Information Processing and Limited Liability*

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Abstract

We study how limited liability affects the behavior of an agent who has to process information concerning what to do in a contingent event. Limited liability reduces the amount of information processed, particularly about very bad, highly unlikely events.

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How dire the consequences of an adverse event are depends in part on whether people are prepared to take good actions in that event. People seemed unprepared to take good actions in several recent adverse events (the global financial crisis, the European debt crisis, and the Fukushima nuclear accident) and these events have had dramatic consequences. In other work, we study what determines how carefully people think ex ante about what to do in a contingent event (Maćkowiak and Wiederholt, 2011). Here we consider how limited liability affects the extent to which people think ex ante about optimal actions in a contingent event. Limited liability may have played a role in the recent events. For example, an executive may have had little incentive to think about what to do when subprime mortgages underperform because that executive had known that his or her loss would be bounded in that event.

We model an agent who chooses how much information to process – how carefully to think – about his or her optimal action in a contingent event. The agent's optimal action in a contingent event depends on a random fundamental. The agent has a prior about the fundamental. In addition, the agent can acquire a signal about the fundamental. The agent chooses the properties of this signal subject to the constraint that more informative signals are more costly.

The model predicts that limited liability reduces the agent's incentive to process information. Hence the agent chooses to be less informed about what to do in a contingent event compared to the case of unlimited liability. Furthermore, limited liability tilts the agent's allocation of attention toward thinking less about the optimal action in "bad times" and thinking more about the optimal action in "good times". Lastly, limited liability tilts the agent's allocation of attention toward thinking less about the optimal action in "unusual times" and thinking more about the optimal action in "normal times". In sum, an agent with limited ability will be less prepared for a contingent event compared with an agent with unlimited liability, and the difference between the two agents will be particularly large for a very bad, highly unlikely event.

This paper belongs to the literature on rational inattention building on Sims (2003), in the sense that we model information processing as uncertainty reduction, where uncertainty is measured by entropy.[1] However, our results concerning how limited liability affects information acquisition are valid under general conditions made precise below.

[1] See Maćkowiak and Wiederholt (2011) for a list of other references in this literature.

1 Model

We study the decision problem of a single agent who decides how much information to acquire and process in order to take a good decision. The agent has to take an action $a \in \mathbb{R}$. The agent's payoff depends on the action a and the fundamental z. The agent's payoff function is

$$U(a,z) = \max\{u(a,z), 0\}.$$

The function $u(a,z)$ is quadratic, concave in its first argument ($u_{11} < 0$), and has a nonzero cross-derivative ($u_{12} \neq 0$). The max operator formalizes the notion of limited liability. The agent is uncertain about the payoff-maximizing action. The reason is that the agent is uncertain about the fundamental z. The agent has a normal prior $z \sim N(\mu_z, \sigma_z^2)$. The agent can acquire a signal s about the fundamental z before taking an action. The agent chooses the properties of this signal. Timing is as follows: (1) The agent chooses the properties of the signal. This decision problem is stated formally below. (2) The agent observes a realization of the signal s and takes the action $a \in \mathbb{R}$ that maximizes $E[U(a,z)|s]$.

For tractability, we impose a restriction on the signal and a restriction on the payoff function. We assume that the fundamental and the signal have a multivariate normal distribution. The signal has the form

$$s = z + \varepsilon,$$

where the noise ε is independent of the fundamental z and $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. The agent chooses the precision of the signal, $1/\sigma_\varepsilon^2$, but takes as given that the fundamental and the signal have a multivariate normal distribution. Next, since $u(a,z)$ is a quadratic function, we have

$$u(a,z) = u(a^*, z) + \frac{u_{11}}{2}(a - a^*)^2,$$

where $a^* = -\frac{u_1}{u_{11}} - \frac{u_{12}}{u_{11}} z$ denotes the payoff-maximizing action at the fundamental z. For tractability, we assume that $u(a^*, z)$ is independent of z and thus equals some constant $\bar{u}$. For ease of exposition, we assume that $a^* = z$. The last equation then becomes

$$u(a,z) = \bar{u} + \frac{u_{11}}{2}(a - z)^2.$$

These restrictions allow us to derive simple, transparent results concerning how limited liability affects information acquisition.

We now state the agent's information choice problem. The agent chooses the precision of the signal so as to maximize the expected payoff minus the cost of information. The agent anticipates that for a given realization of the signal he or she will take the best action given his or her posterior. The agent understands that there is limited liability. The cost of information is assumed to be linearly increasing in the amount of information contained in the signal. The agent's trade-off is that a more precise signal improves the quality of the action but is also more costly. Formally, the agent's decision problem reads

$$\max_{1/\sigma_\varepsilon^2 \in \mathbb{R}_+} \left\{ E\left[\max_{a\in\mathbb{R}} E\left[\max\left\{ \bar{u} + \frac{u_{11}}{2}(a-z)^2, 0\right\} |s\right]\right] - \lambda I(z;s)\right\}, \tag{1}$$

subject to

$$s = z + \varepsilon, \tag{2}$$

and

$$I(z;s) = \frac{1}{2}\log_2\left(2\pi e\sigma_z^2\right) - \frac{1}{2}\log_2\left(2\pi e\sigma_{z|s}^2\right), \tag{3}$$

where $z \sim N\left(\mu_z, \sigma_z^2\right)$, $\varepsilon \sim N\left(0, \sigma_\varepsilon^2\right)$, and z and ε are independent. The constant $\lambda > 0$ is the marginal cost of information. The term $I(z;s)$ quantifies the amount of information that the signal contains about the fundamental. Following Sims (2003), we quantify information as uncertainty reduction where uncertainty is measured by entropy. The inner max operator formalizes limited liability. The middle max operator is the assumption that the agent takes the best action given his/her posterior. The outer max operator is the assumption that the agent chooses the optimal precision of the signal given the payoff function and the cost of information.

2 Solution

To solve the decision problem (1)-(3), we apply the three max operators. First, the expected payoff associated with action a after the agent has received signal s equals

$$\begin{aligned} E[U(a,z)|s] &= \int_{-\infty}^{\infty} \max\left\{\bar{u} + \frac{u_{11}}{2}(a-z)^2, 0\right\} f(z|s)\,dz \\ &= \int_{a-\Delta}^{a+\Delta} \left[\bar{u} + \frac{u_{11}}{2}(a-z)^2\right] f(z|s)\,dz, \end{aligned} \tag{4}$$

where $f(z|s)$ is the conditional density of the fundamental given the signal and

$$\Delta = \sqrt{\frac{2\bar{u}}{-u_{11}}}.$$

To understand equation (4), note that limited liability kicks in if and only if the distance between the payoff-maximizing action z and the actual action a is at least Δ.

Second, to find the best action after the agent has received the signal, set the partial derivative of $E[U(a,z)|s]$ with respect to a equal to zero. Applying the Leibniz integral rule and using the definition of Δ yields the first-order condition

$$\frac{\partial E[U(a,z)|s]}{\partial a} = \int_{a-\Delta}^{a+\Delta} u_{11}(a-z) f(z|s)\, dz = 0.$$

The unique action satisfying this first-order condition is the conditional expectation of the payoff-maximizing action

$$a = \mu_{z|s}.$$

Furthermore, at this action $\partial^2 E[U(a,z)|s]/\partial^2 a < 0$. The best action given the agent's posterior is the conditional mean of the payoff-maximizing action (i.e., certainty equivalence holds). The maximum expected payoff therefore equals

$$\max_{a\in\mathbb{R}} E[U(a,z)|s] = \int_{\mu_{z|s}-\Delta}^{\mu_{z|s}+\Delta} \left[\bar{u} + \frac{u_{11}}{2}\left(\mu_{z|s} - z\right)^2\right] f(z|s)\, dz. \tag{5}$$

Note that in our model limited liability does not affect the best action given the agent's posterior. Certainty equivalence holds without and with limited liability. The reasons are the objective and the symmetry of the conditional distribution of z given s. However, limited liability does affect the expected payoff associated with this action. To see this, rewrite the last equation as

$$\begin{aligned}\max_{a\in\mathbb{R}} E[U(a,z)|s] &= \bar{u} + \frac{u_{11}}{2}\sigma^2_{z|s} \\ &\quad -2\int_{-\infty}^{\mu_{z|s}-\Delta} \left[\bar{u} + \frac{u_{11}}{2}\left(\mu_{z|s} - z\right)^2\right] f(z|s)\, dz. \end{aligned}\tag{6}$$

The first term on the right-hand side of (6) is the expected payoff if there were no limited liability. The second term is the expected benefit from limited liability.

Third, we are now almost in the position to solve the decision problem (1)-(3). Before doing that, it is useful to study in some detail the expected benefit from limited liability. Standard formulas for the moments of a truncated normal distribution yield

$$-2\int_{-\infty}^{\mu_{z|s}-\Delta}\left[\bar{u}+\frac{u_{11}}{2}\left(\mu_{z|s}-z\right)^{2}\right]f\left(z|s\right)dz=-2\Phi\left(-\frac{\Delta}{\sigma_{z|s}}\right)\left[\bar{u}+\frac{u_{11}}{2}\sigma_{z|s}^{2}\left(1+\frac{\Delta}{\sigma_{z|s}}\frac{\phi\left(-\frac{\Delta}{\sigma_{z|s}}\right)}{\Phi\left(-\frac{\Delta}{\sigma_{z|s}}\right)}\right)\right], \tag{7}$$

where $\phi\left(\cdot\right)$ and $\Phi\left(\cdot\right)$ denote the pdf and cdf of the standard normal distribution. It follows that the expected benefit from limited liability depends on the conditional variance of the payoff-maximizing action ($\sigma_{z|s}^{2}$) but not on the conditional mean of the payoff-maximizing action ($\mu_{z|s}$). Thus, without loss in generality, we can set $\mu_{z|s}=0$ when we study the expected benefit from limited liability. Furthermore, equation (7) can be used to compute the expected benefit from limited liability without numerical integration. Next, it is useful to study several derivatives of the expected benefit from limited liability. The derivative of the expected benefit from limited liability with respect to $\sigma_{z|s}^{2}$ equals

$$-2\int_{-\infty}^{-\Delta}\left[\bar{u}+\frac{u_{11}}{2}z^{2}\right]\frac{\partial f\left(z|s\right)}{\partial\sigma_{z|s}^{2}}dz. \tag{8}$$

This expression is always strictly positive. The term in square brackets is strictly negative for all $z\in\left(-\infty,-\Delta\right)$. The derivative of the density function with respect to its variance is strictly positive for all $z\in\left(-\infty,-\sigma_{z|s}\right)$. Thus, expression (8) is strictly positive when $-\Delta\leq-\sigma_{z|s}$. Furthermore, taking the derivative of expression (8) with respect to $\bar{u}$ (and taking into account that Δ depends on $\bar{u}$) yields

$$-2\int_{-\infty}^{-\Delta}\frac{\partial f\left(z|s\right)}{\partial\sigma_{z|s}^{2}}dz. \tag{9}$$

This expression is strictly negative. Lowering $\bar{u}$ (and thereby reducing Δ) increases expression (8). Thus, expression (8) is strictly positive also when $-\Delta\in\left(-\sigma_{z|s},0\right]$. In summary, the derivative of the expected benefit from limited liability with respect to $\sigma_{z|s}^{2}$ is always strictly positive, and this derivative is larger when $\bar{u}$ is lower.

Finally, we turn to the decision problem (1)-(3). Using equations (6)-(7) and noting that choosing $\left(1/\sigma_{\varepsilon}^{2}\right)\in\mathbb{R}_{+}$ is equivalent to choosing $\sigma_{z|s}^{2}\in\left(0,\sigma_{z}^{2}\right]$ one can write the decision problem

(1)-(3) as

$$\max_{\sigma^2_{z|s}\in(0,\sigma^2_z]}\left\{\bar{u}+\frac{u_{11}}{2}\sigma^2_{z|s}+BLL\left(\bar{u},u_{11},\sigma^2_{z|s}\right)-\lambda\frac{1}{2}\log_2\left(\frac{\sigma^2_z}{\sigma^2_{z|s}}\right)\right\}, \tag{10}$$

where $BLL\left(\bar{u},u_{11},\sigma^2_{z|s}\right)$ is the expected benefit from limited liability which is given by equation (7). One can also state the decision problem (10) in terms of uncertainty reduction. Defining $\kappa=\frac{1}{2}\log_2\left(\sigma^2_z/\sigma^2_{z|s}\right)$ the problem can be stated as

$$\max_{\kappa\in\mathbb{R}_+}\left\{\bar{u}+\frac{u_{11}}{2}\sigma^2_z 2^{-2\kappa}+BLL\left(\bar{u},u_{11},\sigma^2_z 2^{-2\kappa}\right)-\lambda\kappa\right\}. \tag{11}$$

The first-order condition for this problem is

$$\frac{-u_{11}}{2}\sigma^2_z 2^{-2\kappa}-\frac{\partial BLL\left(\bar{u},u_{11},\sigma^2_{z|s}\right)}{\partial\sigma^2_{z|s}}\sigma^2_z 2^{-2\kappa}=\frac{\lambda}{2\ln(2)}. \tag{12}$$

Without limited liability the solution would be

$$\kappa^*=\begin{cases}\frac{1}{2}\log_2\left(\frac{-u_{11}\sigma^2_z\ln(2)}{\lambda}\right) & \text{if } \frac{-u_{11}\sigma^2_z\ln(2)}{\lambda}\geq 1\\ 0 & \text{otherwise}\end{cases}. \tag{13}$$

With limited liability the equilibrium uncertainty reduction κ is strictly smaller than κ^* so long as $\kappa^*>0$. This follows from the fact that the second term on the left-hand side of (12) is strictly negative. Limited liability reduces equilibrium information processing. The reason is that processing information reduces the expected benefit from limited liability. Furthermore, the difference between equilibrium information processing with and without limited liability is larger when $\bar{u}$ is smaller. This follows from the fact that the second term on the left-hand side of (12) is more negative when $\bar{u}$ is smaller. Limited liability reduces in particular the incentive to acquire information about the optimal action in bad times.

References

[1] Maćkowiak, Bartosz and Mirko Wiederholt (2011): "Inattention to Rare Events." CEPR Discussion Paper 8626.

[2] Sims, Christopher A. (2003): "Implications of Rational Inattention." Journal of Monetary Economics, 50(3), 665-690.