



# WORKING PAPERS

RESEARCH DEPARTMENT

**WORKING PAPER NO. 12-1**  
**THE MACROECONOMICS OF FIRMS' SAVINGS**

Roc Armenter  
Federal Reserve Bank of Philadelphia

Viktoria Hnatkovska  
University of British Columbia and the Wharton School

December 14, 2011

RESEARCH DEPARTMENT, FEDERAL RESERVE BANK OF PHILADELPHIA

Ten Independence Mall, Philadelphia, PA 19106-1574 • [www.philadelphiafed.org/research-and-data/](http://www.philadelphiafed.org/research-and-data/)

# The Macroeconomics of Firms' Savings\*

Roc Armenter

Federal Reserve Bank of Philadelphia

Viktoria Hnatkovska

University of British Columbia and the Wharton School

December 14, 2011

## Abstract

We document that the U.S. non-financial corporate sector became a net lender in the 2000s, using aggregate and firm-level data. We develop a structural model with investment, debt, and equity. Debt is fiscally advantageous but subject to a no-default borrowing constraint. Equity allows the firm to suspend dividends when the cash flow is negative. Firms accumulate financial assets for precautionary reasons, yet value equity as partial insurance against shocks. The calibrated model replicates the prevalence of net savings in the period 2000-2007 and attributes the rise in corporate savings over the past 40 years to lower dividend taxes.

**Keywords:** Corporate savings, debt, equity, dividend taxation.

## 1 Introduction

In the last 40 years a number of developed economies have experienced large changes in the level and composition of private savings. For the U.S., the private savings rate dropped from 10 percent in the 1970s and 1980s to less than 4 percent at the beginning of the 2000s. The composition of private savings has undergone even more dramatic changes. While U.S. households have set out on a path of lower and lower savings, the corporate sector has emerged as a large saver. These changes have led to a secular shift in the financial position of U.S. firms. In the 1970s and 1980s the corporate sector was a net debtor, borrowing between 15 and 20 percent of the value of its tangible assets (capital henceforth) from the rest of the economy.<sup>1</sup> The corporate

---

\*We thank Paul Beaudry, Mitch Berlin, Michael Devereux, Burcu Eyigungör, Amartya Lahiri, Ellen McGrattan and the audiences at the University of British Columbia, Wharton School, FRB Philadelphia, Sauder School, Banco de España, Philadelphia Fed-NBER conference “Macroeconomics across time and space,” SCE at San Francisco, the NBER Summer Institute (EFACR), Berkeley, FRB San Francisco, FRB Minneapolis, Fordham University and the Federal Reserve Board for their comments and suggestions. The views expressed here do not necessarily reflect the views of the Federal Reserve Bank of Philadelphia or the Federal Reserve System. This paper is available free of charge at <http://www.philadelphiafed.org/research-and-data/publications/working-papers/>

<sup>1</sup>We define the corporate sector as non-farm, non-financial corporations. Data are from the U.S. Flow of Funds (FoF henceforth).

sector then switched to being a net *lender* in the 2000s, following a rapid transition phase in the 1990s. The corporate net financial asset (NFA) position, defined as the difference between financial assets and debt liabilities, has exceeded the value of capital by over 5 percent for the period 2003-2007.

As investment has remained stable in relationship to output, equity has sharply outgrown capital since the 1970s, suggesting an overall shift in the capital structure of firms away from debt financing and toward equity financing. The ratio of equity to capital, which was about 85 percent in the 1970s and 1980s, rose to well above 100 percent in the 2000s.

In this paper we study the emergence of the U.S. corporate sector as a net lender to the rest of the economy. In order to understand better which firms have become net savers and why, we work with the Compustat database. It contains detailed historical information about balance sheets of publicly traded firms in the U.S. We show that the trends in the aggregate data for the period 1970-2007 also emerge in the disaggregated firm-level data.<sup>2</sup> The average firm saw its NFA to capital ratio increase from -0.12 in the 1970s to 0.07 in the 2000s. Net lending became quite widespread, with about 44 percent of the firms in our sample carrying positive average NFA during 2000-2007—almost double the share observed in the 1970s. The median NFA to capital also rose over the sample period, from -0.17 in the 1970s to -0.07 in the 2000s. We also show that the rise of NFA is particularly pronounced for the manufacturing sector, small and medium size firms, and younger firms.

Why do we observe so many firms with positive NFA—even among those that rely on public equity? Internal funds appear to be preferable to external funds and, if the latter are needed, debt offers several advantages over equity. Interest payments are tax deductible, while dividends and capital gains are taxed. In addition, equity has significant floatation costs and can worsen agency problems by bringing external ownership into the company.<sup>3</sup> Thus from a cost perspective firms should adhere to a hierarchy of financing sources: first they should rely on internal funds; if external finance is needed, debt should be preferred to equity, which becomes a finance source of last resort. Surprisingly, the data suggest the opposite pattern, with firms relying on equity even though internal funds are available.

We argue that firms accumulate financial assets to avoid being financially constrained in the future—a precautionary motive—and simultaneously value equity as it provides partial insurance against negative cash flows. There are two key premises to our theory. First, firms face a borrowing constraint, so on occasion they must resort to costly equity to finance their investment needs. As a result, the firms' value function is strictly concave even if the underlying

---

<sup>2</sup>We restrict our sample to non-financial companies, excluding the technology and regulated utilities sectors. More details on the data are provided in Section 2 and in Appendix A.

<sup>3</sup>This is a necessarily very short list of the main advantages and disadvantages of debt and equity. Frank and Goyal (2008) offer an overview of the corporate debt literature, noting that existing theories struggle to explain the low demand for debt observed in the data.

objective function is linear. Second, dividend payments are suspended whenever a firm faces a negative cash flow, thus offering insurance against this possibility. The concavity of the value function implies that firms are willing to pay an insurance premium for equity and, at the same time, strive to accumulate net worth—in the form of financial assets.

In fact, we show that firms will find it optimal to fund additional financial asset holdings with equity revenues, despite their higher cost. This policy increases the internal funds available to the firm in the event of operational losses, safeguarding the firm from having to issue further equity. The intuition is as follows. A firm with low net worth has no choice but to issue equity to satisfy its financing needs. Since a large fraction of the cash flow is then committed to shareholders, the firm’s net worth increases only very slowly subsequently, requiring equity issuance again in the following period, and so on. Thus, one additional dollar of internal funds allows the firm to reduce equity issuance in the present *and future periods* after a negative cash flow shock. In other words, the firm values internal funds above the one-time cost of equity and is thus willing to raise equity revenues to build its financial asset holdings.

More precisely, we set up a partial-equilibrium model where risk-neutral entrepreneurs own firms operating a decreasing-returns-to-scale technology using labor and capital. Firms are heterogeneous regarding their net worth and productivity, which evolves stochastically. Labor can be contracted in spot markets, but capital is determined by the firm’s investment in the previous period.

There are two external sources of finance: risk-free debt and equity. Firms face a borrowing constraint—derived from the model primitives—which ensures that debt repayment is feasible in all states. Whenever the constraint is binding, firms must resort to equity to finance the desired level of investment. We model equity as a one-period claim on net revenues subject to a non-negativity constraint on dividends. Whenever the firm faces a negative cash flow—an event we label an “operational loss”—shareholders receive no distributions, providing some financial relief to the firm. The firm decides how much equity to issue and accumulates the net revenues not committed to shareholders.

We focus on fiscal considerations to calibrate the cost of equity relative to debt. The price of equity is such that the *after-tax* return of debt and equity are equalized for a risk-neutral household. From the firm’s point of view, this implies that shareholders demand a higher expected return than creditors. Debt is also fiscally advantageous since interest payments are deductible from corporate tax liabilities.<sup>4</sup>

---

<sup>4</sup>Our decision to focus on fiscal considerations is driven by the availability of independent estimates of tax rates across time. We recognize there are other important factors influencing the relative costs and benefits of equity, such as floatation costs, agency considerations, and deadweight losses associated with liquidation. Unfortunately, there are no independent, reliable measurements across time for these factors. For a review of empirical and theoretical work, see Frank and Goyal (2008) and Tirole (2006).

We show that our model can quantitatively match the data in the period 2000-2007. The model predicts a large share of firms with positive NFA, very close to the data: 43 percent versus 44 percent in the data. It also matches the mean, standard deviation and various percentiles of NFA to capital distribution. Importantly, the model generates the fat right tail of the NFA distribution found in the data.

The quantitative success of our model rests on its ability to generate realistic levels of financing needs. Motivated by the data, we introduce two novel features to our calibration. First, firms suffer infrequent but costly operational losses that reduce their net worth. Second, firms occasionally learn of investment opportunities, which we model as movements up a productivity ladder in the next period. These increase the desired investment by the firm without contemporaneously increasing its cash flow. We calibrate the parameters governing operational losses and investment opportunities to match, respectively, the observed transition probability into losses and the fraction of firms with investment expenditures exceeding their cash flow. Both investment opportunities and operational losses lead firms to accumulate financial assets while simultaneously keeping net worth growth in check.<sup>5</sup>

Our model also provides a structural framework to explore the factors behind the rise in corporate net savings in the past 40 years. More precisely, we test the hypothesis that changes in dividend taxation have played a prominent role. According to our calculations, reductions in dividend taxes in the 1980s and 1990s, up to the tax reform of 2003, reduced by half the cost of equity relative to debt.<sup>6</sup>

We re-calibrate the cost of equity using the dividend tax rates observed in the 1970s, leaving all other parameters intact. We find that the model predicts the mean ratio of NFA to capital to be negative, at  $-0.10$  — just slightly above the value observed in the data for the 1970s. The predicted share of firms with positive NFA in the model is also quite close to the data: 32 percent in the model compared to 27 percent in the data. Thus the change in dividend taxation can explain, virtually by itself, the corporate sector’s switch from net borrower to net lender. Intuitively, the higher cost in the 1970s limited the firms’ demand of equity to accumulate financial assets, and thus both equity and NFA were lower. We also discuss other factors that could have affected the costs and benefits of equity relative to debt in the 1970s relative to the 2000s. In particular, we show that operational losses have increased dramatically

---

<sup>5</sup>Standard specifications in the literature are calibrated to match revenue dynamics. These specifications do not generate enough finance demand because investment expansions are driven by positive productivity shocks, which also bring a cash flow windfall. It is thus too easy for the firms to self-finance. The role of negative cash flows is also emphasized in Gorbenco and Strebulaev (2010). In the data, the importance of such shocks for firms’ cash holdings has been documented by Opler et al. (1999) and Bates et al. (2009). We confirm that our calibration for productivity does not predict counterfactual distribution for firms’ revenues.

<sup>6</sup>See also Poterba (2004) for further discussion on the taxation of corporate distributions. McGrattan and Prescott (2005) link changes in the U.S. tax and regulatory system regarding corporate distributions to large secular movements in corporate equity.

in the Compustat sample over the past 40 years. After incorporating the lower incidence of operational losses, together with the higher dividend tax, in our calibration we find that the model’s predictions for the 1970s are substantially improved.

Finally, we extend the model to allow for young and old firm types. In the data, firms deplete their NFA positions in the years following an IPO. The model qualitatively reproduces these dynamics once we match the higher incidence of operational losses and higher expected growth for younger firms, as given by the age-size relationship.

Our paper is most closely related to two separate strands of the literature. The first strand focuses on dividend taxation and is represented by McGrattan and Prescott (2005), who argue that changes in dividend taxes and regulations can explain the large increase in corporate equity relative to GDP. McGrattan and Prescott (2005), as well as most of the related research, assume equity is the only source of financing for firms. Thus they are not equipped to study corporate NFA, by construction. In addition, evaluating dividend taxation changes in the absence of equity’s closest substitute, debt, is necessarily an incomplete analysis.<sup>7</sup>

The second strand of the related literature is on dynamic corporate finance, represented by Hennessy and Whited (2005, 2007), among others.<sup>8</sup> A key insight from this literature is that dynamic structural models can explain many “puzzling” findings in empirical corporate finance.<sup>9</sup> For example, Hennessy and Whited (2005) propose a model that generates a negative relationship between leverage and lagged measures of cash-flows, debt hysteresis, and path-dependence in financing policy. While our model also replicates these facts qualitatively, our main focus is on characterizing the aggregate distribution of firms’ financing choices. We therefore choose to dispense with several structural features typically used in this literature to match firm-level elasticities, such as adjustment costs or ad-hoc liquidation costs. As a result, our framework is substantially simpler and more parsimonious while it still delivers on the quantitative side.<sup>10</sup>

The paper is organized as follows. Section 2 documents the key facts regarding corporate NFA for the period 1970-2007. Section 3 describes the model setup and defines the industry equilibrium. We discuss how our model generates a simultaneous demand for equity and net

---

<sup>7</sup>Following the Jobs and Growth Tax Relief Reconciliation Act of 2003 there has been a renewed interest in how dividend and capital gains taxes affect capital structure and investment. See, for example, Chetty and Saez (2005, 2006), Gourio and Miao (2010), and Gourio and Miao (forthcoming).

<sup>8</sup>Papers with similar focus include Gomes (2001), Whited (2006), Gamba and Triantis (2008), and DeAngelo et al. (2011).

<sup>9</sup>There is an extensive empirical literature that focuses on cross-sectional determinants of corporate leverage and aims to distinguish among various capital structure theories (e.g. Titman and Wessels (1988), Rajan and Zingales (1995), Fama and French (2002), Shyam-Sunder and Myers (1999), and Welch (2004) among others).

<sup>10</sup>Our work is also related to Cooley and Quadrini (2001), who use a model of industry dynamics to study the role of financial frictions and persistent productivity shocks for firm dynamics and their dependence on firms’ characteristics, such as initial size and age. Cooley and Quadrini (2001), however, do not allow for capital accumulation and abstract from the role of taxes. Other papers that feature endogenous dynamic financing and investment policies include Brennan and Schwartz (1984), Titman and Tsyplakov (2007), and Riddick and Whited (2009).

savings in Section 4. We then turn to our quantitative analysis. Section 5 documents our calibration and Section 6 discusses the model fit and the key quantitative determinants of positive NFA. Section 7 answers what is behind the rise in corporate net savings from 1970 to 2007. Finally the association of NFA with age is discussed in Section 8. We conclude in Section 9. The Appendix contains a more detailed description of the data as well as several technical results regarding the model.

## 2 The rise of corporate net savings 1970-2007

In this section we document the key empirical developments in the capital structure of the U.S. corporate sector. To set the stage for our firm-level analysis we start with the aggregate data, drawn from the Flow of Funds (FoF) accounts of the United States. We focus on the non-farm, non-financial corporate business data on the levels of financial assets, tangible assets, liabilities and net worth during 1970-2007 period. We compute net financial assets as the difference between financial assets and liabilities, while equity is obtained as net worth. In all cases, we scale the variables by tangible assets, which provide a measure of the firms' capital stock. All variables are measured at market value.<sup>11</sup>

Figure 1 presents the dynamics of the NFA to capital ratio during the 1970-2007 period. It shows that aggregate NFA to capital was relatively stable at -0.15 during the 1970s and 1980s, experienced a dramatic run-up during the 1990s, and stabilized again at around 0.03 in the 2000s.<sup>12</sup> These developments highlight the transition of the U.S. corporate sector from a net debtor into a net creditor at the turn of the century. The increase in NFA was also accompanied by a rise in equity financing, where the net worth of the U.S. corporate sector as a share of its capital has increased from 0.85 in the 1970s and 1980s to 1.03 in the 2000s.<sup>13</sup>

What is behind these aggregate changes? Which firms have become net lenders and why? To answer these questions we turn to disaggregated firm-level data from Compustat. We focus on U.S. firms only; we exclude technology and financial firms, as well as regulated utilities. We also drop the firms whose capital is below 50,000 USD, those with negative equity, and zero

---

<sup>11</sup>The Flow of Funds data set also contains the value of non-financial assets at historical cost. We find that using these variables does not change the trends in the ratios of NFA and equity to capital but raises their (absolute) levels.

<sup>12</sup>Interestingly, during the 1950s and 1960s, the NFA to capital ratio in the FoF was above its level in the 1970s and 1980s. However, it remained negative throughout that period, making the qualitative switch of the NFA position in the 2000s unprecedented.

<sup>13</sup>Both asset and liability positions of the corporate sector rose over the period, with assets rising faster than liabilities. Unfortunately, the Flow of Funds data provide only a few disaggregated components for both assets and liabilities, preventing us from an in-depth look into the factors behind the rise in aggregate NFA in the U.S. We include a discussion of the trends and conduct some decompositions based on the available Flow of Funds data in the online Appendix available at <http://faculty.arts.ubc.ca/vhnatkovska/research.htm>.

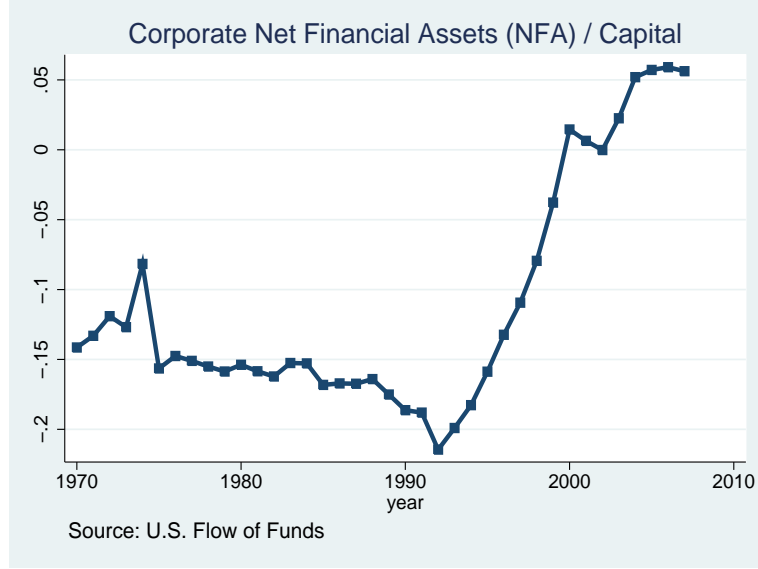


Figure 1: U.S. non-farm, non-financial corporate NFA to K

sales.<sup>14</sup> This selection leaves us with a sample of 5400 firms in the 1970s, 7212 firms in the 1980s, 8174 firm in the 1990s, and 6535 firms in the 2000s.<sup>15</sup> In line with the definitions used in the Flow of Funds data, we construct our measure of net financial assets in the Compustat database. Financial assets are obtained as the sum of cash and short-term investments, total other current assets, and account receivables. Liabilities are computed as the sum of current and long-term debt, accounts payable, and taxes payable. Our measure of tangible assets, or capital, includes firms' gross property, plant and equipment, investment and advances, intangible assets, and inventories.

We begin by reporting the mean and median of the NFA to capital ratio. We focus on these central moments to control for the outliers in our data set.<sup>16</sup> Figure 2 presents our results.

It is easy to see that Compustat firms show a pronounced increase in NFA ratios, mirroring the trends we uncovered in the aggregate data. Both the mean and median NFA to capital are rising steadily over time. The mean turns positive in the mid-1990s, reaching about 12 percent in 2006-2007. The median NFA to capital ratio, although it has risen sharply over the past

<sup>14</sup>We exclude technology firms from our analysis due to a potentially serious mismeasurement of their capital stock, which is predominantly intangible.

<sup>15</sup>In the last 20 years U.S. firms have also increasingly relied on *public* equity. According to Moskowitz and Vissing-Jorgensen (2002), the ratio of public to private equity increased threefold from 1989 to 1998. The number of U.S. firms listed on the NYSE or NASDAQ grew 33 percent over the same period.

<sup>16</sup>We also looked at the ratio of mean net savings to mean capital, and the same ratio for medians. We found that the ratio of medians exhibits the same trends as presented here, while the ratio of means does not exhibit any pronounced trends, suggesting that small and medium-size firms, as opposed to large firms, are behind the rise of net savings in the Compustat data set. Those results can be found in Appendix A.



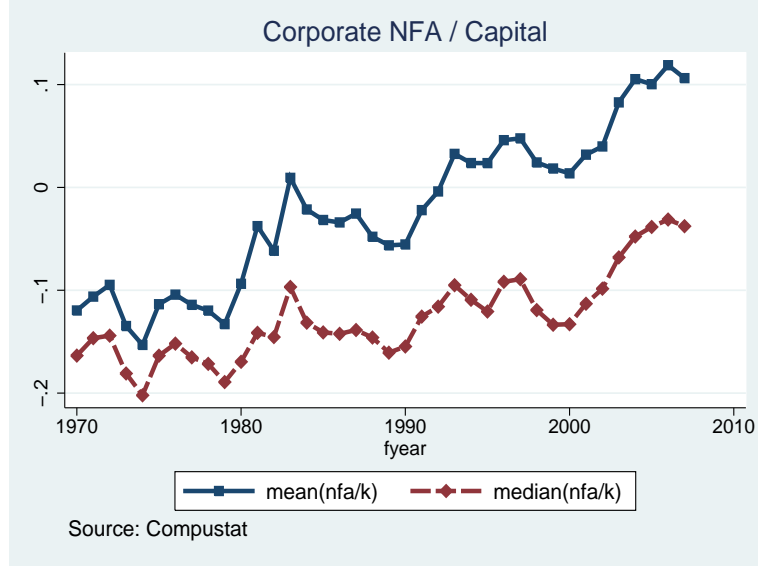


Figure 2: U.S. Non-financial, non-utilities, non-technology corporate NFA to K

40 years, does not turn positive in the 2000s.<sup>17</sup> While publicly traded companies included in Compustat are not a representative sample of the U.S. firms, it is very reassuring that the central moments are roughly similar to the aggregate data and exhibit similar trends. In addition, our Compustat sample resembles economy aggregates on another important dimension – the capital-output ratio in our sample is equal to 2 across all industries and is equal to 3 for the largest sector, manufacturing. In terms of overall size, non-financial Compustat firms employ about 36 percent of the aggregate U.S. labor force and hold 60 percent of the aggregate U.S. capital stock during the 2000s.<sup>18</sup>

Figure 3 takes a closer look at the distributions of the NFA to capital ratio in the 1970s and 2000s. Several features stand out on that figure: (i) there is a rightward shift in the distribution of NFA to capital in the 2000s relative to the 1970s; (ii) the majority of this shift is due to an increase in the share of firms with positive NFA, resulting in a fatter right tail of the distribution; (iii) at the same time, the left tail of the distribution almost did not change over time. The moments of NFA to capital distributions during the two decades are summarized in Table 1.

<sup>17</sup>The Compustat data do not exhibit as dramatic a run-up in the NFA to capital ratio in the 1990s that we observed for the FoF series. In the FoF, the run-up is driven by the “Miscellaneous” component – the largest of all components of assets and liabilities reported in those data. We attempt to decompose the “Miscellaneous” NFA in the online Appendix. Another potential reason for the difference in NFA dynamics in the 1990s between the FoF and Compustat is the exclusion of the technology sector from the Compustat sample. In Appendix A we show that while the rise in the NFA to capital ratio is characteristic of all industries in the U.S., the technology sector has shown the sharpest increase. In fact, starting in the late 1980s, technology firms have had the largest median NFA to median capital ratio of all sectors, with this ratio remaining high ever since. Combined with the growing share of that sector in the U.S. economy, this could have contributed to the sharp rise in corporate savings in the 1990s in the aggregate FoF data.

<sup>18</sup>See the online Appendix for details.

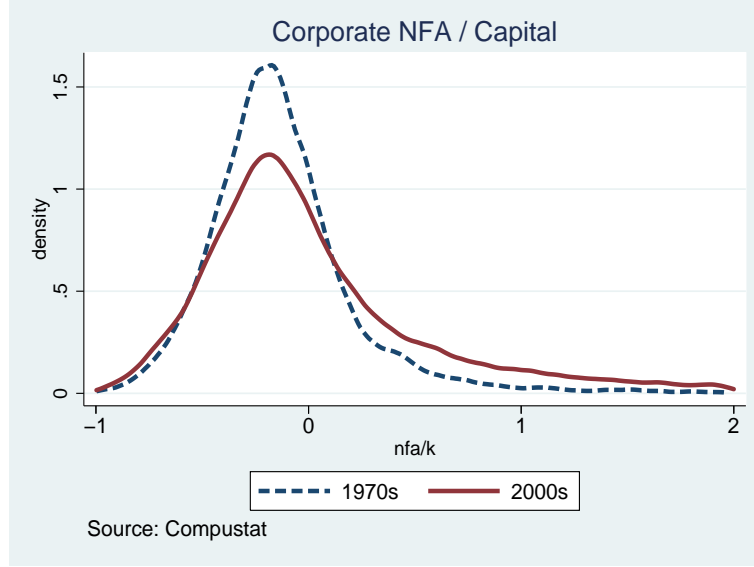


Figure 3: NFA to capital density, 1970s and 2000s

Table 1: Moments of corporate NFA/capital distribution

	share of firms with nfa2k >0	mean	median	skeweness	nfa2k std dev	10pct	25pct	75pct	90pct
1970s	26.86	-0.12	-0.17	2.31	0.39	-0.50	-0.34	0.02	0.29
2000s	43.55	0.07	-0.07	1.81	0.65	-0.51	-0.31	0.35	1.38

Note: The table reports moments of the distribution of corporate net financial assets to capital using Compustat data for the U.S. Column labeled “share of firms with nfa2k >0” refers to the share of firms in the sample with positive net financial assets to capital ratio.

Both the mean and median of the NFA to capital ratio show a dramatic increase over time in our sample. The median has increased from -0.17 in the 1970s to -0.07 in the 2000s, while the mean has almost doubled during the same period. The upper percentiles of the NFA to capital distribution have shown an even more pronounced increase, while the bottom percentiles have barely changed over time. Finally, the share of firms in our data set with positive NFA has increased from 27 percent in the 1970s to 44 percent in the 2000s.

Which firms are driving the rise in net financial assets? To address this question, we study NFA positions conditional on firm size, age, industry, and entry cohort. Our findings are as follows.<sup>19</sup>

First, we find a substantial amount of heterogeneity in the levels of NFA across sectors. Firms in the manufacturing sector have one of the highest NFA to capital ratios throughout the sample period, while firms in trade, transportation and warehousing have the lowest NFA to

<sup>19</sup>Detailed results and discussion of these findings are provided in Appendix A.

capital levels. At the same time, the increase in NFA to capital over time has been characteristic of all sectors, with technology and manufacturing firms switching to positive positions in the 2000s.

Second, we find that small and medium-size firms (as measured by the number of employees) have experienced the largest increase in their NFA to capital ratios during the 1970-2007 period. In fact, firms with up to median employment have seen their NFA to capital ratio turn from being negative in the 1970s to well above zero in the 2000s. In contrast, larger firms (with the number of employees above the sample median) were net debtors in the 1970s and remained such in the 2000s, experiencing relatively minor changes in their NFA to capital positions during this period.

Third, we find that younger firms, as measured by the time since their initial public offering (IPO), tend to have higher NFA to capital than older firms in the 2000s, while there is no association of NFA with age in the 1970s. This result is not surprising given NFA's relationship to firm size discussed above and the fact that age and size tend to be positively correlated in our sample.

Finally, we also consider how the NFA to capital ratio varies across entrants and incumbent firms. Here we find that entrants tend to have higher NFA to capital ratios relative to incumbents, and that this tendency has become more pronounced over time.

Overall, our findings suggest that small to medium size firms, younger firms, those in manufacturing, and entrants into Compustat are responsible for the dramatic rise in NFA we uncovered in the aggregate data.

### 3 Set-up

We set up a partial-equilibrium model of the corporate sector. There is a continuum of entrepreneurs. Each entrepreneur has risk-neutral preferences and seeks to maximize the expected present value of consumption,

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t c_t \right\}$$

where  $\beta \in (0, 1)$  is the intertemporal discount factor and  $c_t$  is non-negative consumption.

Each entrepreneur owns a firm that combines capital  $k$  and labor  $l$  into final output according to the production function

$$f(l, k; \sigma) = \frac{z(\sigma)^{\nu+\eta} k^{\nu} l^{1-\nu-\eta}}{\nu + \eta},$$

where  $z(\sigma) \in Z$  is an idiosyncratic productivity shock governed by the exogenous state  $\sigma \in \Sigma$ ,

which follows a first-order Markov stochastic process. Parameters  $\nu, \eta > 0$  satisfy  $\nu + \eta < 1$  and determine the income shares of labor, capital, and the entrepreneur's rents.

Labor is hired at a spot market at exogenously given wage rate  $w$ . The firm pays a corporate tax rate  $\tau^c$  on earnings minus capital depreciation expenses,  $\delta k$ , where  $\delta > 0$  is the depreciation rate of capital. Investment  $k$  is set one period in advance. In addition we introduce the possibility that a firm suffers a cash flow loss by allowing for additional after-tax expenses  $c^f(k; \sigma)$ . Then, the firm's after-tax net revenues are given by

$$\pi(k; \sigma) = \max_l (1 - \tau^c) (f(l, k; \sigma) - wl - \delta k) + k - c^f(k; \sigma). \quad (1)$$

The additional expenses may be due to overhead costs, minimum scale requirements, product obsolescence, or, more exceptionally, liabilities or accidents. We must note that operational losses play an important role in our model. Firms will periodically have to use finance to cover the cash shortfall, possibly in states of the world where their immediate revenue prospects are poor.

In order to obtain finance, an entrepreneur may rely on internal funds, debt, or equity issuance. Let us start with the latter. We model equity as one-period claims to net revenues subject to a non-negativity constraint.<sup>20</sup> Let  $s_{t+1}$  be the number of claims on period  $t + 1$  net revenues, or shares, issued at date  $t$ . Given investment  $k_{t+1}$  and state realization  $\sigma_{t+1}$ , each share  $s_{t+1}$  will produce a dividend equal to

$$d(k_{t+1}, \sigma_{t+1}) = \max\{\pi(k_{t+1}; \sigma_{t+1}), 0\}$$

at date  $t+1$ . Dividends are non-negative: in the event of operational losses,  $\pi(k_{t+1}; \sigma_{t+1}) < 0$ , the firm just suspends dividend payments. Investors price shares according to function  $p(k_{t+1}, \sigma_t) : \mathbb{R}_+ \times \Sigma \rightarrow \mathbb{R}_+$ . We will derive the price schedule later from the arbitrage condition of a stand-in household. Let  $e_{t+1} = p(k_{t+1}, \sigma_t)s_{t+1}$  be total equity revenues. We assume that entrepreneurs cannot short themselves,  $s_{t+1} \geq 0$ .

Our specification for equity requires some further discussion. By modeling equity as a contract we abstract from governance or agency issues, as the entrepreneur—akin to a controlling group of shareholders—retains full control of the firm's decision-making. Loosely, we interpret the entrepreneur's residual claims to the firm as “inside equity,” while additional issuances are to be considered as “outside equity.” Note that the additional issuances are an active decision by the firm, which chooses which share of net revenues to commit to shareholder distributions next period.

---

<sup>20</sup>It is possible to price longer-lived assets in our model or keep track of net equity issuance over time. However, for tractability, we have to limit the maturity of the financing instruments at hand.

Since each share is a claim to net revenue, dividends depend on the firm's performance. The tight link between net revenues and shareholder distributions is broken by the non-negativity constraint on dividends.<sup>21</sup> Thus equity is effectively a partial insurance asset. While admittedly ad-hoc, our specification is a parsimonious representation of shareholder payout policies.<sup>22</sup>

Debt is risk-free with exogenous pre-tax gross return  $1 + \tilde{r} > 1$ . Since interest expenses are deductible from corporate taxes due, the after-tax gross return is  $1 + r = 1 + (1 - \tau^c)\tilde{r}$ . The firm can also save at the same rate. Let  $a_t$  denote financial asset position at date  $t$ , that is,  $a_t > 0$  denotes positive net savings (and thus internal funds), and  $a_t < 0$  denotes debt.

Since debt is risk-free, we must ensure it is feasible to repay outstanding debt with probability one. This no-default condition implies the following borrowing constraint:

$$a_{t+1} \geq -\alpha, \quad (2)$$

where  $\alpha$  is derived from the primitives of the model. In the Appendix we discuss the steps to derive the borrowing constraint, as well as conditions such that  $\alpha$  is strictly positive and constant across firms.

We are now ready to set up the entrepreneur's problem. Entrepreneurs choose plans for asset holdings  $a_t$ , capital  $k_t$ , equity  $s_t$ , and consumption  $c_t$  to maximize

$$\sum_{t=0}^{\infty} \beta^t E\{c_t\},$$

subject to budget constraint

$$\frac{c_t}{1 - \tau^d} + a_{t+1} + k_{t+1} \leq \pi(k_t; \sigma_t) + (1 + r)a_t - d_t s_t + p(k_{t+1}, \sigma_t)s_{t+1} \quad (3)$$

as well as

$$\begin{aligned} c_t &\geq 0 \\ a_{t+1} &\geq -\alpha \\ s_{t+1} &\in [0, 1] \end{aligned}$$

at all dates  $t \geq 0$ . Proceeds from the firm to the entrepreneur are taxed at the dividend tax rate  $\tau^d$ .

---

<sup>21</sup>In our model, share issuance is not equivalent to negative dividends. The former remains a decision of the firm, which can seek further equity financing if it so chooses.

<sup>22</sup>Our simplicity comes at a cost, as our model is not well equipped to match micro-facts such as the frequency and size of equity issuance, among others. The literature routinely assumes adjustment costs to portfolio decisions in order to match these facts. Since we focus on aggregate data, we prefer to evaluate the main mechanism of the model with as few ad-hoc assumptions as possible.

The entrepreneur's problem can be stated recursively by defining net worth,

$$\omega_{t+1} = \pi(k_{t+1}; \sigma_{t+1}) + (1 + r)a_{t+1} - d_{t+1}s_{t+1},$$

as the endogenous state variable for the firm's problem. Net worth summarizes all the cash inflows as well as payment obligations of the firm entering in period  $t + 1$ . It is thus a concise summary of the internal funds the entrepreneur can tap into. Since cash flow and net financial assets are bounded below, we can show that net worth is bounded below,  $\omega \geq \omega^b$ . There is no upper bound for net worth, and thus the support for net worth is  $\Omega = \{\omega \geq \omega^b\}$ .

We proceed by splitting the recursive problem into two stages. Given state  $\{\omega, \sigma\}$ , the entrepreneur decides how much to invest:

$$V(\omega, \sigma) = \max_{k' \in \Gamma(\omega, \sigma)} J(k', \omega, \sigma),$$

where  $V : \Omega \times \Sigma \rightarrow \mathbb{R}_+$  is bounded and  $\Gamma(\omega, \sigma) : \Omega \times \Sigma \rightrightarrows \mathbb{R}_+$  is a correspondence with a non-empty compact image.<sup>23</sup> With  $k'$  as given, the entrepreneur decides the best way to finance investment, and whether to consume

$$J(k', \omega, \sigma) = \max_{c, a', s'} c + \beta E_\sigma V(\omega'(\sigma'), \sigma')$$

subject to the following constraints

$$\begin{aligned} \frac{c}{1 - \tau^d} + a' + k' &\leq \omega + p(k'; \sigma)s', \\ c &\geq 0, \\ a' &\geq -\alpha, \\ s' &\in [0, 1], \end{aligned}$$

where

$$\omega'(\sigma') = \pi(k'; \sigma') + (1 + r)a' - d(k', \sigma')s'$$

for all  $\sigma' \in \Sigma$ . We denote by  $\psi^x : \Omega \times \Sigma \rightarrow \mathbb{R}$  the resulting policy functions for  $x \in \{c, k', a', s'\}$ . We also obtain a law of motion for net worth,  $\psi^\omega(\omega, \sigma, \sigma')$ .

### 3.1 Equity prices

Ours being a partial equilibrium model, the equity price schedule  $p(k, \sigma)$  is taken as exogenous. However, we want to choose a specification that allows flexibility while relating the cost of equity

---

<sup>23</sup>See the Appendix for a derivation of  $\Gamma(\omega, \sigma)$  as well as a detailed discussion of the recursive formulation.

to the dividend process as well as fiscal considerations. To this end we postulate

$$p(k, \sigma) = \xi(1 + r)^{-1} E \{d(k, \sigma') | \sigma\} \quad (4)$$

where  $\xi > 0$  is a “markdown” parameter that summarizes the relative cost of equity and debt from the point of view of the firm.<sup>24</sup> If  $\xi = 1$  the firm is indeed indifferent between debt and equity, and the Miller-Modigliani theorem holds. If  $\xi < 1$ , equity is relatively costly, so, absent any other friction, the firm prefers to use debt to finance itself.

While we have assumed equity is one-period lived for computational reasons, firms will renew the stock of equity period to period. It is thus possible to use (4) to price longer-lived assets. Special care, though, is then needed when setting the markdown parameter. In Section 5 we show how to map the fiscal treatment of an infinitely lived asset into parameter  $\xi$ .

### 3.2 Industry equilibrium

We finally close the model description. Let  $F_t(\omega, \sigma)$  be the cumulative distribution function of firms defined over net worth and productivity, with support  $\Omega \times \Sigma$ . Firms exit exogenously at rate  $\varkappa > 0$ —the entrepreneur gets zero revenue flow from the exit date onwards.<sup>25</sup> The borrowing constraint indeed ensures that a firm retains positive value at all dates, and thus liquidation is never optimal. We assume that each period there is a flow of new entrants replacing exiting firms. The joint distribution of net worth and productivity is given by c.d.f.  $G(\omega, \sigma)$ , with support  $\Omega \times \Sigma$ .

To obtain the law of motion for the firm distribution, we combine the exit and entry dynamics with the law of motion for net worth,

$$F_{t+1}(\omega', \sigma') = \varkappa G(\omega', \sigma') + (1 - \varkappa) \sum_{\sigma \in \Sigma} \mu(\sigma' | \sigma) F_t(\phi(\omega', \sigma, \sigma')) \quad (5)$$

for all  $\omega', \sigma'$ , where  $\phi(\omega', \sigma, \sigma') = \sup \{\omega \in \Omega : \psi^\omega(\omega, \sigma, \sigma') \leq \omega'\}$ .

Finally, we assume there is a fixed entry cost,  $f_e$ , that takes the form of an initial investment necessary to start up production. To be clear, there is no equilibrium condition associated with entry. However, the initial investment is carried on the balance sheet as an asset.<sup>26</sup>

Our focus in this paper is a stationary industry equilibrium with  $F_t = F_{t+1}$ .

**Definition 1** *A **stationary industry equilibrium** is a stationary distribution  $F$  and policy functions  $\{\psi^a, \psi^c, \psi^s, \psi^k, \psi^\omega\}$  such that policy functions solve the entrepreneur’s problem given*

<sup>24</sup>Recall that interest rate  $r$  is the after-tax rate.

<sup>25</sup>The exit probability is embedded in their intertemporal discount rate  $\beta$ .

<sup>26</sup>For simplicity, we assume the initial investment does not depreciate.

prices and taxes, and  $F$  satisfies the law of motion (5).

A sufficient condition for a stationary industry equilibrium to exist is that  $1 + r = \beta^{-1}$  and  $\varkappa > 0$ . For the remainder of the paper we assume those two parameter conditions.

## 4 Net Savings and Equity

As simple as our model is, it can generate strong demand for both net savings and equity. To understand how the model works, we first roll back the borrowing constraint and let the entrepreneur tap into as much debt or equity as needed. We then explore how the firm chooses to finance itself as we vary the cost of equity.

Consider first the case with  $\xi = 1$ , that is, the after-tax expected return on debt and equity is the same. The Miller-Modigliani theorem applies and thus the capital structure of the firm is undetermined as the entrepreneur is indifferent between the two financing sources. If  $\xi \neq 1$ , then the risk-neutral entrepreneur will rely exclusively on the cheaper asset. For our case of interest, equity is relatively costly,  $\xi < 1$ , and thus the entrepreneur would finance investment exclusively with debt.<sup>27</sup>

We now re-introduce the borrowing constraint for the case of costly equity,  $\xi < 1$ . At first pass this seems of little help to generate a demand for net savings and additional equity. Debt-holders require a lower return, and the entrepreneur prefers to finance fully with debt. Only if the firm is at debt capacity the entrepreneur would have to resort to equity for additional funding. Thus the firm would follow a “pecking order” among finance sources, where internal funds would be preferred to external funds and, among the latter, debt would be preferred to equity. We would observe most firms relying heavily on debt and resorting to equity issuance only if they are at debt capacity.

However, this argument misses a key observation: the entrepreneur’s problem becomes strictly concave, and thus risk considerations come into play, due to the interplay between the borrowing constraint and costly equity. Consider a firm following the pecking order described above to finance a given amount of investment. If the firm has a high net worth, investment can be financed by the firm’s own savings or debt. Thus the firm values an additional dollar of net worth at the risk-free return  $1 + r$ . A firm with low net worth, though, will hit debt capacity and will have to rely on equity. The higher finance cost not only reduces the value of the firm, but it also increases the value of an additional dollar of net worth: now one dollar allows the firm to save the expected return to equity,  $(1 + r)/\xi$ . Thus the firm values a dollar more when

---

<sup>27</sup>If  $\xi > 1$ , then the return on equity is lower than the return on debt (and thus savings). The entrepreneur would engage in arbitrage in this case: it would raise as much funds as possible from shareholders and simply save the proceeds.



it has low net worth than when it has high net worth. Indeed, the differences in the value of an additional dollar get larger once the full dynamic program is considered. A firm with low net worth will find a large share of its cash flow committed to shareholders and will build its net worth only slowly, and thus may need to repeatedly tap into equity financing. Hence, one more dollar of net worth allows the firm not only to save equity issuance in the present period but also in future periods, and thus the firm values the additional dollar well above  $(1+r)/\xi$ .

We are now in place to tackle the main mechanism in the model. Firms will strive to accumulate net financial assets for precautionary reasons, that is, to avoid finding themselves at debt capacity at future dates. Simultaneously firms will be willing to pay an insurance premium for equity because dividend distributions and net worth are positively correlated. In fact, firms will find it useful to fund additional financial asset holdings with equity revenues. This large deviation from the pecking order is indeed crucial for the model to match the high levels of net financial assets observed in the 2000s. Furthermore, the reliance of firms on equity to fund purchases of financial assets will also be at the core of our explanation for the rise of corporate savings between the 1970s and the 2000s.

The precautionary motive resembles closely the one found in models of household finance. Firms want to build their net worth up rapidly in order to decrease the probability that they find themselves at debt capacity at future dates. Indeed, the entrepreneur delays any distributions to herself until the firm can self-finance at all future dates. Consider the first-order condition associated with the risk-free asset,

$$\lambda \geq \beta(1+r)E\{V'(\omega'(\sigma'), \sigma') | \sigma\}$$

with strict equality if the firm is not at debt capacity,  $a' > -\alpha$ , where  $\lambda$  is the Lagrangian multiplier associated with the budget constraint and thus the marginal benefit of net savings. The first-order condition associated with consumption implies that  $\lambda \geq 1$ . Using the envelope theorem, we can rewrite the previous first-order condition as

$$\lambda \geq E\{\lambda' | \sigma\}$$

where we have also used the condition  $(1+r)\beta = 1$ . Thus  $\lambda$  is a supermartingale, and  $\lambda$  converges almost surely to its lower bound. Whenever the firm is at debt capacity, one more dollar would allow it to relax the borrowing constraint, and thus it is more valuable,  $\lambda > 1$ . Thus the firm seeks to save as much net worth as possible in anticipation of states of the world where the debt capacity will bind. Only when there is zero probability that the borrowing constraint is ever binding, that is, when

$$\lambda = E\{\lambda' | \sigma\} = 1$$

for all  $\sigma \in \Sigma$ , there will be distributions to the entrepreneur.<sup>28</sup> Financial assets allow firms to build up net worth over time without introducing further risk or incurring decreasing returns to capital.

Simultaneously, firms find equity valuable due to its insurance properties: dividend payments decrease when the firm has a bad productivity shock and are zero when the firm experiences losses. Thus equity delivers some financial relief in the states where the firm will have lower net worth and thus is likely to face a higher finance cost. In other words, firms are willing to pay an insurance premium for equity.

Let us take a closer look at how the demand for both net savings and equity coexists. Consider the first-order condition associated with equity issuance,

$$p(k', \sigma)\lambda = \beta E \{ V'(\omega'(\sigma'), \sigma') d(\sigma') | \sigma \},$$

where we have assumed positive issuance,  $s' > 0$ . We can rewrite this expression in terms of the covariance,

$$p(k', \sigma)\lambda = \beta E \{ V'(\omega'(\sigma'), \sigma') \} E \{ d(\sigma') \} + \beta \text{Cov} (V'(\omega'(\sigma'), \sigma'), d(\sigma')) ,$$

where we dropped the arguments where there is no confusion possible. Now assume that the firm is not at debt capacity,  $a > -\alpha$ , and thus the last dollar of equity revenues is effectively funding the financial assets of the firm. Combining the equity price, (4), with the first-order condition for  $a$ , we have that

$$E \{ V'(\omega'(\sigma'), \sigma') \} E \left\{ \frac{d(\sigma')}{p(k', \sigma)} \right\} = \xi^{-1}(1 + r) > 1 + r.$$

Both optimality conditions can be satisfied only if

$$\text{Cov} \left( V'(\omega'(\sigma'), \sigma'), \frac{d(\sigma')}{p(k', \sigma)} \right) < 0.$$

This requires both that the value function  $V$  is strictly concave, and dividends are positively correlated with net worth. As discussed earlier, the concavity arises naturally in our model due to the borrowing constraint and the cost of equity. One can also argue that any realistic depiction of dividend policies will feature the right covariance between shareholder distributions and cash flows.

It perhaps remains counterintuitive that firms find it useful to issue equity, at a cost, to insure

---

<sup>28</sup>There exists a level of financial assets,  $a^*$ , such that the net return  $ra^*$  is sufficient to cover all finance needs in all states. Thus the entrepreneur can maintain the financial asset position  $a^*$  with probability one and consume the excess cash flow.

themselves against having to issue equity in future periods. The key is that one additional dollar available for a firm with low net worth allows the firm to reduce equity issuance in the present *and future periods*. A firm with low net worth has no choice but to commit a large share of its cash flow to shareholder distributions in order to raise enough equity finance. Thus the firm finds itself crawling very slowly from debt capacity and resorting to equity repeatedly. One more dollar of net worth allows the firm to reduce equity issuance in the present period, which in turn frees additional cash flow in the next period and again reduces equity issuance in that period, and so on.

We should note that the precautionary motive would remain even if we had specified equity as a full state-contingent contract; firms would still tolerate some residual risk because of the additional cost of equity  $\xi < 1$ . In our model the non-negativity constraint in dividends, as well as the ad-hoc relationship with cash flows, further limits the insurance properties of equity. As a result, equity issuance decreases as the firm accumulates net worth, which in turn lowers its chances of hitting the borrowing constraint in the near future.<sup>29</sup>

The logic of the model highlights the idea, emphasized by Hennessy and Whited (2005), that it is essential to view the capital structure decision in the context of a fully specified dynamic problem. Firms with a moderate level of net worth may have no chance of being at debt capacity *next* period or, more generally, in the short term. A model with a short horizon would need huge cash flow shocks in order to induce demand for equity among firms with some net savings. In a fully forward-looking model, even firms that can self-finance in the short term strive to accumulate further NFA and value the insurance properties of equity.

There remains the question, though, of whether our model can generate the observed *positive* net savings among firms that rely on equity. We answer this question with a quantitative evaluation of our model.

## 5 Calibration

We turn now to the core question of the paper: can our model generate positive NFA as observed for the period 2000-2007? As the model is taken to the task, we have to take a stand on two crucial aspects of the calibration. First, we have to quantify the relative cost of equity to debt. Second, we have to decide which moments to target with the productivity process. The remaining parameters regarding technology and entry are set to standard or straightforward values.

---

<sup>29</sup>The total value of the firm, though, increases monotonically with net worth.

## 5.1 The fiscal cost of equity

We start with the relative cost of equity,  $\xi$ . We choose to base our calibration on fiscal considerations alone. There is no question those are significant and, more important, fiscal considerations can be observed and quantified reliably from statutory rates and estimates from the public finance literature. We recognize that there are many factors affecting the costs and benefits of equity, e.g., floatation costs and agency problems, among others. However, it is not easy to quantify any of these factors. By focusing exclusively on fiscal considerations, we do not need to infer the equity markdown from the very same facts we seek to explain. This indeed becomes invaluable once we explore the model predictions for the decade of the 1970s.

We derive the price of an infinitely lived equity claim after accounting for all taxes, then compare it with the cost of debt financing for the firm. As a summary statistic the markdown parameter is defined as the ratio of the equity prices implied by the household and firm arbitrage conditions. We then impute the markdown to the one-period equity claim in the model.<sup>30</sup>

We first derive a risk-neutral household's arbitrage condition between equity and debt.<sup>31</sup> Tax liabilities are calculated in nominal terms. Let  $\gamma_p$  be the inflation rate, assumed to be deterministic. The pre-tax nominal interest rate is

$$1 + \tilde{R} = (1 + \tilde{r})(1 + \gamma_p),$$

where  $\tilde{r}$  is the real interest rate, before taxes. The real gross after-tax return to debt is then

$$\rho^b = \frac{1 + (1 - \tau^i)\tilde{R}}{1 + \gamma_p} \quad (6)$$

where  $\tau^i$  is the marginal tax rate on interest income and  $\tilde{R}$  the nominal interest rate before taxes, defined above.

Now we turn to equity. We need to specify the payment schedule of the asset. First, the asset is bought ex-dividends at date  $t$ . Let  $D_{t+1}$  be the dividend distribution at date  $t + 1$  and  $P_t$  the asset price at date  $t$ . The asset price grows at gross nominal rate  $1 + \gamma_a$ .<sup>32</sup>

One dollar invested in equity obtains  $\frac{1}{P_t}$  shares, which generate pre-tax revenues

$$\frac{D_t}{P_t} + \frac{P_{t+1}}{P_t}$$

---

<sup>30</sup>We need to compute the fiscal cost in an infinitely lived equity claim to properly account for the taxation of the net return and capital gains.

<sup>31</sup>We must emphasize that we do not model the household side: we just take the household's optimality condition in order to price equity and debt.

<sup>32</sup>This formulation allows for real growth in addition to inflation, as  $\gamma_a$  may be larger than  $\gamma_p$ .

at date  $t + 1$ . Taxes to be paid are

$$\tau^d \frac{D_t}{P_t} + \tau^g \frac{P_{t+1} - P_t}{P_t}$$

where  $\tau^d$  is the marginal tax rate in dividends and  $\tau^g$  the tax on capital gains. Note capital gains are accrued—unlike in the actual U.S. tax code, where they are only imposed on realization.<sup>33</sup>

The real gross after-tax return to equity is then

$$\rho^d = (1 - \tau^d) \frac{d}{p_t} + \frac{1 + \gamma_a(1 - \tau^g)}{1 + \gamma_p} \quad (7)$$

where  $d = \frac{D_{t+1}}{1 + \gamma_p}$  and  $p = P_t$  are the dividend and asset price in real terms.

We now equate the returns (6) and (7) to solve for  $p$ ,<sup>34</sup>

$$p = \frac{(1 - \tau^d)d}{\frac{1 - \tau^i}{1 + \gamma_p} \tilde{R} - (1 - \tau^g) \frac{\gamma_a}{1 + \gamma_p}}. \quad (8)$$

In order to compute the markdown, we have to derive the relative cost of debt and equity for the firm. The cost of debt in gross after-tax terms is

$$\frac{1 + (1 - \tau^c) \tilde{R}}{1 + \gamma_p},$$

where  $\tau^c$  is the corporate tax rate. Consider now the equity price that would leave a risk-neutral, unconstrained firm indifferent. The cost of equity is

$$\frac{D_t + Q_{t+1}}{Q_t}$$

where  $Q_t$  is the equity price. Equating the equity return with the cost of debt, we obtain

$$q = \frac{d}{\frac{(1 - \tau^c) \tilde{R} - \gamma_a}{1 + \gamma_p}}. \quad (9)$$

Note that  $\gamma_a$  is an increased cost for equity, so if  $\gamma_a > (1 - \tau^c) \tilde{R}$ , that is, if the net growth rate of the asset is more than the net after-tax nominal interest rate, no positive asset price can induce

---

<sup>33</sup>As long as we maintain the assumption of linear tax rates, the same calculations go through for an equity claim with stochastic return.

<sup>34</sup>For the equity price to be positive, it must be that

$$(1 - \tau^i) \tilde{R} - (1 - \tau^g) \gamma_a > 0.$$

Otherwise the asset price appreciation would, by itself, pay a higher return than NFA.

the firm to issue equity.

Finally, we compute the markdown by taking the ratio of  $p$  from (8) and  $q$  from (9), that is  $p = \xi q$ , to obtain:

$$\xi = \frac{(1 - \tau^d) \left( (1 - \tau^c) \tilde{R} - \gamma_a \right)}{(1 - \tau^i) \tilde{R} - (1 - \tau^g) \gamma_a}$$

Note the dividend  $d$  cancels, so the markdown is independent of how shares are defined. While the inflation rate does not enter the expression explicitly either,  $\tilde{R}$  is the nominal interest rate and thus the relative cost of equity will vary with the level of expected inflation.

## 5.2 Taxes and interest rate

We pick tax rates and interest rates representative of the period 2000-2007 for the U.S. Our choices are summarized in Table 2. Let us start with the corporate tax rate,  $\tau^c$ . Due to investment not being expended for tax purposes, the corporate tax rate directly impacts the firm's decision beyond its implications for the relative cost of equity. In the U.S. the corporate tax code specifies a flat tax rate of 34 percent from \$335,000 to \$10 million, and caps the marginal rate at 35 percent.<sup>35</sup> The literature has an ample consensus on setting  $\tau^c = .34$ , and we follow suit.

Interest income is taxed at the federal income tax rate and thus varies across investors. Wealth, though, is heavily concentrated on the right tail, so we choose a tax rate close to the top rate,  $\tau^i = .34$ , which is slightly higher than estimates of the average marginal tax rate across households.<sup>36</sup> The pre-tax nominal interest rate is set at 7 percent, while the inflation rate is at 2 percent. This results in an after-tax real rate of 2.5 percent.

Now we turn to the taxation of equity. The period 2000-2007 includes an important tax reform, the Jobs and Growth Tax Relief Reconciliation Act of 2003. The act equated dividend and capital gains tax rates at 15 percent, although there are several caveats. First, Poterba (1987) argues that the effective capital-gains tax rate is one fourth of the statutory rate, due to the gain referral and step-up basis at death. Second, some low-income households are subject to a lower dividend tax rate of 12 percent, while some other households may end up with a rate above 15 percent due to the alternative minimum tax.<sup>37</sup> Third, some corporate investors do not pay dividend taxes, and the share of equity held by them has increased sharply over time.<sup>38</sup> We

<sup>35</sup>Only small businesses and S corporations get a rate below 30 percent.

<sup>36</sup>Poterba (2002) and NBER TAXSIM estimates tend to be just below 30 percent. Some bonds are tax-exempt, which reduces the average marginal tax rate. However, corporate bonds are always fully taxed.

<sup>37</sup>For example, Poterba (2004) reports an average marginal tax rate on dividends of 18 percent. A similar situation arises regarding capital gains taxes.

<sup>38</sup>For example, pension funds and other fiduciary institutions. See McGrattan and Prescott (2005) for a discussion.

Table 2: Taxes and interest rate — Baseline calibration

	Parameter	Value
Corporate tax	$\tau^c$	0.34
Dividend tax	$\tau^d$	0.15
Interest income tax	$\tau^i$	0.34
Capital gains tax	$\tau^g$	0.15
Pre-tax interest rate	$\tilde{R}$	0.07
Equity markdown	$\xi$	0.82

note, though, that most estimates track closely the statutory rates in the decade of the 2000s. We thus decide to go with the statutory rates,  $\tau^d = .15$  and  $\tau^g = .15$ . If anything, these rates are likely to overstate slightly the fiscal cost of equity.

### 5.3 Technology and entry parameters

We first discuss the parameters governing technology, which are set to match standard values in the literature. We postpone the calibration of the productivity process for the next subsection. We start with the parameterization of the production function. We set  $\eta$  to equate the entrepreneurs' rents to the share of dividends over GDP, roughly 12 percent. Parameter  $\nu$  is set to .3. Assuming entrepreneur rents are roughly split 50-50 between capital and labor income accounts, this results in the standard total capital income share of 36 percent. We normalize the wage to  $(\eta + \nu)/(1 - \eta - \nu)$  so employment is equal to net revenues. The depreciation rate is set to 10 percent.

Next we turn to our calibration of the entry parameters. As we work with a stationary distribution, the entry rate in the model also serves as exit rate. In the data there is a slight upward trend in the number of firms, so the entry rate is slightly above the exit rate. We set our exit/entry parameter at 5 percent, closer to the exit rate in the data. For the net worth distribution of entrants we use a Pareto distribution with curvature parameter  $\varsigma$  equal to 1.3, which matches the relative capital holdings of entrants to incumbents. The entry cost  $f_e$  is set to match the 10th percentile of the distribution of NFA over capital.<sup>39</sup>

Finally the discount rate  $\beta$  is pinned down by our choice of the interest rate and the condition that  $\beta R = 1$ . The resulting value .96 is right on the standard values. Table 3 summarizes the parameter choices reported in this subsection.

<sup>39</sup>Parameters  $\varsigma$  and  $f_e$  are matched to moments that require us to evaluate the full model, and thus it would be more correct to say that they are jointly calibrated with the productivity process. However the relationship between the parameters and the moments is very tight, so we feel comfortable linking them at this point.

Table 3: Technology and entry parameters — Baseline calibration

	Parameter	Value
Discount factor	$\beta$	0.96
Entrepreneur rent	$\eta$	0.14
Depreciation rate	$\delta$	0.10
Capital elasticity	$v$	0.20
Exit rate	$\varkappa$	0.05
Entry distribution	$\varsigma$	1.3
Entry cost	$f_e$	5.15

## 5.4 Productivity process

The productivity process is the other crucial aspect of the calibration. As our primary interest lies in the corporate finance decisions, it is important that we match the firms' observed financing needs. Looking at the data, we identify two key drivers of the firms' financing needs: negative cash flows and large investment expenses.

First, we observe that a substantial fraction of firms experience a negative cash flow.<sup>40</sup> Firms must balance the operating loss with either a decrease in assets or an increase in liabilities. In particular, cash flow shortfalls will provide the basis of the precautionary demand for financial assets.<sup>41</sup>

Second, firms occasionally have opportunities to expand their operations, perhaps by acquiring a foundering competitor or by upgrading their production process because a new technology has become available. These opportunities often present themselves without any relationship to the contemporaneous cash flow of the firm and usually require investment expenditures that are larger than the firm's net revenues.<sup>42</sup> Firms that want to take advantage of these opportunities need to finance their increase in assets without having the benefit of an immediate increase in cash flows.

Unfortunately, we find that the standard specification used in the literature does not allow either for operational losses or for forward-looking investment opportunities and thus does not generate a realistic level of financing needs. Under the usual autoregressive process, firms' investment is driven by contemporaneous positive productivity shocks. Investment can then

<sup>40</sup>In any given year during the 2000-2007 period, about 25 percent of the firms in our sample had a negative cash flow, defined as operating income before depreciation expenses. The transition rate from positive to negative cash flow is also quite high at 6 percent.

<sup>41</sup>Lins et al. (2010) document that CFOs use cash to guard against future negative cash flow shocks. Lines of credit, due to financial covenants, are not a good substitute, as documented by Sufi (2009).

<sup>42</sup>For the period 2000-2007, we find that about 22% of the firms with positive cash flow incurred investment expenditures in excess of their cash flow in a given year. Among those, more than half had investment expenditures totaling 150% or more of their cash flow.



be easily financed from the firm's own net revenues, since the latter also increase with the productivity shock. In short, it is quite easy for firms to self-finance under the usual productivity specifications, as financing needs arise only when the firm is experiencing a cash-flow windfall.

We instead propose a productivity process that directly incorporates the possibility of operational losses and investment opportunities, and it is thus capable of generating realistic levels of financing needs in the model. More precisely, we assume productivity takes one of  $n$  levels,  $\{z_1, z_2, \dots, z_n\}$ . We capture operational losses with state  $n = 1$ , setting  $z_1 = 0$ , so for simplicity there are zero net revenues in that state, and cost expenses  $c^f(k, z_1)$  are such that

$$\pi(k, z_1) = -\kappa$$

for all  $k$ , with  $\kappa \geq 0$ . We set  $c^f(z, k) = 0$  for all other states and levels of investment, thus ensuring that net revenues are non-negative everywhere but in state 1. The probability of operational losses is  $\phi > 0$ , which we assume to be i.i.d. across firms. Our specification for operational losses, while stark, is very parsimonious and keeps the portfolio decision in the firm's problem simple. It also implies that the no-default borrowing constraint is constant across firms, as it suffices to show that the firm can repay the outstanding debt in the event of operational losses.

Investment opportunities are modeled as a movement along the productivity ladder. Each period a fraction  $\iota$  of firms receive an investment opportunity shock. These firms will either transition to operational losses (with probability  $\phi$ ) or will upgrade their productivity by one level. That is, a firm with productivity level  $z_t = z_i$  that receives an investment opportunity will transition to productivity level  $z_{t+1} = z_{i+1}$  next period with probability  $1 - \phi$ , or  $z_{t+1} = z_1$  with probability  $\phi$ . A firm without an investment opportunity remains at the same productivity level,  $z_{t+1} = z_i$  next period with probability  $1 - \phi$ , or  $z_{t+1} = z_1$  with probability  $\phi$ .<sup>43</sup>

Finally, we set productivity levels  $z_2, z_3, \dots, z_n$  to be equally log-spaced, with growth rate  $\gamma_z$ , that is,  $z_i = \gamma_z^{i-2} z_2$ . This guarantees that there is no hard-wired relationship between firm size and growth rates.<sup>44</sup>

Let us now discuss our numerical choices for the productivity parameters  $\phi, \iota, \gamma_z$ . First we set the transition probability into operational losses  $\phi$  to 6%, which is the transition rate from positive to negative cash flows that we observe in the data. Since the operational losses shock is i.i.d., the calibration slightly underestimates the fraction of firms reporting a negative cash flow

---

<sup>43</sup>Firms at state  $z_1$  automatically have an investment opportunity, so they transition to  $z_2$  unless they suffer operational losses again. Firms with the highest productivity level,  $z_n$ , do not receive further investment opportunities.

<sup>44</sup>In Section 8 we consider a simple extension that relates both firm size and growth rates to age. This can explain some interesting facts regarding NFA positions in the years immediately after an IPO.

on a given date.<sup>45</sup>

For the investment opportunities,  $\iota$ , we match the share of firms with investment expenditures exceeding their cash flow, about 22 percent of firms with positive cash flow. The resulting parameter value is  $\iota = .28$ . Both rates do not exactly coincide because investment is an endogenous variable in the model. In particular, firms with very low net worth may not be able to expand investment significantly due to their higher cost of capital.

Finally we set the growth rate of productivity,  $\gamma_z$ , to reproduce an average growth rate in revenues of about 5 percent among firms with positive cash flow. The level  $z_2$  is normalized to 1. We use six states for the productivity process, enough to generate a right tail in revenues, yet keep the computational time in check. In order to reduce the degrees of freedom, we set  $\kappa = 0$ . Note that this still implies that a firm experiencing operational loss has a negative cash flow.<sup>46</sup>

Table 4 reports all the parameter choices concerning productivity.

Table 4: Productivity process — Baseline calibration

	Parameter	Value
Operational losses	$\phi$	0.06
Investment opportunity	$\iota$	0.28
Growth rate	$\gamma_z$	1.18
States	$n$	6
Level	$z_2$	1

Since we are targeting facts for publicly traded firms, we look only at firms in our model that have a positive probability of issuing equity. In our model firms with very high net worth can rely exclusively on self-financing for investment—and thus have no need to tap outside investors. We consider these firms to be private equity and drop them from our sample.<sup>47</sup>

## 6 The corporate sector as a net lender

Does our model replicate the positive level of NFA observed during 2000-2007? Yes, it does. Table 5 reports the model predictions along with the corresponding data moments. Our model

<sup>45</sup>We explored relaxing the i.i.d. assumption, which allows the model to match the hazard rate of operational losses as a function of size as well as the persistence of operational losses. The extension did not significantly alter the model’s aggregate implications, so we decided to keep the productivity specification to a minimal structure.

<sup>46</sup>We should note that our interest in firms’ financing choices necessitates the use of cash flows, as opposed to revenues or value added, when calibrating the productivity process. However, we find that with our calibration the model generates the distribution of revenues, which is well approximated by a log-normal distribution, in line with the data. Overall, we believe our calibration is broadly consistent with Midrigan and Xu (2010).

<sup>47</sup>Note the model’s sample includes all firms with debt. Thus the censoring from the model does not help to generate positive NFA in the sample. The fraction of firms dropped is usually very small, less than 5 percent.

reproduces the large fraction of firms with a positive NFA position, 43.5 percent in the data versus 42.9 percent in the model. The model’s performance regarding the central moments is also very good. The median NFA to capital is just a tad below the data, and the mean is matched exactly.<sup>48</sup>

Table 5: Model and Data - Baseline Calibration

NFA to K	2000s	
	Data	Model
mean	0.07	0.07
median	-0.07	-0.08
$\Pr(a > 0)$	43.6%	42.9%
std dev	0.65	0.64
10pct	-0.51	-0.50
25pct	-0.31	-0.35
75pct	0.35	0.31
90pct	1.38	1.88

Overall, the model does a remarkable job at matching the full distribution of NFA over K in the data. The standard deviation in the model and in the data is very close, so we are confident that our simple productivity process is capable of generating enough variation in corporate finance portfolios. Both the first and third quartiles are very close to the data.<sup>49</sup> We overshoot the 90th percentile, albeit not by a large margin.

We should emphasize that our model can rationalize the corporate sector as a net lender only through the mechanism highlighted in Section 4. No productivity process would generate positive NFA if we were to drop the borrowing constraint or the positive covariance between dividend payments and cash flows in the model. Without a borrowing constraint firms would finance only with debt, as it is the cheaper finance source. Without dividends payments providing partial insurance, only firms at debt capacity would resort to equity, and we would not observe firms with positive NFA actively relying on equity. Conversely, if equity had no cost and provided full insurance, all firms would spurn debt.

Quantitatively, though, the key to the model’s fit is our specification for productivity. After exploring several alternatives, we realized that it is necessary to generate realistic levels of financing needs in the data in order to match the level and dispersion of NFA. Motivated by the data, we modeled operational losses and investment opportunities as the two key drivers

<sup>48</sup>We compute the moments from a simulation of 50,000 firms drawn from the stationary distribution. To ensure consistency we treat the simulated data as we treated the data in Section 2.

<sup>49</sup>Recall we used the fixed entry parameter  $f_e$  to directly target the 10th percentile, although this has surprisingly little effect on the overall shape of the distribution.

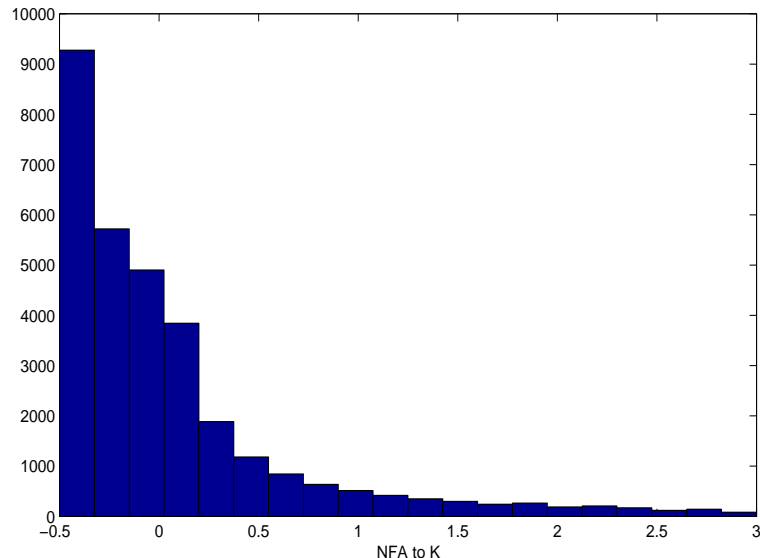


Figure 4: NFA to capital histogram, model

of the firms' demand for finance. We imposed a minimal structure with a very parsimonious specification and calibrated the parameters to the frequency of operational losses and large investment expenditures—so we did not target any moment of the NFA distribution. The fact that the model performs very well suggests that the link between financing needs and balance sheets is very tight, and that operational losses and investment opportunities effectively capture the relevant shocks for corporate finance purposes.

We now take a further look at the overall distribution of NFA over capital. Figure 4 contains the histogram of the NFA to capital as generated by the model. As in the data, the distribution is skewed to the right and features a long right tail, with a small number of firms having very large NFA holdings relative to their productive assets. The model, though, does not generate a left tail and the overall distribution resembles more closely a power law distribution rather than the lognormal distribution apparent in the data. The reason is twofold. First, all firms have the same borrowing constraint and thus all firms close to the borrowing constraint have very similar NFA positions. Second, firms are at or close to the borrowing constraint only if they suffer operational losses or receive an investment opportunity while their net worth is very low. In both cases, they end up with similar low investment levels. Both factors combine to create a large mass of firms with very similar NFA to capital ratios in the lower end. We are thus not able to generate enough dispersion among firms that rely heavily on debt.

While our calibration focuses on the firms' financing needs, we also hope that the model's performance regarding employment or revenues is not far from the data. Figure 5 reports the histogram for employment in the model. As in the data, the employment distribution is skewed

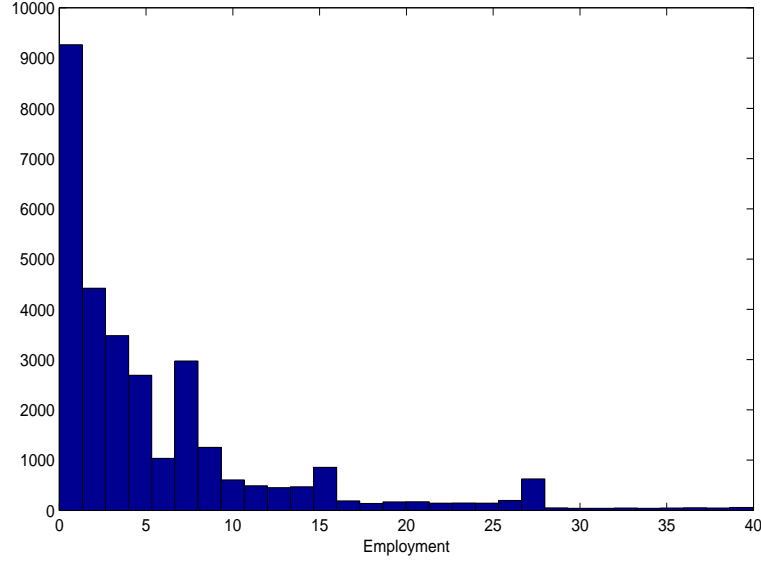


Figure 5: Employment histogram, model

to the right, although we fail to generate a left tail. The distribution for total and net revenues display similar features. Firm-level employment is highly persistent and dispersed in the model, with an autocorrelation coefficient of .98 and a log-employment standard deviation of 1.3. Cash flow is more volatile at higher frequencies, with an autocorrelation coefficient of .55 and only a slightly lower standard deviation than employment levels. Regarding investment, we picked the parameters to match the share of firms with investment expenses in excess of cash flows. The resulting distribution of investment is bimodal, with a substantial share of firms doing zero or very small investment.

Figure 6 plots the policy functions for NFA, investment, the NFA to K ratio, as well as the law of motion for net worth, all as functions of the net worth of a firm in state  $z_3$ , without an investment opportunity. The law of motion for net worth is plotted for the two possible realizations of next period state: “operational losses” (dashed line), returning to state  $z_1$ ; and staying at state  $z_3$  (solid line). As we would expect, firms with very low net worth have low NFA and low investment, and the corresponding equity price is low. Note, though, that even firms with very low net worth are not at the borrowing constraint, that is,  $a > -\alpha$ . This allows them to improve the net worth available to the firm in the event of operational losses, as can be seen from the top right picture.

Firms with higher net worth have higher investment, as they can leave spare debt capacity, and this reduces the cost of capital. Eventually the investment level approaches its optimal level. Note how the realizations for net worth diverge, since the larger the investment, the larger the drop in net worth associated with operational losses. In this sense firms with higher net worth

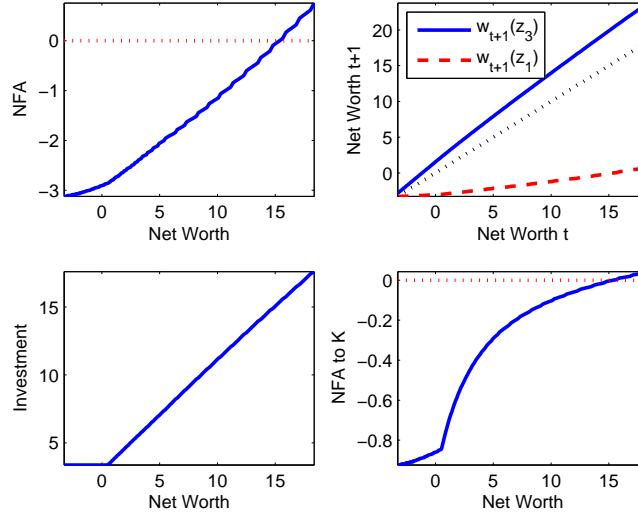


Figure 6: Policy functions

can take on more “investment risk.” Since both NFA and capital are increasing as a function of net worth, it is an open question whether NFA to capital increases with net worth. The lower-right plot displays the ratio of NFA to capital, which is clearly increasing.

Finally we discuss the effect of our two shocks. Figure 7 displays the policy functions for a firm without an investment opportunity (solid line) and a firm with an investment opportunity (dashed line). Since the firm with an investment opportunity expects productivity to increase next period, its investment is higher. How does it finance the additional investment expenses? Since both firms have the same contemporaneous cash flow, the differences in investment must come from adjusting the balance sheet. The top panel shows that the NFA holdings, for the same level of net worth, are lower for the firm with an investment opportunity. Thus investment is partially financed by increasing debt (or reducing financial assets for higher net worth firms).

Note, though, that the firms are quite cautious in their use of debt. Indeed, firms with very low net worth expand their investment in response to the investment opportunity shock more than firms with higher net worth. The reason is that whenever the firm is at or very close to the borrowing constraint, it relies more aggressively on equity. Investment is limited due to the high cost of equity, but there are no risk considerations anymore: if the firm suffers operational losses next period it will find itself against the borrowing constraint again. As the net worth of the firm increases, investment increases because the cost of capital declines, yet only slowly because the firm with more investment is now increasing its exposure to a loss.

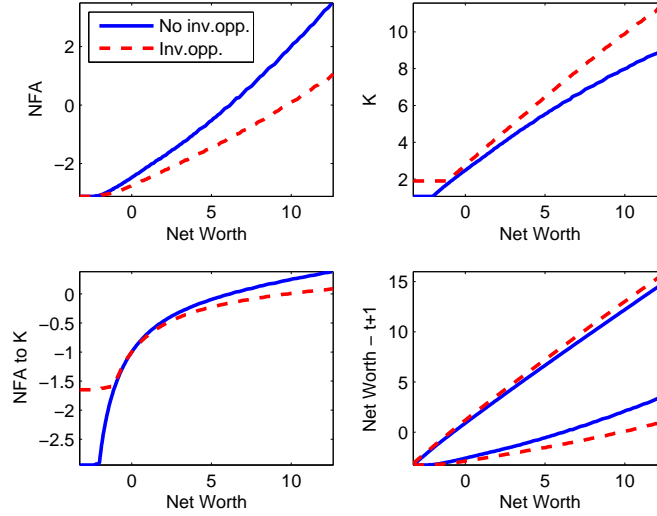


Figure 7: Investment opportunity

## 7 What is behind the rise of corporate financial assets?

In this section we take the model for a trip back in time to ask what was behind the rise of the corporate NFA between the 1970s and 2000s. As reported in Section 2, the corporate sector was a net debtor in the 1970s and turned into a net lender in the 2000s. Looking at our sample of firms from Compustat, we highlighted that the fraction of firms with positive NFA increased sharply, as did the mean and median NFA to capital positions.

We argue that the decline in effective dividend tax rates from the late 1970s can explain, by itself, most of the observed rise in the aggregate NFA position. There is no question that the fiscal and regulatory burden on equity has eased up over the past 40 years. There have been two main forces behind this change. First, there were significant cuts in top marginal income tax rates in the 1980s and, starting in 2003, dividend income was taxed separately from income and at a rate significantly below income tax rates.<sup>50</sup> The second force has been emphasized by McGrattan and Prescott (2005), who argue that changes in regulation have had an important impact on the effective marginal tax rates by increasing the share of equity held by fiduciary institutions that pay no taxes on dividend income or capital gains.<sup>51</sup>

What is the link between dividend taxation and net financial assets? Based solely on cost considerations, a fall in effective dividend taxes will have no direct effect on the demand for debt

<sup>50</sup>The public finance literature has documented this shift extensively as early as in Poterba (1987). The latter change was brought up by the Jobs and Growth Tax Relief Reconciliation Act of 2003, which spurred a large literature that we cannot hope to summarize here.

<sup>51</sup>See Kristian Rydqvist and Strebulaev (2011) for cross-country evidence on the role of tax policies on the decline of direct stock ownership by households.

as long as equity remains fiscally disadvantageous. The connection between the cost of financing and investment is clear, but why should cheaper financing lead to more internal funds?

As discussed in section 4, firms in our model fund additional financial asset holdings with equity in order to insure against cash flow shocks. As the cost of equity relative to debt falls, raising equity to fund financial assets becomes less expensive, and firms seek to acquire further insurance—driving up both their equity financing and their holdings of net financial assets. While this mechanism is at the center of our result, the cost of equity also has some additional effects through investment decisions, the curvature of the value function, and the stationary distribution. To sort out the net effect, we resort again to a quantitative evaluation of the model.

We rely on Poterba (1987) for effective tax rate estimates and set the dividend tax rate  $\tau^d$  corresponding to the 1970s at 0.28. Our baseline calibration for the 2000s used a tax rate of  $\tau^d = 0.15$ , the statutory rate for most of the period. There is no statutory rate for the 1970s, since dividend income was not taxed separately. The effective tax rate is instead estimated from marginal income tax rates and the distribution of income across households.<sup>52</sup> Thus according to our calibration, the decline in dividend taxation during the 1980s and 1990s, up to the Jobs and Growth Tax Relief Reconciliation Act of 2003, halved the effective dividend tax rate. We recompute our markdown parameter for the 1970s with the higher tax rate, which renders equity more expensive relative to debt,  $\xi = 0.69$ . The estimates for the effective dividend tax in the 1970s from McGrattan and Prescott (2005) are significantly higher: if anything, we may be underestimating the decline in the fiscal burden on equity.

We keep all the remaining parameters of the model unchanged in order to evaluate the role of dividend taxation independently of other changes.<sup>53</sup> We should mention that tax rates on capital gains and interest income were also slightly higher in the 1970s. However, the effect of these two tax rates in the relative cost of equity to debt is quite small, and we feel comfortable abstracting from them and focusing on dividend taxes.

Table 6 reports the moments from the distribution of NFA to capital from the model evaluated at  $\tau^d = .28$  and compares them with the data. The shift toward debt is remarkably close to the data. The model predicts the mean NFA to capital in the 1970s at  $-0.10$  – a dramatic drop relative to the 2000s – while the corresponding number in the data is  $-0.12$ . Thus, according to the model, the higher dividend taxes of the 1970s explain virtually all of the shift in the mean. Similarly, just above 32 percent of the firms in the model have a positive NFA in the 1970s, down from the 44 percent in the 2000s, and very close to the 27 percent in the data in the 1970s.

For such a stark exercise as ours, the overall fit of the distribution is surprisingly good. The

---

<sup>52</sup>See Poterba (2002) for further details and an updated time series.

<sup>53</sup>For the exercise, we treat the borrowing constraint as a parameter. As the support for the net worth distribution changes, we also adjust the entry distribution to replicate the entrants' characteristics in the 2000s.



Table 6: Dividend tax  $\tau^d = .28$ 

NFA to K	1970s	
	Data	Model
mean	-0.12	-0.10
median	-0.17	-0.26
$\Pr(a > 0)$	26.9%	32.2%
std dev	0.39	0.58
10pct	-0.50	-0.58
25pct	-0.34	-0.51
75pct	0.02	0.07
90pct	0.29	1.07

10th percentile is within range, and the third quartile is very close to the data. The model correctly predicts that the overall dispersion in balance sheets is lower with the higher dividend tax. However, the observed standard deviation for the 1970s is significantly lower than predicted by the model. This is mainly driven by a longer right tail in the model than in the data, as one can see by comparing the 90th percentile. At the same time the model overshoots the shift in the data regarding the median NFA to capital. Clearly, the distribution is too skewed to the right compared with the data.<sup>54</sup>

Interestingly, the higher dividend tax rates imply only a slight decline in the capital-to-output ratio. This fits well with the experience of the U.S., where the capital-to-output ratio in the data has been broadly stable during the same period. However, our model can offer only an incomplete picture of the growth experience of the U.S. as we lack an explicit formulation for intangible investment. McGrattan and Prescott (2005) show how intangible investment plays a key role to reconcile the large movements in corporate equity values relative to GDP with the (large) changes in dividend taxation and (small) changes in the capital-to-output ratio.

We recognize that there are a number of other changes over time that are likely to have affected the costs and benefits of equity. We return to the key inputs in our calibration in section 5 and check for differences in the 1970s. One of the key developments we uncover in the Compustat sample during the 1970-2007 period is a large increase in the incidence of operational losses over time. Figure 8 plots the share of firms experiencing a negative cash flow in the Compustat sample. The pattern is clearly increasing, with a significant run-up in the 1980s and the second half of the 1990s. We did not see any clear time series pattern regarding investment expenditures in excess of cash flows, the moment we targeted for the calibration of the investment opportunities.

<sup>54</sup>Overall, the skewness in the data is much lower in the 1970s than in the 2000s.

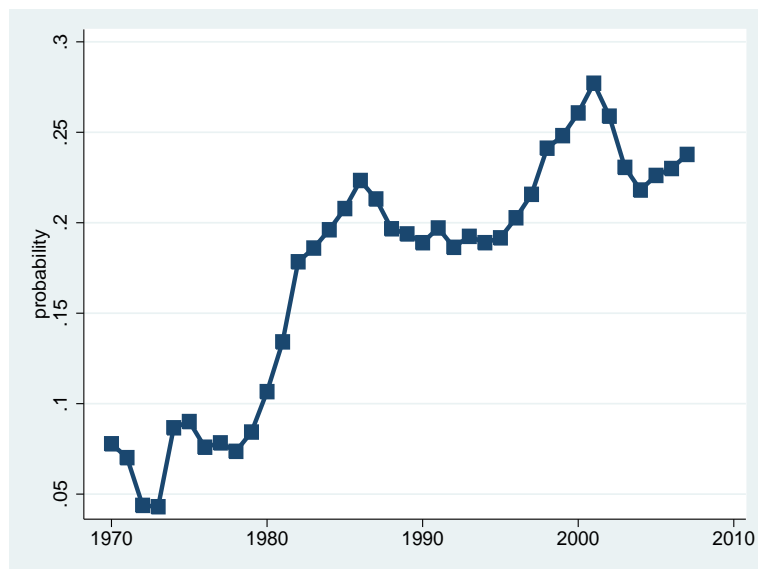


Figure 8: Share of firms with negative cash flows in Compustat

The increase in firms with negative cash flow lines up with the evidence on listed firms reported in other studies. Comin and Philippon (2006) document how volatility of sales and employment growth for Compustat firms sharply increased. Irvine and Pontiff (2009) also report similar increases in volatility of firm-level returns and cash flows. However, Davis et al. (2007) argue that privately held firms display the opposite behavior. See also Thesmar and Thoenig (2011).

We incorporate this observation in a second run of our model for the 1970s. We set  $\phi = .04$ , so the probability that a firm incurs operational losses is reduced to approximately match the transition rate from positive to negative cash flows in the 1970s. The dividend tax remains at .28. As in the previous exercise, we do not change any other parameters in the model. Table 7 displays the results.

Incorporating the lower incidence of operational losses improves the fit of the model. In particular, the median is much closer to the data, and the excessive skewness to the right that the model displayed before has been somewhat tempered. Reducing the probability of operational losses has two counterbalancing effects. Mechanically, firms can accumulate NFA more rapidly since they simply suffer negative shocks less frequently. More subtly, though, the lower probability of losses makes debt more attractive and equity less so, as the precautionary motive is weaker, and partial insurance is less valuable.

Overall, our results suggest that a decline in dividend taxes and regulations over the past 40 years goes a long way in explaining the rise of corporate NFA to capital in the U.S. during the same period.

NFA to K	1970s	
	Data	Model
mean	-0.12	-0.10
median	-0.17	-0.21
$\Pr(a > 0)$	26.9%	28.7%
std dev	0.39	0.57
10pct	-0.50	-0.55
25pct	-0.34	-0.46
75pct	0.02	0.04
90pct	0.29	0.90

Table 7: Dividend tax  $\tau^d = .28$ , operational losses  $\phi = .04$

## 8 Net Savings and Age

In Section 5 we assumed operational losses and investment opportunities were i.i.d. across firms. This parsimonious approach was ideally suited to test whether the model would replicate the observed distribution of NFA over capital. However, the i.i.d. specification is too stark to capture some interesting relationships in the data, such as that between NFA and firms' age. We briefly discuss here a simple extension of our model that sheds light on the dynamics of the firms' balance sheets in the immediate years after an IPO. Throughout this section we focus on the period 2000-2007.

As mentioned in Section 2, we find that firms have relatively large net financial holdings immediately after an IPO but adjust their NFA position downward in the following years. We also find that younger firms are more prone to suffer operational losses and grow faster than older firms. This is in line with previous research, which has found that younger firms experience higher, more volatile growth rates.

Are the dynamics of NFA related to the specific dynamics of younger firms? We put this question to our model. With a simple extension we are able to match the pattern of operational losses and employment growth in the years after an IPO.<sup>55</sup> We then check the model predictions regarding NFA positions for those same years.

We extend the state space, defining two firm types, labeled young and old. Entrants start as young firms and transition to old with probability  $\mu$ . This simple specification allows us to introduce relationships with age without overburdening the model with a complete age-dependent specification.<sup>56</sup> We preserve the productivity structure of the baseline model, but we

<sup>55</sup>In the model we pin down the IPO to entry.

<sup>56</sup>The stochastic transition also captures the fact that firms' actual age is not pinned down by the initial IPO in the data.

allow the parameters to differ across young and old firms.

The probability of operational losses for young and old firms,  $\phi_y, \phi_o$ , is set jointly with the transition probability  $\mu$  to match the decreasing pattern of operational losses between ages 1 to 10. We find that setting  $\phi_y = .11$ ,  $\phi_o = .055$ , and  $\mu = .15$  delivers the best fit.<sup>57</sup> The transition probability value implies an average duration of youth of about 7 years.

We use the remaining productivity parameters for young firms to fit the positive relationship between size and age. In particular, we target the ratio of the median employment of ages 3 and 14, roughly the 10th and 90th percentiles in the age distribution. In the data, the ratio is just below 4, 3.8 to be precise. The model reproduces the ratio by setting  $z_2^y = .7$ ,  $\gamma_z^y = 1.22$  and  $\iota = .4$ . While we get the gradient between age and size right, the model slightly overestimates the unconditional correlation between age and size: .4 in the model versus .27 in the data.<sup>58</sup>

Overall, the calibration lines up with what we know about younger firms. Due to the increased chance of investment opportunities, young firms have a higher growth rate than old firms in the model. Their revenues are also lower and more volatile, as brought up by the higher incidence of operational losses. The remaining parameters are as reported previously.

We now check the model's implications regarding NFA over the early years of a firm. We find that, in the model as in the data, younger firms initially deplete their financial assets. Given that we just put the question to the model, that is, we did not directly target the balance sheet dynamics in the calibration, this is strongly suggestive that the different productivity process for younger firms can indeed explain life-cycle facts regarding corporate finance choices.

What is the mechanism behind this result? The main determinant is the higher incidence of shocks on younger firms, both leading to operational losses and to investment opportunities. In response to both shocks, firms resort to available NFA, whether to finance the shortfall in cash flow or the expansion of investment, respectively. Note that equity is particularly valuable to younger firms, as they have relatively lower net worth and higher volatility. This combination makes the partial insurance properties of equity very attractive. Relying on equity would suggest that younger firms are able to accumulate NFA faster. However, the smaller cash flow and the higher rate of shocks dominate, and the resulting relationship between NFA and age is decreasing.

Note that new entrants have relatively high NFA over capital, yet they actually start with much lower net worth than older firms. The reason is that younger firms have lower productivity initially and thus start with small investments. Thus their NFA positions tend to be large relative to their capital holdings.

---

<sup>57</sup>The unconditional probability of operational losses in the model also lines up with the data, at around 6 percent.

<sup>58</sup>The correlation is reported for the period 2000-2007. Using the full sample, 1970-2007, we obtain a much higher correlation, .35.

While this simple extension of the model matches qualitatively the pattern of NFA as firms age, it somewhat underperforms quantitatively. In particular, firms in the model rebound too fast and start building up NFA earlier than in the data. The median ten-year firm in the model already accumulates positive NFA. Also the initial NFA to capital ratios are too low for both median and mean new entrants. We conjecture that the lack of fit here is due to the lack of adjustment costs in investment: firms, as they enter the stock market, usually carry large holdings resulting from the IPO, and only over time are the desired investment plans carried out.

## 9 Conclusions

In this paper we document the switch in the financial position of the U.S. corporate sector from a net borrower to net lender during 1970-2007. To explain this fact we develop a model capable of generating simultaneous demand for equity and net savings, despite the fiscal advantages associated with debt. Our hypothesis emphasizes the risk considerations firms face in their capital structure decisions. In particular, demand for net savings is driven by a precautionary motive as firms seek to avoid being financially constrained in future periods. Simultaneously, firms value equity as it provides partial insurance against investment risk. We showed that our model can match quantitatively the net lender position of the corporate sector for the period 2000-2007 and replicates the overall distribution of NFA during that period very well. We then asked what is behind the rise in corporate NFA since 1970 and found that the fall in dividend taxes appears to be the main culprit.

Going forward, we have several questions in mind. First, we would like to set the changes in the saving behavior of the corporate sector in the broader context of the whole economy. For example, the rise of corporate net savings broadly coincides with a fall in the personal savings rate for U.S. households and, more recently, with an increase in the current account deficit. How are these phenomena related? What are the implications for aggregate savings and investment?

We would also like to be more comprehensive regarding the costs and benefits of equity. We have taken a somewhat narrow view of the former by focusing exclusively on fiscal considerations. This allowed us to quantify the cost of equity independently. But no doubt there are other costs associated with equity, and it is possible that they have changed over the last 40 years as well.<sup>59</sup> We hope to be able to encompass several alternative theories of equity and debt in a general framework in future work.

---

<sup>59</sup>Examples are issuance cost, adverse selection, loss of control, etc.

## References

- Bates, Thomas W., Kathleen M. Kahle, and René M. Stulz**, “Why Do U.S. Firms Hold So Much More Cash than They Used To?,” *Journal of Finance*, October 2009, *64* (5), 1985–2021.
- Brennan, Michael J and Eduardo S Schwartz**, “Optimal Financial Policy and Firm Valuation,” *Journal of Finance*, July 1984, *39* (3), 593–607.
- Chetty, R. and E. Saez**, “Dividend Taxes and Corporate Behavior: Evidence from the 2003 Dividend Tax Cut,” *Quarterly Journal of Economics*, 2005, *120* (3), 791–833.
- Chetty, Raj and Emmanuel Saez**, “The effects of the 2003 dividend tax cut on corporate behavior: interpreting the evidence,” *American Economic Review*, 2006, *96* (2), 124–129.
- Comin, Diego and Thomas Philippon**, *NBER Macroeconomics Annual 2005*, Vol. 20, Cambridge, MA: MIT Press,
- Cooley, Thomas F. and Vincenzo Quadrini**, “Financial Markets and Firm Dynamics,” *American Economic Review*, December 2001, *91* (5), 1286–1310.
- Davis, Steven J., John Haltiwanger, Ron Jarmin, and Javier Miranda**, *NBER Macroeconomics Annual 2006*, MIT Press,
- DeAngelo, Harry, Linda DeAngelo, and Toni M. Whited**, “Capital structure dynamics and transitory debt,” *Journal of Financial Economics*, February 2011, *99* (2), 235–261.
- Fama, Eugene F. and Kenneth R. French**, “Testing Trade-Off and Pecking Order Predictions About Dividends and Debt,” *Review of Financial Studies*, March 2002, *15* (1), 1–33.
- Frank, Murray Z. and Vidhan K. Goyal**, *Trade-off and Pecking Order Theories of Debt*, Elsevier Science,
- Gamba, Andrea and Alexander Triantis**, “The Value of Financial Flexibility,” *Journal of Finance*, October 2008, *63* (5), 2263–2296.
- Gao, Huasheng, Jarrad Harford, and Kai Li**, “Determinants of Corporate Cash Policy: A Comparison of Private and Public Firms,” Working Papers 1722123, SSRN 2010.
- Gomes, Joao F.**, “Financing Investment,” *American Economic Review*, 2001, *91* (5), 1263–1285.
- Gorbenko, Alexander S. and Ilya A. Strebulaev**, “Temporary versus Permanent Shocks: Explaining Corporate Financial Policies,” *Review of Financial Studies*, July 2010, *23* (7), 2591–2647.

- Gourio, Francois and Jianjun Miao**, “Firm Heterogeneity and the Long-Run Effects of Dividend Tax Reform,” *American Economic Journal: Macroeconomics*, 2010, *2*, 131–168.
- and —, “Transitional Dynamics of Dividend and Capital Gains Tax Cuts,” *Review of Economic Dynamics*, forthcoming.
- Hennessy, Christopher A. and Toni M. Whited**, “Debt Dynamics,” *Journal of Finance*, 2005, *60* (3), 1129–1165.
- and —, “How Costly Is External Financing? Evidence from a Structural Estimation,” *Journal of Finance*, 08 2007, *62* (4), 1705–1745.
- Irvine, Paul J. and Jeffrey Pontiff**, “Idiosyncratic Return Volatility, Cash Flows, and Product Market Competition,” *Review of Financial Studies*, 2009, *22* (3), 1149–77.
- Lins, Karl, Henri Servaes, and Peter Tufano**, “Wat drives corporate liquidity? An international survey of cash holdings and lines of credit,” *Journal of Financial Economics*, 2010, *98*, 160–176.
- McGrattan, Ellen R. and Edward C. Prescott**, “Taxes, Regulations, and the Value of U.S. and U.K. Corporations,” *Review of Economic Studies*, 2005, *72*, 767–796.
- Midrigan, V. and D.Y. Xu**, “Finance and Misallocation: Evidence from Plant-level Data,” *NBER Working Papers*, 2010.
- Moskowitz, Tobias J. and Annette Vissing-Jorgensen**, “The Returns to Entrepreneurial Investment: A Private Equity Premium Puzzle?,” *American Economic Review*, September 2002, *92* (4), 745–778.
- Niskanen, Mervi and Tensie Steijvers**, “Managerial ownership effects on cash holdings of private family firms,” Working Papers, University of Eastern Finland 2010.
- Opler, Tim, Lee Pinkowitz, Rene Stulz, and Rohan Williamson**, “The determinants and implications of corporate cash holdings,” *Journal of Financial Economics*, April 1999, *52* (1), 3–46.
- Poterba, James**, “Tax Policy and Corporate Saving,” *Brookings Papers on Economic Activity*, 1987, *2*, 454–504.
- , *Handbook of Public Economics*, Vol. 3, Elsevier Science,
- , “Taxation and Corporate Payout Policy,” *American Economic Review*, 2004, *94* (2), 171–175.

- Rajan, Raghuram G and Luigi Zingales**, “What Do We Know about Capital Structure? Some Evidence from International Data,” *Journal of Finance*, December 1995, *50* (5), 1421–60.
- Riddick, Leigh A. and Toni M. Whited**, “The Corporate Propensity to Save,” *Journal of Finance*, 2009, *64*, 1729–1766.
- Rydqvist, Joshua Spizman Kristian and Ilya Strebulaev**, “Government Policy and Ownership of Financial Assets,” October 2011. Working paper.
- Shyam-Sunder, Lakshmi and Stewart C. Myers**, “Testing static tradeoff against pecking order models of capital structure,” *Journal of Financial Economics*, 1999, *51*, 219–244.
- Sufi, Amir**, “Bank Lines of Credit in Corporate Finance: An Empirical Analysis,” *Review of Financial Studies*, 2009, *22* (3), 1057–1088.
- Thesmar, David and Mathias Thoenig**, “Contrasting Trends in Firm Volatility,” *American Economic Journal: Macroeconomics*, 2011, *3*, 143–180.
- Tirole, Jean**, *The Theory of Corporate Finance*, Princeton University Press, 2006.
- Titman, Sheridan and Roberto Wessels**, “The Determinants of Capital Structure Choice,” *Journal of Finance*, March 1988, *43* (1), 1–19.
- and **Sergey Tsyplov**, “A Dynamic Model of Optimal Capital Structure,” *Review of Finance*, 2007, *11* (3), 401–451.
- Welch, Ivo**, “Capital Structure and Stock Returns,” *Journal of Political Economy*, February 2004, *112* (1), 106–131.
- Whited, Toni M.**, “External finance constraints and the intertemporal pattern of intermittent investment,” *Journal of Financial Economics*, September 2006, *81* (3), 467–502.



## Appendix

### A Data

In this section we describe our data work in more detail. Our firm-level analysis uses the Compustat data set for the 1970-2007 period. As in Hennessy and Whited (2005), Gourio and Miao (2010) we use the following criteria to restrict our working sample. First, we focus only on U.S. firms whose capital is above 50,000 USD, whose equity is non-negative, and whose sales are positive. Second, we exclude firms that according to Standard Industry Classification (SIC) belong to finance, insurance and real estate sector (SIC classification is between 6000 and 6799); regulated utilities (SIC classification is between 4900 and 4999); and information technology and telecommunication services firms (SIC classification of 7370-7379, 4800-4899, and 3570-3579).

If the SIC classification is not available, we then use North American Industry Classification System (NAICS) to exclude the firms belonging to the above three industries. In particular, finance, insurance and real estate firms are identified as those under NAICS sector codes 52 and 53; utilities are those with NAICS sector code 22; while information technology and telecommunication services are identified with sector code 51. If both SIC and NAICS classification codes were missing, we allocated the firm into sectors according to its Global Industry Classification Standard (GICS). Thus, we excluded firms with GICS classification of 40 (Financials); 55 (Utilities); 45 and 50 (Information Technology and Telecommunication Services, respectively).

We begin by summarizing the properties of the aggregate net financial assets (NFA) to capital ratio in the Compustat data set. We construct NFA as the difference between financial assets and liabilities. Financial assets are composed of cash and short-term investments, other current assets, and account receivables (trade and taxes). Liabilities are computed as the sum of debt in current (due within one year) liabilities and other current liabilities; long-term debt; and account payable (trade and taxes). Capital stock is obtained as the sum of the firm's gross value of property, plant and equipment; its total investment and advances; unamortized value of intangible assets; and total inventories. Equity is obtained as the value of common and preferred stockholders' equity. All our variables of interest are measured as a ratio of capital.<sup>60</sup>

Figure A1 summarizes our findings. It plots two ratios: the ratio of average NFA to average capital; and the ratio of median NFA to median capital. We must keep in mind that while the ratio of means gives us a measure of NFA to capital that is closest to the Flow of Funds

---

<sup>60</sup>Detailed analysis of the size of the Compustat sample, its industry composition, computation of capital-output ratios, and in-depth decompositions of NFA in both Flow of Funds and Compustat data, etc. are provided in the online Appendix available at <http://faculty.arts.ubc.ca/vhnatkovska/research.htm>

calculation, it is also heavily influenced by the outliers – firms with large capital and/or NFA.<sup>61</sup> It is easy to see from Figure A1 that these large firms are borrowing, on net, 25 percent of their capital, and that this level has remained relatively stable over time. Contrasting this with the Flow of Funds pattern for corporate NFA suggests several possibilities. First, small and medium-sized firms in the Compustat sample are behind the rise in NFA. We verify this conjecture by looking at the median NFA to median capital, which allows us to control for the outliers in both variables. Indeed the ratio of medians exhibits a clear upward trend over time. NFA are rising steadily over time, although they do not turn positive in the 2000s as the Flow of Funds series does. Furthermore, when we explicitly contrast the levels of NFA to capital for small and medium-sized firms with those of large firms (see Figure A4), we find clear support for the idea that small and medium-sized firms are responsible for the increase in NFA to capital over the past 40 years.

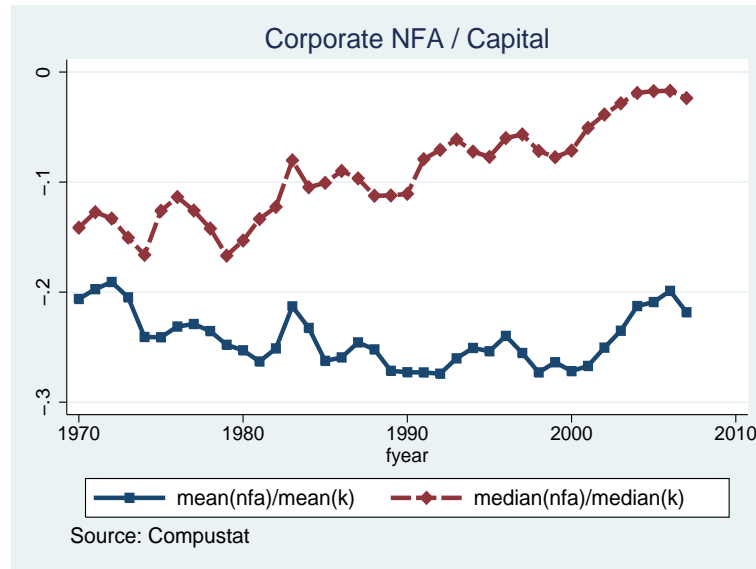


Figure A1: U.S. non-financial, non-utilities, non-technology corporate NFA to K

The second possibility is that private firms, which are not in the Compustat sample, contribute to the increase in NFA to capital. The balance sheet data for private firms, however, is limited, but the recent work by Gao et al. (2010) suggests that these firms may not have contributed much to the rise in NFA to capital in the U.S. corporate sector. In particular, Gao et al. (2010) using a sample of U.S. public and private firms during the 2000-2008 period show that on average private firms hold less than half as much cash as public firms do.<sup>62</sup> While this

<sup>61</sup>For this reason, our preferred aggregate measure of NFA in the Compustat sample is the mean and median of the ratio, which we reported in Figure 2 in the main text.

<sup>62</sup>Niskanen and Steijvers (2010) using a sample of private family firms in Norway find that an increase in firm size is associated with a decrease in cash holdings, a feature that we also document for NFA in our data set of public U.S. firms.

work primarily concerns firms' cash holdings, rather than NFA, it is still informative since, as we show later, an increase in cash holdings and other short-term investments contributed the most to the increase in NFA.

Next, we investigate the gross positions of the firms and their components. Our goal is to isolate the components of financial assets and liabilities that are behind the rise in NFA. Figure A2 shows our results. Panel (a) of that figure presents median financial assets and their components such as cash and short-term investments, other assets, and account receivables, all as a ratio to median capital. Panel (b) presents median liabilities and their components such as short-term and long-term debt and account payables, also as ratios to median capital. From the figures it is easy to see that both assets and liabilities are rising over time, but the increase in assets is more pronounced. Most of the rise in assets is due to higher cash and equivalent holdings of U.S. firms. Other asset categories have been going up as well, but at a much slower pace. Finally, account receivables have declined from about 28 percent of the median capital level in the 1970s to less than 20 percent in the 2000s.

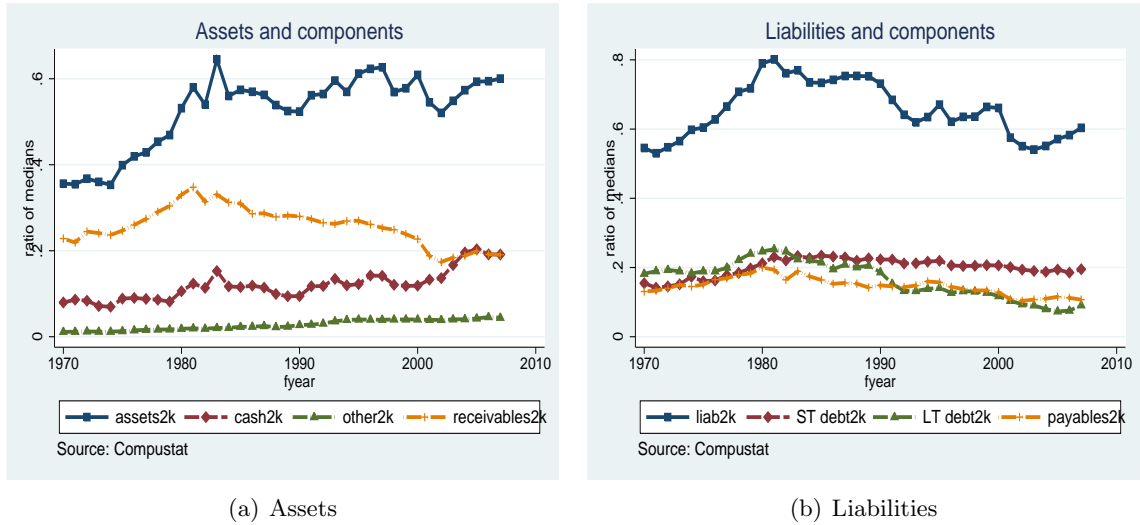


Figure A2: Gross positions and their components

On the liability side, long-term debt and account payables have both fallen over time, while short-term debt has shown a slight increase. Overall, these decompositions suggest a shift in firms' balance sheets away from long-term assets and liabilities toward their short-term counterparts, but with the share of account receivables and payables in the short-term assets and liabilities falling over time. In the model we do not distinguish the maturity structure of debt, and thus in what follows, we focus on the overall NFA position.

What firms are behind the rise in corporate NFA? We turn to this question next and study NFA positions conditional on firm industry, size, age and entry cohort. Figure A3 plots the ratio

of median NFA to median capital in six industries: Agriculture and Mining; Manufacturing; Trade, Transportation and Warehousing; Services; Construction; and Information Technology and Telecommunication Services.<sup>63</sup> Several notable features of the data stand out. First, the increase in NFA to capital is characteristics of all industries, with the exception of construction, which shows a clear break in the series in the late 1980s. However, we have few observations for this industry and thus do not argue that this is a robust finding. The Technology sector, on the other hand, shows the most pronounced increase in NFA over our sample period. In fact, this sector turned into a net lender in the early 1990s and has continued to accumulate net financial assets ever since. Therefore, developments in the Technology sector could have contributed to the run-up in aggregate NFA observed in the Flow of Funds series, especially in the 1990s.

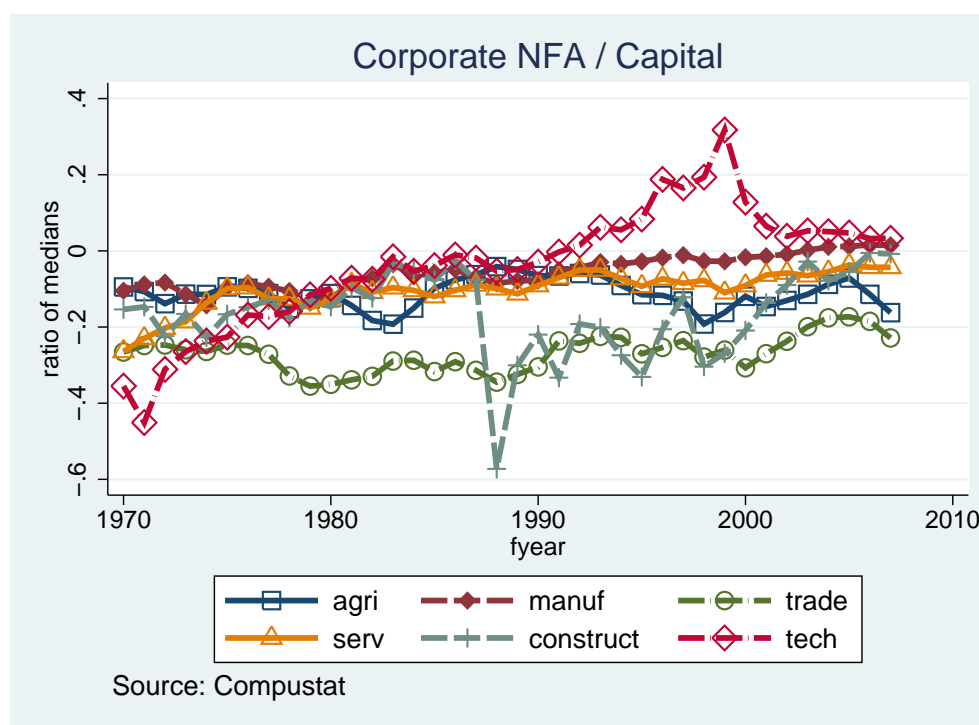


Figure A3: U.S. corporate NFA to capital by industry

Second, there is some heterogeneity in the level of NFA to capital across industries. For instance, firms in the Trade, Transportation and Warehousing industry have consistently had the lowest level of NFA to capital during the 1970-2007 period. The Technology sector was characterized by the lowest level of NFA to capital in the early 1970s, but as discussed above, this has clearly changed in the past 30 years. Firms in the Manufacturing sector (the largest sector in our sample) have exhibited one of the highest levels of NFA to capital throughout the sample period and, in fact, have seen their NFA positions turn positive in the 2000s. Finally,

<sup>63</sup>As mentioned earlier the Technology sector is excluded from our benchmark sample. We include it here for illustration purposes.

agriculture and mining, and services, demonstrate similar levels and dynamics in their NFA to capital ratios during the 1970-2007 period.

Overall, these results suggest that while some of the increase in the NFA to capital ratio over time could be attributed to changes in the industrial structure of the U.S. economy (i.e., expansion of the technology sector), the rise of corporate net savings is characteristic of all industries.

Next we turn to firm-level characteristics and relate them to the rise in NFA. First, we study NFA for firms of different size, as measured by their employment level. Figure A4 reports the median NFA to capital ratio for different employment percentiles, separately for the 1970s and 2000s. It is easy to see that firms of all sizes were net borrowers in the 1970s. In the 2000s the relationship between the NFA to capital ratio and employment became clearly decreasing, with smaller and medium size firms turning into net creditors in that decade. At the same time, larger firms, while increasing their net savings a bit, have remained net debtors. A similar pattern applies at the industry level as well, especially for firms in manufacturing, services, and construction. The increase experienced by agricultural and mining firms, as well as the firms in trade, transportation and warehousing is characteristic of all firms in their respective industries, but is more muted.<sup>64</sup>

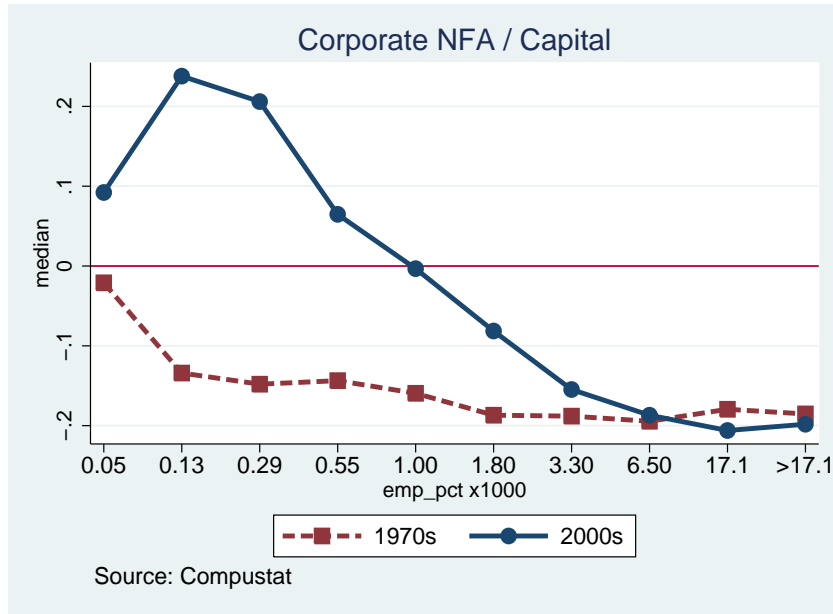


Figure A4: NFA to capital by firm size

Second, we study NFA to capital separately for entrants into Compustat and incumbents for each decade. Table A1 summarizes mean and median of NFA to capital for entrants and

<sup>64</sup>These results are available from the authors upon request.

incumbents in the 1970s and 2000s. A firm is defined as an entrant in a given decade if it appeared in Compustat in any year of that decade.

Table A1: NFA to capital: Entrants and incumbents

	Entrants		Incumbents	
	mean	median	mean	median
	(i)	(ii)	(iii)	(iv)
1970s	-0.12	-0.19	-0.13	-0.16
2000s	0.10	-0.06	0.07	-0.09

Our results indicate that entrants tend to have higher NFA to capital ratios relative to incumbents, and that this tendency has become more pronounced over time.<sup>65</sup> The majority of the differential in NFA to capital ratios between incumbents and entrants is due to the larger cash holdings and short-term investments of the latter. Over time, both cohorts have increased their holdings of cash and short-term investments, but entrants have done so at a significantly faster pace.<sup>66</sup>

Are the differences between entrant and incumbent firms all due to their age differential, or is there an independent cohort effect? We use the number of years since the IPO as a measure of the firm's age. Figure A5 plots median NFA to capital as a function of age, separately for the 1970s and 2000s.

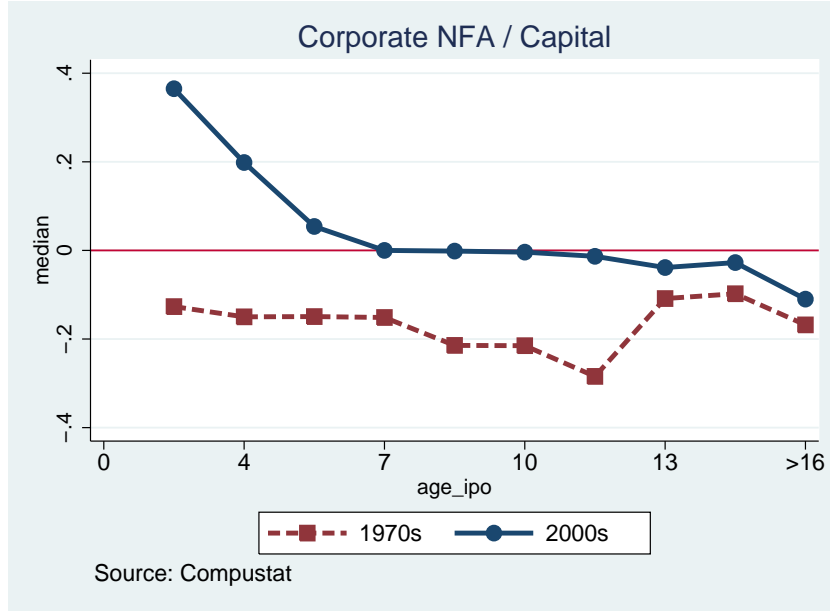


Figure A5: NFA to capital by firm age

<sup>65</sup>Only in the 1970s is the median NFA to capital ratio for entrants somewhat below that for incumbents.

<sup>66</sup>These results are available from the authors upon request.

The figure suggests no association between NFA to capital with age in the 1970s, but the relationship turns negative in the 2000s. The fact that younger firms tend to save more relative to older firms in the 2000s is not surprising given our earlier finding of a negative association of the NFA to capital ratio with size, and the fact that age and size are positively correlated in our sample.

Finally, we investigate the role of all the factors discussed above jointly through a panel regression. In our benchmark specifications that pools firms in Compustat during the 1970-2007 period, we find that after accounting for employment and age, as well as industry and cohort fixed effects, NFA to capital has increased over time and significantly so.<sup>67</sup>

## B Model

### B.1 Feasible investment

We first focus on the set of feasible investment choices,  $\Gamma(\omega, z)$ , for any given values for  $\alpha$  and  $\omega^b$ . Given a choice of investment  $k'$ , there are enough resources to ensure non-negative consumption if and only if

$$\omega + p(k', z) + \alpha \geq +k', \quad (\text{A1})$$

that is, net worth, plus maximum equity issuance  $s' = 1$  and debt  $a' = -\alpha$ , is sufficient to finance investment. The set  $\Gamma(\omega, z) \subset \mathfrak{R}_+$  is thus all the investment  $k'$  such that (A1) is satisfied for given values of  $\omega$  and  $z$ .

To characterize the set, let

$$\psi(k', z) \equiv p(k', z) - k'.$$

This is the maximum amount of equity funds available, net of investment. It can possibly be negative if the firm is not able to raise enough equity to finance all investment. We can then re-write (A1) as

$$\omega + \psi(k', z) \geq -\alpha. \quad (\text{A2})$$

Function  $\psi(k', z)$  is not monotone in  $k'$ . It is easy to check that  $\psi(0, z) = 0$ ,  $\psi(k', z)$  is increasing at first with  $k'$  and has a maximum at point  $\tilde{k}(z) > 0$  where

$$p_k(\tilde{k}(z), z) = 1.$$

Function  $\psi(k', z)$  decreases from then on, eventually crossing zero again. Thus we can charac-

---

<sup>67</sup>The time effect remains positive and significant for the 2000s when we include firm-level fixed effects in the panel regression. These results are available from the authors upon request.

terize the set of feasible investments as

$$\Gamma(\omega, z) = \{k' \geq 0 : \psi(k', z) \geq -\alpha - w\omega\}.$$

Thus the set  $\Gamma(\omega, z)$  is a closed interval, which guarantees that  $\Gamma(w, z)$  is convex and compact, and the resulting recursive problem is well-behaved. However, for arbitrary choice of  $\alpha$  and  $\omega^b$ , the set may be empty. In the next subsection, we ensure that there is always a feasible level of investment .

## B.2 No default condition

We now derive the value of  $\alpha$  that ensures there is no default with probability 1. This is equivalent to saying that at all times there is a feasible level of investment compatible with non-negative consumption—that is,  $\Gamma(\omega, z)$  is not empty. Clearly  $\Gamma(\omega, z) \subset \Gamma(\omega', z)$  if  $\omega < \omega'$  and  $\Gamma(\omega, z) \neq \emptyset$ . We thus evaluate  $\Gamma(\omega, z)$  at the lower bound  $\omega^b$ . Clearly  $\Gamma(\omega^b, z)$  is not empty if

$$\max_{k' \geq 0} \psi(k', z) \geq -\alpha - \omega^b.$$

The right-hand side does not depend on the state; thus, a sufficient condition for  $\Gamma(\omega^b, z)$  to be non-empty at all states is

$$\bar{\psi} \equiv \min_{z \in Z} \max_{k' \geq 0} \psi(k', z) \geq -\alpha - \omega^b. \quad (\text{A3})$$

Note that this is only saying that the firm must be able to raise enough equity and debt, net of investment, to finance its negative net worth position.

Recall that the lower bound  $\omega^b$  is achieved at  $-\kappa - R\alpha$ . Evaluating (A3) with equality, and substituting for the lower bound, we obtain

$$\alpha = \frac{\bar{\psi} - \kappa}{R - 1}.$$

It is, of course, possible to set the borrowing constraint at arbitrary values lower than  $\alpha$  and there would be no default with probability 1.