Information in Cournot: Signaling with Incomplete Control

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Abstract

We embed signaling in the classical Cournot model in which several firms sell a homogeneous good. The quality is known to all the firms, but only to some buyers. The quantity-setting firms can manipulate the price to signal quality. Because there is only one price in a market for a homogeneous good, each firm incompletely controls the price-signal through the quantity decision. We characterize the unique signaling Cournot equilibrium in which the price signals quality to the uninformed buyers. We then compare the signaling Cournot equilibrium with the full-information Cournot equilibrium. Signaling is shown to increase the equilibrium price. Moreover, under certain conditions regarding the composition of buyers, the number of firms, and the distribution of costs across firms, the effects of signaling and market externality cancel each other. In other words, the profits under signaling Cournot equal the profits of a cartel in a full-information environment.

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1 Introduction

Cournot's 1838 model of firms' strategic interactions is the foundation for the analysis of imperfect competition, and has been widely used in a variety of fields in economics. One issue that arises in the Cournot model is the informative role of prices. Previous studies have focused on the learning activity of the firms. Specifically, information flows have been studied in Cournot models of limit-pricing (Harrington, 1987), signal-jamming (Riordan, 1985; Mirman et al., 1993), as well as experimentation (Mirman et al., 1994). However, in a Cournot environment, existence and characterization of the Cournot equilibrium remain open questions for the case in which the buyers are uninformed and engage in learning through prices, i.e., price signaling quality. Moreover, unlike previous signaling models done in the context of price-setting firms (in monopoly or oligopoly), Cournot competition adds the difficulty that there is only one market price and several firms, so that the signal is partially controlled by each firm.

The purpose of this paper is to provide an analysis of signaling in the classical Cournot model in which several firms sell a homogeneous good but face heterogeneous costs. We assume that the quality of the product is known to all the firms, but only to some buyers. The quantity-setting firms manipulate the output, and, therefore, affect the price in order to signal quality. Because there is only one price in a market for a homogeneous good, each firm incompletely controls the price-signal through the quantity decision. In order to study the effect of signaling on the Cournot equilibrium, we retain a standard signaling framework with linear demand in which the quality is related to the reservation price. See Milgrom and Roberts (1986), Bagwell and Riordan (1991), Daughety and Reinganum (1995, 2005, 2007, 2008a,b), Janssen and Roy (2010), Caldieraro et al. (2011), Dubovik and Jansen (2011), and Mirman and Santugini (2011).

We establish the existence of a unique Cournot equilibrium in which the price transmits information about the quality of the good to uninformed buyers, hereafter referred to as a signaling Cournot equilibrium. While the proof is specific to price signaling quality, it provides a basis for the existence

of a signaling equilibrium in a Cournot framework, in which several firms interact non-cooperatively to signal quality through one price. We assume that quality is a continuum on the real positive line. This yields a unique equilibrium in which every positive price is a possible outcome in equilibrium. Hence, out-of-equilibrium beliefs have no part to play in our analysis. In the unique signaling Cournot equilibrium, signaling and exchanges both occur when there are some informed buyers. However, if all buyers are uninformed, the price corresponding to the signaling Cournot equilibrium equals the reservation price. While information is transmitted through the price, the good is not purchased.

Next, we show that the signaling Cournot equilibrium cannot be the same as the full-information Cournot equilibrium regardless of the composition of buyers, the number of firms, the true quality of the good, and the distribution of costs across firms. Specifically, compared to the full-information case, signaling induces the firms to reduce quantities, which increases the price. This result is consistent with the monopoly case (Bagwell and Riordan, 1991; Daughety and Reinganum, 2008a; Mirman and Santugini, 2011). Moreover, our results are complementary to a similar result found for a signaling Bertrand game with differentiated products. Indeed, Daughety and Reinganum (2008b) show that signaling distorts the price upward when substitutability between differentiated products is sufficiently small whereas we show that it also applies for the case of perfect substitutability.

Finally, the effect of signaling on quantities and price alters profits. Specifically, while the profits of high-quality firms may increase in a differentiated-good Bertrand model when all buyers are uninformed (Daughety and Reinganum, 2008b), we find that signaling increases profits for all firms as long as the fraction of informed buyers is not too low. For instance, if the firms face no cost, then signaling increases profits as long as the fraction of informed buyers is greater than the inverse of the number of firms.[1] The reason for this increase is that signaling mitigates the negative effect of the market externality inherent in the Cournot equilibrium on the profits of the firms.

[1]The positive effect of signaling on profits remains with cost. In fact, a higher cost reduces the fraction of informed buyers needed to yield more profits.

Furthermore, signaling may cancel the negative effect of the market externality. In other words, under certain conditions regarding the composition of buyers, the number of firms, and the distributions of costs across firms, the profits under signaling Cournot equal the profits of a cartel (or a monopolist) in a full-information environment.

Note that an increase in profits has also been found in Caldieraro et al. (2011) in a price-setting and differentiated-product duopoly in which the fraction of informed buyers is endogenized through the inclusion of a disclosure technology.[2] Caldieraro et al. (2011) show that, in a low-price separating equilibrium regime, the low-quality firm has an incentive to disclose its quality prior to setting price so as to increase the fraction of informed buyers, which leads to less price competition, and, thus, increases the profits of the low-quality firm.

The paper is organized as follows. Section 2 presents the model. Section 3 characterizes the signaling Cournot equilibrium. In Section 4, we study the effect of signaling on the Cournot equilibrium. Finally, section 5 concludes and suggests possible extensions.

2 The Model

In this section, we embed signaling in the classical Cournot model in which several firms sell a homogeneous good. We first present the model in which the price transmits information about the quality of the good to uninformed buyers. We then define the Cournot equilibrium under both the (benchmark) full-information and signaling environments.

Consider a market for a homogeneous good of quality $\theta \geq 0$ sold at price P. The demand side is composed of *informed* and *uninformed* price-taking buyers. Informed buyers know θ and have demand $q_I^d = \theta - P$. Uninformed buyers do not know θ, but infer the quality from observing the price.[3] Specifi-

[2]Specifically, in the first stage, the firms decide whether to incur an exogenous cost in order to disclose quality through a trusted certification party. The disclosure is observable to a fraction of buyers who become informed prior to the firms setting prices in the second stage.

[3]Let the prior beliefs about θ be summarized by the p.d.f. $\xi(\theta)$ with mean $\mu \equiv$

cally, upon observing P, the uninformed buyers learn that the quality is $\chi(P)$, where $\chi(P)$ is the inference rule representing posterior beliefs. The only difference between informed and uninformed buyers concerns information, and, thus, the demand of the uninformed buyers is $q_U^d = \chi(P) - P$. Normalizing the mass of buyers to one and letting $\lambda \in [0,1]$ be the fraction of informed buyers, the market demand is $Q^d = \lambda q_I^d + (1-\lambda) q_U^d$, or

$$Q^d = \lambda(\theta - P) + (1-\lambda)(\chi(P) - P). \tag{1}$$

The use of a demand that is linear in the price is standard in the signaling literature. Moreover, the unknown quality is often related to the demand intercept, i.e., the reservation or choke price. See Milgrom and Roberts (1986), Bagwell and Riordan (1991), Daughety and Reinganum (1995, 2005, 2007, 2008a,b), Janssen and Roy (2010), Caldieraro et al. (2011), Dubovik and Jansen (2011), and Mirman and Santugini (2011).[4]

The supply side is composed of J quantity-setting firms who know the quality θ. The firms are heterogeneous because they face different cost functions. Specifically, firm j produces quantity q_j at the total cost $c_j \theta q_j$, $c_j \in [0,1)$.[5] The objective of each firm is to choose q_j so as to maximize profit

$$\pi_j = P q_j - c_j \theta q_j, \tag{2}$$

where, using (1), $P = D\left(\sum_{j=1}^{J} q_j; \theta, \chi(\cdot), \lambda\right)$ is defined by

$$\lambda(\theta - P) + (1-\lambda)(\chi(P) - P) = \sum_{j=1}^{J} q_j. \tag{3}$$

In our model, the quality of the good is potentially undesirable. In other

$\int_{\theta \geq 0} \xi(x) \mathrm{d}x$.

[4] In Bagwell and Riordan (1991), quality can either be low or high. The demand for the high quality is linear, while the low quality product has a unit demand. In Daughety and Reinganum (2008a), the demand is $Q^D = (\alpha - (1-\delta)\theta)/\beta - P/\beta$, where $\alpha, \beta, \delta > 0$ are known parameters and $\theta \in [\underline{\theta}, \overline{\theta}]$ is the unknown parameter for which the price transmits information. As in our case, the demand intercept depends on the unknown parameter.

[5] Quality is assumed exogenous. For quality-choice models, see Wolinsky (1983) and the version by Tirole (1988, chap. 2.3.1.1), as well as Dubovik and Jansen (2011).

words, in a full-information environment with $\lambda = 1$, if $\theta = 0$, then no buyers wants to consume the good and the market does not exist. The zero lower bound for quality implies that the firms are always at risk of having a negative price-cost margin if the market is misperceived as selling the worst possible quality.

Having described the set up, we now define the full-information and signaling Cournot equilibrium. Since signaling is only relevant when there are uninformed buyers, we refer to a signaling environment when $\lambda \in [0, 1)$. The full-information environment applies when all buyers are informed, i.e., $\lambda = 1$.

Definition 2.1 presents the full-information Cournot equilibrium. If $\lambda = 1$, then, from (3), demand is $P = \theta - \sum_{j=1}^{J} q_j$. The superscript C refers to *Cournot*.[6]

Definition 2.1. *A full-information Cournot equilibrium consists of firms' strategies* $\{q_j^C(\theta)\}_{j=1}^J$ *and price* $P^C(\theta)$ *such that*

1. *Given* $\{q_k^C(\theta)\}_{k \neq j}$,

$$q_j^C(\theta) = \arg\max_{q_j} \left(\theta - q_j - \sum\nolimits_{k \neq j} q_k^C(\theta)\right) q_j - c_j \theta q_j. \tag{4}$$

2. *Given* $\{q_j^C(\theta)\}_{j=1}^J$,

$$P^C(\theta) = \theta - \sum\nolimits_{j=1}^{J} q_j^C(\theta). \tag{5}$$

Next, we define the signaling Cournot equilibrium in which the price signals quality. The first two conditions are identical to the full-information Cournot equilibrium except that demand depends on the inference rule. The third condition is specific to the signaling environment and implies that the equilibrium price is fully revealing about θ. The superscript SC refers to *signaling Cournot*.[7]

[6] For the sake of brevity, we drop the letters FI for *full-information*.

[7] We do not analyze Cournot equilibrium in which the price is uninformative about quality, hereafter a non-signaling Cournot equilibrium. Unless all buyers are uninformed

Definition 2.2. *For $\lambda \in [0, 1)$, a signaling Cournot equilibrium consists of firms' strategies $\{q_j^{SC}(\theta)\}_{j=1}^J$, price $P^{SC}(\theta)$, and inference rule $\chi^{SC}(P)$ such that*

1. *Given $\{q_k^{SC}(\theta)\}_{k\neq j}$ and $\chi^{SC}(P)$,*

$$q_j^{SC}(\theta) = \arg\max_{q_j} D\left(q_j + \sum\nolimits_{k\neq j} q_k^{SC}(\theta); \theta, \chi^{SC}(\cdot), \lambda\right) q_j - c_j\theta q_j. \quad (6)$$

2. *Given $\{q_j^{SC}(\theta)\}_{j=1}^J$ and $\chi^{SC}(P)$,*

$$P^{SC}(\theta) = D\left(\sum\nolimits_{j=1}^{J} q_j^{SC}(\theta); \theta, \chi^{SC}(\cdot), \lambda\right). \quad (7)$$

3. *Given $\{q_j^{SC}(\theta)\}_{j=1}^J$ and $P^{SC}(\theta)$, $\chi^{SC}(P^{SC}(\theta)) = \theta$.*

Before proceeding to the characterization of the equilibrium, a few comments are in order. First, the signaling game in a Cournot model is slightly different from that studied in the literature. Signaling occurs when each sender is in full control of the signal, e.g., a monopolist setting the price (Bagwell and Riordan, 1991), or oligopolists selling differentiated products, each setting his own price (Daughety and Reinganum, 2007, 2008b; Caldieraro et al., 2011). In a Cournot setting in which the firms sell the same good, there is only one price. Hence, each firm incompletely controls the price, and, thus, the signal. Information flows in a Cournot setting have been previously studied in models of limit-pricing (Harrington, 1987), signal-jamming (Riordan, 1985; Mirman et al., 1993), as well as experimentation (Mirman et al., 1994). Unlike our model in which the uninformed buyers learn from the price, these models consider situations in which the uninformed firms extract information from the price. Regardless of the type of information flows, a common feature in these Cournot models is that the price-signal is partially controlled by each firm.

Second, in the signaling Cournot model, firms face an informational externality in addition to the usual market (or price) externality. Indeed, in

and the firms face no cost, a non-signaling Cournot equilibrium does not exist. See Appendix A.

both the full-information and signaling Cournot models, each firm faces a market externality because the profit depends on the quantities of the other firms through demand. In the signaling Cournot model, the additional informational externality is due to the learning activity of the uninformed buyers. Indeed, from (6), the inference rule $\chi^{SC}(\cdot)$ alters the profits of all firms through demand.

3 The Cournot Equilibrium

Having presented the model, we now characterize the Cournot equilibrium. Specifically, we first present the (benchmark) full-information Cournot equilibrium, in which every buyer is informed about the quality, and, hence, the firms do not face an informational externality. We then derive the signaling Cournot equilibrium. In order to ensure that quantities are nonnegative in both full-information and signaling environments, Assumption 3.1 holds for the remainder of the paper.[8]

Assumption 3.1. *For* $\overline{c} \equiv \max\{c_k\}_{k=1}^{J}$, $1-(1+J)\overline{c}+\sum_{k=1}^{J} c_k > 0$.

Proposition 3.2 presents the strategies of the firms, aggregate output, and the price in the full-information Cournot equilibrium. Although the price is not used as a signal, it is informative about the quality, i.e., $P^C(\theta)$ is an increasing function of θ.

Proposition 3.2. *Suppose that all buyers are informed, i.e.,* $\lambda = 1$*. Then, there exists a unique full-information Cournot equilibrium, in which, for all* j*, firm* j *supplies*

$$q_j^C(\theta) = \frac{(1-(1+J)c_j+\sum_{k=1}^{J} c_k)}{1+J}\theta > 0. \tag{8}$$

Moreover, total supply is

$$Q^C(\theta) = \frac{J-\sum_{k=1}^{J} c_k}{1+J}\theta, \tag{9}$$

[8]Since $c_j \in [0,1)$ for all j, the condition stated in Assumption 3.1 always holds when the firms face no cost or identical cost.

and the price is

$$P^C(\theta) = \frac{1 + \sum_{k=1}^J c_k}{1 + J}\theta. \tag{10}$$

Proof. From (4), for all j, the profit of firm j is strictly concave in q_j, so that the best reply of firm j to $Q^C_{-j}(\theta) = \sum_{k \neq j} q^C_k(\theta)$ is the unique solution to the first-order condition $\theta - Q^C_{-j}(\theta) - 2q_j - c_j\theta = 0$.[9] Solving the J first-order conditions for individual quantities yields (8). Summing (8) over j yields (9). Plugging (9) into (5) where $\sum_{j=1}^J q^C_j(\theta) \equiv Q^C(\theta)$ yields (10). Since the best-reply for any firm has nonpositive slope larger than -1, the Cournot equilibrium is unique.[10] $\square$

We next consider the signaling environment in which the price is used as a signal by the uninformed buyers. Proposition 3.3 presents the strategies of the firms and the price in the signaling Cournot equilibrium when some buyers (but not all) are uninformed, i.e., $\lambda \in (0, 1)$.[11] Note that the zero lower bound along with the absence of an upper bound on θ removes the need to specify out-of-equilibrium beliefs. Indeed, in the unique signaling Cournot equilibrium, every $P \geq 0$ is a possible outcome in equilibrium.[12]

Proposition 3.3. *Suppose that some buyers are uninformed, i.e., $\lambda \in (0, 1)$. Then, there exists a unique signaling Cournot equilibrium, in which, for all j, firm j supplies*

$$q^{SC}_j(\theta) = \frac{(1 - c_j - B^{SC})(\lambda - B^{SC})}{1 - B^{SC}}\theta > 0. \tag{11}$$

Moreover, total supply is $\sum_{k=1}^J q^{SC}_k(\theta) = B^{SC}\theta$, and the price is

$$P^{SC}(\theta) = (1 - B^{SC})\theta. \tag{12}$$

[9] Assumption 3.1 ensures an interior solution.

[10] For all j, from the first-order condition, the best-reply of firm j is $r_j(Q^C_{-j}(\theta)) = ((1 - c_j)\theta - Q^C_{-j}(\theta))/2$, so that $r'_j(Q^C_{-j}(\theta)) = -1/2$.

[11] When all buyers are uninformed, there is no interior solution. The case of $\lambda = 0$ is handled separately in Remark 3.5.

[12] Note that, in Cournot, it is really the zero bound that removes the need to specify out-of-equilibrium beliefs. Indeed, suppose that $\theta \in [0, \overline{\theta}]$, then the fact that the firms choose quantities imply that $P \in [0, \overline{\theta}]$, i.e., the firms cannot set indirectly a price above the upper bound.

Here,

$$B^{SC} = \frac{1+(1+\lambda)J - \sum_{k=1}^{J} c_k - \sqrt{\left(1+(1+\lambda)J - \sum_{k=1}^{J} c_k\right)^2 - 4\lambda(1+J)\left(J - \sum_{k=1}^{J} c_k\right)}}{2(1+J)} \tag{13}$$

such that $B^{SC} \in (0, \min\{1-\overline{c}, \lambda\})$*,* $\overline{c} \equiv \max\{c_k\}_{k=1}^{J}$*.*

Proof. The proof has three steps. First, we derive the unique aggregate output $Q^{SC}(\theta)$ in a signaling equilibrium using best reply functions. Second, given the unique aggregate output, we show that the price $P^{SC}(\theta)$ and inference rule $\chi^{SP}(P)$ are unique. Third, given the unique aggregate output, we show that the best reply function for any firm has nonpositive slope larger than -1, i.e., there is a unique solution for each firm's supply and, thus, there exists a unique signaling Cournot equilibrium.

1. **Aggregate Output in a Signaling Equilibrium.** For $\theta > 0$ and $\lambda \in (0,1)$, we can exclude the possibility of $Q^{SC}(\theta) = 0$, and, thus, focus on an interior solution when quality is positive, i.e., $Q^{SC}(\theta) > 0$ for $\theta > 0$.

   (a) *Best Reply of Firm j in a Signaling Equilibrium.* For all j, given the inference rule $\chi^{SC}(P)$, the best reply of firm j to $Q_{-j}^{SC}(\theta) \equiv \sum_{k \neq j} q_k^{SC}(\theta)$ is the solution to the first-order condition corresponding to (6), i.e.,

   $$\frac{\partial D\left(q_j + Q_{-j}^{SC}(\theta); \theta, \chi^{SC}(\cdot), \lambda\right)}{\partial q_j} q_j + D\left(q_j + Q_{-j}^{SC}(\theta); \theta, \chi^{SC}(\cdot), \lambda\right) - c_j\theta = 0, \tag{14}$$

   which, in a signaling equilibrium (i.e., $\chi^{SP}(P^{SC}(\theta)) = \theta$), is rewritten as

   $$-\frac{1 - \frac{\mathrm{d}Q^{SC}(\theta)}{\mathrm{d}\theta}}{\lambda - \frac{\mathrm{d}Q^{SC}(\theta)}{\mathrm{d}\theta}} q_j + \theta - q_j - Q_{-j}^{SC}(\theta) - c_j\theta = 0. \tag{15}$$

   To derive (15), we impose the equilibrium informational requirement $\chi^{SC}(P^{SC}(\theta)) = \theta$ on (3), which, evaluated at $Q_{-j} = Q_{-j}^{SC}(\theta)$

and $\chi(\cdot) = \chi^{SC}(\cdot)$, yields

$$D\left(q_j + Q_{-j}^{SC}(\theta); \theta, \chi^{SC}(\cdot), \lambda\right)\Big|_{\chi^{SC}(P^{SC}(\theta))=\theta} = \theta - q_j + Q_{-j}^{SC}(\theta). \tag{16}$$

Next, differentiating (3) with respect to q_j evaluated at $Q_{-j} = Q_{-j}^{SC}(\theta)$ and $\chi(\cdot) = \chi^{SC}(\cdot)$ yields

$$\frac{\partial D\left(q_j + Q_{-j}^{SC}(\theta); \theta, \chi^{SC}(\cdot), \lambda\right)}{\partial q_j} = \frac{1}{-1 + (1-\lambda)\frac{\mathrm{d}\chi^{SC}(P)}{\mathrm{d}P}}. \tag{17}$$

It remains to characterize $\frac{\mathrm{d}\chi^{SC}(P)}{\mathrm{d}P}$. Here, the equilibrium inference rule $\chi^{SC}(P)$ representing posterior beliefs is consistent with Bayes' rule and the firms' strategies, as in any signaling game. In other words, $\chi^{SC}(P)$ is the inverse function of the equilibrium price defined by (7). Specifically, evaluating (3) at $\chi^{SP}(P^{SC}(\theta)) = \theta$ and $\sum_{j=1}^{J} q_j = Q^{SC}(\theta)$ yields

$$P^{SC}(\theta) = \theta - Q^{SC}(\theta). \tag{18}$$

Using (18), for all P, the equilibrium inference rule $\chi^{SC}(P) = \psi$ is implicitly defined by

$$P = \psi - Q^{SC}(\psi), \tag{19}$$

so that

$$\frac{\mathrm{d}\chi^{SC}(P)}{\mathrm{d}P} = \frac{1}{1 - \frac{\mathrm{d}Q^{SC}(\theta)}{\mathrm{d}\theta}}. \tag{20}$$

Plugging (20) into (17) yields

$$\left.\frac{\partial D\left(q_j + Q_{-j}^{SC}(\theta); \theta, \chi^{SC}(\cdot), \lambda\right)}{\partial q_j}\right|_{\chi^{SC}(P^{SC}(\theta))=\theta} = -\frac{1 - \frac{\mathrm{d}Q^{SC}(\theta)}{\mathrm{d}\theta}}{\lambda - \frac{\mathrm{d}Q^{SC}(\theta)}{\mathrm{d}\theta}}. \tag{21}$$

Plugging (16) and (21) into (14) yields (15).

(b) *Characterization of Unique Aggregate Output in a Signaling Equi-*

librium. We now derive the unique aggregate output given the best-reply functions. Summing (15) over j and setting $\sum_{j=1}^{J} q_j = Q^{SC}(\theta)$ yields

$$-\frac{1-\frac{\mathrm{d}Q^{SC}(\theta)}{\mathrm{d}\theta}}{\lambda-\frac{\mathrm{d}Q^{SC}(\theta)}{\mathrm{d}\theta}} Q^{SC}(\theta) + J\theta - JQ^{SC}(\theta) - \sum_{j=1}^{J} c_j\theta = 0. \tag{22}$$

To simplify notation, let $y \equiv Q^{SC}(\theta)$, $y' \equiv \frac{\mathrm{d}Q^{SC}(\theta)}{\mathrm{d}\theta}$, and $C \equiv \sum_{k=1}^{J} c_k$, so that (22) becomes

$$-\frac{1-y'}{\lambda-y'}y + J\theta - Jy - C\theta = 0, \tag{23}$$

which is a differential equation. Rearranging (23) yields

$$y' = \frac{(1+\lambda J)y - \lambda(J-C)\theta}{(1+J)y-(J-C)\theta}. \tag{24}$$

i. Valid Candidates for y. For $\theta > 0$, $P^{SC}(\theta) \in [\underline{c}\theta, \theta)$, $\underline{c} \equiv \min_k\{c_k\}$.[13] Hence, $y \in (0, (1-\underline{c})\theta], \theta > 0$, which implies that the initial condition is $(y_0, \theta_0) = (0, 0)$. Moreover, $P^{SC}(0) = 0$ and $P^{SC}(\theta)$ is increasing in θ. Hence, since posterior beliefs are the inverse of the price function, $\chi^{SC}(P)$ is increasing in P with $\chi^{SC}(0) = 0$, which implies, from (20), that $y' < 1$. Finally, demand must be decreasing for an interior solution to exist, i.e., from (21), $y' < \lambda$. Given that $y' < \lambda$, it follows, from (24) and the fact that $\lambda < 1$, that, for $\theta > 0$, $y < \frac{(J-C)\theta}{1+J}$. In conclusion, the set of valid candidates is[14]

$$S = \left\{(\theta, y) : \theta > 0, 0 < y < \frac{(J-C)\theta}{1+J}, y' < \lambda\right\}. \tag{25}$$

ii. Characterization of a Solution for y. We now show that $y = B^{SC}\theta$, where B^{SC} is defined by (13) satisfies (23). Further-

[13] Suppose rather that $P^S(\theta') < \underline{c}\theta'$ for some $\theta' > 0$. Then, no firm has an incentive to produce, which is inconsistent with an interior solution for aggregate output.

[14] From Assumption 3.1, $(J-C)\theta/(1+J) < (1-\underline{c})\theta$ for $\theta > 0$.

more, we show that $\{\theta, B^{SC}\theta\} \in S$. To see this, plugging $y = B^{SC}\theta$ into (23) yields

$$-\frac{1-B^{SC}}{\lambda - B^{SC}} B^{SC}\theta + J\theta - JB^{SC}\theta - \sum\nolimits_{k=1}^{J} c_k\theta = 0, \qquad (26)$$

Rearranging (26) shows that B^{SC} is the solution to the polynomial

$$(1+J)x^2 - \left(1 + (1+\lambda)J - \sum\nolimits_{k=1}^{J} c_k\right) x + \lambda\left(J - \sum\nolimits_{k=1}^{J} c_k\right) = 0. \qquad (27)$$

The left-hand side of (27) is convex in x. Moreover, since $c_k \in [0,1)$ for all k, it follows that $J > \sum_{k=1}^{J} c_k$, which implies that both roots of (27) are positive. If $x = \min\left\{\frac{(J-C)\theta}{1+J}, \lambda\right\}$, then the left-hand side of (27) is negative. Hence, the smallest root of (27), defined in (13), is the only root for B^{SC} such that $\{\theta, B^{SC}\theta\} \in S$.

iii. Uniqueness of the Solution for y. Note that the right-hand side and the derivative of the right-hand side of (24) are both continuous for $(\theta, y) \in S$, where S is defined by (25). By the Fundamental Theorem of Differential Equation, there exists a unique solution $y = \phi(\theta)$ for any initial condition $(\theta_0, y_0) \in S$. However, our initial condition $(0,0) \notin S$. Therefore, for uniqueness, we need to show that there is no other $y = \phi(\theta)$ with initial condition $(\theta_0, y_0) \in S \backslash \{\theta, B^{SC}\theta\}$ such that $\phi(0) = 0$, which satisfies (23). From (24),

$$\frac{\mathrm{d}y'}{\mathrm{d}y} = -\frac{(1-\lambda)(J-C)\theta}{((J-C)\theta + (1+J)y)^2} < 0 \qquad (28)$$

for $(\theta, y) \in S$, which implies that any solution $y = \phi(\theta)$ above $y = B^{SC}\theta$ has a flatter slope and any solution $y = \phi(\theta)$ below $y = B^{SC}\theta$ has a steeper slope. Hence, no solution $y = \phi(\theta), (\theta, y) \in S \setminus \{\theta, B^{SC}\theta\}$ converges toward the origin.

2. **Equilibrium Price and Inference Rule.** Given $Q^{SC}(\theta)$ and $\chi^{SC}(P^{SC}(\theta)) = \theta$, the equilibrium price is

$$P^{SC}(\theta) = \theta - Q^{SC}(\theta), \tag{29}$$
$$= \left(1 - B^{SC}\right)\theta, \tag{30}$$

as in (12). Given $Q^{SC}(\theta)$ and $P^{SC}(\theta)$, the equilibrium inference rule is

$$\chi^{SC}(P) = \frac{P}{1 - B^{SC}}, \tag{31}$$

which satisfies $\chi^{SC}(P^{SC}(\theta)) = \theta$.

3. **Firms' Strategies.** Given $\chi^{SC}(P)$, the profit of firm j is strictly concave so that the best reply of firm j is the *unique* solution to (15). Evaluating (15) at $Q^{SC}(\theta) = B^{SC}\theta$ and $\mathrm{d}Q^{SC}(\theta)/\mathrm{d}\theta = B^{SC}$ and solving for individual supply yields

$$q_j^{SC}(\theta) = \frac{\left(1 - c_j - B^{SC}\right)\left(\lambda - B^{SC}\right)\theta}{1 - B^{SC}}, \tag{32}$$

for all j, as in (11). Recall that the left-hand side of (27) evaluated at $x = \lambda$ is strictly negative, so that $B^{SC} < \lambda$. In addition, combining Assumption 3.1 with the fact that B^{SC} is increasing in λ, and that $B^{SC}|_{\lambda=1} = (J - \sum_{k=1}^{J} c_k)/(1 + J)$ implies that $B^{SC} < 1 - \overline{c}$, $\overline{c} \equiv \max\{c_k\}_{k=1}^{J}$.[15] Hence, from (11), for all j, $q_j(\theta) > 0$.

4. **Uniqueness of Signaling Cournot Equilibrium.** Since $Q^{SC}(\theta) = B^{SC}\theta$ is unique, (32) is unique. In other words, since the best reply for any firm has nonpositive slope larger than -1, the signaling Cournot equilibrium is unique. Indeed, given $Q_{-j}^{SC}(\theta) \equiv \sum_{k \neq j} q_k^{SC}(\theta)$

[15] Note that the condition stated in Assumption 3.1 is only sufficient in a signaling environment. In other words, a signaling environment allows for higher costs for all the firms to supply the good. Indeed, from (11), it must be that $B^{SC} \leq 1 - \overline{c}$, $\overline{c} \equiv \max_{k=1}^{J}\{c_k\}$. Since, from (13), $B^{SC}|_{\lambda=1} = \left(J - \sum_{k=1}^{J} c_k\right)/(1 + J)$ and B^{SC} is increasing in λ, the condition stated in in Assumption 3.1 is necessary for $\lambda = 1$ and sufficient for $\lambda \in (0, 1)$.

and $\chi^{SP}(P)$, (15) yields

$$r_j(Q_{-j}^{SC}(\theta)) = \frac{(1-c_j)\theta - Q_{-j}^{SC}(\theta)}{1+\frac{1-B^{SC}}{\lambda - B^{SC}}}, \tag{33}$$

$B^{SC} \in (0,\lambda)$, so that

$$r_j'(Q_{-j}^{SC}(\theta)) = -\frac{1}{1+\frac{1-B^{SC}}{\lambda - B^{SC}}}, \tag{34}$$

$r_j'(Q_{-j}^{C}(\theta)) \in (-1,0)$. Note that $\lambda = 1$ is consistent with Footnote 10.

□

Before proceeding with the effect of signaling on the Cournot equilibrium, three special cases are presented. First, evaluating the signaling Cournot equilibrium at $\lambda = 1$ yields the full-information Cournot equilibrium. In other words, for all j, $q_j^{SC}(\theta)|_{\lambda=1} = q_j^C(\theta)$, and $P^{SC}(\theta)|_{\lambda=1} = P^C(\theta)$. Second, when $c_j = 0$ for all j, (i.e., the firms are symmetric and consumer net surplus is always non-negative), the signaling Cournot equilibrium has a simple solution. Indeed, from (13), $B^{SC}|_{\forall j, c_j=0} = J\lambda/(1+J)$.

Remark 3.4. *If $c_j = 0$ for all j, then firm j supplies*

$$q_j^{SC}(\theta) = \frac{\lambda\theta}{1+J}, \tag{35}$$

and the price is

$$P^{SC}(\theta) = \frac{(1+(1-\lambda)J)\theta}{1+J}. \tag{36}$$

Third, in a signaling Cournot equilibrium, if the fraction of informed buyers goes to zero, then the limiting price is equal to the reservation or choke price, i.e., there is no interior solution. In other words, even though the price signals quality, no exchanges occur in the market.

Remark 3.5. *From (11) and (12), for all j, $\lim_{\lambda\to 0} q_j^{SC}(\theta) = 0$ and $\lim_{\lambda\to 0} P^{SC}(\theta) = \theta$.*

The absence of informed buyers implies that profit is independent of quality. Moreover, profit is always equal to zero, which is the profit of the lowest possible quality, i.e., $\theta = 0$. Observe that the learning activity of the uninformed buyers induce the firms to set quantities to zero. Indeed, from (1), in the absence of informed buyers, given the inference rule $\chi^{SC}(P) = P$ and the strategies of the other firms, the best response for each firm is to set quantity to zero. If there is a cost of production, producing would lead to negative profits. Without cost, the firm is indifferent between producing or not producing, as both yield zero profit. Hence, not to produce is a weakly dominant strategy. This result is not specific to the Cournot environment. Mirman and Santugini (2011) show that, without informed buyers, the price-setting monopolist generates no profit regardless of the level of quality when the lowest possible quality is zero. Our finding is in contrast to previous studies in monopoly and differentiated-good Bertrand models, in which profits are decreasing in quality (Bagwell and Riordan, 1991; Daughety and Reinganum, 2005, 2008a,b). The difference in results is due to different lower bounds about quality.

4 The Effect of Signaling

Using Propositions 3.2 and 3.3, we study the effect of signaling on the Cournot equilibrium. Specifically, we compare the strategies of the firms, the price, as well as the profits corresponding to the full-information and signaling Cournot equilibrium.

4.1 Firms' Strategies and Price

Proposition 4.1 states that, in a Cournot market with perfect substitutability, signaling distorts the price upward through a decrease in the quantities supplied. The reason is that signaling steepens the effective demand curve, which induces each firm to decrease quantity.[16]

[16]Note that $Q^{SC}(\theta) = B^{SC}\theta \in (0, \lambda\theta)$ is increasing in λ since $\partial B^{SC}/\partial\lambda > 0$. Hence, from (21), signaling steepens the demand curve, i.e., the slope is smaller than -1.

Proposition 4.1. *If $\theta > 0$, then, for all j, $q_j^{SC}(\theta) < q_j^C(\theta)$ and $P^{SC}(\theta) > P^C(\theta)$.*

Proof. From (8), let $\beta_j^C \equiv \frac{1-(1+J)c_j+\sum_{k=1}^{J} c_k}{1+J}$. The left-hand side of (27) evaluated at $x = \sum_{j=1}^{J} \beta_j^C$ is negative, which implies that the total quantity supplied under signaling Cournot is less than the total quantity supplied under full-information Cournot. In other words, $B^{SC}|_{\lambda\in[0,1)} < B^{SC}|_{\lambda=1}$.[17] From (11) and (12), Proposition 4.1 follows immediately. □

The increase in the price due to signaling is analogous to the monopoly case (Bagwell and Riordan, 1991; Daughety and Reinganum, 2008a; Mirman and Santugini, 2011). Moreover, Proposition 4.1 complements a similar result found for a signaling Bertrand game with differentiated products (Daughety and Reinganum, 2008b). Indeed, Proposition 3 in Daughety and Reinganum (2008b) holds when substitutability between differentiated products is sufficiently small, whereas our Proposition 4.1 applies for the case of perfect substitutability.

In a signaling environment, the presence of informed buyers generates a positive externality on the demand side. From Remark 3.5, the firms produce only where there are informed buyers. Moreover, increasing the fraction of informed buyers weakens the effect of signaling, i.e., production is increased, which reduces the market price.

Proposition 4.2. *If $\theta > 0$, then, for all j, $\partial q_j^{SC}(\theta)/\partial\lambda > 0$ and $\partial P^{SC}(\theta)/\partial\lambda < 0$.*

Proof. From (11), (12) and (13), $\partial q_j^{SC}(\theta)/\partial\lambda > 0$ and $\partial P^{SC}(\theta)/\partial\lambda < 0$. □

Note that Proposition 4.2 is consistent with other studies on the informational role of prices (Wolinsky, 1983; Bagwell and Riordan, 1991; Mirman and Santugini, 2011).[18] As noted, an increase in the fraction of informed buyers flattens the demand curve, which leads to a lower market price. On the other

[17]Recall that we can calculate variables of the full-information Cournot equilibrium by evaluating the signaling Cournot equilibrium at $\lambda = 1$.

[18]See also Tirole (1988, chap. 2.3.1.1).

hand, Anderson and Renault (2000) shows, in a search model with information acquisition, that the informed buyers generate a negative externality on the demand side. In Anderson and Renault (2000), the mass of informed buyers are the source of inelasticity, which implies that the more informed buyers, the higher the price. This difference with our result depends on the way information acquisition is modeled.

4.2 Profits

Next, we consider the effect of signaling on profits $\pi_j = Pq_j - c_j\theta q_j$. In order to clarify the analysis, we begin with the case in which the cost is zero, i.e., $c_j = 0$ for all j. We then study separately the cases of homogeneous cost (i.e., $c_j = c$ for all j), and heterogeneous cost.

No Cost. From Remark 3.4, if $c_j = 0$ for all j, then the profit of a firm in a signaling Cournot equilibrium is

$$\pi_j^{SC}(\theta) = \frac{(1+(1-\lambda)J)\lambda\theta^2}{(1+J)^2}. \tag{37}$$

The profit of a firm in a full-information Cournot equilibrium $\pi^C(\theta) = \pi^{SC}(\theta)|_{\lambda=1}$ is

$$\pi_j^C(\theta) = \frac{\theta^2}{(1+J)^2}. \tag{38}$$

Proposition 4.3 provides the condition for which signaling strictly increases the firms' profits. While the profits of high-quality firms may increase in a differentiated-good Bertrand model when all buyers are uninformed (Daughety and Reinganum, 2008b), we find that, in the Cournot model, signaling increases profits as long as the fraction of informed buyers is not too low. The condition stated in Proposition 4.3 is found by comparing (37) and (38).[19]

[19] In a two-stage game in which price-setting firms decide whether to disclose quality in the first stage (which determines the fraction of informed buyers), and then compete in price in the second stage, Caldieraro et al. (2011) show that, in a low-price separating equilibrium regime, the low-quality firm has an incentive to disclose its quality prior to setting price so as to increase the fraction of informed buyers, which leads to less price competition, and, thus, increases the profits of the low-quality firm.

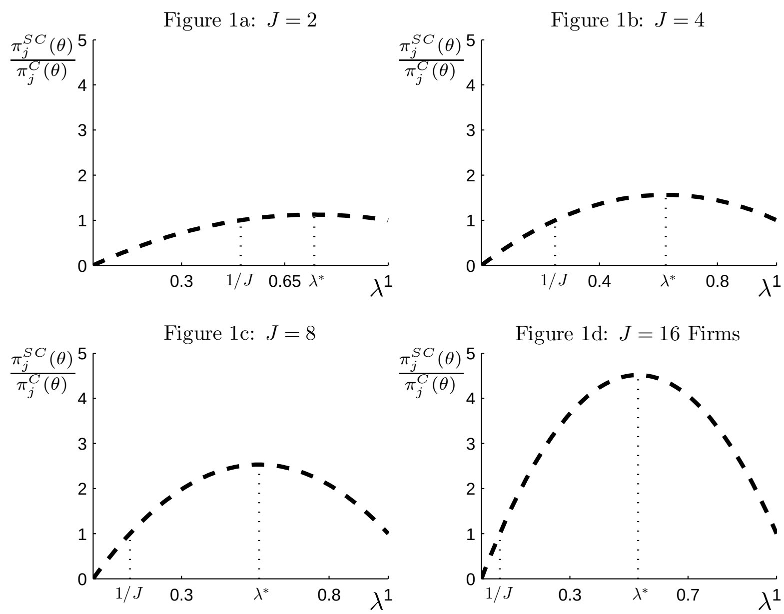

Figure 1: The Effect of Signaling on Profits without Cost

Proposition 4.3. *Suppose that* $c_j = 0$ *for all* j*. Then, for* $\lambda \in [0, 1)$ *and* $\theta > 0$, $\pi_j^{SC}(\theta) > \pi_j^C(\theta)$ *if and only if* $1/J < \lambda < 1$.

From Proposition 4.3, the number of firms and the fraction of informed buyers are both determinants of the effect of signaling on profits. Specifically, the higher the number of firms, the lower the fraction of informed buyers required to obtain higher profits under signaling. Figure 1 depicts the effect of the fraction λ of informed buyers on the ratio of signaling to full-information profits. Consistent with Proposition 4.3, signaling strictly increases profit for all sufficient high levels of informed buyers, i.e., $\pi_j^{SC}(\theta)/\pi_j^C(\theta) > 1$ when $\lambda \in (1/J, 1)$.

Next, Proposition 4.4 states that when the effect of signaling is strongest (i.e., at $\lambda = \lambda^*$ in Figure 1), there is no market externality. That is, the maximum profits for each firm under a signaling Cournot equilibrium, obtained at $\lambda = \lambda^*$, is equal to the profits corresponding to the firms' share in a full-information cartel.

Proposition 4.4. *Suppose that $c_j = 0$ for all j. Then, for $\theta > 0$, $\pi_j^{SC}(\theta) = (\pi_j^C(\theta)|_{J=1})/J$ if and only if the fraction of informed buyers is equal to*

$$\lambda^* = \frac{1+J}{2J}. \tag{39}$$

Proof. Evaluating (38) at $J = 1$ and dividing by J yields the profit of a firm participating in a cartel in a full-information environment, i.e., $\frac{\theta^2}{4J}$. Setting (38) equal to $\frac{\theta^2}{4J}$ and solving for $\lambda \in [0,1]$ yields (39). Moreover, using (37) and (38), $\pi_j^{SC}(\theta)/\pi_j^C(\theta) = (1 + (1-\lambda)J)\lambda$ attains the maximum when λ is equal to (39). $\square$

Before proceeding with the case of homogeneous and heterogeneous cost, we make two further remarks. First, while informed buyers always generate a positive externality on the uninformed buyers (through a lower price), the effect on profits is ambiguous. Indeed, from (37), $\pi_j^{SC}(\theta)$ is concave in the fraction of informed buyers, profits under signaling is concave in λ. While more informed buyers reduces the usual price distortion in a signaling game, thereby increasing profits, an increase in the fraction of informed buyers also yields more competition, which reduces profits. The two effects pull in opposite directions and the overall effect depends on the value of λ. Specifically, an increase in the fraction of informed buyers increases profits under signaling if and only if $\lambda < \lambda^*$.

Second, from (37) and (38), signaling alters the effect of Cournot competition on profits. Specifically, signaling mitigates the negative impact of Cournot competition on profits, i.e., $\partial\pi_j^C(\theta)/\partial J < \partial\pi_j^{SC}(\theta)/\partial J < 0$ for $\lambda \in [0,1)$ and $\theta > 0$. Under signaling, the effect of competition on profits is two-fold. First, more firms decreases profits directly, which is the usual effect also found in the full-information environment. This partial effect is captured through the denominator of (37). Second, an increase in J increases profits through the nominator of (37), as long as there are uninformed buyers, i.e., $\lambda \in [0,1)$. This is the indirect effect of competition on profits through signaling. This indirect effect arises because the learning activity of some of the buyers alters the demand, which induces each firm to decrease quantity,

thereby reducing the negative effect of the market externality on profits.[20]

Homogeneous Cost. We next show that the results stated in Propositions 4.3 and 4.4 are robust to the presence of cost. Proposition 4.5 provides the condition for which signaling strictly increases the profits of all firms.

Proposition 4.5. *Suppose that $c_j = c$ for all j. Then, for $\lambda \in [0, 1)$ and $\theta > 0$, $\pi_j^{SC}(\theta) > \pi_j^C(\theta)$ if and only if*[21]

$$(1 - c - B^{SC})^2(\lambda - B^{SC})(1 + J)^2 - (1 - c)^2(1 - B^{SC}) > 0 \tag{40}$$

where

$$B^{SC}|_{\forall j, c_j = c} = \frac{1 + (1 + \lambda)J - Jc - \sqrt{(1 + (1 + \lambda)J - Jc)^2 - 4\lambda(1 + J)J(1 - c)}}{2(1 + J)}. \tag{41}$$

Proof. From (8) and (10), $\pi_j^C(\theta)|_{\forall j, c_j = c} = \frac{(1-c)^2\theta^2}{(1+J)^2}$, while, from (11) and (12), $\pi_j^{SC}(\theta)|_{\forall j, c_j = c} = \frac{(1-c-B^{SC}|_{\forall j, c_j=c})^2(\lambda - B^{SC}|_{\forall j, c_j=c})}{1-B^{SC}|_{\forall j, c_j=c}}\theta^2$, where $B^{SC}|_{\forall j, c_j = c}$ is defined in (41). Comparing $\pi_j^C(\theta)|_{\forall j, c_j = c}$ and $\pi_j^{SC}(\theta)|_{\forall j, c_j = c}$ yields (40). □

Because condition (40) is difficult to analyze, we proceed with a graphical analysis. Figure 2 provides a contour plot of the ratio of signaling to full-information profits of a firm for values of the composition of buyers and the common cost when there are $J = 4$ firms. In other words, the lines on the graph are equal-value lines. They represent the combination of values of λ and c for which the ratio $\pi_j^{SC}(\theta)/\pi_j^C(\theta)$ is the same. We make two observations. First, consistent with Proposition 4.5, signaling increases profits (i.e., $\pi_j^{SC}(\theta)/\pi_j^C(\theta) \geq 1$) when the fraction of informed buyers is not too low.[22]

[20] Specifically, from (21), $\partial P/\partial q_j = -(1 + (1 - \lambda)J)/\lambda$. Hence, more competition via an increase in J steepens the demand, which induces the firms to decrease quantities.

[21] Note that condition (40) holds in general, i.e., for different cost.

[22] Note that, using (40), signaling increases profits as long as $B^{SC} \in (0, x)$, where x is the solution to

$$\begin{aligned} &- (1 + J)^2x^3 + (2(1 - c) + \lambda)(1 + J)^2x^2 \\ &+ (1 - c - (1 - c + 2\lambda)(1 + J)^2)(1 - c)x + ((1 + J)^2\lambda - 1)(1 - c)^2 = 0. \end{aligned} \tag{42}$$

Here, (42) is decreasing in x, first concave, then convex. A necessary condition for $B^{SC} \in$

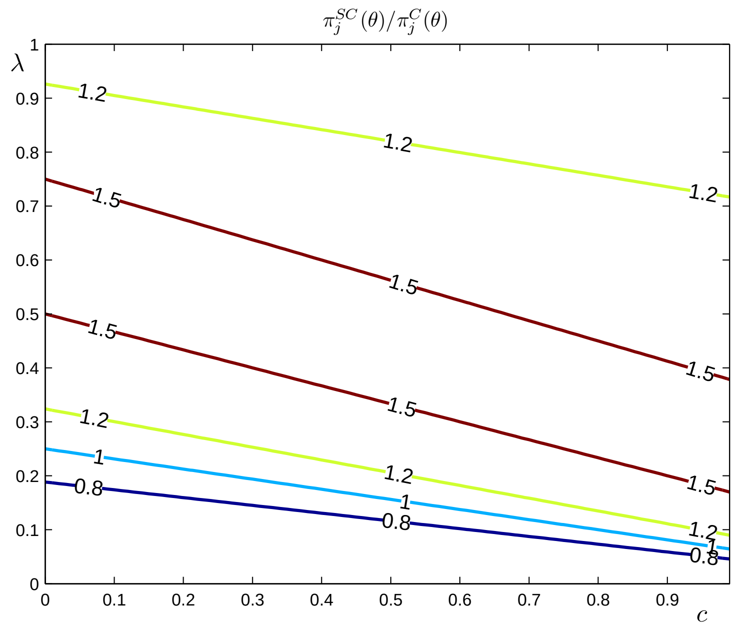

Figure 2: The Effect of Signaling on Profits with Homogeneous Cost, $J = 4$

The necessity for a high enough fraction of informed buyers is consistent with Remark 3.5. Indeed, as $\lim_{\lambda \to 0} \pi^{SC}(\theta) = 0$, while $\pi^C(\theta) > 0$ is independent of λ. Second, an increase in the cost relaxes the constraint, i.e., a lower number of informed buyers is needed to yield an increase in profits under signaling. Note that these two observations hold regardless of the number of the firms in the market, although an increase in competition strengthens the positive effect of signaling on profits. See Figure 4 in Appendix B.

The result stated in Proposition 4.4 remains true with the inclusion of cost. However, the optimal composition of buyers λ^* does change with cost. Indeed, Figure 3 shows that for any $c \in [0, 1)$, there exists a composition of buyers such that signaling Cournot profits equal the full-information cartel profits. Specifically, the lines in Figures 3a and 3b represent the pair $\{\lambda, c\}$ for which full-information cartel profits are reached in a market with 2 and 3

$(0, x)$ is that $x > 0$, which occurs when $(1 + J)^2\lambda - 1 > 0$ or $\lambda > 1/(1 + J)^2$. Hence, even in the presence of cost, the fraction of informed buyers cannot be too low in order for signaling to increase profits.

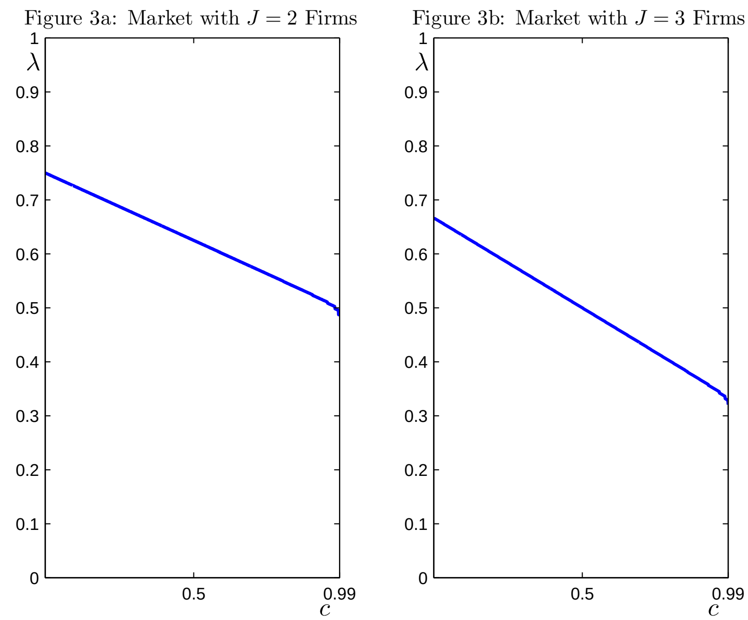

Figure 3: Full-Information Cartel Profits

firms, respectively. A higher cost requires a lower number of informed buyers in order to reach such profits.

Heterogeneous Cost. The results stated in Propositions 4.3 and 4.5 continue to hold when the firms are asymmetric. Figures 5 and 6 in Appendix B provide a contour plot of the ratio of signaling to full-information profits of firm 1 for values of the composition of buyers and the cost of firm 1, given the competition and the cost structure of the other firms. In other words, the lines on the graph are equal-value lines. They represent the combination of values of λ and c_1 for which the ratio $\pi_j^{SC}(\theta)/\pi_j^{C}(\theta)$ is the same. Hence, given Proposition 4.3, the curve for which $\pi_j^{SC}(\theta)/\pi_j^{C}(\theta) = 1$ hits the λ-axis at $1/J$ (i.e., when the cost is zero for all firms) for Figures 5b and 6b. Figure 5 considers the case of a market with two firms in which firm 1 has the lowest cost (Figure 5a) or has the highest cost (Figure 5b).[23] Figure 6 repeats the analysis for the case of three firms, two of which are identical. In

[23]In order for Assumption 3.1 to hold for $J \in \{2, 3\}$, $c_j \in [0, 0.25]$ for all j.

both cases, signaling might increase profits, i.e., $\pi_j^{SC}(\theta)/\pi^C(\theta)_j > 1$.

5 Final Remarks

This paper applies the analysis of information flows in a Cournot equilibrium with heterogeneous firms (through the cost) to the case of uninformed buyers extracting information from the price. In a Cournot model with a homogeneous good, each firm incompletely controls the price, and, thus, the signal. We show that there exists a unique signaling Cournot equilibrium, in which the behavior of the firms is altered and profits may increase along with an increase in the fraction of *uninformed* buyers. Moreover, under conditions regarding the composition of buyers, the number of firms, and the distribution of costs, signaling and the market externality pull in opposite directions and cancel each other out. In other words, there are conditions under which the profits under signaling Cournot equal the profits of a cartel in a full-information environment. The fact that asymmetric information in a non-cooperative game yields the cartel outcome is of interest in itself and should be investigated in other models of strategic interaction.

In order to study signaling in Cournot competition, we have focused on a noiseless environment. Indeed, there is no uncertainty in our model beyond the unknown quality. Extending the study of the signaling role of prices to a noisy environment would lessen the informational requirement of uninformed buyers about the structure of the market. It would also further our understanding of signaling in a dynamic model. In particular, in a dynamic context with repeated purchases, the presence of noise implies that the buyers' beliefs about quality evolve due to information acquisition not only from observing prices, but also from past consumption, which may lead to passive learning or experimentation. How the presence of different sources of information (e.g., price and experience) affects the behavior of the firms is a question left for future research. Indeed, combining experimentation (Grossman et al., 1977; Aghion et al., 1991; Fusselman and Mirman, 1993) or passive learning (Koulovatianos et al., 2009) with signaling could further our understanding on how different types of learning affect the information role of price.

How buyers form their beliefs could also be studied in a noiseless environment through the endogenization of the fraction of informed buyers. Specifically, in a two-period model, all the buyers are uninformed in the first period and purchase the good based on their prior beliefs. The buyers who purchase the good learn from experience and become informed in the second period, which then determines the fraction of informed buyers.[24] In period 2, the firms set quantities given the composition of the buyers as in our paper.[25] Another approach suggested by Caldieraro et al. (2011) in a price-setting and differentiated-product duopoly is to endogenize the fraction of informed buyers through the inclusion of a disclosure technology. Specifically, in the first stage, the firms decide whether to incur an exogenous cost in order to disclose quality through a trusted certification party. The disclosure is observable to a fraction of buyers who become informed prior to the firms setting prices in the second stage.

[24] This approach would also bring forward-looking behavior on the part of both the firms and the consumers.

[25] In addition to price and own experience, the uninformed buyers can learn from past aggregate sales, as in Caminal and Vives (1996).

A Non-Signaling Cournot Equilibrium

In this appendix, we consider the existence and characterization of a Cournot equilibrium in which the price is uninformative about quality. Such equilibrium is hereafter referred to as a non-signaling Cournot equilibrium.

Definition A.1 presents the non-signaling Cournot equilibrium in which the price transmits no information about quality, and, thus, the uninformed buyers revert to their prior mean beliefs $\mu \geq 0$ for any P. The superscript NSC refers to *non-signaling Cournot*.

Definition A.1. *For $\lambda \in [0, 1)$, a non-signaling Cournot equilibrium consists of firms' strategies $\{q_j^{NSC}(\theta)\}_{j=1}^J$, price $P^{NSC}(\theta)$, and inference rule $\chi^{NSC}(P)$ such that*

1. *Given $\{q_k^{NSC}(\theta)\}_{k \neq j}$ and $\chi^{NSC}(P)$,*

$$q_j^{NSC}(\theta) = \arg\max_{q_j} D\left(q_j + \sum\nolimits_{k \neq j} q_k^{NSC}(\theta); \theta, \chi^{NSC}(\cdot), \lambda\right) q_j - c_j \theta q_j. \tag{43}$$

2. *Given $\{q_j^{NSC}(\theta)\}_{j=1}^J$ and $\chi^{NSC}(P)$,*

$$P^{NSC}(\theta) = D\left(\sum\nolimits_{j=1}^J q_j^{NSC}(\theta); \theta, \chi^{NSC}(\cdot), \lambda\right). \tag{44}$$

3. *Given $\{q_j^{NSC}(\theta)\}_{j=1}^J$ and $P^{NSC}(\theta)$, $\chi^{NSC}(P^{NSC}(\theta)) = \mu$.*

Proposition A.2 states the conditions for the existence of a non-signaling Cournot equilibrium.

Proposition A.2. *There exists a non-signaling Cournot equilibrium if and only if $\lambda = 0$ and $c_j = 0$ for all j.*

Proof. Suppose to the contrary that there exists in general a non-signaling Cournot equilibrium. Given $\{q_k^{NSC}(\theta)\}_{k \neq j}$, $\chi^{NSC}(P) = \mu$ and (3), (43) is rewritten as

$$q_j^{NSC}(\theta) = \arg\max_{q_j} \left(\lambda\theta + (1-\lambda)\mu - q_j - \sum\nolimits_{k \neq j} q_k^{NSC}(\theta)\right) q_j - c_j \theta q_j. \tag{45}$$

The strategy of firm j corresponding to the non-signaling Cournot equilibrium is

$$q_j^{NSC}(\theta) = \frac{\lambda\theta + (1-\lambda)\mu - (1+J)c_j\theta + \sum_{k=1}^{J} c_k\theta}{1+J}, \tag{46}$$

and the price is

$$P^{NSC}(\theta) = \frac{\lambda\theta + (1-\lambda)\mu}{1+J} + \frac{\sum_{k=1}^{J} c_k\theta}{1+J}. \tag{47}$$

If $\lambda \in (0,1)$ or $c_k \neq 0$ for some k, then (47) is increasing in θ, and, thus, informative about quality. However, if $\lambda = 0$ and $c_k = 0$ for all k, then a non-signaling Cournot equilibrium exists as the price is indeed uninformative about quality. □

B Figures

Figure 4 provides a contour plot of the ratio of signaling to full-information profits of a firm for values of the composition of buyers and the common cost under different levels of competition, $J \in \{2, 4, 8, 16\}$. Figures 5 and 6 provide a contour plot of the ratio of signaling to full-information profits of firm 1 for values of the composition of buyers and the cost c_1 under different levels of competition, $J \in \{2, 3\}$, and different cost structure of the other firms.

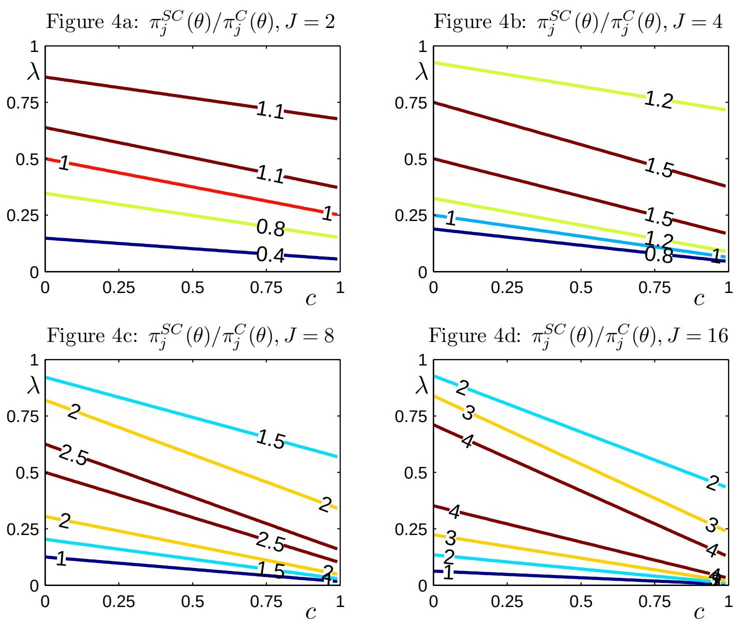

Figure 4: The Effect of Signaling on Profits with Homogeneous Cost, $J = \{2, 4, 8, 16\}$.

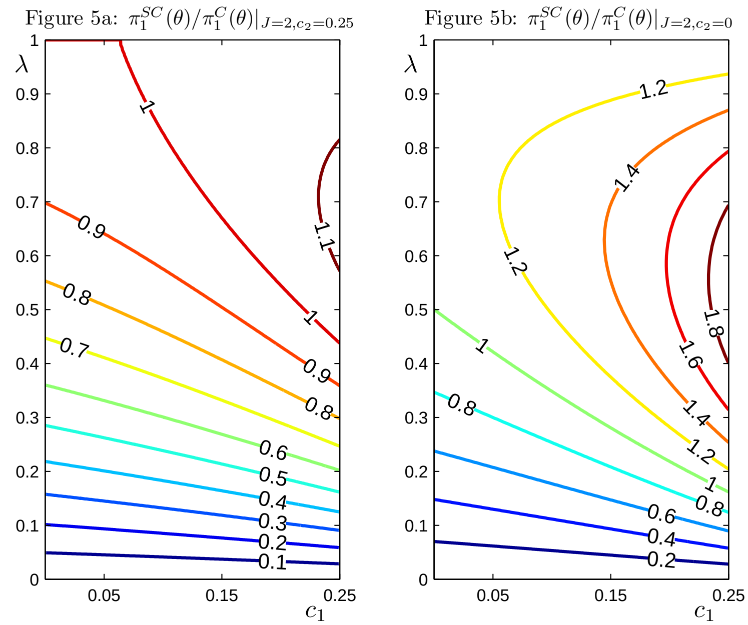

Figure 5: The Effect of Signaling on Profits with Cost, $J = 2$

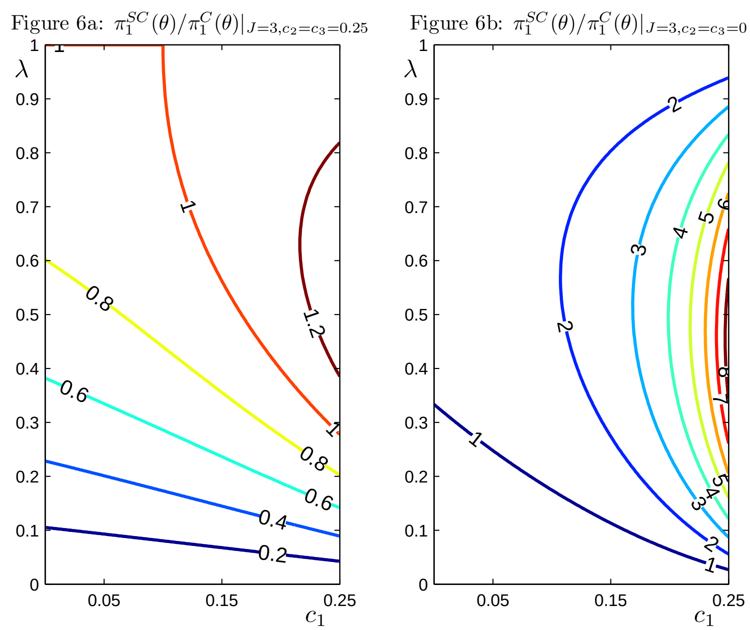

Figure 6: The Effect of Signaling on Profits with Cost, $J = 3$

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