A UNIFIED MODEL OF THE ILLEGAL IMMIGRATION SYSTEM

By

Joseph Chang,[1] Allison Reilly,[1] and Dean Judson[2]

[1]Homeland Security Studies and Analysis Institute, Arlington, Virginia
[2]Decision Analytics, Accokeek, Maryland

Presented at the 2012 American Economic Association Annual Meeting, Chicago, Illinois.

Acknowledgments: This work was supported by the Department of Homeland Security's Office of Program Analysis and Evaluation. The results presented in this paper do not necessarily reflect official DHS opinion or policy.

1. INTRODUCTION

Immigration and border security undoubtedly have been major issues confronting all levels of government and society. These issues constitute an extremely complex system with many dimensions, such as law enforcement, public policy, socio-economics, etc. They came even more to the forefront after the September 11, 2001, terrorist attacks and the resulting creation of the Department of Homeland Security (DHS). For example, "securing and managing our borders" is one of the DHS's missions (DHS 2010a).

There is a consensus that the current immigration and border security policies need an overhaul, but the political climate has thus far prevented a comprehensive reform. Nevertheless, DHS's Office of Program Analysis and Evaluation (PA&E) sponsored a study to investigate analytic tools that can analyze the impacts associated with changes to immigration and border security policies. These impacts can potentially include factors such as resource allocation, performance measurement, stocks and flows of immigration, macroeconomic outcomes, etc.

As immigration (both legal and illegal) and border security as a whole is an exceedingly complex, multi-faceted problem, a decision was made early on to first focus on illegal immigration between ports of entry (POEs). While many studies exist that focus on some particular aspects (e.g., the flow of illegal aliens between POEs, effectiveness of border enforcement, migration decisions, etc.), we identified only two existing comprehensive models that address many related issues in an integrated manner. These two models are the Wein model (Wein et al. 2009; Liu and Wein 2008) and the Secure Border Initiative (SBI) model (MITRE 2008).

The Wein model consists of four submodels that deal with various aspects of the illegal immigration system between ports of entry (POEs), including (1) a multinomial logit submodel (Ben-Akiva and Lerman 1985) describing the choice made by illegal aliens; (2) a border apprehension submodel accounting for the interaction among Border Patrol (BP) agents, surveillance technology, and illegal aliens along the border between POEs; (3) a removal submodel accounting for the probability of removing an apprehended alien subject to available detention bed space; and (4) an illegal wage model including worksite enforcement and the supply of and demand for unskilled labor. All these submodels are further interconnected via feedback mechanisms so that a change in one submodel will propagate throughout the system. These feedback mechanisms require the solving of a number of nonlinear equation systems.

The SBI model is more comprehensive, as it attempts to address both illegal and legal immigration, and both at POEs and between POEs. Even though the model is not as mathematically complex as the Wein model (i.e., not solving any nonlinear equations), it is rather tedious as it tries to simulate, using the system dynamics approach (see Forrester 1994), the stocks and flows of about two dozen "states" in the overall immigration system. A stock represents an accumulation of population at a state, while a flow transitions people in or out of the state. An inflow increases a stock, and an outflow depletes a stock. As a result, the SBI model requires numerous engineering assumptions about the stocks and even more assumptions about the flows. These assumptions are rather ad hoc and do not take migrants' behaviors into consideration.

As a result, the Wein model appears to provide the best analytic framework for the illegal immigration system, as it does have appealing technical attributes and is built upon well-tested operations research methodologies (e.g., multinomial logit model, game theory, queueing theory, etc.). Chang et al. (2011) describe in detail how we re-implemented the Wein model in a much more computationally efficient software platform. We updated the data used in the model so that it better reflects the current landscape of border security. For example, the catch-and-release practice for other-than-Mexican (OTM) aliens has ended (DHS 2006), the number of detention bed spaces has increased (ICE 2008), the number of BP agents has increased (DHS 2010b), and the number of apprehensions between POEs has decreased (CBP 2011). We also updated some of the formulations to increase the model's robustness.

This paper primarily highlights the results of the model. The reader should refer to Wein et al. (2009) and Chang et al. (2011) for other details concerning the model. Section 2 gives a brief overview of the Wein model. Section 3 describes the model results. Conclusions appear in section 4.

2. Introduction to the Wein Model

Wein et al. (2009) and Liu and Wein (2008) describe the model's formulations and assumptions in detail. This section provides only a high-level description. Additional technical details on the model's solution procedure, software implementation, updates, and instructions can be found in Chang et al. (2011).

The Wein model is a sophisticated mathematical model consisting of four interconnected submodels that deal with various aspects of illegal immigration. These four submodels are:

- a discrete choice border crossing submodel based on the utility (i.e., economic gains) of an illegal migrant deciding whether and where to cross;
- a border apprehension submodel similar to a Stackelberg game—i.e., migrants responding to U.S. Border Patrol (BP) agents' moves—and a single-server loss queueing system;
- a removal submodel based on a single pooled queueing system for the allocation of Enforcement and Removal Operations (ERO) bed spaces according to detention priority, with an ultimate goal that sufficient beds are available to detain and eventually remove all apprehended illegal migrants; and
- an illegal wage submodel based on economic equilibrium theory that accounts for labor supply and demand and worksite enforcement.

Figure 1 shows the schematic of the Wein model, with the four shaded rectangles representing the main output of each submodel. The ovals show the key decision variables of each submodel. These key decision variables were varied in this study to simulate different policy options. As described in Wein et al. (2009) and Chang et al. (2011), many other parameters are also required by each submodel. The double blue arrows indicate feedback between submodels. For example, the probability of apprehension affects the crossing rate, which in turn affects the probability of apprehension. The figure shows that most of the relationships among submodels entail feedback mechanisms. This property makes the model more realistic, but requires significantly more computational resources

The four submodels are briefly described in the following sections.

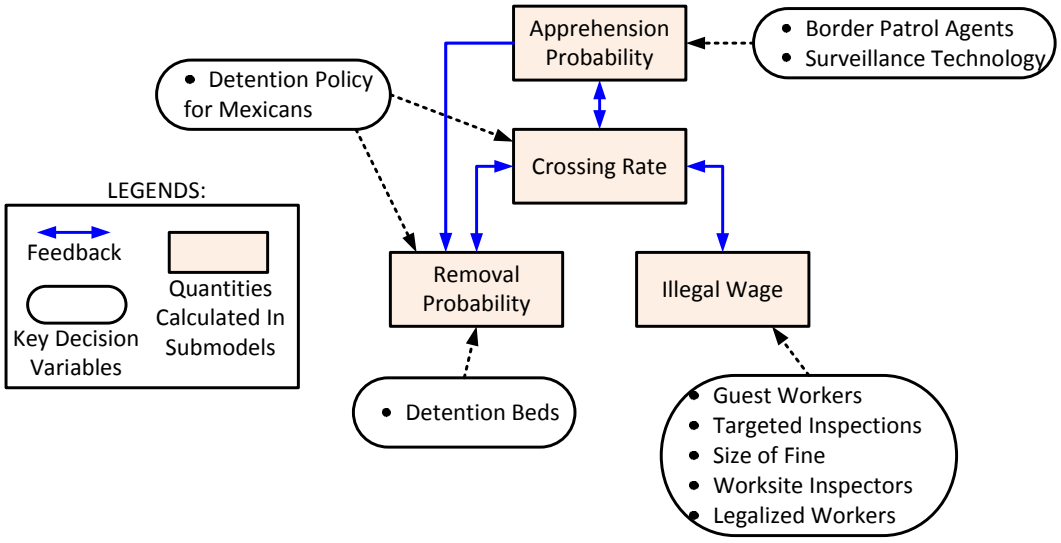

Figure 1. Schematic of the Wein model.

2.1. Discrete Choice Border Crossing Submodel

The Wein model uses a multinomial logit model (see Ben-Akiva and Lerman 1985) to describe the discrete choice made by a migrant. The likelihood that a potential migrant decides to illegally cross the southwestern border into the United States is described by the following expression for the crossing probability:

$$P_{cross} = \frac{e^{\theta \cdot u_1}}{e^{\theta \cdot u_1} + e^{\theta \cdot u_2}}, \qquad (1)$$

where u_1 and u_2 are the utilities (i.e., economic gains) for crossing and not crossing the border, respectively, and θ is a scale variable that measures population heterogeneity (i.e., not everyone will make the same choice given the same utilities). The crossing utility includes the illegal wage gain if the entry is successful minus the cost of traveling to the border (including hiring a smuggler, or a "coyote") and the cost of detention if the entry is unsuccessful. The not-crossing utility simply includes the wage gain in home countries. The utility formulations for Mexican and other-than-Mexican (OTM) migrants are necessarily different due to the voluntary return program for apprehended Mexican migrants (see Wein et al. 2009).

The scale variable θ measuring population heterogeneity can be illustrated by considering two limiting cases. When θ approaches zero, the discrete choice becomes purely random, i.e., the probability is always one half, like a coin toss, regardless of utilities. When θ approaches infinity, the discrete choice becomes purely deterministic, i.e., the crossing probability is always one, even if the crossing utility is just marginally greater than the not-crossing utility. This scale variable is typically estimated by surveys or by numerical calculations based on certain integral constraints. The Wein model uses the latter approach

In addition to the decision of whether to cross, the same discrete choice model is also used in (1) the decision of where to cross (i.e., a migrant arriving at location x may choose to cross at location y due to the government's border enforcement posture) and (2) the decision of whether an illegal worker is willing to work for a firm targeted for worksite inspection at a lower wage (see section 2.4). The utility formulations for these cases are structurally different, but the Wein model assumes that the same scale variable applies to all these choices. Ideally, each choice should probably have its own scale variable. However, this would significantly increase the model's complexity.

This submodel's primary outputs are the crossing rates of Mexican and OTM migrants at each of the discrete locations along the border (see section 2.2). As shown in figure 1, the crossing rates further impact the probability of apprehension, the equilibrium illegal wage, and the probability of removal. Similarly, through feedback mechanisms allowed by the model, these three factors also impact the crossing rates.

2.2. Between-POEs Border Apprehension Submodel

The border apprehension submodel first discretizes the ~2,000 miles of the southwestern border into 60 line segments. The interaction among illegal crossers, BP agents, and surveillance technology between POEs is accounted for at each of these segments similar to a sequential Stackelberg (i.e., leader-follower) game (see Osborne and Rubinstein 1994). The government chooses the number of BP agents, their locations, and portions and locations of the border with surveillance technology. The illegal crossers in turn arrive at the border at a rate proportional to BP agents (as the government is likely to deploy BP agents based on the arrival of illegal crossers), and determine where to cross. This decision process is simulated through the discrete choice model mentioned above. There are busy and quiet areas along the border in terms of where crossers arrive and where BP agents are deployed. To account for this spatial heterogeneity in the simplest way, Wein et al. (2009) assume that both crosser arrivals and BP agents follow sinusoidal functions with the same frequency.

The interaction between illegal crossers and BP agents is further described by a single-server loss queueing system. In this case, a BP agent is patrolling back and forth along the border segment for which he or she is responsible. At the same time, an illegal crosser may arrive randomly at any location of that segment according to a Poisson process

(i.e., their inter-arrival times follow an exponential distribution; see Banks et al. 2001). The distance between the agent and the crosser can then be formally described by a probability density function. It is further assumed that a BP agent has a certain radius of influence, depending on whether surveillance technology is present. A BP agent aided by technology is assumed to have a larger radius of influence than an agent who is not. Additionally, when an agent is busy with an apprehension, he or she cannot make another apprehension. In other words, in queueing theory's terminology, each newly arriving customer (i.e., crosser) immediately goes into service (i.e., being apprehended) if a server (i.e., BP agent) is available, and that customer is lost (i.e., successful entrance) if the server is busy (i.e., being occupied with an ongoing apprehension).

The primary outputs of this submodel are the probability of apprehension (depending solely on a BP agent's ability to apprehend) at each of the 60 border segments and the aggregate probabilities of apprehension of unauthorized Mexican and OTM aliens for the entire border. Mathematically, the aggregate probabilities of apprehension of Mexican and OTM aliens are simply weighted averages of the BP apprehension probability according to the crossing rates given by the discrete choice submodel.[1] As result, the aggregate probabilities of apprehension of unauthorized Mexican and OTM aliens might be slightly different because their respective crossing rates are different.

2.3. Removal Submodel

The removal submodel is based on a single pooled queueing system for the allocation of Enforcement and Removal Operations (ERO, formerly called Detention and Removal Operations or DRO) bed spaces to apprehended illegal migrants based on established detention priority. The ultimate goal is to have sufficient bed space so that all apprehended aliens will be detained and removed. The reason why it is a "pooled" queueing system is because the model assumes that all ERO detention facilities share their resources. The submodel assumes that apprehended illegal aliens arriving at ERO detention facilities are in two classes: mandatory and nonmandatory.[2] A DHS memorandum (Hutchinson 2004) articulates the detention policy by defining the types of aliens for whom mandatory detention is required, and a prioritized list for the remaining nonmandatory detainees. In the removal submodel, priority is given to mandatory detainees. Nonmandatory detainees can be either blocked from entering a detention facility if bed space is unavailable and if they are not already in the system, or preempted (released) when they are already in a full detention facility and bed space is needed for a mandatory detainee. As a result, when bed space is insufficient, it is possible that nonmandatory detainees will be released and subsequently not removed from the United States.

The submodel assumes a base case where all unauthorized OTM migrants apprehended at the border are nonmandatory, as most of the criminal aliens entering ERO detention facilities come directly from U.S. prisons and not from the border. The base case also assumes that all unauthorized Mexican migrants apprehended at the border are directly returned to Mexico and not detained. As described later, we devised a simple approach where this assumption concerning Mexican migrants was relaxed.

The primary output of this submodel is the probability that a detained alien will be successfully removed. The probability of removal further impacts the utility calculation required by the discrete choice submodel, as illustrated in the feedback mechanism shown in figure 1. Furthermore, the probability of removal clearly depends on the detainee arrival rates, which in turn depend on the probability of apprehension described in the previous section. It is important to point out that this removal submodel plays a more prominent role if bed space is insufficient, leading to

---

[1] For example, suppose the BP apprehension probability and the Mexican alien crossing rate at each border segment, y, are $P_a(y)$ and $\lambda_{c1}(y)$, respectively, then the aggregate probability of apprehension of Mexican aliens is $P_{a1} = \int P_a(y) \bullet \lambda_{c1}(y)dy / \int \lambda_{c1}(y)dy$.

[2] Detainees can also be classified as criminal and noncriminal. All criminal aliens by definition are mandatory, but some noncriminal aliens are also mandatory.

a probability of removal less than one. However, once bed space becomes sufficient and the probability of removal is close to one, the removal submodel becomes less important (see Chang et al. 2011).

2.4. Illegal Wage Submodel

The illegal wage submodel consists of two interconnected components: an economic equilibrium model balancing the supply of and demand for unskilled labor, including both legal and illegal workers; and a worksite enforcement model accounting for the impacts (i.e., reduced wages) of enforcement on the labor market.

The economic equilibrium model uses a Cobb-Douglass production function to describe the demand for unskilled labor (Cobb-Clark et al. 1995), where labor demand decreases as legal wage increases. The model considers four sources of unskilled labor:

- Legal U.S. workers, as described by a neo-classical labor supply function (Deaton and Muellbauer 1980), where labor supply increases as legal wage increases.
- Illegal aliens in the United States who have jobs but with less pay, assuming that employers pass along penalties resulting from worksite enforcement in the form of depressed wages. Behaviors of illegal aliens in the U.S. labor market are assumed to follow these rules: (1) illegal aliens decide whether to keep jobs based on the discrete choice model described above; (2) it is assumed that worksite enforcement follows a hybrid targeted-random strategy, where the wage at targeted firms is naturally lower than that at untargeted firms; (3) illegal aliens who quit their jobs from untargeted firms will go home as the wage for working for targeted firms will be even lower; and (4) those who quit jobs from targeted firms will enter a matching process of idle workers and vacated jobs at untargeted firms, where those illegal workers who are not matched will return to their home countries.
- Illegal workers in the United States who become legalized and are paid regular wages (e.g., via some form of immigration reform).
- New guest workers from other countries (e.g., via a temporary worker program).

The worksite enforcement model assumes that the number of illegal workers per firm follows an exponential distribution in that many illegal workers are concentrated in a small number of firms. Worksite inspections can be both targeted (for certain selected industries) and random, and employers pass expected penalties onto illegal workers in the form of lower wages.

The primary output of this submodel is the equilibrium wage of an illegal worker. This wage rate is further used in the discrete choice submodel by contributing to the expected utility gained from entering the United States, as depicted in the feedback mechanism shown in figure 1.

3. MODEL RESULTS

This section discusses the model results. As described in Wein et al. (2009) and Chang et al. (2011), the Wein model is complex, with many input and output variables. Hence, the model results can be studied in a plethora of ways. We will first present more straightforward results, followed by those based on increasingly innovative ways to apply the model. It is important to point out that the model's numerical outputs are primarily intended to capture qualitative impacts of various government decisions, hence over-interpretation of numerical accuracy should be avoided.

3.1. Base Case Values

It is necessary to first provide some supporting information to facilitate subsequent discussions. As described in Chang et al. (2011), the Wein model's solution procedure first assumes the base case values of various parameters, including the key decision variables. Once the rest of the model parameters are estimated with this information, we then conduct "what-if" analysis to see the dependence of model results on the key decision variables while keeping the estimated model parameters fixed.

Table 1 lists the values of the key decision variables for the base case, together with the corresponding costs of those decision variables that are related to enforcement. We retained the same base case values as the original Wein et al. (2009) model for key decision variables a, s_b, r_w, f_w, Δ_l, and N_g; and used updated values for key decision variables n_b, s, and m_w.

Table 1. Key decision variables and their base case values and the corresponding costs for those variables related to enforcement.

Symbol	Description	Value (Reference)	Unit Enforcement Cost
a	Number of apprehensions until detention of Mexicans	∞ (Espenshade 1995; Turner 2004)	N/A
n_b	Number of BP agents deployed at the border on a 24/7 basis[3]	2,947 (DHS 2010b)	$1,173,000/agent-year
s_b	Length of border covered by surveillance technology	322 miles (~1/6 of the border; Wein et al. 2009)	$30,000/mile-year
s	Number of ERO beds	32,000 (ICE 2008)	$32,600/bed-year
r_w	Fraction of worksite inspections that are random	0.4 (Bosher 1987)	N/A
m_w	Number of worksite inspectors	400 (informal FY10 ICE data)	$243,900/inspector-year
f_w	Fine per illegal worker-hour	$5 (Wein et al.	N/A

[3] Based on DHS (2010b), the number of BP agents is around 20,500. A "24/7" BP agent mentioned here refers to an agent deployed at the border at *all* hours to conduct line watch. Due to such factors as work rules, non-line-watch activities, personal leave, etc., on average, a 24/7 agent deployed at the border roughly equals seven BP agents. The much higher cost ($1,173,000/agent-year) for each 24/7 agent also reflects this.

		2009)	
Δ_l	Number of illegal workers in the United States who become legalized	0	N/A
N_g	Number of new guest workers from outside the United States	0	N/A

The table suggests that the base case considered by the Wein model has an annual budget around $4.7B. These costs represent our best-effort estimates, and are probably subject to additional adjustments. Moreover, we considered only the costs of enforcement-related decision variables (i.e., n_b, s_b, s, and m_w), but not those policy-related decision variables (i.e., a, r_w, f_w, Δ_l, and N_g).

3.2. Behavior of Basic Model Solutions

The following series of figures show the behavior of model solutions in terms of:

- the Mexican alien arrival and crossing rates at location y (λ_{b1} and λ_{c1}, respectively);
- the OTM alien arrival and crossing rates at location y (λ_{b2} and λ_{c2}, respectively); and
- the BP apprehension probability at location y (P_a, which is independent of the crosser being a Mexican or an OTM);

where model assumptions are sequentially changed. Figure 2 shows the model solutions for the base case, where one sixth of the border is covered by surveillance technology, whose deployed locations coincide with the peak alien arrival locations. Figure 3 shows the base case when the entire border is covered by surveillance technology. Figure 4 further assumes a uniform distribution of BP agents instead of the original sinusoidal distribution. Finally, figure 5 assumes more effective technology with a fivefold increase in its radius of influence.

Figure 2 for the base case shows Mexican aliens arriving at the border according to a sinusoidal distribution that is consistent with the distributions of BP agents and surveillance technology. As these unauthorized crossers try to avoid agents, the discrete choice border crossing submodel leads to a crossing distribution that is out of phase with the arrival distribution, e.g., crossings are highest where arrivals (and thus agents and technology) are lowest. The aggregate probabilities of apprehension of Mexicans and OTMs (i.e., the weighted averages of the BP apprehension probability according to the Mexican and OTM crossing rates) are 0.26 and 0.38, respectively.

Figure 3 shows that the phase shift between arrivals and crossings still persists even when the entire border is covered with technology. This is because the BP agents' distribution is still sinusoidal and aliens still try to exploit the areas with fewer agents. Nevertheless, the BP apprehension probability is now higher as a result of more technology. The aggregate probabilities of apprehension of Mexicans and OTMs are now 0.50 and 0.60, respectively.

The phase shift between alien arrivals and crossings disappears in figure 4, when BP agents are now assumed to be uniformly distributed. At the same time, the BP apprehension probability is almost uniform along the border and at a higher value. The aggregate probabilities of apprehension of Mexicans and OTMs are now both 0.64.

Figure 5 is similar to figure 4, except that surveillance technology is assumed to be five times more effective, i.e., having a radius of influence that is five times larger. In this case, the BP apprehension probability is even higher. The aggregate probabilities of apprehension of Mexicans and OTMs are now both 0.82.

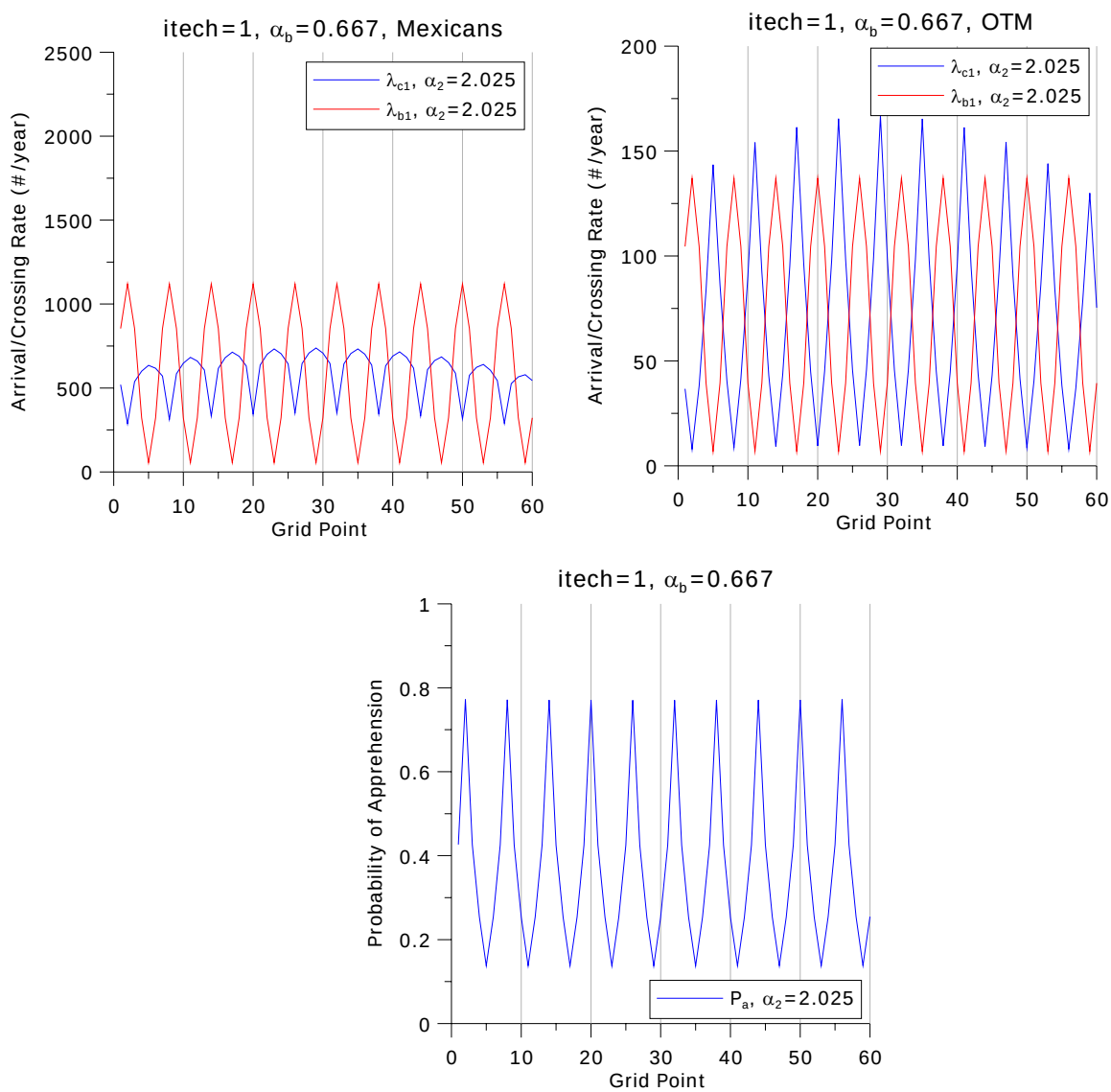

Figure 2. (Upper left) Mexican alien arrival and crossing rates (λ_{b1} and λ_{c1}, respectively), (upper right) OTM alien arrival and crossing rates (λ_{b2} and λ_{c2}, respectively), and (bottom) the BP apprehension probability given by the Wein model for the base case, assuming one sixth of the border (itech=1) is covered by surveillance technology (aligned with the peak locations of alien arrivals) and BP agents are deployed according to a sinusoidal distribution (with a relative amplitude α_b of 0.667, see Chang et al. (2011)). Variable α_2 is the exponential apprehension parameter of technology; alternatively, its reciprocal gives the radius of influence of technology.

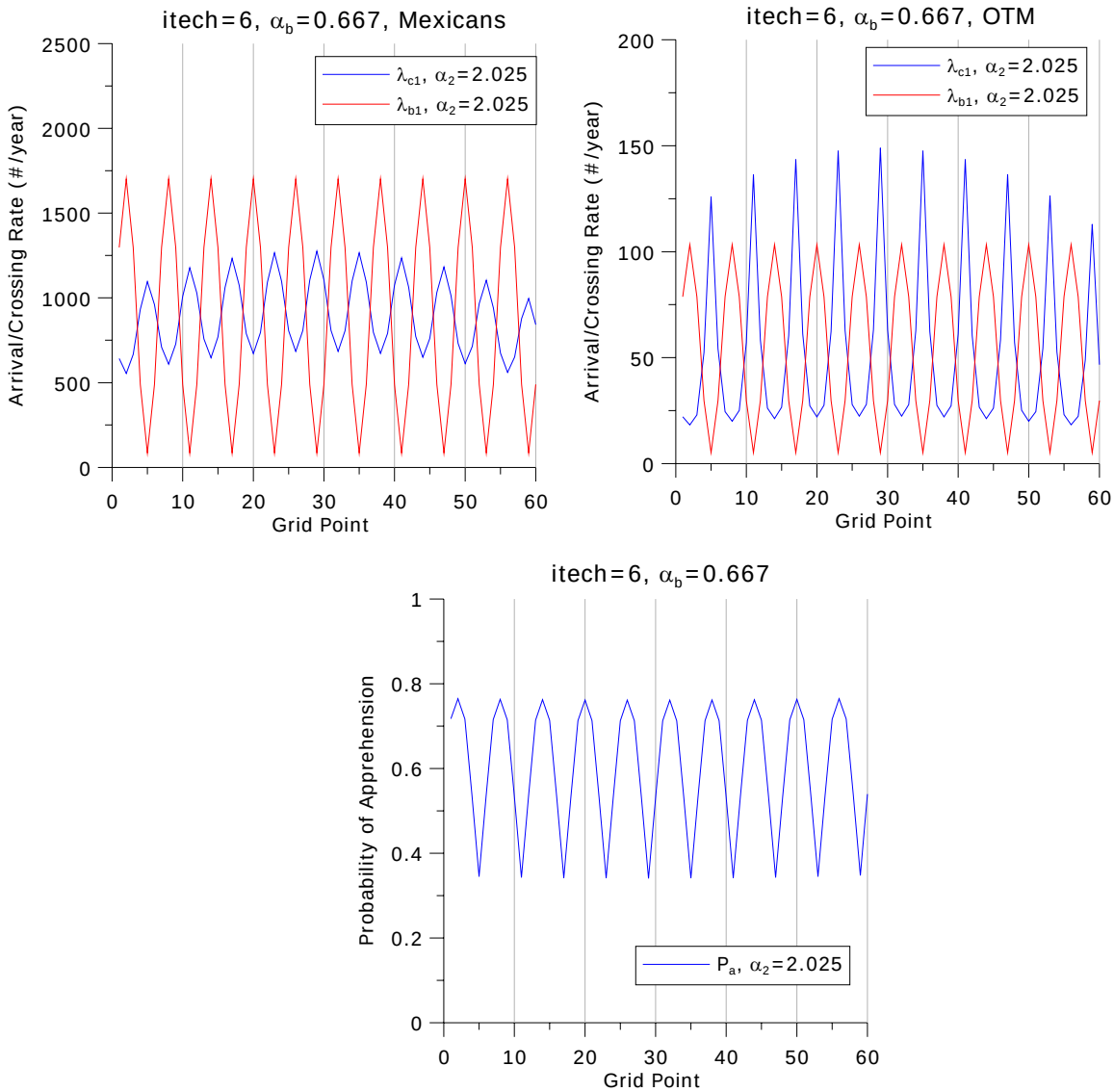

Figure 3. (Upper left) Mexican alien arrival and crossing rates (λ_{b1} and λ_{c1}, respectively), (upper right) OTM alien arrival and crossing rates (λ_{b2} and λ_{c2}, respectively), and (bottom) the BP apprehension probability given by the Wein model for the base case, assuming the entire border (itech=6) is covered by surveillance technology and BP agents are deployed according to a sinusoidal distribution (with a relative amplitude α_b of 0.667, see Chang et al. (2011)). Variable α_2 is the exponential apprehension parameter of technology; alternatively, its reciprocal gives the radius of influence of technology.

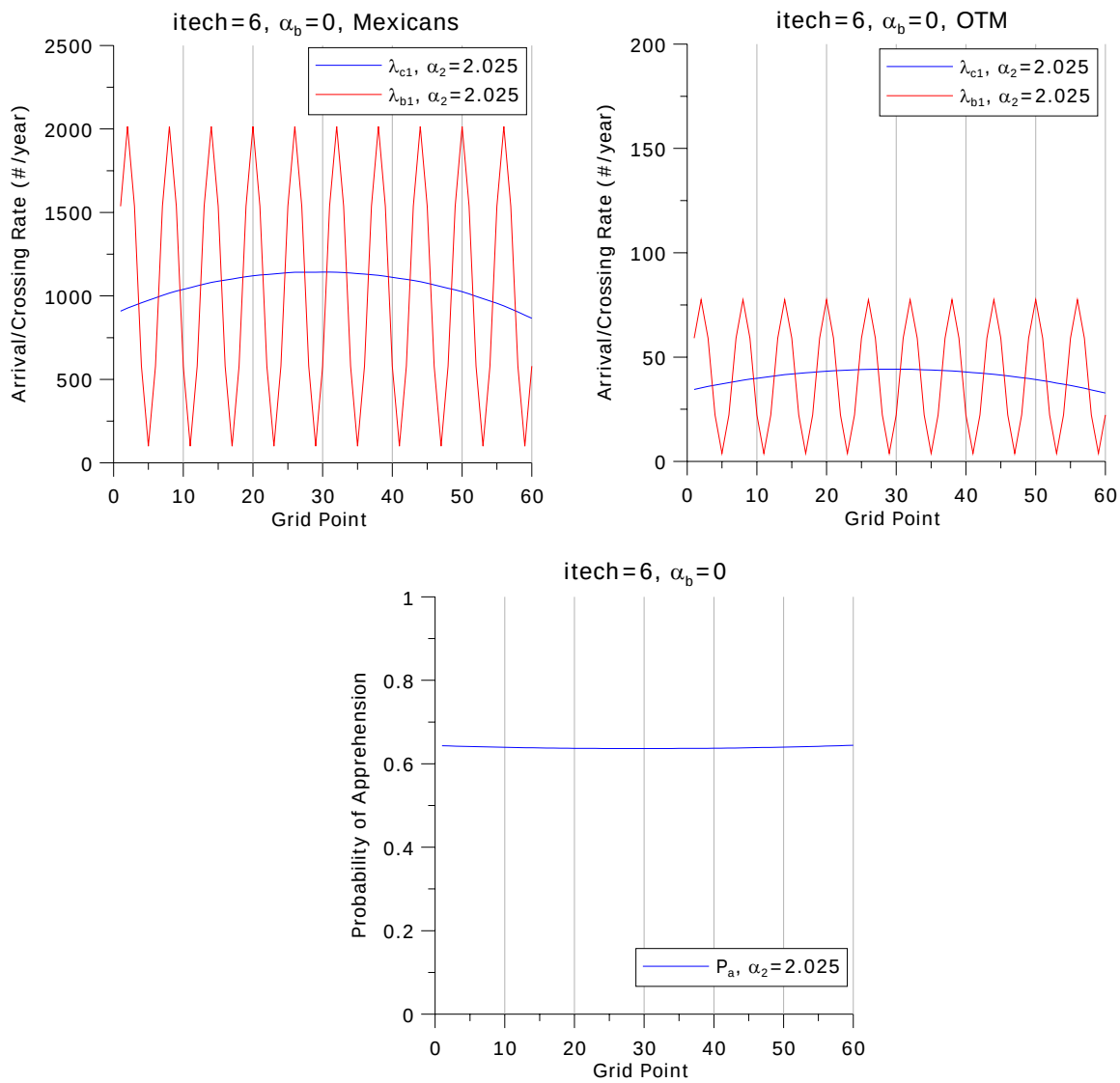

Figure 4. (Upper left) Mexican alien arrival and crossing rates (λ_{b1} and λ_{c1}, respectively), (upper right) OTM alien arrival and crossing rates (λ_{b2} and λ_{c2}, respectively), and (bottom) the BP apprehension probability given by the Wein model for the base case, assuming the entire border (itech=6) is covered by surveillance technology and BP agents are uniformly deployed (with a zero relative sinusoidal amplitude α_b). Variable α_2 is the exponential apprehension parameter of technology; alternatively, its reciprocal gives the radius of influence of technology.

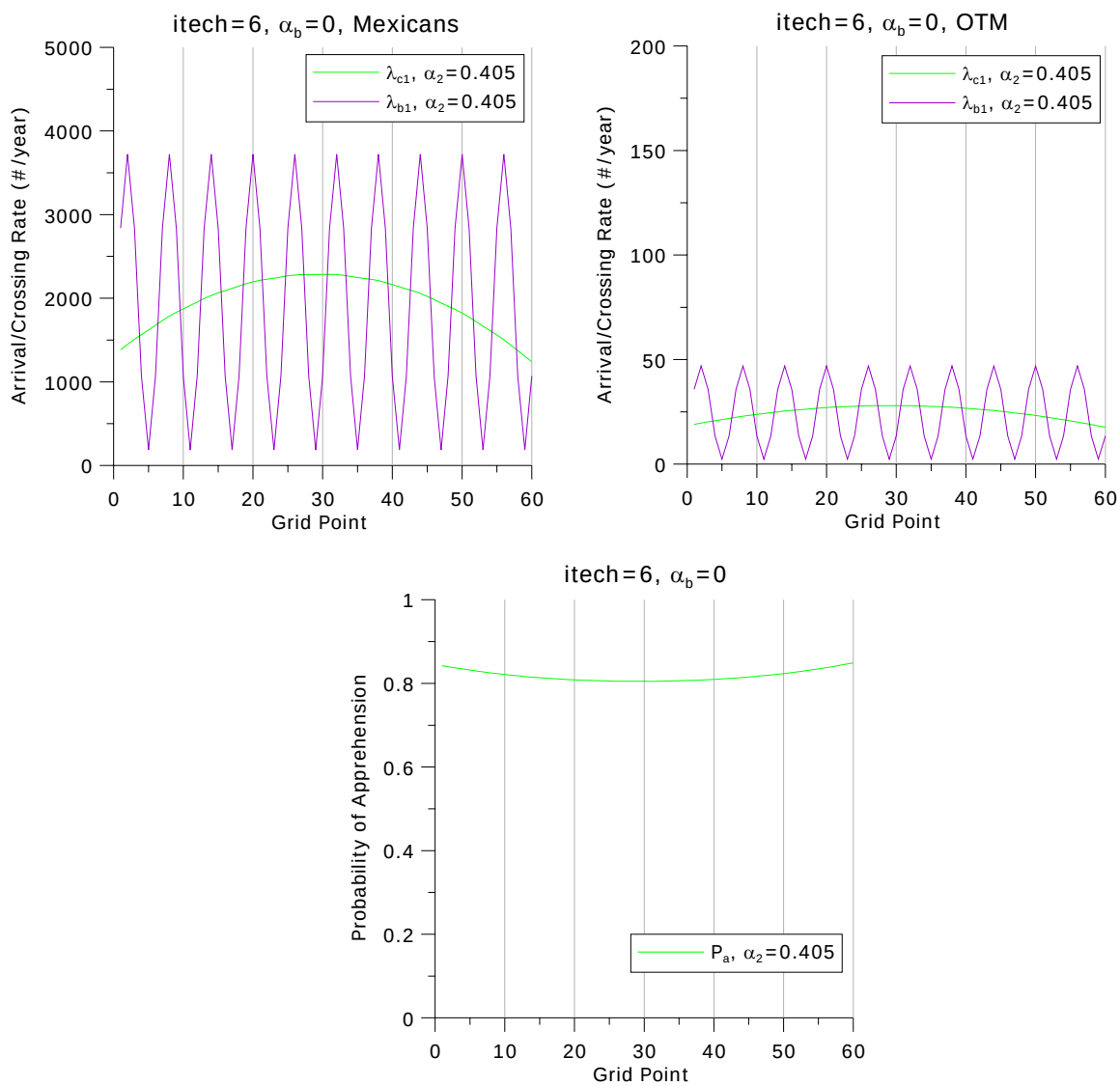

Figure 5. (Upper left) Mexican alien arrival and crossing rates (λ_{b1} and λ_{c1}, respectively), (upper right) OTM alien arrival and crossing rates (λ_{b2} and λ_{c2}, respectively), and (bottom) the BP apprehension probability given by the Wein model for the base case, assuming the entire border (itech=6) is covered by surveillance technology with a five-fold increase in its radius of influence and BP agents are uniformly deployed (with a zero relative sinusoidal amplitude α_b). Variable α_2 is the exponential apprehension parameter of technology; alternatively, its reciprocal gives the radius of influence of technology. The value of α_2 is 1/5 of that used in preceding figures, i.e., a five times larger radius of influence.

3.3. Relationships of Equilibrium Illegal Wage to Key Decision Variables

The illegal wage submodel (see section 2.4 and Wein et al. 2009) accounts for the supply of and demand for unskilled labor and the impact of worksite enforcement on wages. This submodel gives the equilibrium illegal wage (w_u) as a function of five decision variables:

- the number of worksite inspectors (m_w);
- the fraction of worksite inspections that are random (r_w);

- the fine per illegal worker-hour (f_w);
- the number of illegal workers who become legalized inside the United States (Δ_l); and
- the number of new guest workers from outside the United States (N_g).

The equilibrium illegal wage in turn influences the discrete choice submodel. In other words, the influence of the five key worksite enforcement decision variables (m_w, r_w, f_w, Δ_l, and N_g) is absorbed into a single variable, w_u. Hence, the Wein model implicitly assumes that any combinations of m_w, r_w, f_w, Δ_l, and N_g leading to the same resulting w_u will have the same impact on the model, with the only exception that different budgets might be involved because of different values of m_w (see table 1). This suggests that the five policy instruments (i.e., m_w, r_w, f_w, Δ_l, and N_g) all have the same goal of making working in the United States less attractive. As a result, policy makers should closely monitor the equilibrium illegal wage, for it summarizes the effectiveness of these instruments. More importantly, they should be seen as alternative means to the same end, rather than independent policies that should each be ramped up to the maximum value possible.

Figure 6 through figure 8 summarize the dependence of w_u on m_w, r_w, f_w, Δ_l, and N_g. Figure 6 suggests that m_w has the biggest impact on w_u. Stronger dependence of w_u on r_w and f_w exists only for higher m_w. Figure 7 focuses on a subset of the data used to create Figure 6 by inspecting the dependence of w_u on r_w for two values of m_w (400 and 3,000) combined with two values of f_w (5 and 15). (Both Figure 6 and Figure 7 assume Δ_l=0 and N_g=0.) The stronger dependence of w_u on r_w and f_w for higher m_w is again clear in Figure 7. It further shows an interesting dependence (also evident in Figure 6 upon a careful examination) of w_u on r_w. When the fine for hiring illegal workers is relatively low (i.e., f_w=\$5/worker-hour), more random worksite inspections lead to a lower w_u. However, when the fine is relatively high (i.e., f_w=\$15/worker-hour), more random worksite inspections lead to a higher w_u. In other words, in a low-fine regime, more random worksite inspections are more effective in reducing the illegal wage, whereas in a high-fine regime, more targeted worksite inspections are more effective.

A close inspection of figure 8 shows rough equivalence between Δ_l and N_g in terms of their impacts on w_u. For example, the long blue dashed curve (Δ_l=0 and N_g=4M) almost completely overlaps the solid red curve (Δ_l=4M and N_g=0). This is intuitive because Δ_l and N_g provide undifferentiated unskilled labor.

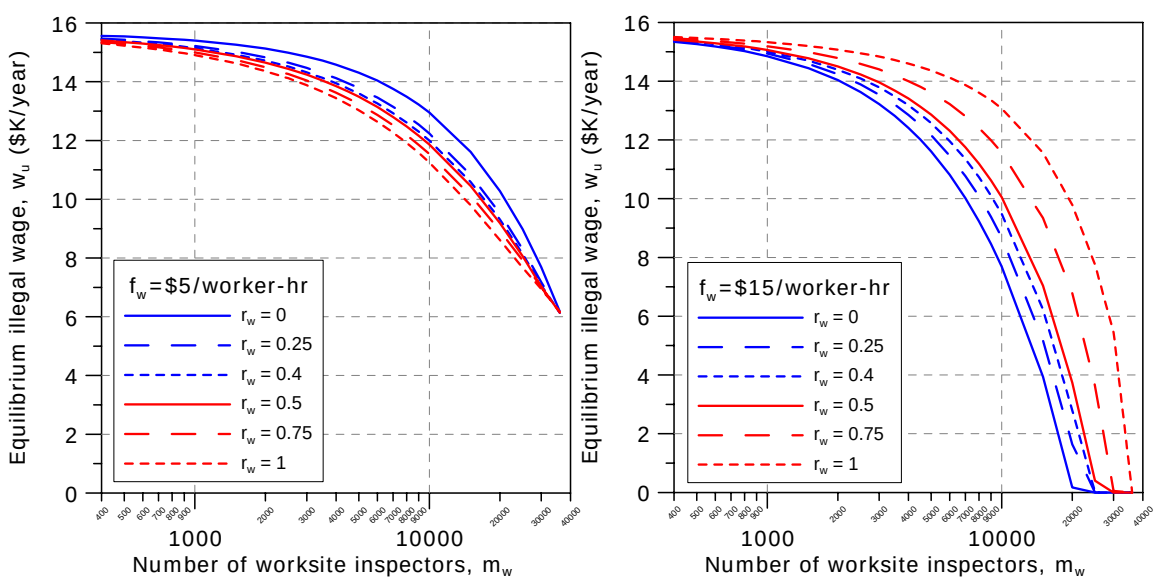

Figure 6. The equilibrium illegal wage (w_u, in \$K/year) as a function of the number of worksite inspectors (m_w), the fraction of worksite inspections that are random (r_w), and the fine per illegal worker-

hour (f_w). These charts assume the number of illegal workers who become legalized inside the United States (Δ_l) and the number of new guest workers from outside the United States (N_g) are at their base case values of zero.

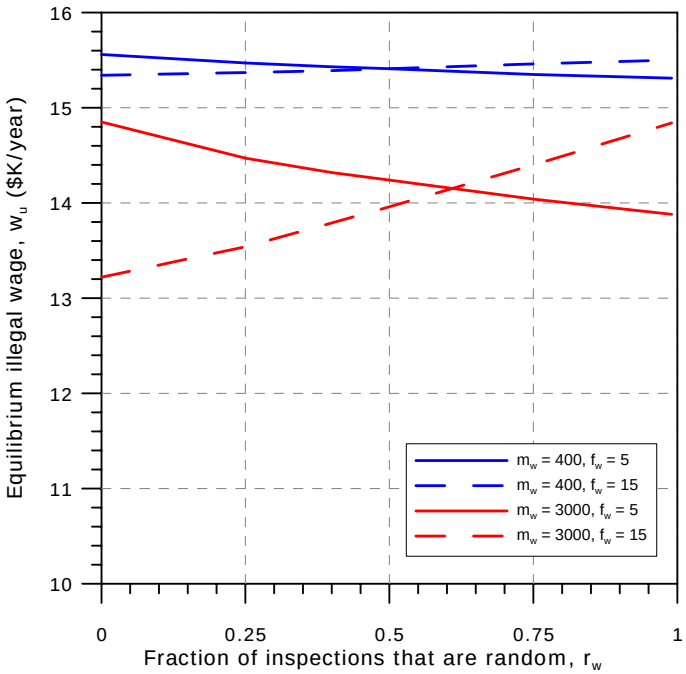

Figure 7. The equilibrium illegal wage (w_u, in $K/year) as a function of the fraction of worksite inspections that are random (r_w) for different combinations of the number of worksite inspectors (m_w; 400 or 3,000) and the fine per illegal worker-hour (f_w; 5 or 15). The figure assumes the number of illegal workers who become legalized inside the United States (Δ_l) and the number of new guest workers from outside the United States (N_g) are at their base case values of zero.

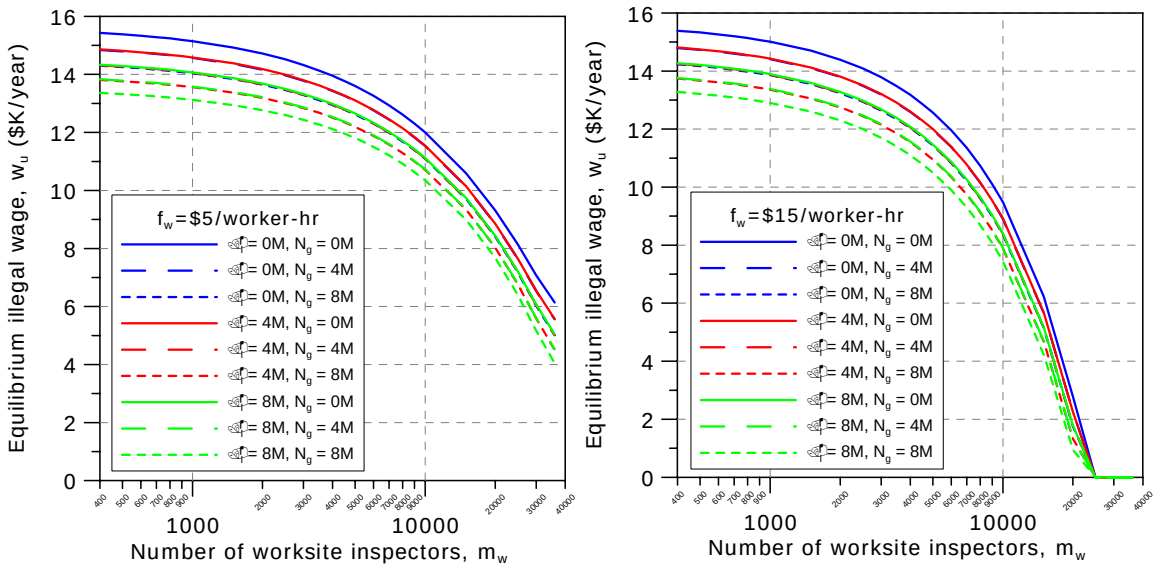

Figure 8. The equilibrium illegal wage (w_u, in $K/year) as a function of the number of worksite inspectors (m_w), the fine per illegal worker-hour (f_w), the number of illegal workers who become legalized inside the United States (Δ_l), and the number of new guest workers from outside the United States (N_g).

These charts assume the fraction of worksite inspections that are random (r_w) is at its base case value of 0.4.

Figure 9 jointly displays the dependence of the equilibrium illegal wage (w_u) on the number of worksite inspectors (m_w, solid black curve), the number of illegal workers in the United States who become legalized (Δ_l, dashed blue curve), and the number of new guest workers from outside the United States (N_g, dashed red curve), while all the remaining worksite enforcement decision variables are held at their base case values. For example, the dashed blue curve shows the relationship between Δ_l and w_u, while $m_w = 400$, f_w = \$5/worker-hour, $r_w = 0.4$, and $N_g = 0$. The figure has two noteworthy features. First, the curves for N_g and Δ_l almost overlap, consistent with an earlier observation that these two variables have roughly the same impact on w_u. Second, the figure can be used to graphically establish equivalence among m_w, Δ_l, and N_g by drawing a hypothetical vertical line (e.g., the thin green line, which corresponds to \$14.3K/year for w_u). For example, the figure suggests that hiring 3,000 worksite inspectors, legalizing 8 million illegal workers, and granting 8 million work visas have roughly the same effect in reducing the equilibrium illegal wage from the base case value of \$15.4K/year to \$14.3K/year. While these results should not be interpreted with absolute accuracy, the model does offer a framework to consider the trade-off between hiring more worksite inspectors and various immigration policies.

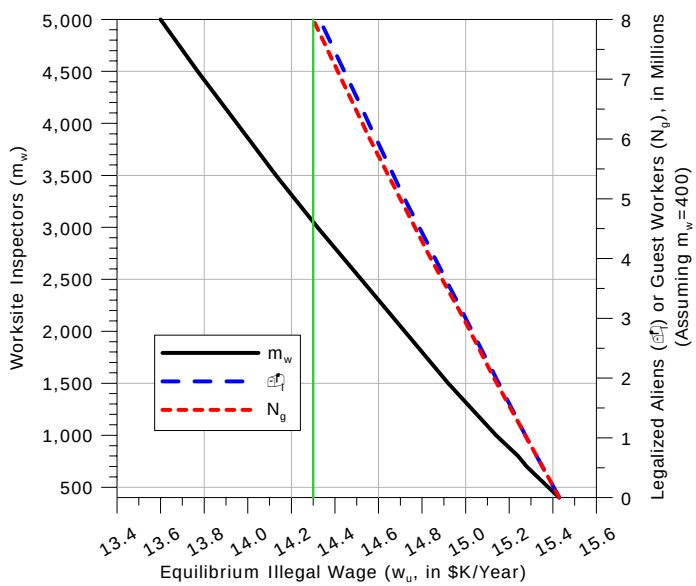

Figure 9. Dependence of the equilibrium illegal wage (w_u) on the number of worksite inspectors (m_w, solid black curve), the number of illegal workers in the United States who become legalized (Δ_l, dashed blue curve), and the number of new guest workers from outside the United States (N_g, dashed red curve), while all the remaining worksite enforcement decision variables are held at their base case values. The hypothetical green line corresponds to w_u = \$14.3K/year, and illustrates how the equivalence among m_w, Δ_l, and N_g can be established. Note different ordinates for m_w, Δ_l, and N_g.

To further study the relationships of m_w, r_w, f_w, Δ_l, and N_g to w_u, we took the logarithm of the model-generated data used to create figure 6 through figure 8, and then applied linear regression on the transformed data, similar to a standard econometric approach. That is, we considered the following estimating equation:

$$\ln w_u = \alpha_0 + \beta_1 \ln m_w + \beta_2 \ln r_w + \beta_3 \ln f_w + \beta_4 \ln \Delta_l + \beta_5 \ln N_g + \varepsilon \qquad (2)$$

where β's are the elasticities of w_u with respect to m_w, r_w, f_w, Δ_l, and N_g; α_0 is the intercept, and ε is the error term. The values of α_0 and β's are given in the table below, together with the t-statistics for hypothesis testing. The correlation coefficient, R, of the resulting regression equation is 0.597, meaning that R^2 is 0.357. It is not surprising that β_1 (the coefficient for m_w) has the largest magnitude of -0.271, which means, as suggested by the illegal wage

submodel, a 1 percent increase in m_w will lead to roughly 0.3 percent decrease in w_u, if all other variables are held constant. On the other hand, β_4 and β_5, the coefficients for Δ_l and N_g, are almost ten times smaller in magnitude and of equal value. This means that w_u is much less sensitive to the two decision variables related to the number of illegal workers who become legalized in the United States and the number of new guest workers from outside the United States, and that these two variables have similar impacts on w_u.

Table 2. Coefficients of the estimating equation where w_u is regressed against m_w, r_w, f_w, Δ_l, and N_g after a logarithmic transformation.

Coefficient	α_0	β_1	β_2	β_3	β_4	β_5
Associated with:		m_w	r_w	f_w	Δ_l	N_g
Value	4.987	-0.271	0.132	-0.104	-0.034	-0.034
t-statistics	236.2	27.1	-140.6	-25.6	-9.7	-9.7

3.4. Influence of Equilibrium Illegal Wage on Model Solutions

Figure 6 through figure 9 primarily show how the equilibrium illegal wage, w_u, varies with respect to the five worksite enforcement decision variables, m_w, r_w, f_w, Δ_l, and N_g. The influence of these five decision variables is then propagated through w_u to the discrete choice submodel, and the rest of the Wein model. Hence, the next step is to investigate the impacts of w_u on other model outputs, such as the aggregate probabilities of apprehension of unauthorized Mexican and OTM aliens (P_{a1} and P_{a2}, respectively), the propensities of unauthorized Mexicans and OTM aliens to migrate (P_{11} and P_{21}, respectively), and the probability of removal (P_r). The results are shown in figure 10, where it is assumed that the number of 24/7 BP agents and the number of detention beds are at their base case values of ~3,000 and 32,000, respectively, and that the entire border is covered with surveillance technology. Due to the fact that there is already sufficient bed space, the removal probability, P_r, remains close to one. The aggregate probabilities of apprehension along the border, P_{a1} and P_{a2}, only marginally depend on w_u. This makes sense as the probability of apprehension depends more on border enforcement than conditions inside the United States. On the other hand, the propensities, P_{11} and P_{21}, show greater dependence on w_u. It is interesting to note that P_{21} is consistently lower than P_{11}, primarily due to the higher cost for OTMs to cross the border.

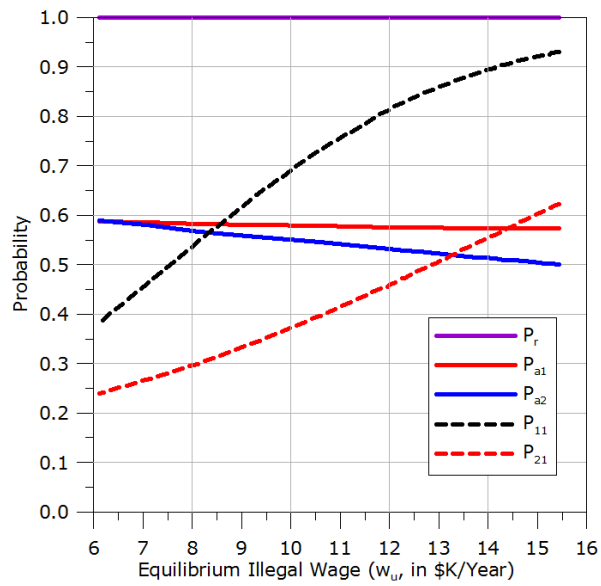

Figure 10. Dependence of the aggregate probabilities of apprehension of unauthorized Mexican and OTM aliens (P_{a1} and P_{a2}, respectively), the propensities of unauthorized Mexicans and OTM aliens to migrate (P_{11} and P_{21}, respectively), and the probability of removal (P_r) on the equilibrium illegal wage (w_u).

3.5. Effects of Increased Border Patrol Agents vs. Worksite Inspectors

Table 1 suggests the base case annual budget considered by the Wein model to be about $4.7B. Changing that budget—by increasing the number of worksite inspectors or BP agents—may impact outcomes along the border. Figure 11 shows how increasing the budget for either worksite inspectors (m_w) or BP agents (n_b) affects the aggregate probabilities of apprehension of unauthorized Mexican and OTM aliens (P_{a1} and P_{a2}, respectively) and the propensities of unauthorized Mexican and OTM aliens to migrate (P_{11} and P_{21}, respectively). Figure 11 assumes that the entire border is covered by surveillance technology, and all curves in the figure start from the annual base case budget of $4.7B. The figure shows that more worksite inspectors primarily impact the propensities, whereas more BP agents primarily impact the aggregate probabilities of apprehension. For example, P_{11} has a dramatic decrease from 0.93 to 0.39 (the solid blue curve) if the increased money beyond $4.7B is entirely spent on worksite inspectors, and P_{a1} increases from 0.57 to 0.75 (the dashed back curve) if the increased money beyond $4.7B is entirely spent on BP agents. This makes sense, as BP agents mainly impact the border apprehension submodel (section 2.2); and worksite inspectors mainly impact the equilibrium illegal wage (section 2.4), which affects the utility for crossing, which affects the propensity for crossing. This finding is consistent with the previous section, where the equilibrium illegal wage—on which the number of worksite inspectors has the strongest influence—was found to have a larger effect on the propensity to migrate.

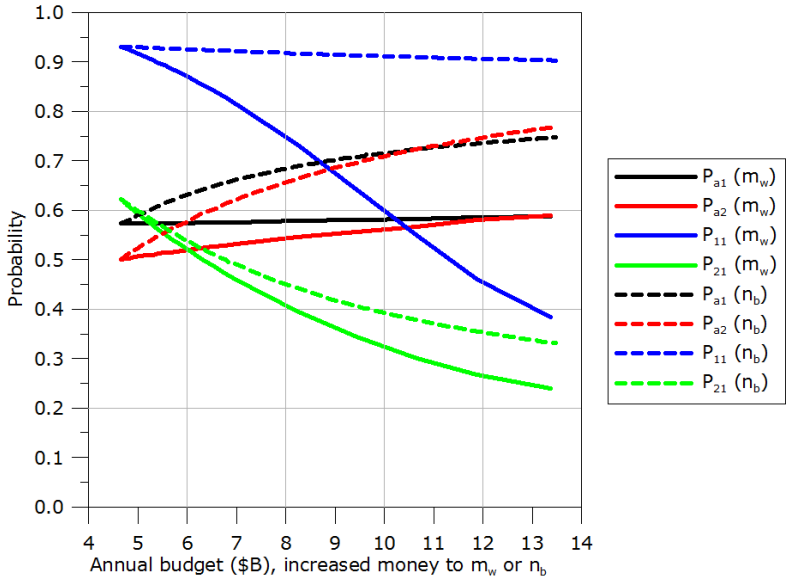

Figure 11. Dependence of the aggregate probabilities of apprehension of unauthorized Mexican and OTM aliens (P_{a1} and P_{a2}, respectively) and the propensities of unauthorized Mexican and OTM aliens to migrate (P_{11} and P_{21}, respectively) on the increased budget to either worksite inspectors (m_w) or BP agents (n_b).

3.6. Deterrence Due to E-Verify

E-Verify has recently emerged as an important component of potential comprehensive immigration reform (Rosenblum 2011). It is an Internet-based DHS program that allows an employer, using the information reported on a new hire's Form I-9, to determine whether the employee is eligible to work in the United States. The program is voluntary, with the exception that it is mandatory for federal contractors and subcontractors. The program is operated by DHS in partnership with the Social Security Administration. Congress has considered whether the program should be made mandatory for all employers. Nevertheless, in 2007 Arizona took the lead to pass a state law called the Legal Arizona Worker Act (LAWA) that, among other things, requires all employers to use the E-Verify work authorization system for all new hires. Lofstrom (2011) finds that LAWA was largely successful in meeting its goals of deterring unauthorized immigration to Arizona and preventing employment of unauthorized workers. The Wein model may help demonstrate the outcomes of a program that deters employment of illegal workers, such as E-Verify.

Within the model context, we assumed that E-Verify decreased the demand for illegal labor. We represented this decrease in demand as a multiplier less than one. Figure 12 shows the change in the equilibrium illegal wage (w_u) from its base case value of $15.4K/year due to reduced labor demand. In this simple example, we considered a labor demand multiplier ranging between 0.5 and 1.0, i.e., a reduction in labor demand up to 50 percent. The figure suggests an almost linear relationship between w_u and reduction in labor demand. Figure 10 and figure 11 can be used to further study the impacts of a reduced w_u on other model outputs.

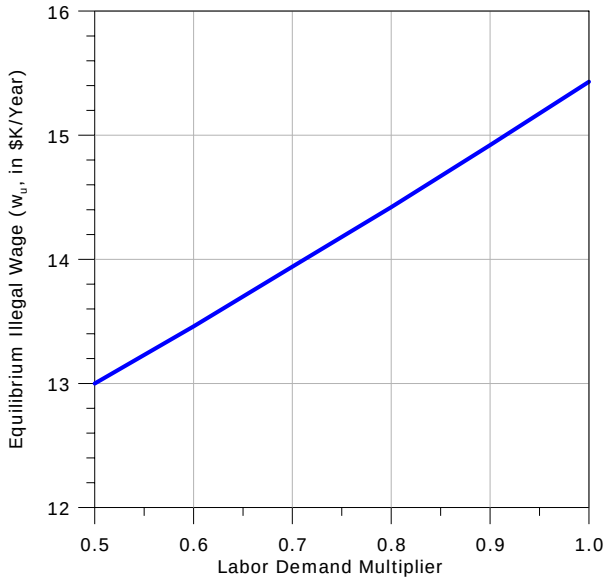

Figure 12. Change in the equilibrium illegal wage (w_u) as labor demand is decreased by up to 50 percent due to the deterrent effects resulting from such program as E-Verify.

3.7. Effects of Increased Post-Apprehension Consequence

One of the basic assumptions of the Wein model is that most unauthorized Mexican crossers are offered voluntary return upon apprehension. However, to break the smuggling cycle and to reduce recidivism, DHS has developed the Consequence Delivery System (CDS) designed to deter further illegal activities (Fisher 2011). The CDS includes program components such as Operation Against Smugglers Initiative on Safety and Security, Operation Streamline, Alien Transfer Exit Program, Mexican Interior Repatriation Program, Expedited Removal, etc. (see also Fisher 2011). It will be difficult to study the impacts of these individual components as they are inherently different. Therefore, we decided to study deterrence due to the CDS in a generic fashion as described below.

To account for this deterrence, we focused on the utility calculation of an unauthorized Mexican alien arriving at location x and choosing to cross at location y in the discrete choice border crossing submodel. One of the terms included in the utility is the detention cost (i.e., negative contribution to the utility)

$$\tilde{d}_1 = \psi\sqrt{2\,\text{days}} \qquad (3)$$

where ψ is the toll factor of apprehension and it is assumed that two days of salary are lost during the apprehension process. The square root function is used to crudely capture the psychological toll incurred regardless of the length of detention. As described in Chang et al. (2011), we used a simple scaling argument to estimate ψ to be in the order of ~$1K/√day. Therefore, a natural way to account for increased consequence after apprehension is to lengthen the days lost (i.e., the 2 days in eq. (3)).

Figure 13 shows the propensity of unauthorized Mexican aliens to cross as the number of days lost to the CDS is increased. Of course, it would be unreasonable to assume the days lost to be as long as 400, as assumed in the figure. However, we decided to consider up to 400 days lost as it would be equivalent to doubling the value of ψ, which was measured in a crude way as just mentioned, and 100 days lost. The figure shows that the model predicts the propensity to decrease from 0.95 to 0.75, which is still relatively high. This is probably due to the fact that the scale variable θ that measures population heterogeneity was estimated to be 0.16 (see Chang et al. (2011)), a value that is still relatively high to make the discrete choice model more deterministic, i.e., a more determined migrant population. In the future, it is necessary to study more adequate ways to treat the impacts of these post-apprehension consequence programs.

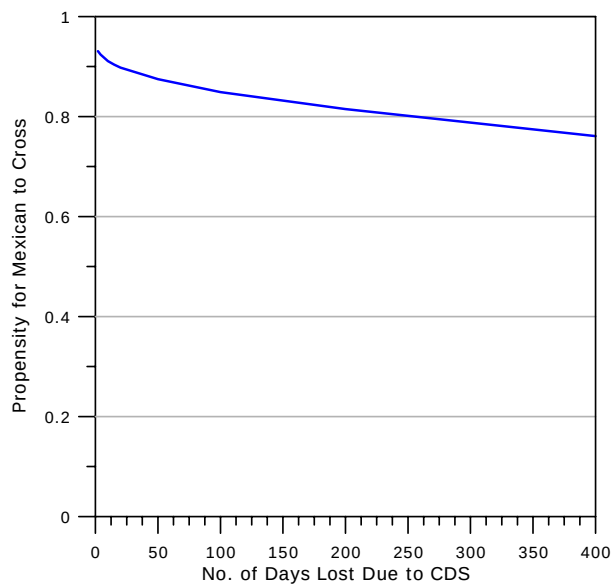

Figure 13. Propensity of unauthorized Mexicans to cross versus the number of days lost to the Consequence Delivery System (CDS).

3.8. Resource Allocation

The Wein model provides an optimization framework to study resource allocation. For example, the analysis can be configured to answer such questions as, "given a budget level, what is the optimal resource allocation to achieve a certain objective?" Many objectives can be considered. The objective function we used here is slightly different from Wein et al. (2009). Specifically, we chose to minimize the probability that an unauthorized alien (either Mexican or OTM) can successfully enter the United States, and even if apprehended will not be subsequently removed. This probability is given by the following equation:

$$P = 1 - P_a + P_a\left(1 - P_r\right) \tag{4}$$

where P_a is the aggregate probability of apprehension of *all unauthorized aliens* and is calculated as the weighted average of P_{a1} and P_{a2} by the crossing rates of Mexicans and OTMs (λ_{c1} and λ_{c2}, respectively). That is,

$$P_a = \frac{P_{a1}\lambda_{c1} + P_{a2}\lambda_{c2}}{\lambda_{c1} + \lambda_{c2}} \tag{5}$$

Equations (4) and (5) ensure a balanced treatment of unauthorized Mexican and OTM aliens, not just OTMs as originally considered in Wein et al. (2009). The optimization framework works as follows. A certain budget level, say $5B/year, can be achieved by different combinations of BP agents, worksite inspectors, surveillance technology, and bed space (see table 1). Each of these combinations leads to different values P_{a1}, P_{a2}, λ_{c1}, λ_{c2}, P_r, and ultimately P. It is then possible to find the "optimal" combination that minimizes the value of P. Figure 14 presents the results of resource allocation for an annual enforcement budget up to $10B. Recall that the budget for the base case is roughly $4.7B, where for simplicity it is assumed that the entire border is covered by surveillance technology, as its cost is relatively low compared to other resources. It can be seen that the value of P gradually decreases from 0.57 to 0.29 as the budget increases from $3B to $10B. The model suggests that the current number of beds (~32,000) is already sufficient to remove all detainees. As a result, the number of BP agents essentially drives the evolution of resource allocation. This is different from the results reported in Wein et al. (2009), as a few years ago a regime prevailed with a regime where there was a shortage of bed space, thus sometimes it was better to add more beds instead of more agents (i.e., the red and blue dashed curves would cross). The cross in the figure indicates the

current DHS resource allocation for the base case budget is essentially at an optimal level, in terms of minimizing P defined in eq. (4). However, as mentioned above, since BP agents and worksite inspectors have different impacts on the probability of apprehension and the propensity to migrate, the current allocation is not necessarily optimal if the objective is to minimize, say, the propensity to migrate

It is important to recognize the many assumptions and simplifications associated with these resource allocation calculations. For example, the unit cost of surveillance technology needs refinement, a better treatment of the effectiveness and cost of fencing is necessary, and the cost associated with other immigration policy options needs to be included. Therefore, a more realistic analysis will probably be much more nuanced. Nevertheless, these calculations demonstrate a rigorous, analytic framework to study resource allocation.

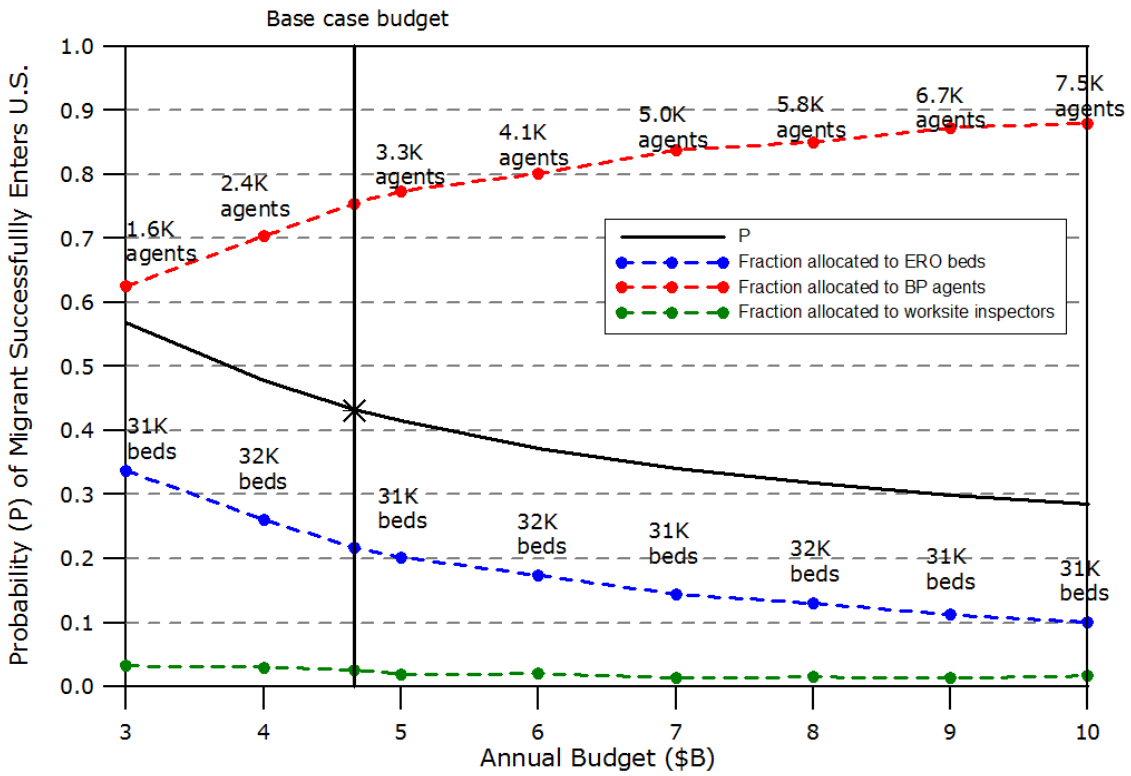

Figure 14. Resource allocation with an objective to minimize the probability (P) that an unauthorized alien can successfully enter the United States, and even if apprehended will not be subsequently removed. The black curve indicates the optimal P that can be achieved at different budget levels. The dashed blue, red, and green curves indicate the fractions of budget allocated to ERO beds, 24/7 BP agents, and worksite inspectors, respectively, for the optimal resource allocation. The cross indicates the current resource allocation for the base case budget.

4. CONCLUSIONS

Border security and immigration are an exceedingly complex system. Even by limiting scope to just a subset, i.e., illegal immigration between POEs, and by making many assumptions and simplifications, we are still led to a rather sophisticated model. Nevertheless, the unified model of the illegal system originally developed by Wein et al. (2009) and further updated by us provides a good framework to consider the important trade-offs of different policy options. The model has many desirable attributes, such as migrants' behavior, economic equilibrium, and feedback mechanisms that are based on well-established modeling techniques.

Some of the issues that we investigated include the efficacy of different distributions of BP agents, the dependence of the equilibrium wage that an illegal alien might receive as a function of worksite enforcement and immigration policy variables, the influence of this equilibrium illegal wage on the propensity to migrate, the trade-off between more BP agents vs. more worksite inspectors, deterrence due to the Consequence Delivery System, and optimal resource allocation based on a formal optimization framework.

We caution that due to the many assumptions and simplifications involved, the model's numerical outputs are primarily intended to capture qualitative impacts. Hence, over-interpretation of numerical accuracy should be avoided.

The Wein model can be further improved in a number of areas. For example, the model requires a large number of parameters, some of which are derived from data sources and some estimated by solving complex equation systems. Using more accurate data is always desirable. Some parameters that are estimated by solving equation systems might be better informed by surveys instead. All submodels are still simple representations of complex phenomena. For example, more robust and realistic treatments of surveillance technology, fencing, terrain, agent and alien arrival distributions, and the Consequence Delivery System are still necessary. Finally, the costs associated with immigration policy-related decisions (e.g., legalization of illegal aliens in the United States and a temporary worker program) should also be included.

REFERENCES

Banks, J., J.S. Carson II, B.L. Nelson, and D.M. Nicol, 2001: *Discrete-Event System Simulation, Third Edition*. Prentice Hall, 594 pp.

Ben-Akiva, M., and S.R. Lerman, 1985: *Discrete Choice Analysis: Theory and Application to Travel Demand*. MIT Press, Cambridge, Massachusetts, 390 pp.

Bosher, C.A., 1987: Immigration Reform: Status of Implementing Employer Sanctions after One Year. Government Accountability Office. Report GAO/GGD-88-14.

Chang, J.C., A. Reilly, D. Judson, and B. Streett, 2011: *A Unified Model of the Illegal Immigration System*. Prepared for the Office of Program Analysis and Evaluation, U.S. Customs and Border Protection, Department of Homeland Security, by the Homeland Security Studies and Analysis Institute, Arlington, Virginia, 82 pp.

Cobb-Clark, D.A., C.R. Shiells, and B.L. Lowell, 1995: Immigration reform: the effects of employer sanctions and legalization on wages. *J. Labor Economics*, **13**, 472-498.

CBP, 2011: *U.S. Border Patrol Statistics*. U.S. Customs and Border Protection, Department of Homeland Security, Washington, D.C. Last accessed at http://www.cbp.gov/xp/cgov/border_security/border_patrol/usbp_statistics/ on 30 December 2011.

Deaton, A., and J. Muellbauer, 1980: *Economics and Consumer Behavior*. Cambridge University Press, Cambridge, UK, 450 pp.

DHS, 2010a: *Quadrennial Homeland Security Review Report: A Strategic Framework for a Secure Homeland.* Department of Homeland Security, Washington, D.C., 108 pp.

DHS, 2010b: *U.S. Department of Homeland Security Annual Financial Report, Fiscal Year 2010*. Department of Homeland Security, Washington, D.C., 316 pp.

Espenshade, T.J., 1995: Using INS border apprehension data to measure the flow of undocumented migrants crossing the U.S.-Mexico frontier. *International Migration Review*, **29**, 545-565.

Fisher, M.J., 2011: 4 October 2011 Testimony of Michael J. Fisher, Chief, U.S. Border Patrol, U.S. Customs and Border Protection, Department of Homeland Security, before the House Committee on Homeland Security, Subcommittee on Border and Maritime Security. Last accessed at http://www.cbp.gov/xp/cgov/newsroom/congressional_test/fisher_testimony.xml on 11 November 2011.

Forrester, J.W., 1994: System dynamics, systems thinking, and soft OR. *System Dynamics Review*, **10**, 245-256.

Hutchinson, A., 2004: U.S. Department of Homeland Security Detention Priorities-a memorandum by Under Secretary Asa Hutchinson to Robert Bonner, Commissioner of U.S. Custom and Border Protection, 18 October 2004.

ICE, 2008: *ICE Fiscal Year 2008 Annual Report*. U.S. Immigration and Customs Enforcement, Department of Homeland Security, Washington, D.C., 51 pp.

Liu, Y. and L.M. Wein, 2008: A Queueing Analysis to Determine How Many Additional Beds Are Needed for the Detention and Removal of Illegal Aliens. *Management Science*, **54**, 1-15.

Lofstrom, M.J., 2011: Employment effects of Arizona's legislation against the hiring of unauthorized immigrants. 86th Western Economic Association International annual meetings, San Diego, CA.

MITRE, 2008: *Secure Border Initiative (SBI), Decision Support Model of Immigration Systems: Detailed Model Description*. Spiral 3, Version 0.07. Prepared for the Department of Homeland Security by the MITRE Corporation, McLean, Virginia.

Osborne, M.J. and A. Rubinstein, 1994: *A Course in Game Theory*. MIT Press, Cambridge, Massachusetts, 352 pp.

Rosenblum, M., 2011: E-Verify: strengths, weakness, and proposals for reform. Migration Policy Institute, Washington, D.C.

Turner, J., 2004: *Transforming the U.S.-Mexico Border: Providing Security and Prosperity in the Post 9/11 World*. House Select Committee on Homeland Security, Washington, D.C.

Wein, L.M., Y. Liu, and A. Motskin, 2009: Analyzing the Homeland Security of the U.S.-Mexico Border. *Risk Analysis*, **29**, 699-713, with appendix.