left(I\right)Q$.

At $t = 0$, the government has some expectations about the realization of θ. I denote the probability that the production is damaging by $\Pr\left(\theta = 1\right) = \gamma$. γ is public knowledge. Also at $t = 0$, the government announces a future tax rate, τ_0, and sets the cost of changing it in the

[2] See Dasgupta et al. (1979); Myerson (1979)

future, c, to maximize its objective function. This c can be thought of as political capital that the government has to incur to pass legislation, the probability that a particular legislation passes (although it is not explicitly modeled as that) or simply the resources that have to be expended to implement a particular tax rate. Although I treat it as a real cost in the government's objective function, I am agnostic as to whether it is a cost from a social welfare standpoint.

The model has N producers who decide how much to invest in lowering the marginal production cost of the possibly harmful good. They observe the government's time 0 decision, and they know that the demand for the good will be $Q\left(p\right)$, where p is the gross of tax price to consumers. At $t=1$, firms choose how much to invest in lowering the marginal cost of production, given a convex investment cost. The total investment cost for each firm is $\mu(I)$, and $\kappa\left(I\right)$ is the subsequent marginal cost of production. For simplicity, firms produce nothing at $t=1$, and the solution is assumed to be symmetric.

At $t=2$, the uncertainty about θ is resolved. The government can deviate from the promised tax rate and set it equal to τ^{θ} by incurring a cost of $c\left(\tau_0-\tau^{\theta}\right)^2$. Production also takes place at $t=2$. The marginal cost of production, $\kappa\left(I\right)$, for each firm is a function of the investment it made. Firms compete in quantities and are assumed to have some market power.[3]

To review the notation, $Q\left(p,I\right)$ is total production, $p=r+\tau^{\theta}$ is the price paid by consumers for the good, r is the revenue received by the firm, and $\theta\phi\left(Q\left(p,I\right)\right)$ is the damage of producing $Q\left(p,I\right)$. The total investment when the government sets the cost of changing legislation equal to c is $I(c)$. $c\left(\tau_0-\tau^{\theta}\right)^2$ is the cost of setting the tax rate equal to τ^{θ} in period 2. Finally, $P\left(q\right)$ is the inverse of the demand function $Q\left(p\right)$. I assume that all these functions are continuously differentiable. I also make standard assumptions about the demand, price, and investment functions: $P'(q)<0$, $Q'(p)<0$, $\mu'(I)>0$, $\mu''(I)>0$, $\kappa'(I)<0$, and $\phi'(Q)>0$. I also assume that the quantity produced is linear in the price, tax, and marginal cost of production. Namely, $Q_{\kappa}^{S}=Q_{\kappa}^{D}=Q_{\kappa}$, $Q_{\tau}^{D}=Q_{\tau}^{S}=Q_{\tau}$, and $Q_{\tau\tau}=0$. This assumption implies that $\int_0^{Q(p,I)}P\left(q\right)dq$ is a quadratic, which in turn implies that its second derivative is a constant.

[3]Without market power or convex costs of production, firms have no incentive to make costly investments because they cannot recoup their costs.

At $t = 0$, prior to investment costs being sunk, the government is assumed to solve the following maximization problem:

$$\max_{c,\tau_0} E_\theta \left[\int_0^{Q(p,I)} P(q)\, dq - \theta\phi\left(Q(p,I)\right) - \kappa(I)\, Q(p,I) - N\mu(I) - c\left(\tau_0 - \tau^\theta\right)^2 \right] \tag{1}$$

$$s.t.\ c \geq 0$$

Equation (1) may not be exactly equal to the social welfare function. The government may care partially about its own utility and partially about social welfare. Thus, while $c\left(\tau_0 - \tau^\theta\right)^2$ is a cost from the government's standpoint, it is not necessarily a welfare loss. One could have also allowed c to be negative. In that case, the government would be committing itself to deviating from the promised tax rate. This possibility may be interesting to consider in other settings.

At $t = 2$, θ is known, the investment costs are sunk, and c cannot be changed. The government's problem now is to choose the optimal tax rate given the constraint it has placed on itself:

$$\max_{\tau^\theta} \int_0^{Q(p,I)} P(q)\, dq - \theta\phi\left(Q(p,I)\right) - \kappa(I)\, Q(p,I) - c\left(\tau_0 - \tau^\theta\right)^2 \tag{2}$$

$$s.t.\ \tau^\theta \geq 0$$

Equations (1) and (2) demonstrate the fundamental commitment problem. Suppose there is no possibility for the government to impose a second-period cost on itself (in other words, $c = 0$). The government could then still announce a second-period tax rate in period 0. However, once firms have decided on the level of investment, and the investment costs are sunk, then the government has no incentive to keep its promise. If $c = 0$, it is easy to show that the government would equate the marginal social benefit of consuming the good to the marginal social cost. However, once the

government has made it costly to change the tax rate and pollution is damaging, this will no longer be true.

The condition that $\tau \geq 0$ is not innocuous. Because firms have market power, it is possible that the government would want to set a negative tax rate to bring production closer to the socially optimal level. I assume that the government is prevented from doing this by factors that are not modeled here. To the extent that subsidies justified by the presence of market power are not observed in practice, this is a reasonable assumption.

I proceed to use backward induction to derive (a) the government's choice of τ, (b) the firms' choice of production and investment, and (c) the government's choice of c. I then present and simulate an example that demonstrates the tradeoff between reducing uncertainty to stimulate investment and maintaining flexibility to address potential externalities.

3.2 Government's choice of τ at $\mathrm{t} = 2$

At $t = 2,\ \theta$ is known, investment I and cost of regulation c are fixed, and the investment costs are sunk. The first order condition corresponding to equation (2) can be written as:

$$-2c\left(\tau_0 - \tau^\theta\right) = \left(S_Q^\theta - \theta\phi_Q - \kappa\left(I\right)\right) Q_\tau \tag{3}$$

where the subscripts represent the partial derivative with respect to the given variables.

Lemma 1. $\tau^D \geq \tau^S$.

See appendix for a rough proof. Intuitively, conditional on the level of commitment, c, and the announced tax rate, τ_0, the optimal level of production is lower in the state of the world where production has lower marginal utility ($\theta = 1$). But because firms don't internalize the harm potentially caused by production, the government will set a higher tax rate in the state of the world where production is less beneficial.

Lemma 2. $1 > \tau^\theta_{\tau_0} > 0$. $\tau^\theta_c < 0$ if and only if $\tau^\theta > \tau_0$.

See appendix for proof. Although the second result is not fully intuitive, some explanation can be provided. Recall that the government incurs a cost of $c\left(\tau_0 - \tau^\theta\right)^2$ when setting the second period tax rate equal to τ^θ. Thus, one component in the marginal cost of changing the tax rate by one unit is $-2c\left(\tau_0 - \tau^\theta\right)$. When $\tau^\theta > \tau_0$, this quantity is positive and increasing c raises the marginal cost of changing the tax rate, thus causing τ^θ to fall with an increase c. When $\tau^\theta < \tau_0$, the opposite is true: increasing c lowers the marginal cost of changing the tax rate by making $-2c\left(\tau_0 - \tau^\theta\right)$, which implies that τ^θ increases with a rise in c.

3.3 Firms' investment and production choices

Recall that N firms, where N is large, compete in quantities. Each firm faces two decisions: (1) How much to invest in the first period and (2) How much to produce in the second period, conditional on the investment. The firms' problem can also be solved using backward induction.

Production at $\mathbf{t} = \mathbf{2}$. Conditional on the level of investment, each firm i solves the following problem:

$$\max_{q_i} \left(P\left(Q\right) - \tau\right) q_i - \kappa(I) q_i$$

where $Q = \sum_j q_j$, and $P\left(Q\right)$ is the inverse of the demand function $Q\left(p\right)$. I focus on the case of a symmetric equilibrium, where $q_i^* = q_j^* = q^*$ for all i and j.

The simplified first order condition relates the equilibrium quantity to the investment level as well as the tax rate:

$$\kappa(I) = P\left(Nq^*\right) + P_Q q^* - \tau$$

Let $\pi\left(\tau, I\right) = \left(P\left(Q\right) - \tau - \kappa\left(I\right)\right) q^*$ denote the equilibrium profit of each firm when investment level is I and the tax rate is τ.

Investment. The investment decision faced by the firms in the first period can be writted as:

$$max_I E_\theta \left[\pi\left(\tau, I\right)\right] - \mu(I)$$

$\pi(\tau, I)$ implicitly depends on the level of c announced by the government at $t = 0$ because the tax rate depends on c. However, I assume that the firms take the tax rate as given when making their investment decisions.

Denoting $Pr(\theta = 1) = \gamma$, we can write this as:

$$E[\pi(\tau, I)] = \gamma\pi\left(\tau^D, I\right) + (1-\gamma)\,\pi\left(\tau^S, I\right)$$

It can be shown through simple algebra (see Appendix) that the first order condition for investment simplifies to:

$$\mu_I = -\kappa_I E_\theta[q] \tag{4}$$

where

$$E_\theta[q] = \left(\gamma q^D + (1-\gamma)\, q^S\right)$$

Furthermore, a necessary and sufficient condition for the change in investment with each tax rate (I_{τ^θ}) to be negative is:

$$\kappa_{II} E_\theta[q] + \kappa_I^2 q_\kappa > -\mu_{II}$$

See Appendix for more details. Correspondingly, the relationship between investment and the announced τ_0 and c can be expressed as:

$$I_{\tau_0} = I_{\tau^D}\tau^D_{\tau_0} + I_{\tau^S}\tau^S_{\tau_0}$$

and

$$I_c = I_{\tau^D}\tau^D_c + I_{\tau^S}\tau^S_c$$

If the conditions for $I_{\tau^\theta} < 0$ are met, then $I_{\tau_0}$is unambiguously negative, while the sign of I_c is ambiguous.

3.4 Government setting c

The government's problem at $t = 0$ can be written as:

$$\max_{c,\tau_0} E_\theta \left[S\left(Q^\theta\right) - \theta\phi\left(Q^\theta\right) - \kappa\left(I\right)Q^\theta - c\left(\tau_0 - \tau^\theta\right)^2 \right] - N\mu\left(I\right)$$

The first order condition with respect to c can be shown to equal:

$$\gamma\left(\tau_0 - \tau^D\right)^2 + (1-\gamma)\left(\tau_0 - \tau^S\right)^2 = \kappa_I I_c \left(E_\theta\left[SS_Q\right]Q_\kappa - E_\theta\left[Q\right]\right) - N\mu_I I_c$$

Where SS_Q^D denotes the net marginal social surplus in the "damaging" state of the world, $S_Q^D - \phi_Q - \kappa\left(I\right)$. SS_Q^S denotes the net marginal surplus in the "safe" state of the world, $S_Q^S - \kappa\left(I\right)$. Finally, $E_\theta\left[SS_Q\right]$ denotes the expected net marginal social surplus, $\gamma SS_Q^D + (1-\gamma)SS_Q^S$. See appendix for details.

Similarly, the first order condition with respect to τ_0 is equal to:

$$0 = \kappa_I I_{\tau_0}\left(E_\theta\left[SS_Q\right]Q_\kappa - E_\theta\left[Q\right]\right) - N\mu_I I_{\tau^0} + E_\theta\left[SS_Q\right]Q_\tau$$

See appendix for mathematical details.

3.5 Comparison with other solutions

Several important questions remain at this point. First, how do the investment levels and welfare with partial commitment compare to (a) the socially optimal investment levels and (b) the case where government has full commitment power? Second, what happens if the government can subsidize or tax investment directly? Although I have yet to derive formal results, I present how the government's problem changes across all these scenarios.

3.5.1 Socially Optimal Investment

First, suppose the government can choose the investment level directly and sets the second period taxes as before. The government's problem for choosing investment is:

$$\max_I E_\theta \left[S\left(Q^\theta\right) - \theta\phi\left(Q^\theta\right) - \kappa\left(I\right)Q^\theta\right] - N\mu\left(I\right)$$

The first order condition when the government is setting investment is equal to:

$$\begin{aligned} N\mu_I &= \gamma\left(SS_Q^D Q_\kappa^D \kappa_I - \kappa_I Q^D\right) + \\ &+ \left(1-\gamma\right)\left(SS_Q^S Q_\kappa^S \kappa_I - \kappa_I Q^S\right) \end{aligned}$$

Recall that the firms' first order condition for the optimal level of investment is:

$$\mu_I = -\kappa_I\left(\gamma q^D + \left(1-\gamma\right)q^S\right)$$

Multiplying the firms' first order condition by N results in a first order condition comparable to that of the government:

$$N\mu_I = -\kappa_I\left(\gamma Q^D + \left(1-\gamma\right)Q^S\right)$$

Because κ does not depend on N and q^θ is simply $\frac{Q^\theta}{N}$, these equations differ in that the firm considers its own revenue when deciding how much to invest while the government also considers the marginal social utility. Clearly, the firm does not take the existence of the externality into account when it decides on its investment levels. The extra component in the government's problem is

$$\gamma SS_Q^D Q_\kappa^D \kappa_I + \left(1-\gamma\right)SS_Q^S Q_\kappa^S \kappa_I$$

$\kappa_I < 0$ and $Q_\kappa < 0$. Because the government can set the tax rate in each world, the net marginal social utility in each state of the world will be greater than or equal to 0. Thus, this quantity is greater than or equal to 0. Because we assume that investment costs are convex, this implies that, *conditional on the tax rate*, $I_G^* > I_F^*$, where the subscript G represents the government and F represents the firms. However, as the simulations show in later sections, once the tax rate is allowed to vary, the relationship between socially optimal investment and investment with partial commitment is ambiguous.

3.5.2 Taxation with full commitment power

Next, suppose the government has full commitment power. In other words, the government can credibly announce τ^D and $t = 0$. The government now maximizes:

$$\max_{\tau^\theta} E_\theta \left[S\left(Q^\theta\right) - \theta\phi\left(Q^\theta\right) - \kappa\left(I\right) Q^\theta - N\mu(I) \right]$$

The government's first order condition with respect to τ^D is:

$$\begin{aligned} N\mu_I I_\tau \;\; = \;\; & \gamma\left(\left(S_Q^D - \phi_Q^D - \kappa\left(I\right)\right)\left(Q_\tau^D + Q_\kappa^D \kappa_I I_\tau\right) - \kappa_I I_\tau Q^D\right) + \\ & (1-\gamma)\left(\left(S_Q^S - \kappa\left(I\right)\right) Q_\kappa^S \kappa_I I_\tau - \kappa_I I_\tau Q^S\right) \end{aligned} \tag{5}$$

Recall that when the government has limited commitment power,[4] the equation determining the second period tax rate is:

$$-2c\left(\tau_0 - \tau^\theta\right) = \left(S_Q^\theta - \theta\phi_Q^\theta - \kappa\left(I\right)\right) Q_\tau^\theta \tag{6}$$

In the first equation, the government takes into account the cost of investment, the impact that the investment will have on the marginal cost of production, and the effect of the announced

[4] At the extreme, when the government has no commitment power, $c = 0$.

tax rate on the investment levels made by the firms. In the second equation, the government only cares about the effect of the tax on output, taking investment as given. As in the previous section, simulation results show that the relationship between taxation with full and with partial commitment is ambiguous.

3.5.3 Direct investment tax/subsidy

Now suppose that the government can subsidize or tax investment at the rate λ (but has no commitment power). The firms' first order condition for investment will then become:

$$\mu_I - \lambda = -\kappa_I \left(\gamma q^D + (1-\gamma)\, q^S\right)$$

The optimal level of output, conditional on investment, is still given by the equation:

$$\kappa(I) = P\left(Nq^*\right) - \tau + P_Q q^*$$

Government's initial problem is now given by:

$$\max_{\lambda} E_\theta \left[S\left(Q\left(\tau^D, I\right)\right) - \phi\left(\theta Q\left(p, I\right)\right) - \kappa\left(I\right) Q\left(p, I\right) - N\mu(I(\lambda))\right]$$

In the second period, the government will set the tax rate to satisfy:

$$S_Q^D = \phi_Q^D + \kappa\left(I\right)$$

This is potentially another way to bring investment in line with the socially optimal level.

3.6 Example

Suppose consumers have a linear demand function given by:

$$Q(p) = c - \frac{1}{b}p$$

Firms. The above demand function implies that the revenue of firms producing Q is $r = a - bQ - \tau$, where $a = bc$ and τ is the tax rate. The firms' maximization problem is then:

$$\max_{q_i} (a - bQ - \tau) q_i - \kappa(I) q_i$$

This leads to the following first order condition:

$$0 = (a - bQ - \tau) - bQ_{q_i} q_i - \kappa(I)$$

Assuming the solution is symmetric ($q_i^* = q^*$), the aggregate market supply Q as a function of the per-firm investment in the market, I, and the tax rate, τ is then:

$$Q = Nq^* = \frac{N(a - \tau - \kappa(I))}{b(N+1)}$$

Letting τ^S denote the tax rate in the "safe" state of the world where production is not polluting and τ^D denote the tax rate in the "damaging" state of the world, the first-order condition for investment is given by:

$$\mu_I = -\kappa_I \left(\gamma \left(\frac{a - \tau^D - \kappa(I)}{b(N+1)} \right) + (1-\gamma) \left(\frac{a - \tau^S - \kappa(I)}{b(N+1)} \right) \right)$$

$\kappa_I < 0$. Thus, the right hand side of the equation is decreasing in τ^θ. Because μ is convex, μ_I is increasing in I. A higher τ^θ implies a lower right hand side, which in turn implies a lower level of investment in equilibrium. In general, it is infeasible to find an analytic solution for I when $\mu(I)$ is positive, increasing and convex in I and $\kappa(I)$ is positive and decreasing. Thus, I proceed without explicitly solving for I, although I do numerically solve for it in the simulation section.

Government at t $= 2$. Denote the solution to the firms' investment problem by I^*. At $t=2$, the government then solves:

$$\max_{\tau} S(Q) - \theta\phi(Q) - \kappa(I^*)Q - c\left(\tau_0 - \tau^\theta\right)^2$$

where

$$\begin{aligned} S(Q) &= \int_0^{\frac{N\left(a-\tau^\theta-\kappa(I^*)\right)}{b(N+1)}} (a-bq)\,dq \\ &= a\frac{N\left(a-\tau^\theta-\kappa(I^*)\right)}{b(N+1)} - \frac{b}{2}\left(\frac{N\left(a-\tau^\theta-\kappa(I^*)\right)}{b(N+1)}\right)^2 \end{aligned}$$

and

$$\theta\phi(Q) = \rho\frac{N\left(a-\tau^\theta-\kappa(I^*)\right)}{b(N+1)}$$

The first order condition for τ^θ is:

$$-2c\left(\tau_0-\tau^\theta\right) = \frac{N\left(-a+\kappa(I^*)+\theta\rho(N+1)-N\tau^\theta\right)}{b^2(N+1)^2}$$

Thus:

$$\tau^\theta = \frac{N\left(-a+\kappa(I^*)+\theta\rho(N+1)\right)+2cb^2(N+1)^2\tau_0}{2cb^2(N+1)^2+N^2}$$

The derivatives of τ^θ with respect to c and τ_0 are:

$$\tau_c^\theta = \frac{2b^2(N+1)^2N^2\tau_0}{\left(2cb^2(N+1)^2+N^2\right)^2}$$

$$\tau_{\tau_0}^{\theta} = \frac{2cb^2 (N+1)^2}{2cb^2 (N+1)^2 + N^2}$$

Clearly, $\tau_c^{\theta} \geq 0$ (strictly greater if $\tau_0 > 0$) and $1 > \tau_{\tau_0}^{\theta} \geq 0$ (strictly greater if $c > 0$).

Government at t $= 0$. At $t = 0$, the government solves its welfare maximization problem for c and τ_0.

$$\max_{c,\tau_0} \gamma \left[S\left(Q^D\right) - \rho Q^D - \kappa(I) Q^D - c\left(\tau_0 - \tau^D\right)^2 \right] + (1-\gamma)\left[S\left(Q^S\right) - \kappa(I) Q^S - c\left(\tau_0 - \tau^S\right)^2 \right] - N\mu(I)$$

The resulting first order conditions are algebraically complicated and are not reproduced here. Because there is no analytic solution for investment in this example, an analytic solution for c and τ_0 likewise does not exist.

Socially optimal investment. If the government could choose investment levels directly, it would do so by solving:

$$max_I \gamma \left(S^D - \phi^D - \kappa(I) Q^D\right) + (1-\gamma)\left(S^S - \kappa(I) Q^S\right) - N\mu(I)$$

The first order condition is given by:

$$\begin{aligned} N\mu_I &= \gamma\left(\left(S_Q^D - \phi_Q^D - \kappa(I)\right) Q_\kappa^D \kappa_I - \kappa_I Q^D\right) + \\ &+ (1-\gamma)\left(\left(S_Q^S - \kappa(I)\right) Q_\kappa^S \kappa_I - \kappa_I Q^S\right) \end{aligned}$$

This results in the following relationship:

$$\mu_I = \kappa_I \frac{\gamma(\rho + N\rho + \tau) - a(N+2) + \kappa(I)(N+2)}{b(N+1)^2} \tag{7}$$

Because $\kappa_I < 0$, the right hand side is falling in τ. μ_I is increasing in I, which implies that investment is also falling in τ. Because the government will also choose a different tax rate in this case, the relationship between equilibrium investment levels chosen by the government and by the firms is ambiguous.

4 Simulations

In this section, I present simulation results following the example from Section 3.6. Recall that the inverse demand function is assumed to be $p = a - bQ$. Firm investment costs are given by $\mu(I) = \frac{\omega}{2}I^2$ and the marginal production cost is $\kappa(I) = \eta e^{-I}$. I choose a, b, ω, and η such that the equilibrium level of firms' investment and production is non-negative. The number of firms, N, ranges from 51 to 100. The cost of investment, ω, ranges from 1 to 6. The marginal cost parameter, η, ranges from 0.01 to 1. Demand parameters a and b range from 20 to 30 and from 0.1 to 1.5, respectively.

Each variable is drawn independently from a discrete uniform distribution, which is scaled to units of 0.1 for γ, ρ, b and η. For each simulated economy, I compute the socially optimal investment, the firms' investment decision, the optimal ex-ante and ex-post taxes, and the optimal cost of deviating from the promised tax rate.

4.1 Ex-ante commitment and ex-post taxation

Figures 1 and 2 show how the announced tax, τ_0, the commitment level, c, and the realized taxes τ^D and τ^S vary with the probability of damage γ and the extent of damage ρ. In this scenario, $\omega = 6$, $\eta = 0.91$, $a = 30$, $N = 58$, and $b = 1$. These values were randomly chosen. I let the probability of damage,γ, range from 0.05 to 0.75. The damage parameter, ρ, ranges from 0.6 to 5.1.

Figure 1 shows the two ex-post tax rates, τ^D and τ^S plotted against the probability and extent of damage. In this scenario, $\tau^S = 0$ for all combinations of γ and ρ. τ^D increases with the extent

of damage and this increase appears to be independent of the probability of damage (left top and left bottom graphs). Counterintuitively, it decreases slightly with the extent of damage (right top and right bottom graphs).

Figure 1: Taxation vs. the extent and probability of damage.

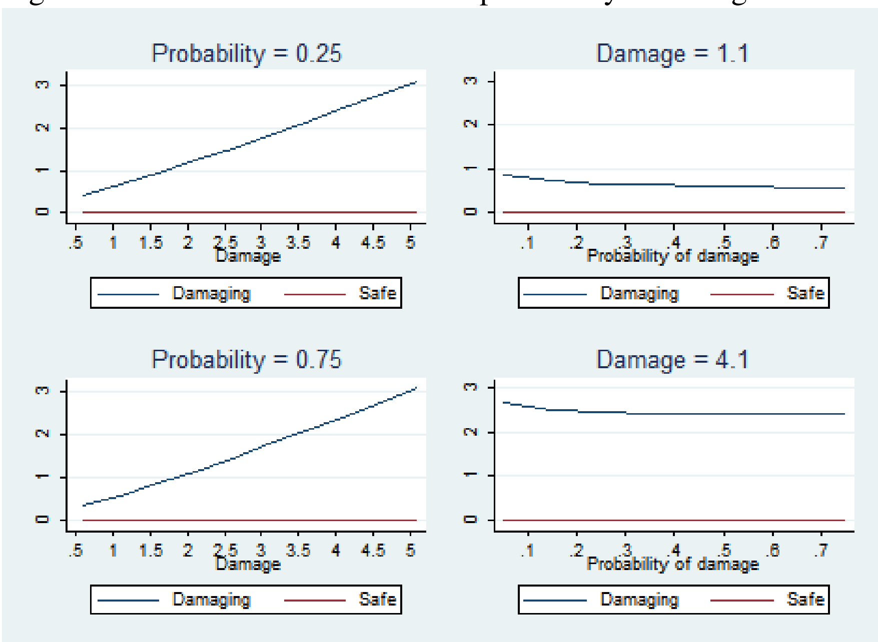
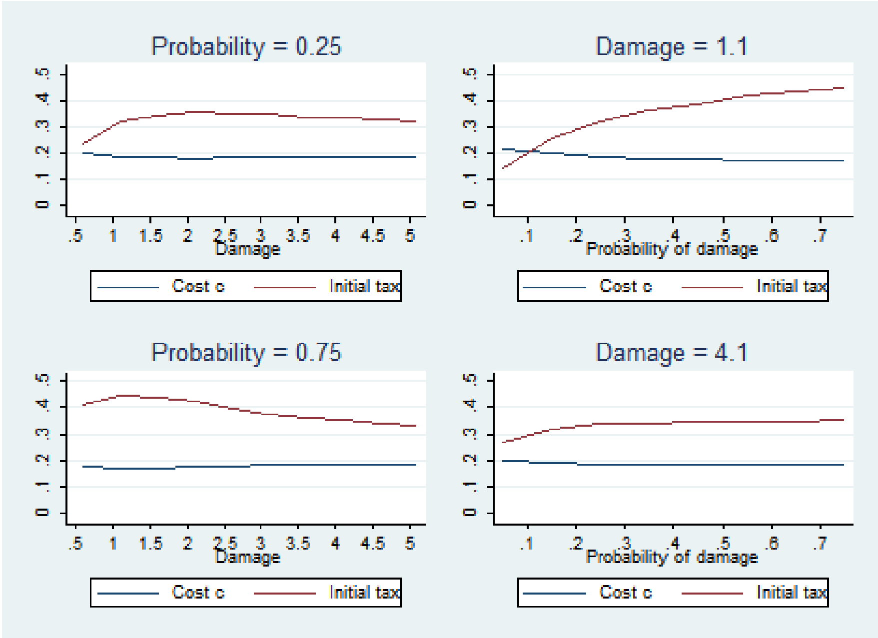


Figure 2 shows how the announced tax rate, τ_0, and the marginal cost of deviating from this tax rate, c, change with the probability and extent of damage. The marginal cost of deviating from the tax rate changes very little with either γ or ρ. The initial tax rate, on the other hand, increases with the probability of damage and is non-monotonic with respect to the extent of damage. At very low damage levels, the announced tax rate increases with damages but then appears to decrease.

Figure 2: Announced tax rate and c vs. the extent and probability of damage.



Finally, I use simulation results to compare investment and tax rates when the government has partial commitment power to the case where it has full or no commitment power. 577 observations were generated from 7 different demand scenarios drawn randomly from the distribution of parameters described above. For each scenario, γ and ρ were then allowed to vary from 0.05 to 0.75 and from 0.6 to 5.1, respectively.

Table 1 shows the ratios of tax rates when there is full commitment, no commitment and partial commitment. The first row shows the relationship between the partial commitment tax rate when $\theta = 1$, τ^D, and the extent of damage, ρ. On average, this ratio is equal to 0.68. As expected, the government never sets a tax above the marginal damage rate. There are two forces that cause the government to set the tax rate below the marginal damage. The first is the cost of changing the announced tax rate. The second is the market power of the firms, which makes production inefficiently low in the case where there is no externality. To detangle the two effects, I also summarize the ratio of τ^D to the announced tax rate τ_0, shown in the second row. Relative to the ratio of τ^D to ρ, this ratio is much more volatile, with a mean of 9.77 and a standard deviation of 9.89. Note that although τ^D is greater than τ_0 most of the time, this is not always the case.

The third row of Table 1 summarizes the ratio of the tax rate with full commitment, τ_{fc}, to

the tax rate with partial commitment, τ^D. On average, τ_{fc} is smaller than τ^D, although this is not always the case. Finally, the last row of Table 1 summarizes the ratio of the tax rate with no commitment, τ_{nc}, to τ^D. τ_{nc} is on average larger than τ^D. The average ratios are consistent with the intuition that the government would like to have a lower tax but cannot commit to it credibly in the world where changing the tax rate is costless. However, this table also demonstrates that there is no clear relationship between the three scenarios and there are cases in which the full commitment tax is smaller than the tax with partial commitment.

Table 2 summarizes the ratios of investment rates in the three commitment scenarios as well as the ratio of investment in the partial commitment case to socially optimal investment. As with tax rates, there is no clear relationship between investment levels in the different commitment cases. Investment in the partial commitment case is on average 0.8% higher than investment levels in both the no commitment and partial commitment scenario, but this ratio ranges from 0.994 to 1.045. On average, investment with partial commitment is identical to the socially optimal investment, but there is a variation of several percent around that. These simulation results show that, contrary to intuition, partial commitment power does not have an unambiguous effect on tax rates or investment levels.

5 Conclusion

The predictability of policy is thought to be highly important for costly-to-reverse decisions. Example range from the effect of monetary policy on bank lending decisions to companies deciding how much to invest into coal power plants when there is the possibility of a carbon tax to individuals deciding whether to put their money into a Roth or traditional retirement account. Much of the literature on dynamic policy has ignored the possibility of endogenous commitment and instead focused on two polar cases: no commitment and perfect commitment. In this paper, I create a simple model of endogenous commitment, where the government announces a promised tax rate and the cost it is willing to impose on itself for deviating from it.

I show that the government is able to remove some, but not all, of the issues associated with time inconsistency by setting a cost of changing the tax rate in the first period. However, contrary to popular wisdom, this does not always lead to lower tax rates than in the case where the government is unable to commit. Similarly, the effect of allowing the government to impose a penalty on itself for deviating from a promised tax rate, as modeled here, has an ambiguous effect on investment, both relative to the case where the government is able to commit fully and the case where is government has no commitment power.

Simulation results also demonstrate that this model has nice properties: the tax rate in the non-polluting state of the world is always zero, while the tax rate in the polluting state of the world rises with the extent of the pollutant's damage. Counterintuitively, the tax rate in the polluting state of the world falls with the probability of damage. Future investigations of this model will focus on formally deriving the relationship between the tax rate and the probability and extent of damage, as well as on formally comparing the full, partial, and no commitment scenarios.

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Tables

Table 1: Tax ratios

	Mean	Standard deviation	Minimum	Maximum	N
t_d/ρ	0.68	0.10	0.53	1.00	577
t_d/t_0	9.77	9.89	0.49	55.80	577
t_{fc}/t_d	0.95	0.69	0.65	4.64	521
t_{nc}/t_d	1.02	0.92	0.66	5.10	521

Table 2: Investment ratios

	Mean	Standard deviation	Minimum	Maximum	N
I_{pc}/I_{nc}	1.008	0.009	0.994	1.043	577
I_{pc}/I_{fc}	1.008	0.010	0.995	1.045	577
I_{pc}/I_{opt}	1.000	0.011	0.984	1.039	577

Appendix

Proof of Lemma 1. Subtract the first order condition for τ^θ when $\theta = 0$ from the first order condition when $\theta = 0$:

$$-2c\left(\tau_0 - \tau^D\right) + 2c\left(\tau_0 - \tau^S\right) = \left(S_Q^D - S_Q^S - \phi_Q\right) Q_\tau$$

This simplifies to:

$$2c\left(\tau^D - \tau^S\right) = \left(S_Q^D - S_Q^S - \phi_Q\right) Q_\tau$$

Because $Q_\tau < 0$, $\tau^D - \tau^S \geq 0$ if and only if $S_Q^D - S_Q^S - \phi_Q \leq 0$ or $S_Q^D - \phi_Q \leq S_Q^S$. Without the wedge created by the cost of deviating from the announced tax rate, the government would set $S_Q^D - \phi_Q = S_Q^S$=$\kappa\left(I\right)$. Because the marginal benefit of production is lower when $\theta = 1$ at every quantity, the optimal production when $\theta = 1$ is lower than when $\theta = 0$. This implies that $S_Q^D > S_Q^S$. However, if $S_Q^D - \phi_Q > S_Q^S$, the government has reduced production more than is socially optimal relative to the state of the world where $\theta = 0$. Because I have assumed that the government cannot subsidize production and firms have market power, production cannot be inefficiently high when $\theta = 0$. This implies that, if $S_Q^D - \phi_Q > S_Q^S$, production is too low when $\theta = 1$ and the government would want to lower τ^D. Thus, at the optimal tax levels, $S_Q^D - \phi_Q \leq S_Q^S$.

Proof of Lemma 2. Recall that the first order condition for $\tau^\theta$is

$$-2c\left(\tau_0 - \tau^\theta\right) = \left(S_Q^\theta - \theta\phi_Q - \kappa\left(I\right)\right) Q_\tau \tag{8}$$

Because the right-hand-side quantities in equation (8) may also be functions of τ^θ, solving for τ^θ is not straightforward. However, the relationship between τ^θ and τ_0 can be examined by taking the derivative of (8) with respect to τ_0:

$$-2c + 2c\tau^{\theta}_{\tau_0} = \left(S_Q - \theta\phi_Q - \kappa\left(I\right)\right) Q_{\tau\tau}\tau^{\theta}_{\tau_0} + \left(S_{QQ} - \theta\phi_{QQ}\right) Q^2_{\tau}\tau^{\theta}_{\tau_0}$$

Solving for $\tau^{\theta}_{\tau_0}$and recalling that $Q_{\tau\tau} = 0$:

$$\tau^{\theta}_{\tau_0} = \frac{2c}{2c - \left(S_{QQ} - \theta\phi_{QQ}\right) Q^2_{\tau}} > 0$$

Because $S_{QQ} - \theta\phi_{QQ} < 0$ and $Q^2_{\tau} > 0$, it is easy to see that $1 > \tau^{\theta}_{\tau_0} > 0$.

Similarly, taking the derivative of equation (8) with respect to c yields:

$$-2\left(\tau_0 - \tau^{\theta}\right) + 2c\tau^{\theta}_{c} = \left(S_Q - \theta\phi_Q - \kappa\left(I\right)\right) Q_{\tau\tau}\tau^{\theta}_{c} + \left(S_{QQ} - \theta\phi_{QQ}\right) Q^2_{\tau}\tau^{\theta}_{c}$$

Solving for τ^{θ}_{c} and recalling that $Q_{\tau\tau} = 0$:

$$\tau^{\theta}_{c} = \frac{2\left(\tau_0 - \tau^{\theta}\right)}{2c - \left(S_{QQ} - \theta\phi_{QQ}\right) Q^2_{\tau}}$$

Because $c \geq 0$, $S_{QQ} < 0$, and $\phi_{QQ} \leq 0$, this expression clearly shows that $\tau^{\theta}_{c} < 0$ if and only if $\tau^{\theta} > \tau_0$.

First order condition for investment and I_{τ}. The first order condition for investment can be written as:

$$\mu_I = \gamma\pi_I\left(\tau^D, I\right) + \left(1 - \gamma\right)\pi_I\left(\tau^S, I\right)$$

Writing the profit function out fully and rearranging, we have:

$$\mu_I = \gamma\left[\left(\underbrace{P\left(Q^D\right)-\tau^D+P_Q q^D-\kappa(I)}_{=0}\right)q_\kappa\kappa_I-\kappa_I q^D\right]$$
$$+(1-\gamma)\left[\left(\underbrace{P\left(Q^S\right)-\tau^S+P_Q q^S-\kappa(I)}_{=0}\right)q_\kappa\kappa_I-\kappa_I q^S\right]$$

We can see that the first order condition for investment also contains the first order condition for production, which is equal to 0. Thus, the first order condition simplifies to:

$$\mu_I = -\kappa_I\left(\gamma q^D+(1-\gamma)\,q^S\right) \tag{9}$$

Denote

$$E_\theta[q] = \gamma q^D+(1-\gamma)\,q^S$$

To see how investment changes with the tax rate in each state of the world, τ^D, differentiate equation (9) with respect to τ^D:

$$\mu_{II}I_{\tau^D} = -\kappa_{II}I_{\tau^D}E_\theta[q]-\kappa_I\left(q_\kappa\kappa_I I_{\tau^D}+\gamma q_\tau\right)$$

Solving for I_{τ^θ}, we get:

$$\left(\mu_{II}+\kappa_{II}E_\theta[q]+\kappa_I^2 q_\kappa\right)I_{\tau^D} = -\kappa_I\gamma q_\tau$$

$$I_{\tau^D} = \frac{\overbrace{-\kappa_I\gamma q_\tau}^{<0}}{\mu_{II}+\kappa_{II}\underbrace{E_\theta[q]}_{>0}+\underbrace{\kappa_I^2 q_\kappa}_{<0}}$$

The corresponding equation for τ^S is similar:

$$I_{\tau^S} = \frac{\overbrace{-\kappa_I\left(1-\gamma\right)q_\tau}^{<0}}{\mu_{II} + \kappa_{II}\underbrace{E_\theta\left[q\right]}_{>0} + \underbrace{\kappa_I^2 q_\kappa}_{<0}}$$

The fact that$\kappa_I < 0$ and $q_\tau < 0$ implies that the numerator is less than 0. Overall, this expression makes it clear that investment does not always fall with the tax rate - the exact relationship depends on the parameters of the model. Specifically, a necessary and sufficient condition for investment to fall with the tax rate is:

$$\kappa_{II}E_\theta\left[q\right] + \kappa_I^2 q_\kappa > -\mu_{II}$$

The response of investment to c and τ^0.

$$I_{\tau_0} = \frac{-\kappa_I q_\tau}{\mu_{II} + \kappa_{II}E_\theta\left[q\right] + \kappa_I^2 q_\kappa}\left(\frac{2c}{2c - \left(S_{QQ} - \phi_{QQ}\right)Q_\tau^2} + \frac{2c}{2c - S_{QQ}Q_\tau^2}\right)$$

If $\kappa_{II}E_\theta\left[q\right] + \kappa_I^2 q_\kappa > -\mu_{II}$, then $I_{\tau_0} < 0$

$$I_c = \frac{-\kappa_I q_\tau}{\mu_{II} + \kappa_{II}E_\theta\left[q\right] + \kappa_I^2 q_\kappa}\left(\frac{2\left(\tau_0 - \tau^D\right)}{2c - \left(S_{QQ} - \phi_{QQ}\right)Q_\tau^2} + \frac{2\left(\tau_0 - \tau^S\right)}{2c - S_{QQ}Q_\tau^2}\right)$$

Because the relationship between τ^D, τ^S, and c is ambiguous, so is the net relationship between I and c.

First order conditions with respect to c and τ_0.

Writing out the expectation:

$$\max_{c,\tau_0}\gamma\left[S\left(Q^D\right) - \phi\left(Q^D\right) - \kappa\left(I\right)Q^D - c\left(\tau_0 - \tau^D\right)^2\right]$$

$$
\begin{aligned}
&+ (1-\gamma)\left(S\left(Q^S\right) - \kappa(I)\,Q^S - c\left(\tau_0 - \tau^S\right)^2\right) \\
&- N\mu(I)
\end{aligned}
$$

Taking the first order condition with respect to c yields:

$$
\begin{aligned}
0 &= \gamma\left[\left(S_Q^D - \phi_Q - \kappa(I)\right)\left(Q_\tau \tau_c^D + Q_\kappa \kappa_I I_c\right) - \kappa_I Q^D I_c - \left(\tau_0 - \tau^D\right)^2 + 2c\left(\tau_0 - \tau^D\right)\tau_c^D\right] \\
&\quad + (1-\gamma)\left(\left(S_Q^S - \kappa(I)\right)\left(Q_\tau \tau_c^S + Q_\kappa \kappa_I I_c\right) - \kappa_I Q^S I_c - \left(\tau_0 - \tau^S\right)^2 + 2c\left(\tau_0 - \tau^S\right)\tau_c^S\right) \\
&\quad - N\mu_I I_c
\end{aligned}
$$

Let SS_Q^D denote the net marginal social surplus in the "damaging" state of the world, $S_Q^D - \phi_Q - \kappa(I)$; SS_Q^S denotes the net marginal surplus in the "safe" state of the world, $S_Q^S - \kappa(I)$. Finally, let $E_\theta[SS_Q]$ denote the expected net marginal social surplus, $\gamma SS_Q^D + (1-\gamma)\,SS_Q^S$. Then the first order condition with respect to c simplifies to:

$$
\begin{aligned}
0 &= \kappa_I I_c\left(E_\theta[SS_Q]\,Q_\kappa - E_\theta[Q]\right) - N\mu_I I_c \\
&\quad + \gamma\left[SS_Q^D Q_\tau \tau_c^D - \left(\tau_0 - \tau^D\right)^2 + 2c\left(\tau_0 - \tau^D\right)\tau_c^D\right] \\
&\quad + (1-\gamma)\left(SS_Q^S Q_\tau \tau_c^S - \left(\tau_0 - \tau^S\right)^2 + 2c\left(\tau_0 - \tau^S\right)\tau_c^S\right)
\end{aligned}
$$

Recall that the solution to the second period problem results in the first order condition$-2c\left(\tau_0 - \tau^\theta\right) = \left(S_Q^\theta - \theta\phi_Q - \kappa(I)\right)Q_\tau = SS_Q^\theta Q_\tau$. Substituting for $2c\left(\tau_0 - \tau^D\right)$ in the above equation yields:

$$
\begin{aligned}
0 &= \kappa_I I_c\left(E_\theta[SS_Q]\,Q_\kappa - E_\theta[Q]\right) - N\mu_I I_c \\
&\quad + \gamma\left[SS^D Q_\tau \tau_c^D - \left(\tau_0 - \tau^D\right)^2 - SS_Q^D Q_\tau \tau_c^D\right] \\
&\quad + (1-\gamma)\left(SS^S Q_\tau \tau_c^S - \left(\tau_0 - \tau^S\right)^2 - SS_Q^D Q_\tau \tau_c^S\right)
\end{aligned}
$$

This simplifies to:

$$\begin{aligned}0 &= \kappa_I I_c \left(E_\theta \left[SS_Q\right] Q_\kappa - E_\theta \left[Q\right]\right) - N\mu_I I_c \\ &\quad -\gamma \left(\tau_0 - \tau^D\right)^2 - (1-\gamma)\left(\tau_0 - \tau^S\right)^2\end{aligned}$$

Similarly, the full first order condition with respect to τ_0 is equal to:

$$\begin{aligned}0 &= \gamma \left[\left(S_Q^D - \phi_Q - \kappa\left(I\right)\right)\left(Q_\tau \tau_{\tau_0}^D + Q_\kappa \kappa_I I_{\tau_0}\right) - \kappa_I Q^D I_{\tau_0} - 2c\left(\tau_0 - \tau^D\right)\left(1 - \tau_{\tau_0}^D\right)\right] \\ &\quad + (1-\gamma)\left(\left(S_Q^S - \kappa\left(I\right)\right)\left(Q_\tau \tau_{\tau_0}^S + Q_\kappa \kappa_I I_{\tau_0}\right) - \kappa_I Q^S I_{\tau_0} - 2c\left(\tau_0 - \tau^S\right)\left(1 - \tau_{\tau_0}^S\right)\right) \\ &\quad -N\mu_I I_{\tau^0}\end{aligned}$$

Substituting the above notation into this first order condition results in the following expression:

$$\begin{aligned}0 &= \kappa_I I_{\tau_0} \left(E_\theta \left[SS_Q\right] Q_\kappa - E_\theta \left[Q\right]\right) - N\mu_I I_{\tau^0} \\ &\quad +\gamma \left[SS_Q^D Q_\tau \tau_{\tau_0}^D - 2c\left(\tau_0 - \tau^D\right)\left(1 - \tau_{\tau_0}^D\right)\right] \\ &\quad + (1-\gamma)\left(SS_Q^S Q_\tau \tau_{\tau_0}^S - 2c\left(\tau_0 - \tau^S\right)\left(1 - \tau_{\tau_0}^S\right)\right)\end{aligned}$$

Substituting for $2c\left(\tau_0 - \tau^D\right)$ in the above equation yields:

$$\begin{aligned}0 &= \kappa_I I_{\tau_0} \left(E_\theta \left[SS_Q\right] Q_\kappa - E_\theta \left[Q\right]\right) - N\mu_I I_{\tau^0} \\ &\quad +\gamma \left[SS_Q^D Q_\tau \tau_{\tau_0}^D + SS_Q^D Q_\tau \left(1 - \tau_{\tau_0}^D\right)\right] \\ &\quad + (1-\gamma)\left(SS_Q^S Q_\tau \tau_{\tau_0}^S + SS_Q^S Q_\tau \left(1 - \tau_{\tau_0}^S\right)\right)\end{aligned}$$

This expression, in turn, simplifies to:

$$
\begin{aligned}
0 &= \kappa_I I_{\tau_0} \left(E_\theta \left[SS_Q \right] Q_\kappa - E_\theta \left[Q \right] \right) - N \mu_I I_{\tau^0} \\
&\quad + E_\theta \left[SS_Q \right] Q_\tau
\end{aligned}
$$