

# The Fiscal Limit and Non-Ricardian Consumers<sup>\*</sup>

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## ABSTRACT

The U.S. federal government faces the prospect of exponentially rising entitlement obligations that threaten to push the debt-to-GDP ratio to historically unprecedented levels. I introduce a fiscal limit into a Perpetual Youth model to assess how intergenerational redistributions of wealth and the maturity structure of government debt impact the economic consequences of fiscal stress. Growing entitlement commitments require taking a stand on how future monetary and fiscal policies may adjust. When the economy hits its fiscal limit—the point at which increases in taxes are no longer feasible—either the fiscal authority must renege on its promised entitlement benefit or the monetary authority must adjust its policy to stabilize debt. I find that intergenerational transfers of wealth strengthen the expectational effects of the fiscal limit and magnify the likelihood of stagflation. A longer average maturity of government debt weakens these effects in the short and medium runs but still increases the risk of stagflation when taxes come due in the long-run. Dire scenarios never transpire, but delaying reform—legislation that places entitlement spending on a stable path—lengthens the stagflationary period.

*Keywords:* Finite lifetime, Debt maturity structure, Policy uncertainty, Fiscal limit

*JEL Classifications:* E62, E63, H60, E43

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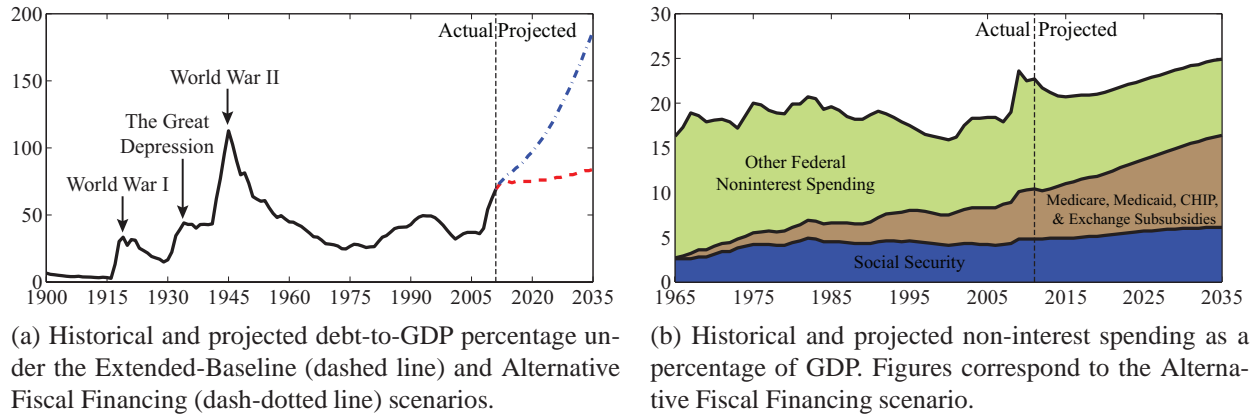


Figure 1: Characteristics of fiscal stress. See [appendix A](#) for details. Source: Congressional Budget Office (2011)

## 1 INTRODUCTION

The volume of public discourse surrounding fiscal policy is unprecedented. Despite the heightened concern about current deficits, many policymakers have begun to recognize that the real problem is projected future deficits. The driving force behind these projections is the growth in spending on the largest three entitlement programs—Medicare, Medicaid, and Social Security ([figure 1](#)). By 2035, the Congressional Budget Office (CBO) projects total spending on entitlement programs will rise from 10.4 to 15.5 percent of GDP, of which nearly 75 percent is attributable to growth in Medicare spending. These projections, which are the result of an aging population and “excess” growth in health care costs, imply government entitlement programs will become insolvent in the coming decades ([figure 2](#)).<sup>1</sup>

Presidential commissions, bipartisan task forces, and several proposals by elected officials have all been aimed at curbing the growth rate of government debt and ensuring that entitlement programs remain solvent. Although many of their policy proposals point toward a resolution of the problem, Congress has provided no clear message or timetable on how actual policy will unfold. Moreover, even if Congress does act, it is always possible for new Congresses to change policies. As a result, agents possess very little information on which they can base expectations.

There is an extensive and compelling literature that has adopted sophisticated overlapping generations (OLG) models that include features such as inter- and intra-generational heterogeneity, life-cycle and population dynamics, bequest motives, stochastic income levels, and several program-specific components to study the effects of policy adjustments and the consequences of fiscal stress [Auerbach and Kotlikoff (1987); De Nardi et al. (1999); Huggett and Ventura (1999); İmrohoroglu et al. (1995); Kotlikoff et al. (1998, 2007); Smetters and Walliser (2004)]. These models assess distributional and generational effects but do not account for monetary policy or deal with the degree of uncertainty that actually surrounds monetary and fiscal policies.

In models with forward-looking agents, ignoring uncertainty pushes the effects of future policy adjustments toward the present, which is inconsistent with current observations. Recognizing this drawback, another segment of the literature uses a representative agent framework to formally

<sup>1</sup>Many other countries are heading into similar periods of fiscal stress [International Monetary Fund (2009)]. Moreover, the projected increases in the debt-to-GDP ratio are larger than any developed countries have experienced in the post-World War II era [Congressional Budget Office (2009)].

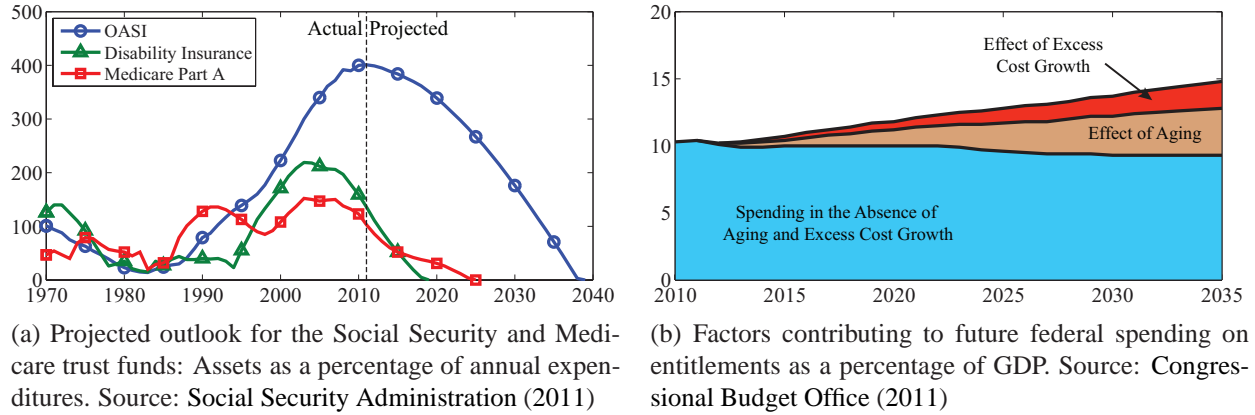


Figure 2: Factors contributing to the fiscal stress of entitlement programs. See [appendix A](#) for details.

model the complex aspects of monetary and fiscal policy uncertainty [Davig and Leeper (2010); Davig et al. (2010, 2011); Eusepi and Preston (2010a); Fernández-Villaverde et al. (2011)] but with the limitation of not being able to account for specific program features or generational effects.<sup>2</sup>

This paper introduces a fiscal limit into a Perpetual Youth model [Blanchard (1985); Yaari (1965)] to assess how intergenerational redistributions of wealth and the maturity structure of government debt impact the economic consequences of fiscal stress. The policy framework allows for a wide range of potential outcomes, which accounts for the uncertain nature of monetary and fiscal policy. At some unknown date, promised government transfers switch from a stable to an explosive trajectory to mimic the demographics underlying the CBO’s debt projections. Each period the economy maintains explosive transfers, there is upward pressure on government debt, which forces tax rates higher. As tax rates rise, policymakers face increasing political resistance and a rising probability of hitting the fiscal limit—the point at which increases in taxes are no longer feasible. Once the fiscal limit is hit, either the fiscal authority must renege on its promised transfers commitments or the monetary authority must adjust its policy to stabilize government debt.

Reneging on government transfers, which is also known as entitlement reform, places policymakers in a bind. On the one hand, they face the economic constraints posed by rising government debt. On the other hand, entitlement recipients (current and prospective) constitute a substantial voting block and any reduction in benefits may be politically toxic. This is why I allow for the possibility that the monetary authority stabilizes debt instead of the fiscal authority reneging on transfers. In this policy mix, the monetary authority adjusts nominal interest rates less than one-for-one with inflation. Without any adjustment in taxes, growing government transfers obligations will then increase the price level until the real value of debt stabilizes. It is the expectation of this surprise increase in inflation that makes the economic consequences of fiscal stress dangerous.

The Perpetual Youth model differs from commonly adopted representative agent models in that it assigns all agents a constant probability of death each period. As agents face a higher probability of death, their expected lifetimes become increasingly misaligned with the government’s infinite planning horizon. This increases the likelihood that current generations, who benefit from increases in (net) government expenditures, will die before taxes come due. The expected shift

<sup>2</sup>Cochrane (2011), Sims (2011), and Daniel and Shiamptanis (2010) also study monetary and fiscal policy interactions in the context of the current fiscal crisis.

of the tax burden onto future generations produces positive wealth effects for current generations, but the threat of rising inflation reduces expected real government liabilities and quickly offsets the positive effects of growing promised transfers. Thus, intergenerational transfers of wealth strengthen the expectational effects of the fiscal limit and magnify the likelihood of stagflation.

Another critical component commonly left out of most policy analysis is a debt maturity structure. Following Woodford (2001) and Eusepi and Preston (2010b), the maturity structure is parameterized to allow longer-term government debt in the baseline model that only includes one-period debt. A longer average maturity of government debt increases the slope of the yield curve and pushes debt and inflation into the future. This weakens the expectational effects of the fiscal limit in the short/medium-runs but still poses economic consequences when taxes come due in the long-run. With long-term debt, stagflation only poses a serious risk several decades into the future.

## 2 ECONOMIC MODEL

I employ a stochastic discrete-time variant of the Blanchard (1985)-Yaari (1965) Perpetual Youth model.<sup>3</sup> This model includes an endogenous labor supply decision and a choice of money holdings. Agents face uncertainty regarding the duration of their lifetimes, the trajectory of their economic variables, and monetary and fiscal policy. Consumption goods are supplied under monopolistic competition, and firms are subject to costly price adjustments. The government finances discretionary spending and delivered lump-sum transfers through seigniorage revenues, short- and long-term nominal debt, and distortionary taxes on capital and labor.

**2.1 INDIVIDUALS** All agents are subject to identical probabilities of death,  $\vartheta$ .<sup>4</sup> Population dynamics are eliminated from the model, since birth and death rates are constant and equalized. The size at birth of generation  $s$  is normalized to  $\vartheta$ , which implies the size of generation  $s$  at time  $t$  is  $\vartheta(1 - \vartheta)^{t-s}$  and the total population size over all generations is one. The average lifetime of a member of generation  $s$  is given by  $\sum_{t=s}^{\infty} \vartheta(1 - \vartheta)^{t-s-1} = 1/\vartheta$ . When  $\vartheta \rightarrow 0$ , this model reduces to the more traditional representative agent setup where agents are infinitely lived.

In period  $t$ , each member of generation  $s \leq t$  maximizes expected lifetime utility of the form<sup>5</sup>

$$E_t \sum_{k=t}^{\infty} [\beta(1 - \vartheta)]^{k-t} \{ \log c_{s,k} + \kappa \log(m_{s,k}/P_k) + \chi \log(1 - n_{s,k}) \}, \quad \kappa, \chi > 0, \quad (1)$$

where  $\beta \in (0, 1)$  is the subjective discount factor,  $P_t$  is the aggregate price index, and  $c_{s,t}$ ,  $m_{s,t}$ , and  $n_{s,t}$  are, respectively, consumption of the final good, nominal money balances, and the quantity of labor supplied at time  $t$  by an agent born at time  $s$ .<sup>6</sup> Following Dixit and Stiglitz (1977),  $c_{s,t} \equiv [\int_0^1 c_{s,t}(i)^{(\theta-1)/\theta} di]^{\theta/(\theta-1)}$  is a consumption bundle composed of a continuum of differentiated goods, where  $\theta > 1$  measures the price elasticity of demand. The demand function for good  $i$ ,  $c_{s,t}(i) = [p_t(i)/P_t]^{-\theta} c_{s,t}$ , corresponds to the agent's maximum attainable consumption bundle

<sup>3</sup>A nonstochastic discrete-time variant of the Blanchard-Yaari model was first developed by Frenkel and Razin (1986). For a stochastic variant see Annicchiarico and Piergallini (2007).

<sup>4</sup>A constant death parameter implies all agents have identical planning horizons, which is required for aggregation.

<sup>5</sup>A nonstochastic continuous-time monetary version of the Blanchard-Yaari model was first introduced by van der Ploeg and Marini (1988). For a discrete-time variant see Cushing (1999). Stochastic monetary models are developed in Annicchiarico et al. (2006), Piergallini (2006), and Annicchiarico et al. (2008).

<sup>6</sup>In general, I denote individual or firm-specific values by lower case letters and aggregate values by capital letters.

given a specific level of expenditures, where  $P_t = [\int_0^1 p_t(i)^{1-\theta} di]^{1/(1-\theta)}$ . Log preferences ensure linearity in wealth and preserve aggregation.

As is conventional in the Perpetual Youth setup, agents have no bequest motive and, instead, sell contingent claims on their assets to insurance companies. Assets are collected each period from  $\vartheta$  agents who died and subsequently transferred to the remaining survivors. With a perfectly competitive life insurance industry, each surviving agent receives a premium payment of  $\vartheta/(1-\vartheta)$ . Incorporating the gross return on the insurance contract,  $1 + \vartheta/(1-\vartheta) = 1/(1-\vartheta)$ , into the per-period budget constraint of a surviving agent yields

$$c_{s,t} + k_{s,t} + \frac{m_{s,t}}{P_t} + \frac{P_t^S b_{s,t}^S}{P_t} + \frac{P_t^M b_{s,t}^M}{P_t} \leq \omega_{s,t} + (1-\vartheta)^{-1} a_{s,t}, \quad (2)$$

where  $k_{s,t}$  is the stock of capital carried into period  $t+1$ . Human income is given by

$$\omega_{s,t} \equiv (1 - \tau_{s,t}) w_{s,t} n_{s,t} + \lambda_t z_{s,t} + d_{s,t}, \quad (3)$$

where  $w_{s,t}$  is the real wage,  $\tau_{s,t}$  is the proportional tax rate levied against capital and labor income,  $z_{s,t}$  are promised real government transfers,  $\lambda_t$  is the fraction of promised transfers received, and  $d_{s,t}$  is the share of real firm profits. Beginning of the period financial wealth is given by

$$a_{s,t} \equiv [(1 - \tau_{s,t}) R_t^k + 1 - \delta] k_{s,t-1} + \frac{m_{s,t-1}}{P_t} + \frac{b_{s,t-1}^S}{P_t} + \frac{(1 + \rho P_t^M) b_{s,t-1}^M}{P_t}, \quad (4)$$

where  $\delta$  is the depreciation rate and  $R_t^k$  is the real rental rate of capital. There are two types of government debt—one-period government bonds,  $b_{s,t}^S$ , in zero net supply with price  $P_t^S$ , and a more general portfolio of government bonds,  $b_{s,t}^M$ , in non-zero net supply with price  $P_t^M$ . The price of short-term nominal bonds satisfies  $P_t^S = R_t^{-1}$ , where  $R_t$  is the gross nominal interest rate. Following Woodford (2001) and Eusepi and Preston (2010b), long-term debt issued at time  $t$  pays  $\rho^j$  dollars  $j+1$  periods in the future, for  $j \geq 0$  and  $0 \leq \rho < \beta^{-1}$ . The payment parameter,  $\rho$ , characterizes the average maturity of government debt,  $1/(1 - \beta\rho)$ , and allows the conventional model with only one-period nominal bonds to be embedded within this more general framework.

Necessary and sufficient conditions for optimality require that each individual's first-order conditions hold in every period, the budget constraint binds, and the transversality condition,

$$\lim_{T \rightarrow \infty} E_t \{ (1 - \vartheta)^{T-t} q_{t,T}(s) a_{s,T} \} = 0, \quad (5)$$

holds, where  $q_{t,t+1} = \beta c_{s,t}/c_{s,t+1}$  is the real stochastic discount factor (SDF) and  $q_{t,T}(s) \equiv \prod_{k=t+1}^T q_{k-1,k}(s) = \beta^{T-t} c_{s,t}/c_{s,T}$ .

To derive the individual law of motion for consumption, first use the individual's first-order conditions to rewrite (2) in terms of the period- $t$  price of the representative agent's portfolio, which has a random value  $a_{s,t+1}$  in the next period. Then solve the resulting budget constraint forward and impose the transversality condition, (5), to obtain<sup>7</sup>

$$c_{s,t} = \xi [a_{s,t}/(1 - \vartheta) + h_{s,t}], \quad (6)$$

where  $\xi \equiv [1 - \beta(1 - \vartheta)]/(1 + \kappa)$  and  $h_{s,t} \equiv \sum_{T=t}^{\infty} (1 - \vartheta)^{T-t} E_t [q_{t,T}(s) \omega_{s,T}]$  is human wealth. An increase in the probability of death,  $\vartheta$ , increases current generations' marginal propensity to consume and the return on financial wealth but reduces the present value of future labor income.

<sup>7</sup>See [appendix B.1](#) for a complete derivation.

**2.2 AGGREGATION** Aggregate values are obtained by summing across all generations and weighting by their relative sizes. Thus, the aggregate counterpart of a generic economic variable,  $x_{s,t}$ , is given by  $X_t \equiv \sum_{s=-\infty}^t \vartheta(1-\vartheta)^{t-s} x_{s,t}$ . Since agents are born with zero assets and government policies are equally distributed, the aggregate budget constraint can be written as

$$C_t + K_t + \frac{M_t}{P_t} + \frac{P_t^S B_t^S}{P_t} + \frac{P_t^M B_t^M}{P_t} = \Omega_t + A_t, \quad (7)$$

where  $\Omega_t \equiv (1 - \tau_t)W_t N_t + \lambda_t Z_t + D_t$  is aggregate human income and

$$A_t = [(1 - \tau_t)R_t^k + 1 - \delta] K_{t-1} + \frac{M_{t-1}}{P_t} + \frac{B_{t-1}^S}{P_t} + \frac{(1 + \rho P_t^M)B_{t-1}^M}{P_t} \quad (8)$$

is aggregate financial wealth. The aggregate counterpart of (6) is given by

$$C_t = \xi(A_t + H_t), \quad (9)$$

where  $H_t = \sum_{T=t}^{\infty} (1 - \vartheta)^{T-t} E_t[Q_{t,T} \Omega_T]$  is aggregate human wealth and  $Q_{t,T}$  is the aggregate SDF.

To derive the dynamic equation for aggregate consumption, follow the techniques applied at the individual level to rewrite (7) and substitute the resulting budget constraint into (9) to obtain a consolidated law of motion for consumption. Then move the original version of (9) forward, apply expectations, and combine with the consolidated law of motion for consumption to obtain<sup>8</sup>

$$C_t = \frac{1}{\beta} E_t\{Q_{t,t+1} C_{t+1}\} + \frac{1}{\beta} \frac{\vartheta \xi}{1 - \vartheta} E_t\{Q_{t,t+1} A_{t+1}\}. \quad (10)$$

When  $\vartheta \neq 0$  higher real government liabilities push the financing of government expenditures onto future generations, which increases consumption by living generations. This relationship between real government liabilities and aggregate consumption, which is not operative in a representative agent model, is critical for understanding the aggregate impacts of fiscal stress.

**2.3 FIRMS** The production sector consists of a continuum of monopolistically competitive intermediate goods producers and a representative final goods producer.

**2.3.1 INTERMEDIATE GOODS PRODUCING FIRMS** Firm  $i \in [0, 1]$  in the intermediate goods sector produces a differentiated good,  $y_t(i)$ , with production function,  $y_t(i) = k_{t-1}(i)^\alpha n_t(i)^{1-\alpha}$ , where  $k(i)$  and  $n(i)$  are the amounts of capital and labor the firm rents and hires. The firm chooses its capital and labor inputs to minimize total cost,  $W_t n_t(i) + R_t^k k_{t-1}(i)$ , subject to its production function. Optimality implies

$$\frac{k_{t-1}(i)}{n_t(i)} = \frac{\alpha}{1 - \alpha} \frac{W_t}{R_t^k}, \quad (11)$$

which shows that the capital-labor ratio is identical across intermediate goods producing firms and equal to the aggregate capital-labor ratio. Hence each firm's marginal cost function, given by,

$$\Psi_t = W_t^{1-\alpha} (R_t^k)^\alpha (1 - \alpha)^{-(1-\alpha)} \alpha^{-\alpha}, \quad (12)$$

is also identical across all intermediate goods.

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<sup>8</sup>See [appendix B.2](#) for a complete derivation.



**2.3.2 PRICE SETTING** The representative final goods producing firm purchases inputs from intermediate goods producers to produce a composite good according to CES technology,  $Y_t \equiv [\int_0^1 y_t(i)^{(\theta-1)/\theta} di]^{\theta/(\theta-1)}$ , where  $Y_t$  denotes aggregate output. Profit-maximization given a specific level of output yields firm  $i$ 's demand function for intermediate inputs,  $y_t(i) = (p_t(i)/P_t)^{-\theta} Y_t$ .

Following Rotemberg (1982), each firm faces a quadratic cost to adjusting its nominal price level, which emphasizes the potentially negative effect that price changes have on customer-firm relationships. Given the functional form used in Ireland (1997), real profits of firm  $i$  are given by

$$d_t(i) = \left[ \left( \frac{p_t(i)}{P_t} \right)^{1-\theta} - \Psi_t \left( \frac{p_t(i)}{P_t} \right)^{-\theta} - \frac{\varphi}{2} \left( \frac{p_t(i)}{\bar{\pi} p_{t-1}(i)} - 1 \right)^2 \right] Y_t, \quad (13)$$

where  $\varphi \geq 0$  parameterizes the adjustment cost and  $\bar{\pi}$  is the steady state gross inflation rate. Each intermediate goods producer chooses their price level,  $p_t(i)$ , to maximize the expected discounted present value of real profits,  $E_t \sum_{k=t}^{\infty} Q_{t,k} d_k(i)$ . In a symmetric equilibrium, all intermediate goods producing firms make identical decisions and the optimality condition reduces to

$$\varphi \left( \frac{\pi_t}{\bar{\pi}} - 1 \right) \frac{\pi_t}{\bar{\pi}} = (1 - \theta) + \theta \Psi_t + \varphi E_t \left[ Q_{t,t+1} \left( \frac{\pi_{t+1}}{\bar{\pi}} - 1 \right) \frac{\pi_{t+1}}{\bar{\pi}} \frac{Y_{t+1}}{Y_t} \right], \quad (14)$$

where  $\pi_t = P_t/P_{t-1}$  is the gross inflation rate. In the absence of costly price adjustments (i.e.  $\varphi = 0$ ), real marginal costs equal  $(\theta - 1)/\theta$ , which is the inverse of the firm's markup factor,  $\mu$ .

**2.4 MONETARY AND FISCAL POLICY** The fiscal authority finances a constant level of real discretionary spending,  $\bar{G}$ , and delivered real government transfers,  $\lambda_t Z_t$ , through a proportional tax on capital and labor income, seigniorage revenues, and by issuing nominal government debt. The government's flow budget constraint is given by

$$\frac{M_t}{P_t} + \frac{P_t^M B_t^M}{P_t} + \tau_t (W_t N_t + R_t^k K_{t-1}) = \bar{G} + \lambda_t Z_t + \frac{M_{t-1}}{P_t} + \frac{(1 + \rho P_t^M) B_{t-1}^M}{P_t}. \quad (15)$$

The model incorporates several layers of policy uncertainty, which follow Davig et al. (2010).<sup>9</sup> Figure 3 illustrates how the uncertainty unfolds. The economy starts with a policy mix ( $S_P = 1$ ) where the monetary authority actively targets inflation by following a simple Taylor rule (AM) and the fiscal authority fully honors its transfers commitments (AT) by passively adjusting the tax rate with the level of real debt (PF).<sup>10</sup>

Real government transfers initially follow a stationary path and evolve according to a first-order two-state Markov chain given by

$$\begin{bmatrix} \Pr[S_{Z,t} = 1 | S_{Z,t-1} = 1] & \Pr[S_{Z,t} = 2 | S_{Z,t-1} = 1] \\ \Pr[S_{Z,t} = 1 | S_{Z,t-1} = 2] & \Pr[S_{Z,t} = 2 | S_{Z,t-1} = 2] \end{bmatrix} = \begin{bmatrix} 1 - p_Z & p_Z \\ 0 & 1 \end{bmatrix},$$

<sup>9</sup>The following analysis omits the possibility of government default. For a detailed analysis of the implications of default within the context of a DSGE model with an imbedded fiscal limit see Bi (2011) and Bi and Leeper (2010).

<sup>10</sup>This terminology follows Leeper (1991). A passive monetary authority weakly adjusts the nominal interest rate with changes in inflation, whereas an active monetary authority targets inflation by sufficiently adjusting nominal interest rates to pin down inflation. Active tax policy implies that the fiscal authority sets the tax rate independently of the size of government debt, while passive tax policy implies that the fiscal authority adjusts taxes to stabilize debt.

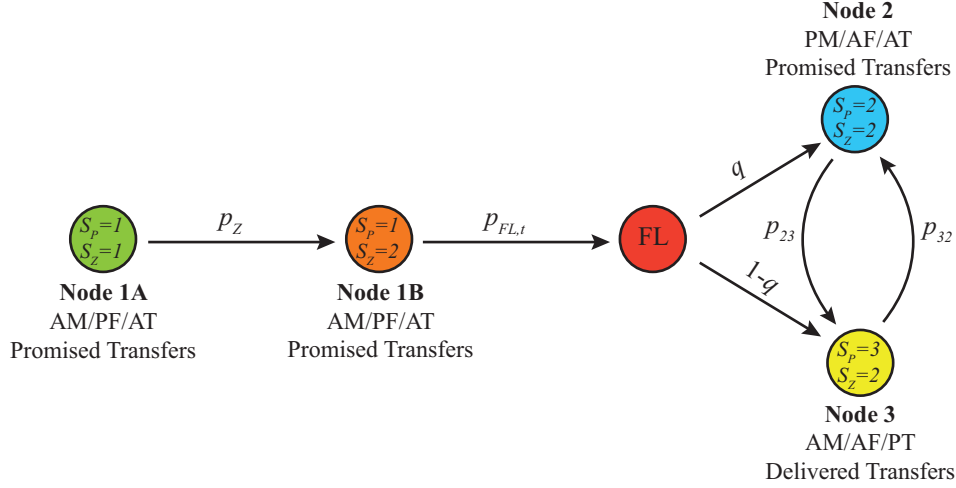


Figure 3: Possible evolution of monetary and fiscal policy regimes

where  $p_Z$  is the time-invariant probability of non-stationary transfers. Each period, the economy faces the dilemma that government transfers may begin to follow a perpetually unsustainable path, as the CBO currently projects. Formally, the transfers process is given by

$$Z_t = \begin{cases} (1 - \rho_Z^S) \bar{Z} + \rho_Z^S Z_{t-1} + \varepsilon_t, & \text{for } S_{Z,t} = 1, \\ \rho_Z^{NS} Z_{t-1} + \varepsilon_t, & \text{for } S_{Z,t} = 2, \end{cases} \quad (16)$$

where  $\bar{Z}$  is the steady state level of promised transfers,  $\rho_Z^{NS} > 1$ ,  $\beta \rho_Z^{NS} < 1$ ,  $|\rho_Z^S| < 1$ , and  $\varepsilon_t \sim i.i.d. N(0, \sigma_Z^2)$ .

Once government transfers start on an unstable path, the economy moves from node 1A to node 1B as shown in figure 3. Government debt mounts and taxes are revised upward. Policymakers face increasing political resistance under this financing scheme and the likelihood of hitting the *fiscal limit* (FL) steadily rises. Eventually, either the resistance becomes so great that higher tax rates are no longer politically feasible or the economy reaches the peak of its Laffer curve—the instance where higher taxes can no longer yield increased revenues—and the fiscal limit is hit.<sup>11</sup> Once this occurs, the post-fiscal limit tax rate binds and either the monetary or fiscal authority is forced to adopt an alternative policy that stabilizes the trajectory of government debt.

The possible policy outcomes are captured by the monetary/tax policy mix ( $S_P \in \{1, 2, 3\}$ ). Specifically, the monetary authority sets the short-term nominal interest rate according to

$$R_t = \begin{cases} \bar{R}(\pi_t/\pi^*)^\phi, & \text{for } S_{P,t} \in \{1, 3\}, \\ \bar{R}, & \text{for } S_{P,t} = 2, \end{cases} \quad (17)$$

while the fiscal authority adjusts tax rates according to

$$\tau_t = \begin{cases} \bar{\tau} \left( \frac{B_{t-1}^M/P_{t-1}}{(B^M/P)^*} \right)^\gamma, & \text{for } S_{P,t} = 1 \text{ (if the FL does not bind),} \\ \tau^{FL}, & \text{for } S_{P,t} \in \{2, 3\} \text{ (if the FL binds),} \end{cases} \quad (18)$$

<sup>11</sup>Trabandt and Uhlig (2010) find that some countries are already at or near the peaks of their Laffer curve.



where an asterisk corresponds to a policy target and a bar corresponds to a steady state value. The parameters  $\phi$  and  $\gamma$  respectively control the response of the nominal interest rate to changes in inflation and the sensitivity of taxes to real debt.  $\tau^{FL}$  is the post-fiscal limit tax rate. The policy specification makes explicit the fact that there exists an upper bound to the degree of financing that taxes can provide. Although agents know the post-fiscal limit tax rate, uncertainty about the trajectory of government transfers implies that agents are unable to predict when this rate will bind.

Agents forecast when the fiscal limit will be hit, which captures some of the uncertainty that surrounds government spending programs when they are funded by future revenue streams, such as with pay-as-you-go financing. Following Davig et al. (2010, 2011), the probability of hitting the fiscal limit,  $p_{FL,t}$ , is endogenously determined by

$$p_{FL,t} = 1 - \frac{\exp(\eta_0 - \eta_1(\tau_{t-1} - \bar{\tau}))}{1 + \exp(\eta_0 - \eta_1(\tau_{t-1} - \bar{\tau}))}, \quad (19)$$

where the parameters  $\eta_0$  and  $\eta_1 > 0$  pin down the intercept and slope of the logistic function.<sup>12</sup> At the fiscal limit, tax policy becomes active (AF), and the policy mix must immediately adjust. If the fiscal authority continues to honor its promised transfers (AT), the monetary authority stabilizes debt by switching from active to passive policy (PM) and the economy moves from node 1B to node 2. Under this policy mix, transfers continue to follow an unsustainable path, which leads to continued increases in debt and, without a central bank response, higher inflation. The higher price level reduces the value of real debt and allows the fiscal authority to stave off any reduction in promised transfers ( $\lambda = 1$ ). If, on the other hand, the monetary authority continues to target inflation (AM), the fiscal authority cannot fully honor its promised transfers (PT) and the economy moves from node 1B to node 3. Reneging on transfers could come in a variety of forms, but regardless of the approach, I assume any modifications (adjustments in  $\lambda$ ) are sufficient to stabilize real government debt, so that both of the post-fiscal limit regimes produce paths that are consistent with a long-run equilibrium. When the fiscal limit is hit, the monetary stabilizes debt with probability  $q$  and the fiscal authority reneges on its transfers commitments with probability  $1 - q$ .

The initial policy adjustment is not permanent. Instead, after the fiscal limit, policy evolves according to a first-order two-state Markov chain given by

$$\begin{bmatrix} \Pr[S_{P,t} = 2 | S_{P,t-1} = 2] & \Pr[S_{P,t} = 3 | S_{P,t-1} = 2] \\ \Pr[S_{P,t} = 2 | S_{P,t-1} = 3] & \Pr[S_{P,t} = 3 | S_{P,t-1} = 3] \end{bmatrix} = \begin{bmatrix} p_{22} & p_{23} \\ p_{32} & p_{33} \end{bmatrix},$$

so that each period either the monetary or fiscal authority can take the lead in stabilizing government debt when the fiscal limit binds. This forces agents to always condition on the possibilities of debt revaluation and entitlements reductions. Post fiscal-limit policy adjustments are marked by movements between nodes 2 and 3 in [figure 3](#).

**2.5 EQUILIBRIUM** The aggregate amounts of labor and capital supplied by the agent are defined as  $N_t = \int_0^1 n_t(i) di$  and  $K_t = \int_0^1 k_t(i) di$ . Equilibrium requires all goods and asset markets to clear each period. The former is satisfied by the aggregate resource constraint,

$$C_t + I_t + \bar{G} = \left[ 1 - \frac{\varphi}{2} \left( \frac{\pi_t}{\bar{\pi}} - 1 \right)^2 \right] Y_t, \quad (20)$$

<sup>12</sup>These restrictions ensure that the probability of hitting the fiscal limit is positive and increases with government debt, since the fiscal authority responds passively to government debt prior to the fiscal limit.

where capital evolves according to  $K_t = I_t + (1 - \delta)K_{t-1}$ . The latter requires that one-period bonds are in zero net supply,  $B_t^S = 0$ , since they are not issued by the government. A competitive equilibrium consists of a sequence of prices,  $\{P_t, W_t, R_t^k, P_t^M, \Psi_t, Q_{t,t+1}\}_{t=0}^\infty$ , quantities,  $\{C_t, K_t, M_t, N_t, B_t^S, B_t^M, Y_t, I_t, A_t\}_{t=0}^\infty$ , and government policies,  $\{R_t, \bar{G}, \tau_t, Z_t\}_{t=0}^\infty$ , that satisfy the aggregate (over all generations) population's optimality conditions, the representative firm's optimality conditions, the government's budget constraint, the monetary and fiscal policy rules, the asset, labor, and goods markets' clearing conditions, and the transversality condition.

### 3 ANALYTICAL INTUITION: MONETARY AND FISCAL POLICY INTERACTIONS

Prior to solving the model outlined in [section 2](#), I examine how monetary and fiscal policy *jointly* determine the equilibrium price level using a simpler setup that permits an analytical solution. Although some of these results are well-known in a representative agent model,<sup>13</sup> they have not been thoroughly examined in a Perpetual Youth model. I first consider a model that retains aggregate uncertainty over government transfers but removes all regime switching. I then introduce a fiscal limit that is hit with certainty at a known future date.<sup>14</sup>

**3.1 ENDOWMENT ECONOMY MODEL** The following model is a cashless version of the Perpetual Youth model where labor is inelastically supplied and agents have identical preferences, receive the same constant endowment, face the same fiscal policies (i.e. government transfers and lump-sum taxes), and are subject to identical probabilities of death,  $\vartheta > 0$ .

Each member of generation  $s \leq t$  chooses sequences  $\{c_{s,t}, b_{s,t}^S, b_{s,t}^M\}_{t=0}^\infty$  to maximize their expected lifetime utility,  $E_t \sum_{k=0}^\infty [\beta(1 - \vartheta)]^k \ln c_{s,t+k}$ , subject to their flow budget constraint,

$$c_{s,t} + \frac{P_t^S b_{s,t}^S}{P_t} + \frac{P_t^M b_{s,t}^M}{P_t} + \tau_{s,t} - z_{s,t} \leq y_{s,t} + \left[ \frac{P_t^S b_{s,t-1}^S}{P_t} + \frac{(1 + \rho P_t^M) b_{s,t-1}^M}{P_t} \right] (1 - \vartheta)^{-1}. \quad (21)$$

After aggregating, the Euler equations for short and longer-term government debt are given by

$$P_t^S = E_t \left\{ Q_{t,t+1} \frac{P_t}{P_{t+1}} \right\} \quad \text{and} \quad 1 = E_t \left\{ Q_{t,t+1} \frac{1 + \rho P_{t+1}^M}{P_t^M} \frac{P_t}{P_{t+1}} \right\}, \quad (22)$$

where  $P_t^S = 1/R_t$ . Following the same procedure described in [section 2](#), the aggregate law of motion for consumption, (10), simplifies to

$$C_t = \frac{1}{\beta} E_t \{ Q_{t,t+1} C_{t+1} \} + \frac{1}{\beta} \frac{\vartheta \xi}{1 - \vartheta} E_t \left\{ Q_{t,t+1} \left[ \frac{B_t^S}{P_{t+1}} + \frac{(1 + \rho P_{t+1}^M) B_t^M}{P_{t+1}} \right] \right\}. \quad (23)$$

Since government spending is zero each period, the government chooses sequences of taxes, lump-sum transfers, and nominal bonds to satisfy

$$\frac{P_t^M B_t^M}{P_t} + \tau_t = Z_t + \frac{(1 + \rho P_t^M) B_{t-1}^M}{P_t}. \quad (24)$$

<sup>13</sup>See Leeper (1991), Sims (1994), and Woodford (2001) for detailed analysis of price level determination in a representative agent model. See Cochrane (2001, 2011) for analysis that incorporates longer-term government debt.

<sup>14</sup>See Leeper (2010a) for price level analysis in a representative agent model that includes a fiscal limit.

In equilibrium, the goods ( $C_t = Y_t = \bar{Y}$ ) and asset markets ( $B_t^S = 0$ ) must clear and the transversality condition, given by,

$$\lim_{T \rightarrow \infty} E_t \left\{ Q_{t,T} \left[ \frac{B_{T-1}^S}{P_T} + \frac{(1 + \rho P_T^M) B_{T-1}^M}{P_T} \right] \right\} = 0, \quad (25)$$

must hold in every period. Moreover, the bond Euler equations imply a no arbitrage condition,  $P_t^M = P_t^S E_t \{1 + \rho P_{t+1}^M\}$ , that when solved forward delivers the term structure of interest rates.

To analytically solve for the equilibrium price level, it is necessary to work with a log-linear approximation of the model around its deterministic steady state. In equilibrium, the log-linear bond Euler equations, given in (22), imply

$$\hat{P}_t^M = \hat{P}_t^S + \rho \bar{P}^S E_t \hat{P}_{t+1}^M = - \sum_{j=0}^{\infty} (\rho P^S)^j E_t \hat{R}_{t+j}. \quad (26)$$

Given (26), the log-linear aggregate law of motion for consumption, (23), is given by

$$\hat{R}_t = E_t \hat{\pi}_{t+1} + \mu (\hat{P}_t^M + \hat{b}_t^M), \quad (27)$$

where a circumflex denotes log-deviations from the deterministic steady state<sup>15</sup> and  $b_t^M = B_t^M / P_t$  is real government debt. The parameter  $\mu \equiv \frac{\vartheta \xi}{1-\vartheta} \frac{(1+\rho \bar{P}^M) \bar{b}^M}{\bar{\pi} Y}$  determines the size of the wealth effect from changes in government debt. When  $\mu > 0$ , the market value of real debt impacts both real and nominal interest rates. The log-linear government budget constraint, (24), is given by

$$\hat{\pi}_t = \hat{b}_{t-1}^M + \rho P^S \hat{P}_t^M - \bar{Q} \left[ \hat{P}_t^M + \hat{b}_t^M + \tilde{\tau} \hat{\tau}_t - \tilde{Z} \hat{Z}_t \right], \quad (28)$$

where  $\tilde{\tau} = \bar{\tau} / (\bar{P}^M \bar{b}^M)$ ,  $\tilde{Z} = \bar{Z} / (\bar{P}^M \bar{b}^M)$ , and  $\bar{Q} = \bar{\pi} / \bar{R} = \beta / (1 + \mu)$ .

**3.1.1 ACTIVE MONETARY AND PASSIVE FISCAL POLICY** Monetary and fiscal policy rules are similar to the initial policy rules described in section 2. The monetary authority targets inflation,

$$\hat{R}_t = \phi \hat{\pi}_t, \quad (29)$$

while the fiscal authority adjusts lump-sum taxes according to

$$\hat{\tau}_t = \gamma (\hat{P}_{t-1}^M + \hat{b}_{t-1}^M), \quad (30)$$

where  $\phi$  is set to ensure price stability and  $\gamma$  is set to ensure that any increases in the market value of debt are met with the expectation that future taxes will rise by enough to service the higher debt and retire it back to its stationary level. Government transfers are exogenous and follow,

$$\hat{Z}_t = \rho_Z^S \hat{Z}_{t-1} + \varepsilon_t, \quad (31)$$

where  $|\rho_Z^S| < 1$  and  $\varepsilon_t \sim i.i.d. N(0, \sigma_Z^2)$ .

<sup>15</sup>Steady state values are denoted by a bar. Thus, for some generic variable  $X$ ,  $\hat{X}_t = \ln X_t - \ln \bar{X} \approx (X_t - \bar{X}) / \bar{X}$ .

To find conditions on monetary policy that stabilize inflation around its target, combine (27)-(29) to obtain the expected evolution of inflation, given by,

$$E_t \hat{\pi}_{t+1} = (\phi + \mu \bar{Q}^{-1}) \hat{\pi}_t - \mu \bar{Q}^{-1} (\hat{b}_{t-1}^M + \rho \bar{P}^S \hat{P}_t^M) + \mu (\tilde{\tau} \hat{\tau}_t - \tilde{Z} \hat{Z}_t). \quad (32)$$

This result reveals that the Taylor principle ( $\phi > 1$ ) is no longer necessary to guarantee a unique bounded solution for inflation. In the special case where only short-term debt is present, a sufficient condition for price level stability is  $\phi > 1 - \mu/\bar{Q}$ , which reduces to the Taylor principle only when  $\vartheta = 0$ .<sup>16</sup> When agents are finitely lived, higher inflation reduces real financial wealth, which imposes negative wealth effects that reduce consumption of current generations. Lower consumption acts as a stabilizer on inflation and implies that the monetary authority no longer needs to adjust nominal interest rates more than one-for-one with inflation to stabilize prices. The presence of longer-term government debt weakens this condition even further. When the monetary authority raises nominal interest rates in response to inflation, the long-term bond price falls. This further reduces consumption demand and acts as an additional stabilizer on inflation.

Active monetary policy implies the unique bounded solution for inflation is given by

$$\hat{\pi}_t = \frac{\mu}{\phi} \sum_{k=0}^{\infty} \left( \frac{1}{\phi} \right)^k (E_t \hat{P}_{t+k}^M + E_t \hat{b}_{t+k}^M). \quad (33)$$

When  $\vartheta > 0$ , deviations of equilibrium inflation from target are proportional to the deviations of the market value of real debt from its target. This shows that even when the monetary authority aggressively targets inflation, fiscal policy still influences equilibrium inflation dynamics. As government debt rises, finitely lived households require higher interest rates to induce them to hold that debt given their finite horizons. The only way this can happen under the monetary policy rule specified in (29) is for inflation to rise. Thus, a Taylor rule induces inflation when accounting for finite planning horizons ( $\vartheta > 0$ ). If, instead, the central bank adjusts the nominal interest rate with fluctuations in government debt by adding  $\mu b_t^M$  to its policy rule, higher levels of debt could be accommodated without compromising their inflation targeting policy, but adding fiscal variables to the monetary feedback rule is anathema to most monetary economists. Moreover, this result is unique to the case where taxes are levied lump-sum and do not distort consumption plans.

The central bank could also mitigate the fiscal authority's influence by increasing its response to inflation (higher  $\phi$ ), but higher debt levels will still increase aggregate demand and cause inflation to deviate from its target level. However, the presence of longer-term government debt, which acts as an automatic stabilizer on inflation, dilutes the interference from the fiscal authority and helps the monetary authority meet its inflation target.

When the monetary authority conducts active monetary policy, a unique bounded equilibrium requires the fiscal authority to respond to disturbances in transfer payments in a manner that stabilizes long-run debt levels. To find conditions on fiscal policy that meet this criteria, combine (28) and (30), apply expectations conditional on information at  $t-1$ , and impose (26) and (27) to obtain the expected evolution of real government debt, given by,

$$E_{t-1} [\hat{P}_t^M + \hat{b}_t^M] = (\tilde{\beta}^{-1} - \gamma \tilde{\tau}) (\hat{P}_{t-1}^M + \hat{b}_{t-1}^M) + E_{t-1} \tilde{Z} \hat{Z}_t,$$

where  $\tilde{\beta} \equiv \beta/(1 + \mu)^2 < 1$  is a discount factor that accounts for the presence of finitely lived agents. The tax rule implies that any increases in the market value of debt will be met by higher

<sup>16</sup>This result is discussed in Leith and Wren-Lewis (2000) and Annicchiarico and Piergallini (2007).

taxes. If the response is not sufficiently strong, disturbances to transfers will lead to explosive debt dynamics that are not consistent with equilibrium. If, on the other hand,  $\tilde{\beta}^{-1} - \gamma\tilde{\tau} < 1$ , the effect of any disturbance to transfers will slowly decay and produce stable debt dynamics.

Further insight regarding the financing of government transfers comes from the intertemporal equilibrium condition, which relates real government debt to the discounted present value of primary surpluses. To derive this condition, first update (28) and impose (26) and (27). Then apply expectations conditional on information at  $t$  and solve forward to obtain

$$\hat{P}_t^M + \hat{b}_t^M = \sum_{k=1}^{\infty} \tilde{\beta}^k E_t \left[ \tilde{\tau} \hat{\tau}_{t+k} - \tilde{Z} \hat{Z}_{t+k} \right]. \quad (34)$$

To see how the price level is pinned down, use the government budget constraint, (28), to decompose the intertemporal equilibrium condition, (34), and obtain

$$\hat{b}_{t-1}^M + \rho \bar{P}^S \hat{P}_t^M - \hat{\pi}_t = \bar{Q} \sum_{k=0}^{\infty} \tilde{\beta}^k E_t \left[ \tilde{\tau} \hat{\tau}_{t+k} - \tilde{Z} \hat{Z}_{t+k} \right]. \quad (35)$$

Since current and future transfers are exogenous and the maturity structure ( $\hat{b}_{t-1}^M$ ) is predetermined, transfers shocks propagate entirely through bond prices, inflation, and future taxes. When agents are infinitely lived, the monetary authority consistently meets its inflation target and any debt financed increase in transfers is met by a commensurate increase in the expected present value of taxes. In this case, agents bear the entire burden of higher future taxes and fully discount any short-run benefits from higher transfers, delivering Ricardian equivalence. When agents are finitely lived, there is a possibility they will die before taxes come due. Thus, a debt financed increase in transfers produces positive wealth effects that cause inflation to temporarily rise above target. This means disturbances to transfers are only partially financed by increases in future taxes in the Perpetual Youth model. An increase in prices, whose timing is determined by the maturity structure of debt, delivers the remaining portion. The interesting question is whether the fiscal authority can exploit this by trading higher government transfers for higher inflation.

**3.1.2 PASSIVE MONETARY AND ACTIVE FISCAL POLICY** Monetary and fiscal authorities do not continuously base policy on the same rules [Davig and Leeper (2006, 2011)]. Policy fluctuates between active and passive regimes as a consequence of both political and economic factors. Under active fiscal policy, the fiscal authority no longer adjusts taxes to stabilize government debt, and instead bases tax policy on exogenous factors such as re-election, stimulus, or hitting the fiscal limit. The most recent evidence for such a regime occurred during the Bush tax cuts of 2001, when income and dividend tax rates were slashed while the debt-to-GDP ratio steadily rose. The defense buildup and tax cuts during the Reagan administration serve as another example.

Under passive monetary policy, the monetary authority no longer aggressively targets inflation and instead focuses on other factors such as output stabilization. This policy typically arises during economic downturns to curtail the severity of recessions. The pre-Volcker era (1960-1979), which experienced high inflation and output volatility, is often characterized by this policy. The most recent example was the monetary authority's response to the financial crisis of 2007-2010, when nominal interest rates were pegged near their lower bound while several unconventional techniques were used to help alleviate the credit crunch and rescue failing financial institutions.



Suppose the fiscal authority fixes lump-sum taxes at a constant level,  $\bar{\tau}$ , while the monetary authority weakly adjusts the nominal interest rate with inflation. To see how the price level gets nailed down, impose active fiscal policy on (35) to obtain

$$\hat{P}_t - \rho \bar{P}^S \hat{P}_t^M = \hat{B}_{t-1}^M + \bar{Q} \sum_{k=0}^{\infty} \tilde{\beta}^k E_t \tilde{Z} \hat{Z}_{t+k}. \quad (36)$$

When fiscal policy is exogenous, the discounted present value of future surpluses is predetermined and fiscal policy determines the overall *change* in prices. To see this, consider two cases. First, suppose there is an unanticipated shock to current government transfers, financed by an increase in government debt,  $B_t^M$ . Without any response from the fiscal authority, at initial prices agents feel wealthier regardless of their planning horizon. Higher consumption demand drives up prices (either  $P_t$  or future prices via  $P_t^M$ ) until agents are content with their initial consumption plan. Now suppose agents expect transfers to increase at some future date. In this case, there is no change in current debt, but without a fiscal response agents still feel wealthier. Once again, prices rise.

The presence of longer-term government debt allows the monetary authority to influence the *timing* of price changes. Consider two extreme cases. The monetary authority can focus on stabilizing current prices,  $P_t$ , but then it must allow expected future inflation to adjust through the price of the maturity,  $P_t^M$ . Alternatively, the monetary authority can focus on stabilizing future prices by pegging the nominal interest rate, but then it must allow the current price level to adjust. This tradeoff between current and future inflation makes clear the important role that longer-term government plays in price level determination. As Cochrane (2001, 2011) emphasizes, a longer average maturity of government debt allows the monetary authority to push inflation into the future.

The presence of finitely lived agents also influences the timing of inflation. Without any response from the fiscal authority, an increase in government debt produces positive wealth effects that drive up current inflation. When agents are finitely lived, higher inflation reduces real wealth and increases real interest rates. Feeling poorer, agents reduce future consumption, which places downward pressure on expected inflation. To see this another way, recall that finitely lived agents require higher returns to induce them to hold additional amounts of government debt. When the monetary authority leans against future price changes by pegging the nominal interest rate, the only way this can occur is if expected inflation falls. Thus, shorter planning horizons push inflation to the present and hinder the monetary authority's ability to delay inflation.

**3.2 FISCAL LIMIT IMPACT** When a fiscal limit is not enforced, the fiscal authority has the option to raise taxes indefinitely in response to increases in real debt. The peak of the Laffer curve imposes an *economic* fiscal limit, but it is possible that the *political* fiscal limit will bind at an even lower tax rate. Using the same model laid out in section 3.1, this section examines how the presence of a fiscal limit—political or economic—impacts the equilibrium price level by imposing an exogenously specified tax rate,  $\tau^{FL}$ , that binds after some known date  $T$ . Government transfers are unaffected by the fiscal limit and follow (31) for all time periods. Since the fiscal authority always honors its transfers commitments, the monetary authority stabilizes debt by switching from active to passive policy when the fiscal limit is hit at date  $T$ .

To solve for the equilibrium market value of debt, first rewrite (34) as a difference equation using the pre-fiscal limit tax rule, (30). Then solve for the current market value of debt, iterate



forward, and use (34) to substitute for the expected market value of debt at time  $T - 1$  to obtain<sup>17</sup>

$$\hat{P}_t^M + \hat{b}_t^M = \begin{cases} - \left[ \left( \frac{1}{1-\gamma\beta\bar{\tau}} \right)^{T-t-1} \frac{(\tilde{\beta}\rho_Z^S)^{T-t}}{1-\beta\rho_Z^S} + \sum_{k=1}^{T-t-1} \left( \frac{\tilde{\beta}\rho_Z^S}{1-\gamma\beta\bar{\tau}} \right)^k \right] \tilde{Z}\hat{Z}_t, & \text{for } t < T, \\ - \frac{\tilde{\beta}\rho_Z^S}{1-\beta\rho_Z^S} \tilde{Z}\hat{Z}_t, & \text{for } t \geq T. \end{cases} \quad (37)$$

Regardless of whether agents are finitely lived, the presence of a fiscal limit prevents the fiscal authority from fully meeting its tax obligations, which makes agents feel wealthier and breaks down Ricardian equivalence. This occurs even though the pre-fiscal limit policy mix exhibits Ricardian equivalence when agents are infinitely lived (section 3.1.1).

Two other results follow through from the representative agent model. First, higher government transfers reduce the market value of government debt. This result follows from the intertemporal equilibrium condition, (34), which shows that any increase in government transfers reduces the discounted present value of primary surpluses, and therefore the market value of debt. Second, the strength of the fiscal response to changes in debt,  $\gamma$ , impacts the value of real debt even if the probability of death is zero. This result follows from the break-down of Ricardian equivalence, but is surprising since the timing of taxes is irrelevant when active monetary/passive fiscal policies are permanent and agents are infinitely lived.

When agents face shorter planning horizons (a higher probability of death), wealth effects from changes in government debt are magnified and the market value of debt is more sensitive to disturbances in government transfers. Monetary policy targets also affect the market value of debt, since both the interest rate and inflation targets impact the value of real government liabilities.

To determine the unique price level, rearrange the government budget constraint, (28), to obtain

$$\hat{P}_t = \hat{B}_{t-1}^M + \rho P^S \hat{P}_t^M + Q[\hat{P}_t^M + \hat{b}_t^M + \tilde{\tau}\hat{\tau}_t - \tilde{Z}\hat{Z}_t]. \quad (38)$$

In general, the sequences of equilibrium prices cannot be solved for analytically, since the long-bond price is dependent on the entire path of future nominal interest rates. However, in the special case where only one-period government debt is issued, the complete trajectory of prices, real debt, and inflation prior to the fiscal limit can be solved for recursively, given  $R_{-1}, B_{-1}^M > 0$ . After the fiscal limit, the economy evolves according to the fixed regime considered in section 3.1.2.

**3.3 SIMULATIONS** It is useful to simulate the equilibrium paths of real debt and inflation to obtain a clearer picture of how the presence of a fiscal limit and intergenerational transfers of wealth affect equilibrium dynamics. Figure 4 isolates the effect of a fiscal limit by comparing the model with permanent passive monetary/active fiscal policy (section 3.1.2) to the model where active monetary/passive fiscal policy holds until a fiscal limit is hit with certainty at date  $T$  and policy permanently switches to passive monetary/active fiscal policy (section 3.2).<sup>18</sup>

Prior to the fiscal limit equilibrium debt and inflation are more volatile when a fiscal limit is present.<sup>19</sup> This follows from the fact that each agent's decisions are governed by long-run policies.

<sup>17</sup>See appendix B.3 for a complete derivation.

<sup>18</sup>The baseline calibration is as follows: The structural parameters are set to  $\beta = .9615$  (4 percent real interest rate) and  $\vartheta = 0.06$  [Leith and Wren-Lewis (2000)]. Prior to the fiscal limit  $\phi = 1.5$  and  $\gamma = 0.15$ . After the fiscal limit is hit,  $\phi = \gamma = 0$ . Steady state values are set to  $\bar{\tau} = 0.19$ ,  $\bar{Z} = 0.17$ ,  $\bar{\pi} = 1.02$ , and  $\bar{b}/\bar{Y} = 0.5$ , which implies  $\mu = 0.0032$ . The parameters of the transfers process are set to  $\rho_Z^S = 0.9$  and  $\sigma_Z = 0.002$ .

<sup>19</sup>The simulations are based on only one realization of the government transfers process, but the qualitative results are not sensitive to the seed. All figures are based on identical realizations of transfers.

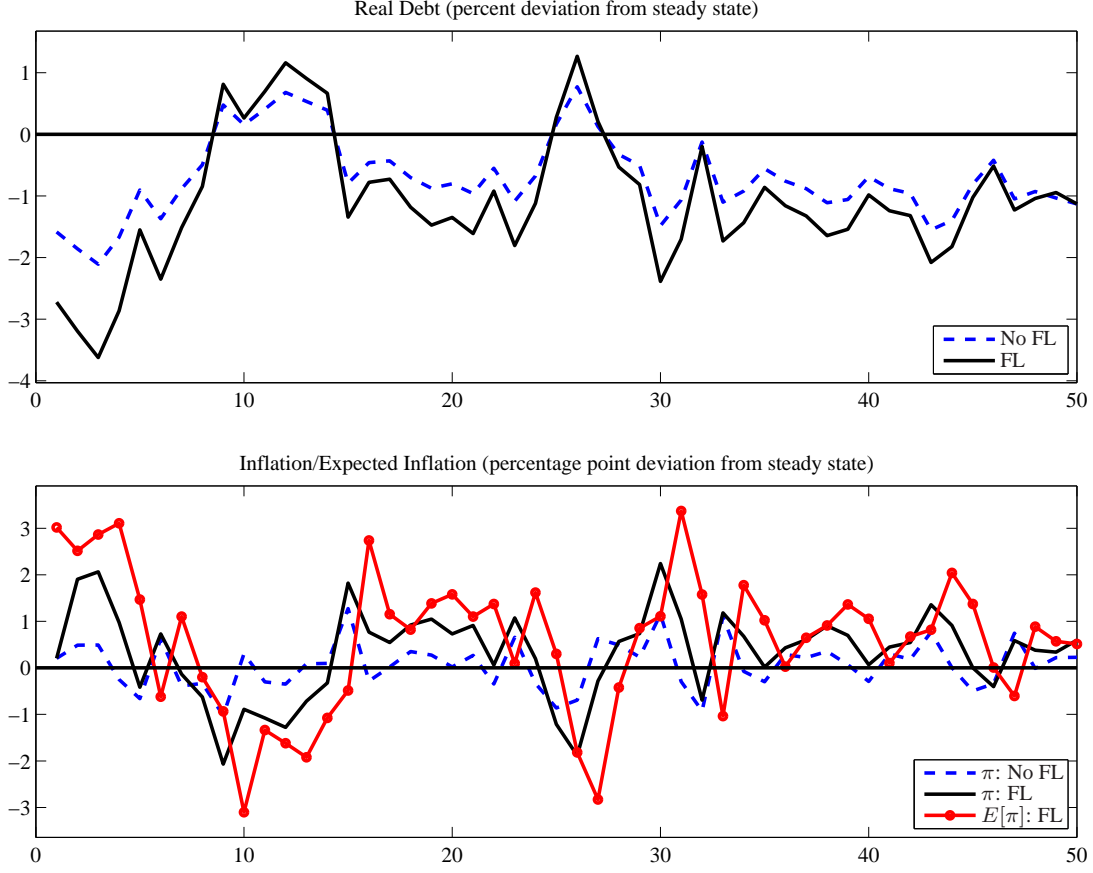


Figure 4: Equilibrium real debt and inflation across two models: one with permanent passive monetary/active fiscal policy (solid lines) and another where active monetary/passive fiscal policy holds until a fiscal limit is hit with certainty at date  $T = 50$  and policy permanently switches to passive monetary/active fiscal policy (dashed lines). The path for expected inflation is marked by circles and corresponds to fiscal limit model. Reported values are based on a particular realization of transfers and are in percent deviations from the deterministic steady state.

In a permanent active monetary/passive fiscal regime, the monetary authority pins down the price level and the fiscal authority adjusts current and future primary surpluses to stabilize government debt. When a fiscal limit is present, the influence of the post-fiscal limit regime, where taxes cannot adjust, prevents the fiscal authority from fully meeting its obligations. Thus, debt deviates widely from target, and since the price level is determined by fluctuations in real debt, inflation also becomes unhinged from target. As the fiscal limit approaches, the discrepancy between the models slowly dissipates, and from time  $T = 50$  onward the equilibrium paths are identical, since forward-looking agents have completely accounted for the anticipated policy adjustment.

Figure 4 also plots expected inflation. Prior to the fiscal limit, expected inflation is given by

$$E_t \hat{\pi}_{t+1} = \phi \hat{\pi}_t - \mu \hat{b}_t. \quad (39)$$

Expected inflation also fluctuates with government debt and is more volatile than realized inflation. When the monetary authority adjusts the nominal interest rate more than one-for-one with inflation ( $\phi > 1$ ), any deviation of realized inflation from target amplifies the deviation of expected inflation from target. Demand-side effects from changes in real debt cause agents to be systematically

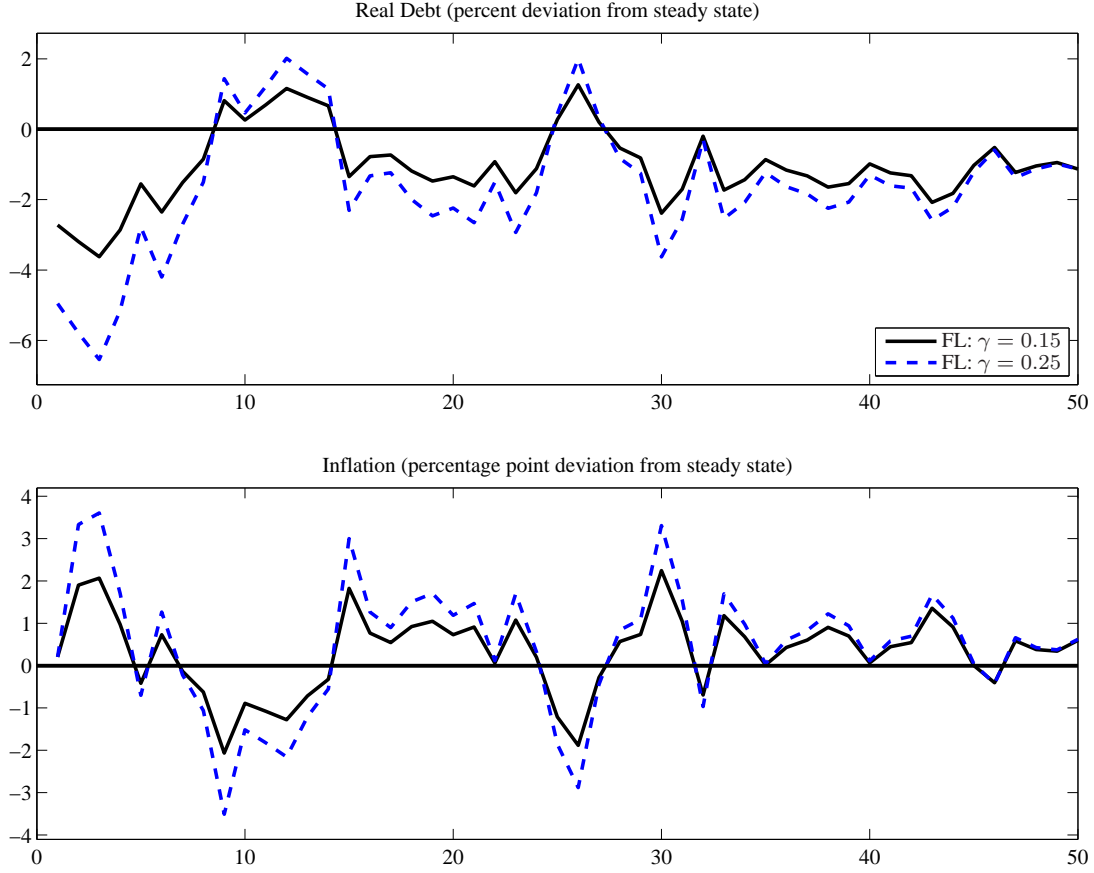


Figure 5: Equilibrium real debt and inflation across two tax policy settings: one where the fiscal authority adjusts taxes *less* aggressively ( $\gamma = 0.15$ ) with real debt and one where the fiscal authority adjusts taxes *more* aggressively ( $\gamma = 0.25$ ) with real debt. All other parameters are identical to the baseline model. Equilibrium paths correspond to the model where the fiscal limit binds at  $T = 50$ . Reported values are based on a particular realization of transfers and are in percent deviations from the deterministic steady state.

incorrect in their inflation forecasts, even though agents are constantly revising their expectations and reacting to changes in realized inflation. Since shorter planning horizons cause agents to demand higher interest rates to hold additional amounts of debt, the presence of finitely lived agents dampens the fluctuations in expected inflation, but this effect is dominated by the fluctuations in realized inflation. At the fiscal limit, the monetary authority stabilizes government debt by pegging the nominal interest rate. At this point, fluctuations in expected inflation are no longer amplified by realized inflation, but still fluctuate (negatively) with real debt.

Figure 5 illustrates the effect of increasing the response of taxes to changes in real debt. In a fixed active monetary/passive fiscal regime, a fiscal authority that more aggressively adjusts taxes with changes in real debt (higher  $\gamma$ ), reduces the volatility of real debt and inflation from transfers shocks. In contrast, when the fiscal authority faces a fiscal limit, (37) shows that a higher  $\gamma$  increases the volatility of real debt, which leads to more volatile realized and expected inflation. The effectiveness of monetary policy is also compromised by the presence of a fiscal limit. In a fixed active monetary/passive fiscal regime, a monetary authority that more aggressively targets inflation (higher  $\phi$ ), helps to stabilize prices more quickly. However, when a fiscal limit is present,

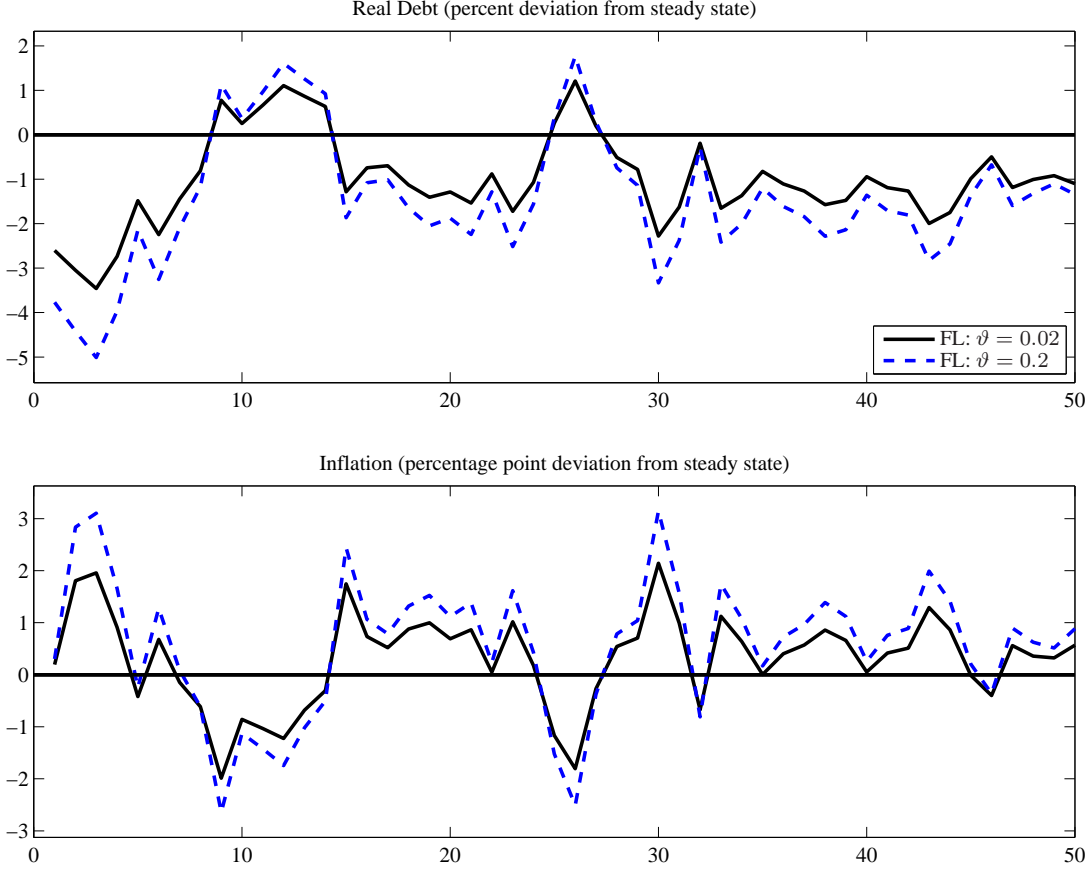


Figure 6: Equilibrium real debt and inflation across two probabilities of death:  $\vartheta = 0.02$  (50 year planning horizon) and  $\vartheta = 0.2$  (5 year planning horizon). All other parameters are identical to the baseline model. Equilibrium paths correspond to the model where the fiscal limit binds at  $T = 50$ . Reported values are based on a particular realization of transfers and are in percent deviations from the deterministic steady state.

more aggressive monetary policy has no effect on real debt or realized inflation and increases the volatility of expected inflation prior to the fiscal limit, as (39) demonstrates. Both of these results are due to the fact that the economy is guided by a passive monetary/active fiscal policy in the long-run. In this regime, prices are not pinned down by the monetary authority and are instead dictated by the fiscal authority. Thus, when the fiscal authority pursues a more passive policy before the fiscal limit, neither authority stabilizes prices, since monetary policy (aside from its targets,  $\pi^*$  and  $R^*$ ) has no influence on the value of debt and inflation.

The probability of death,  $\vartheta$ , pins down agents' planning horizons. Figure 6 illustrates the effect of reducing the planning horizon from fifty ( $\vartheta = 0.02$ ) to five years ( $\vartheta = 0.2$ ). Although this change is extreme, it shows how the volatility of real debt and inflation is impacted by the presence of intergenerational transfers of wealth. When agents' planning horizons are relatively short, any fluctuations from target are stronger and the monetary authority's ability to control current and future inflation is weakened regardless of the presence of a fiscal limit. Increased volatility stems from the wealth effects created by shocks to government transfers. When agents restrict their planning horizons, government liabilities are seen as (net) wealth, and agents require higher real interest rates to induce them to hold additional amounts of government debt. Thus, any increase in

government transfers increases the volatility of debt and inflation. Overall, the results in figures 4-6, which are magnified when accounting for nonlinearities, clearly show that the presence of a fiscal limit and/or intergenerational redistributions of wealth increase the volatility of debt and inflation and compromise the monetary authority's inflation targeting policy.

#### 4 CALIBRATION AND SOLUTION TECHNIQUE

The model laid out in section 2 is calibrated at an annual frequency to characterize the impacts of policy uncertainty over a horizon that extends several decades into the future. The baseline calibration summarized in table 1 is consistent with Rotemberg and Woodford (1997) and Woodford (2003). The steady state markup,  $\mu = \theta/(\theta - 1)$ , is set to 15 percent ( $\theta = 7.666$ ). The annual depreciation rate,  $\delta$ , is set to 10 percent and the cost share of capital,  $\alpha$ , is set to 0.33. Following Sbordone (2002), two-thirds of firms cannot adjust prices each period. Under a quarterly calibration, this implies a costly price adjustment parameter,  $\varphi$ , of approximately 38.<sup>20</sup> Given that prices are roughly 4 times more flexible at an annual frequency,  $\varphi$  is set to 10.

The leisure preference parameter,  $\chi$ , implies a steady state share of time spent working of 0.33, which corresponds to a standard eight hour workday. The transaction services preference parameter,  $\kappa$ , is set so steady state velocity, defined as the ratio of nominal consumption expenditures (less durables) to the M1 money aggregate, corresponds to the average U.S. monetary velocity (1959-2009) of 3.8. The baseline model only includes one-period government debt. When longer-term debt is added, the bond payment parameter,  $\rho$ , corresponds to an average maturity of three years.

The probability of death parameter has many interpretations. Under a strict interpretation, this parameter measures agents' expected lifetime. U.S. life expectancy is roughly 75 years. Restricting attention to the working age population, agents' expected lifetimes are approximately 50 years, which corresponds to a probability of death of 2 percent. Higher values for the probability of death can account for agents being myopic about fiscal policy. Agents may expect to live 50 years but may only consider the next decade in their planning horizon when responding to fiscal policy shocks. Higher values for  $\vartheta$  are also justified to examine how greater deviations from Ricardian equivalence impact equilibrium outcomes. Given these alternative interpretations and this parameter's importance for characterizing how intergenerational transfers of wealth impact equilibrium outcomes, I conduct sensitivity analysis on this parameter. In the baseline model, the probability of death between two consecutive years,  $\vartheta$ , is set to 0.06 as in Leith and Wren-Lewis (2000). Values of  $\vartheta \in \{0.02, 0.1\}$ , which are respectively consistent with average U.S. life expectancy and Freedman et al. (2010), are also considered. I also compare the results from the Perpetual Youth model to the conventional representative agent model where  $\vartheta = 0$ .

The steady state tax rate ensures a debt/output ratio of 0.385, a value consistent with federal U.S. data from 1954-2009. The ratios of government expenditures/output and transfers/output are set to 8 percent and 9 percent, respectively, which matches federal U.S. data over the same period. The steady state gross nominal interest rate,  $\bar{R}$ , and gross inflation rate,  $\bar{\pi}$ , are respectively set to 4 percent and 2 percent. Prior to the fiscal limit, monetary policy is active and tax policy is passive ( $S_P = 1$ ) with parameters  $\phi = 1.5$  and  $\gamma = 0.15$ . In the stationary transfers regime ( $S_Z = 1$ ), government transfers are persistent with an autoregressive coefficient,  $\rho_Z^S$ , of 0.9. The expected duration of the stationary transfers regime,  $1/(1 - p_Z)$ , is set to 5 years, which closely adheres to CBO projections. In the non-stationary transfers regime ( $S_Z = 2$ ), government transfers grow

<sup>20</sup>If  $\omega$  represents the fraction of firms that cannot adjust prices,  $\varphi = \omega(\theta - 1)/[(1 - \omega)(1 - \beta\omega)]$ .

Table 1: Calibration

Baseline Calibration		
Probability of Death	$\vartheta$	0.06*
Price Elasticity of Demand	$\theta$	7.666
Rotemberg Adjustment Cost Coefficient	$\varphi$	10
Capital Depreciation Rate	$\delta$	0.10
Cost Share of Capital	$\alpha$	0.33
Steady state Money Velocity	$V$	3.80
Steady state Gross Inflation Rate	$\bar{\pi}$	0.02
Steady state Gross Nominal Interest Rate	$\bar{R}$	0.04
Steady state Labor	$\bar{N}$	0.33
Steady state Government Spending Share	$\bar{G}/\bar{Y}$	0.08
Steady state Government Transfers Share	$\bar{Z}/\bar{Y}$	0.09
Steady state Debt-to-GDP ratio	$\bar{b}/\bar{Y}$	0.385
Policy Parameters		
Inflation Coefficient: Active MP Rule	$\phi$	1.50
Debt Coefficient: Passive Fiscal Rule	$\gamma$	0.15
Prob. of Moving to PM/AF/AT Regime after FL	$q$	0.50
Initial prob. of the PM/AF/AT Regime after FL	$p_{22}$	0.90
Prob. of staying in the AM/AF/PT Regime after FL	$p_{33}$	0.99
Prob. of Non-stationary Transfers Occurring	$p_Z$	0.20
AR Coefficient: Stationary Transfers Process	$\rho_Z^S$	0.90
Growth Rate: Non-stationary Transfers Process	$\rho_Z^{NS}$	1.01
Standard Deviation of the Transfers Shock	$\sigma_Z$	0.002
Implied Values		
Steady state Tax Rate	$\tau$	0.19953
Tax Rate After Fiscal Limit	$\tau^{FL}$	0.24
Transaction Services Preference Parameter	$\kappa$	0.0101
Leisure Preference Parameter	$\chi$	1.30
Annual Discount Factor	$\beta$	0.9824
Bond Payment Parameter	$\rho$	0 <sup>†</sup>
Logistic Function Slope	$\eta_0$	3.8918
Logistic Function Intercept	$\eta_1$	23.6018

\* Alternative values of the probability of death parameter, given by  $\vartheta = \{0, 0.02, 0.1\}$ , are also considered.

† When a maturity structure is embedded into the model, the bond payment parameter,  $\rho$ , corresponds to an average maturity of three years. Note that the implied parameters change under alternative calibrations.

at 1 percent ( $\rho_Z^{NS} = 1.01$ ), which corresponds to the average projected growth rate of entitlement spending between 2015-2075 [Congressional Budget Office (2011)].

Once government transfers follow an unstable trajectory, the fiscal authority continues to stabilize debt by increasing taxes on capital and labor income while the monetary authority aggressively fights inflation. However, there is a positive probability of hitting the fiscal limit,  $p_{FL}$ , which rises according to the logistic function specified in (19). The parameters of the logistic function are calibrated so that there is a 2 percent chance of hitting the fiscal limit when  $\tau_t = \bar{\tau}$  and a 5 percent chance when  $\tau_t = \tau^{FL}$ . The constant post fiscal-limit tax rate,  $\tau^{FL}$ , is exogenously set in accordance with a steady state debt/output ratio of 2.3. This implies a tax rate and level of debt that are unprecedented in U.S. history and would undoubtedly generate strong political resistance.<sup>21</sup>

With little direction on how Congress might proceed, the potential policy adjustments at the

<sup>21</sup>Rising government deficits and debt are the primary concerns of the Tea Party movement and the source of much of their opposition to current government tax and spending policies (see the July 5, 2010 Gallup poll: “Debt, Government Power Among Tea Party Supporters’ Top Concerns”).



fiscal limit—either debt revaluation or renegeing on government transfers—occur with an equal probability ( $q = 0.5$ ). The transition matrix that governs post-fiscal limit policy is set so the regime where the fiscal authority adjusts policy ( $S_P = 3$ ) has an expected duration of 100 years ( $p_{33} = 0.99$ ,  $p_{32} = 0.01$ ) and the regime where the monetary authority adjusts policy ( $S_P = 2$ ) has an expected duration of 10 years ( $p_{22} = 0.9$ ,  $p_{23} = 0.1$ ). These values reflect the political reality that in the long-run some modifications to entitlement benefits will occur, but because of their politically toxic nature, debt revaluation always remains a possible financing outcome.

**Solution Technique** The prospect of exponential growth in entitlement spending and sudden changes to the policy mix imply that nonlinearity is a crucial component to understanding the distribution of aggregate outcomes that may transpire. Thus, I solve the aggregate *nonlinear* model using policy function iteration based upon the theory of monotone operators, known as the monotone map (MM). The MM has useful theoretical and numerical properties. It was used to prove existence and uniqueness of equilibrium of non-optimal economies by Coleman (1991) and later developed into an algorithm to approximate the solution to models with policy regime switching by Davig (2004). This solution technique discretizes the state space and iteratively solves for updated policy functions that satisfy equilibrium until a specified tolerance criterion is reached. Details of how the algorithm is applied to the model outlined in [section 2](#) are found in [appendix C](#). For additional details and examples of how the algorithm is applied to conventional real business cycle and new Keynesian models see Richter et al. (2011).

## 5 NUMERICAL RESULTS: NO FISCAL LIMIT

To gain insight into the equilibrium dynamics of the Perpetual Youth model, this section first solves the nonlinear model laid out in [section 2](#) *without* a fiscal limit. Conditional on fixed active monetary/passive tax policy and stationary transfers, I show how impulse responses to transfers shocks are impacted by two key adaptations to the model. First, I contrast two methods of fiscal financing—lump-sum taxation and proportional taxation on capital and labor income—to temporarily remove substitution effects and isolate wealth effects from changes in government liabilities. Second, I add longer-term government debt to show how a maturity structure affects the duration and volatility of contractionary periods from government transfers shocks.

**5.1 LUMP-SUM TAXES** [Figure 7](#) displays impulse responses to a 10 percent shock to government transfers, which is consistent with the annual percent change during the Great Recession.<sup>22</sup> Since lump-sum taxes passively respond to changes in government debt, a transfers shock on impact leads to higher expected tax liabilities. When agents are infinitely lived, they fully discount increases in government transfers since they bear the entire burden of higher future taxes. Thus, decision rules are not distorted and Ricardian equivalence holds.

When agents are finitely lived, a positive transfers shock redistributes wealth from future to current generations, since there is a positive probability that living generations will die before taxes come due. This places a higher expected tax burden onto future generations and creates

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<sup>22</sup>Nonlinear impulse responses differ from linear responses. Linear impulse responses report how a shock makes each simulated variable differ from its *deterministic* (calculated) steady state. Nonlinear impulse responses report how a shock makes each simulated path differ from the *stochastic* steady state, defined as a value for the state vector such that  $|\Theta_t - \Theta_{t-1}| < \epsilon$ , where  $\Theta$  is a vector of policy functions and  $\epsilon > 0$  is a tolerance criterion [Gallant et al. (1993)].

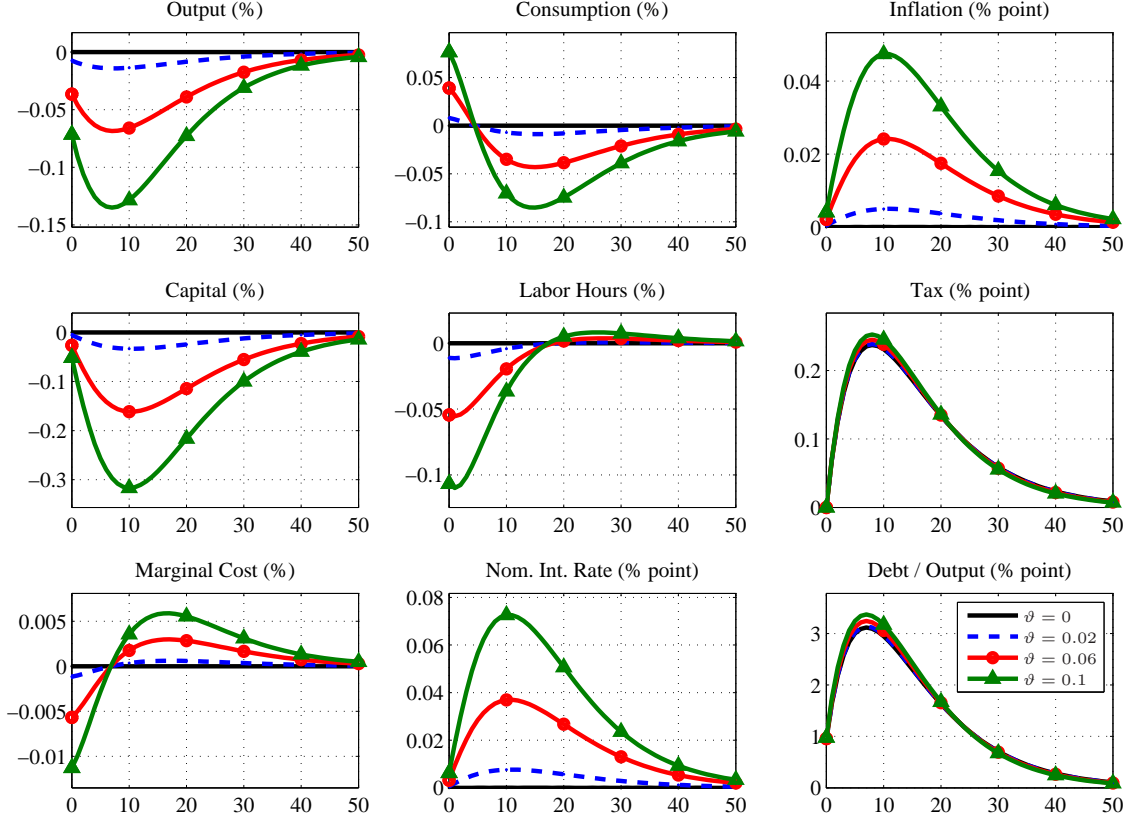


Figure 7: Responses to a 10 percent shock to government transfers conditional on fixed active monetary/passive tax policy and stationary transfers. Policies are financed by *lump-sum* taxes and *one-period* nominal debt. Responses are distinguished by the magnitude of the probability of death. All values represent deviations from the corresponding simulation's stochastic steady state.

an immediate positive wealth effect for current generations. Feeling wealthier, living generations increase consumption and cut back on hours worked on impact. As government debt rises, higher consumption demand crowds out savings and investment in capital, which drives up marginal costs and inflation as price setting firms continually revise their prices. Mounting inflation eventually reduces real government liabilities, causing labor supply to rise and consumption demand to fall.

When agents face a higher probability of death, their expected lifetimes are further misaligned with the government's infinite planning horizon and wealth effects are magnified. The initial positive wealth effects increase consumption demand and further crowd out savings and investment. Inflationary pressures from rising marginal costs mount, and since changes in inflation affect the level of real financial wealth, a higher probability of death imposes more severe stagflation.

As the effect of the transfers shock slowly decays, tax rates and government debt eventually begin to fall. A smaller stock of debt causes private savings to rebound and sends inflation back toward its target rate. Increasingly smaller negative wealth effects from inflation propel consumption, capital, and output back toward their stationary levels. These results confirm the intuition laid out in [section 3.1.1](#)—when agents are finitely lived, shocks to transfers cause the monetary authority to temporarily lose control of inflation even when it is aggressively targeted. Moreover, higher inflation is not associated with higher output, contrary to conventional analysis.

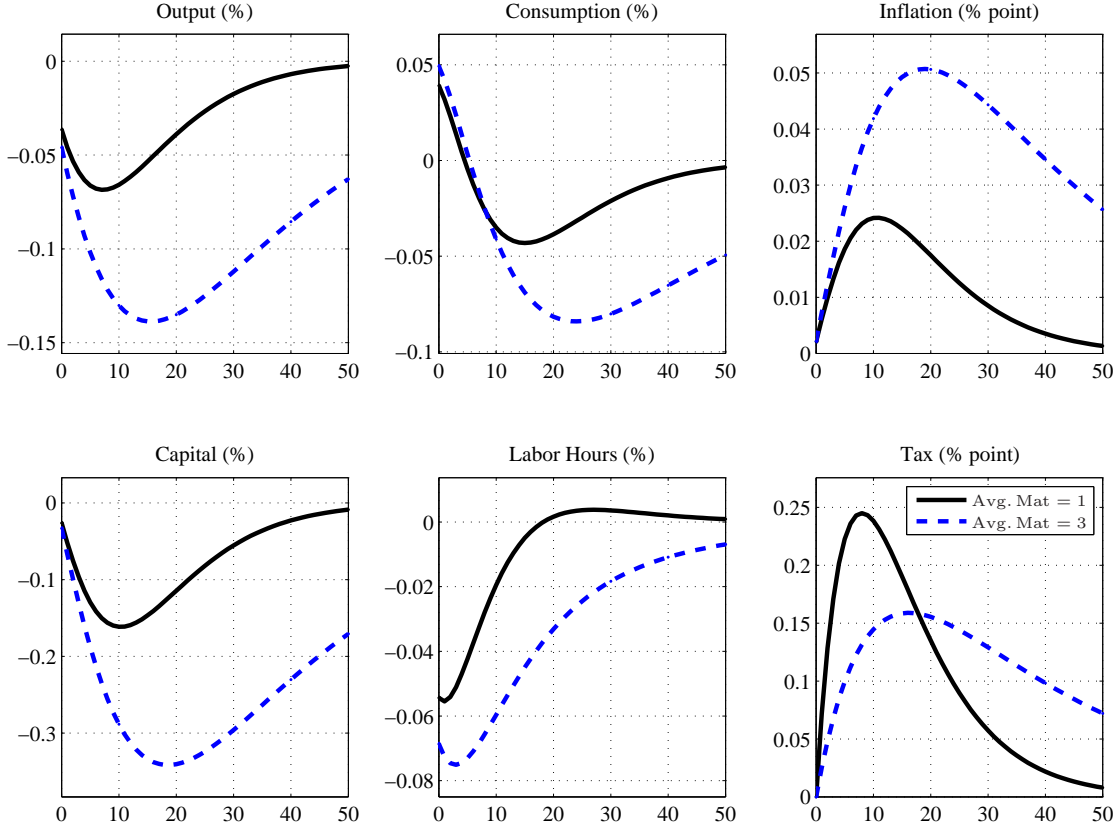


Figure 8: Responses to a 10 percent shock to government transfers conditional on fixed active monetary/passive tax policy and stationary transfers. Policies are financed by *lump-sum* taxes and nominal debt. The probability of death is 6 percent. Responses are distinguished by the average maturity of government debt. All values represent deviations from the corresponding simulation's stochastic steady state.

**Longer-term Government Debt** Longer-term government debt impacts the timing of debt and inflation, which critical for understanding how policy uncertainty affects equilibrium outcomes.

Figure 8 shows the effect of adding a debt maturity structure. When agents are infinitely lived, Ricardian equivalence continues to hold, since the timing of government debt does not impact agents' optimal decision paths when taxes are levied lump-sum. In contrast, when agents are finitely lived, the presence of longer-term government bonds produces higher inflation and a deeper and more persistent contractionary period. As the average maturity of debt is increased, the financing of government liabilities is pushed further into the future. This increases the tax burden for future generations and magnifies the transfer of wealth from future to current generations.

Greater wealth effects lead to larger increases in consumption and further reductions in labor. However, positive short-run effects are quickly erased, as higher aggregate demand and lower aggregate supply crowd out private savings. Marginal costs swell and inflation quickly rises. The central bank responds by sharply increasing nominal interest rates, which drives down long-term bond prices. The loss in net wealth from higher inflation and lower bond prices causes deeper reductions in consumption, capital, and output, and, since real debt remains well above steady state for a protracted period, the contractionary effects are more persistent.

Longer-term government bonds spread the financing of fiscal shocks across future generations.

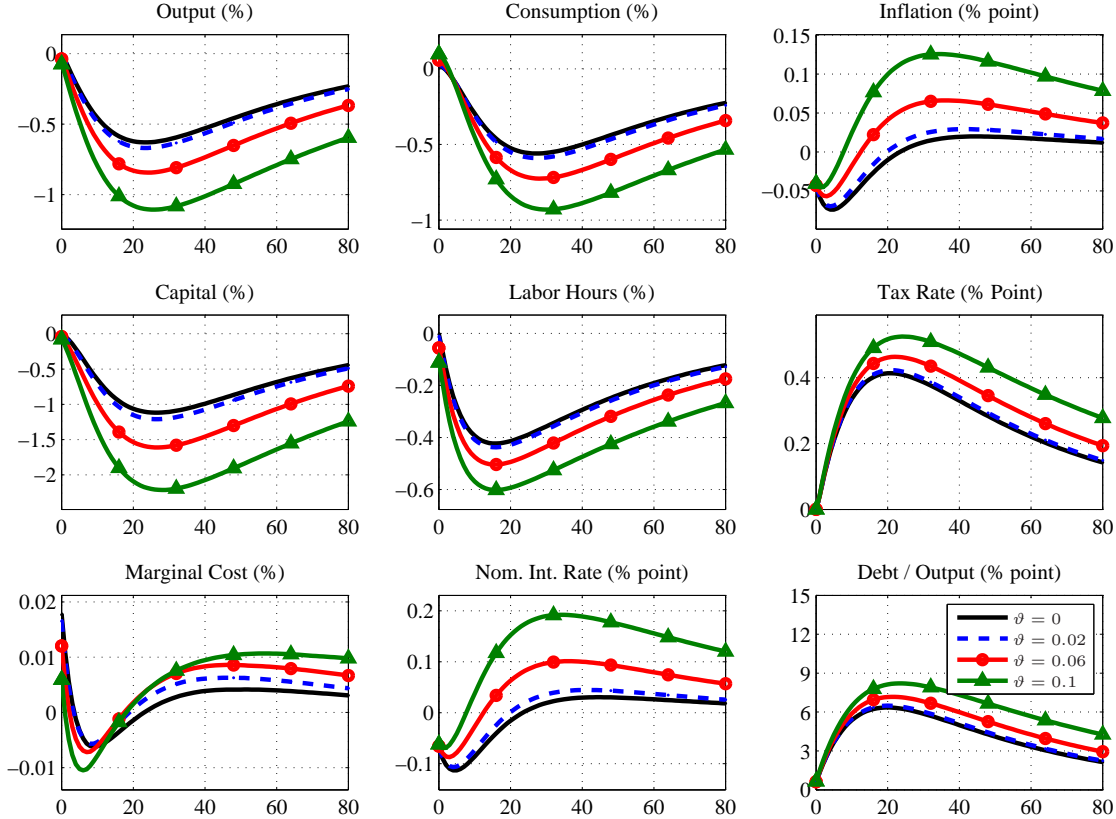


Figure 9: Responses to a 10 percent shock to government transfers conditional on fixed active monetary/passive tax policy and stationary transfers. Policies are financed by *distortionary* taxes (levied against capital and labor income) and *one-period* nominal debt. Responses are distinguished by the magnitude of the probability of death. All values represent deviations from the corresponding simulation's stochastic steady state.

Although debt is less volatile, large wealth effects (which are not captured in a representative agent model) increase macroeconomic volatility and lengthen the period of stagflation. These results under lump-sum taxation illustrate the importance of capturing intergenerational redistributions of wealth and the impact of a more general debt maturity structure.

**5.2 DISTORTIONARY TAXES** Figure 9 shows how proportional taxes levied against capital and labor income affect the responses to a 10 percent shock to government transfers. In contrast to the case where lump-sum taxes finance the increase in transfers, higher expected tax liabilities reduce incentives to work and invest, breaking Ricardian equivalence even if agents are infinitely lived.<sup>23</sup> On impact, this causes agents to substitute out capital in favor of consumption.

When agents face a positive probability of death, each of these effects are magnified, since positive wealth effects from higher real government liabilities increase consumption and reduce labor supply and investment on impact. The reduction in savings reduces the marginal product of labor and lowers the real wage. Since wages constitute two-thirds of firms' costs ( $\alpha = 1/3$ ), the reduction in the real wage initially dominates the rising rental rate of capital. Hence, marginal costs and inflation initially fall.

<sup>23</sup>Note that the responses to a transfers shock with distortionary taxes are almost ten times larger (see output and consumption) than with lump-sum taxes.

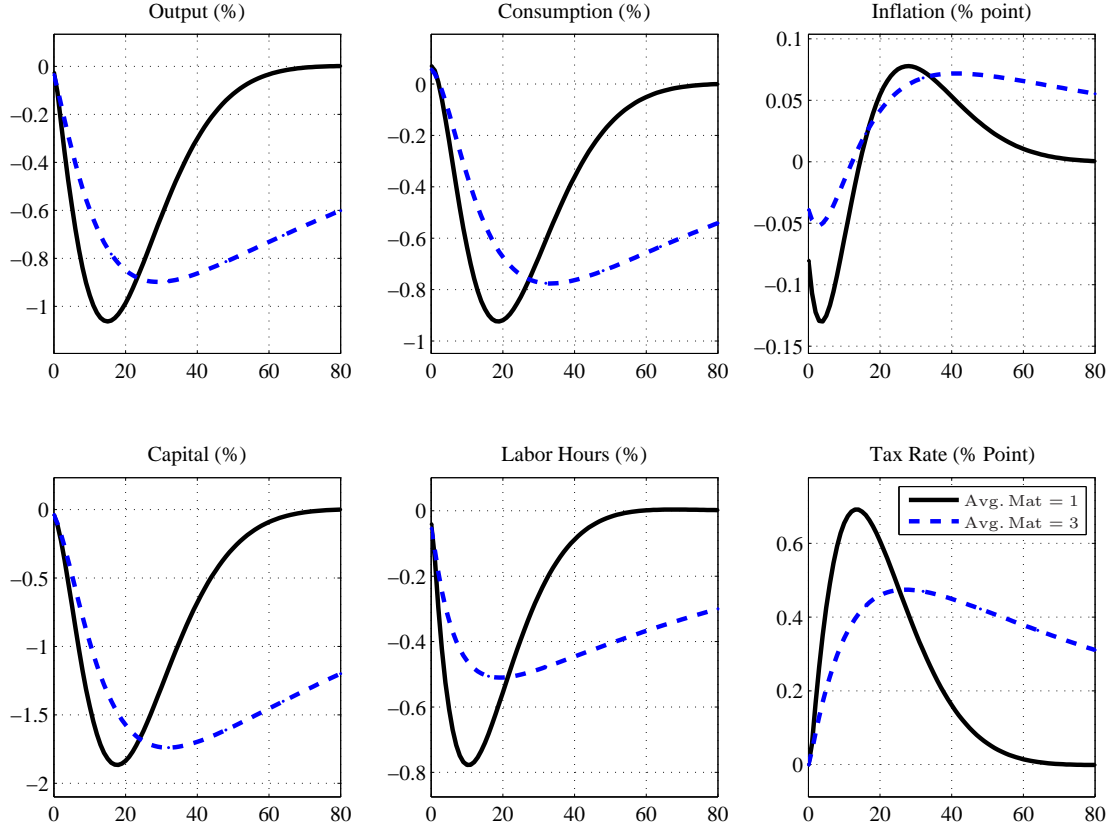


Figure 10: Responses to a 10 percent shock to government transfers conditional on fixed active monetary/passive tax policy and stationary transfers. Policies are financed by *distortionary* taxes (levied against capital and labor income) and nominal debt. The probability of death is 6 percent. Responses are distinguished by the average maturity of government debt. All values represent deviations from the corresponding simulation's stochastic steady state.

Higher distortionary taxes lead to steady reductions in aggregate supply, which eventually drive up marginal costs and inflation. Once again, when agents face a positive probability of death, rising inflation creates a negative wealth effect. As the effect of the transfers shock slowly decays, real debt eventually peaks and starts to fall. This reduces the tax burden and causes labor supply and investment to rise. Moreover, it causes inflation to peak and slowly fall back toward steady state. These results show that even with the presence of substitution effects from distortionary taxes, intergenerational transfers of wealth impact the trajectories of both real and nominal variables. As agents' planning horizons are reduced, the stagflationary period is more severe and the central bank's inflation targeting policy is increasingly compromised.

**Longer-term Government Debt** Figure 10 compares the responses to a 10 percent shock to government transfers with one- and three-period nominal debt.<sup>24</sup> In contrast with the results under lump-sum taxation, the presence of longer-term government debt reduces the volatility of both real and nominal variables. Regardless of the financing mechanism, the presence of longer-term bonds pushes the financing of debt into the future. Once again, this delays tax increases. However, when

<sup>24</sup>Monetary and fiscal policy parameters are held constant across maturity lengths, but the response of taxes to government debt,  $\gamma$ , is increased to 0.40 to guarantee stability. Thus, the results are not comparable to earlier ones.

taxes are proportionally levied against capital and labor income, delayed taxes also imply greater incentives to work and invest, which leads to a smaller reduction in the capital stock and labor supply in the short-run (i.e. the substitution effect dominates in the short-run).

The long-run effect on real variables is more severe. Eventually, taxes come due and the distortionary effects on the capital stock and labor supply lead to a prolonged contractionary period. Even though reduced labor earnings keep aggregate demand low, more persistent reductions in aggregate supply increase marginal costs and keep inflation above target for a longer duration. These results reiterate one of the main points of [section 3](#)—longer-term government debt affects the timing of inflation. With proportional taxes, a longer average maturity of government debt can reduce the volatility of real and nominal variables but at the steep cost of protracted stagflation.

## 6 NUMERICAL RESULTS: FISCAL LIMIT

I now consider a limit to the degree of financing that taxes can provide and solve the complete model laid out in [section 2](#) with a fiscal limit. This model illustrates how agents' expectations alter the aggregate economy by incorporating several layers of policy uncertainty (monetary, tax, transfers). Using counterfactual exercises that condition on a particular monetary/tax/transfers policy regime and Monte Carlo simulations of the model, I show how alternative planning horizons and the maturity of government debt impacts the expectational effects of the fiscal limit and the degree of reneging on government transfers. I also illustrate the consequences of delaying reform by adding the possibility that transfers permanently return to a stable trajectory.

The following results reiterate many of the main points in [section 3.2](#). In that section, however, policy adjustments occur with certainty and their effects are brought into the present, which is inconsistent with current observations. This section, which incorporates several layers of uncertainty, delivers more gradual adjustments and hedging behavior, two commonly seen features.

**6.1 EQUILIBRIUM TRANSITION PATHS** The economy begins in “normal times”, when the monetary authority actively targets inflation and the fiscal authority passively adjusts the tax rate to stabilize debt and fully honor its (stationary) transfers commitments ( $S_P = 1$ ). In period 5, the same policy regime continues to hold, but transfers switch to the non-stationary process ( $S_Z = 2$ ) given in (16). [Figure 11](#) displays counterfactual transition paths, conditional on the initial policy mix and non-stationary transfers remaining in place even after the fiscal limit is hit.<sup>25</sup>

Steadily rising government transfers push real debt and taxes continually higher. Higher proportional tax rates levied against capital and labor income decrease incentives to work and invest, reducing labor supply and savings in capital.<sup>26</sup> When agents are finitely lived, growing debt produces positive wealth effects, but as [figure 9](#) shows, these effects are dominated by the sharp reduction in aggregate supply, which reduces consumption. Although lower aggregate demand

<sup>25</sup>This simulation is different from the impulse response functions shown in [section 5](#). Those simulations produce responses to a one-time shock to government transfers. This simulation is based on a sequence of *policy* regime shocks, where transfers set off on a non-stationary path in period 5 ( $S_Z = 2$ ) and the active monetary/passive tax/active transfers regime ( $S_P = 1$ ) remains in place even after the fiscal limit is hit. Although the initial policy mix is absorbing and no transfers shocks are realized, agents continue to base expectations on the true probability distributions described in [section 2](#). Thus, this exercise highlights how expectational effects alter equilibrium outcomes.

<sup>26</sup>In the baseline model, the tax rates levied against capital and labor income are identical. [Appendix D](#) differentiates between these tax rates and shows how the expectational effects of the fiscal limit are altered when capital and labor taxes hit their respective limits at different dates.



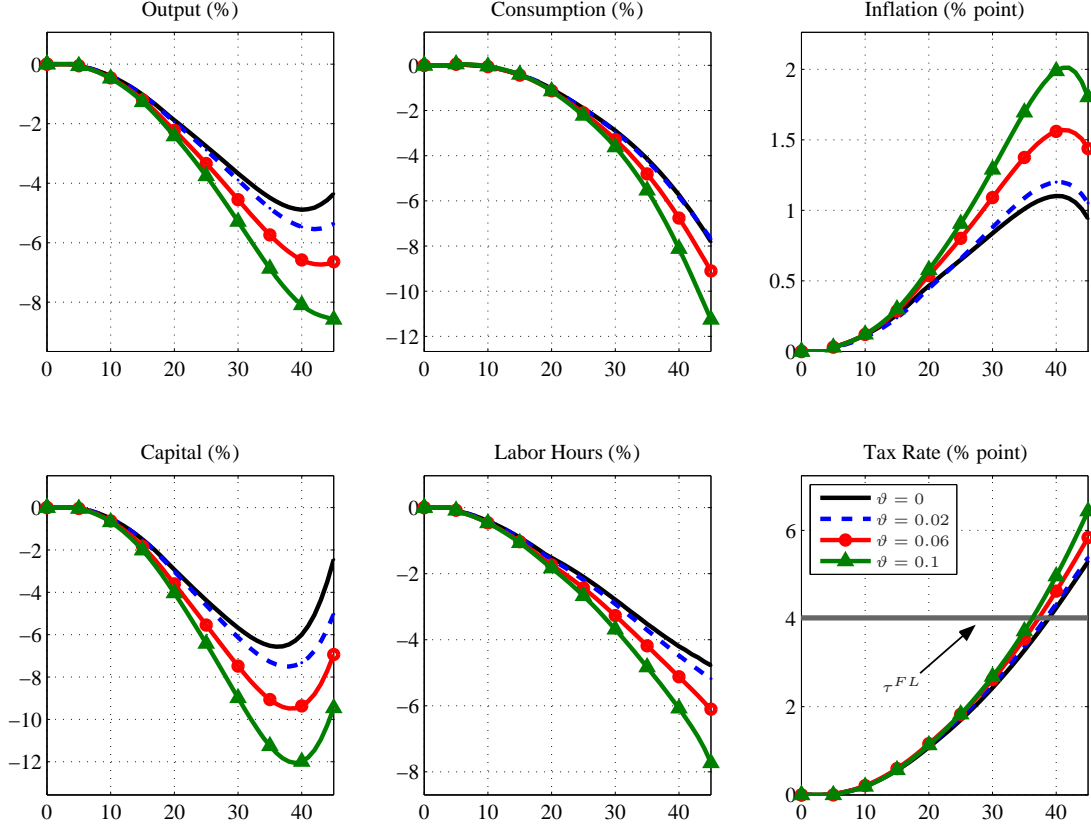


Figure 11: Responses to government transfers switching to a non-stationary path ( $S_Z = 2$ ) in period 5, conditional on active monetary/passive tax/active transfers policy remaining in place even after the fiscal limit is hit. Responses are distinguished by the magnitude of the probability of death. The grey line represents the post-fiscal limit tax rate. Reported values are in deviations from the stochastic steady state.

tends to push inflation downward, higher marginal costs and the expectational effects of moving to a regime where debt is revalued ( $S_P = 2$ ), steadily increase expected and realized inflation.<sup>27</sup>

In the Perpetual Youth model, expectational effects are stronger than in the conventional representative agent model. When agents are finitely lived, steadily rising inflation shifts real wealth from current to future generations, since it decreases real government liabilities and lowers the tax burden of future generations. Faced with a negative wealth effect, current generations further reduce consumption and investment. Feeling poorer, agents would typically work more, but higher debt levels from a smaller tax base force higher taxes, which suppresses incentives to work. Higher marginal costs and a greater likelihood of hitting the fiscal limit, produces higher inflation. Thus, the severity of the stagflationary period rises with the probability of death.<sup>28</sup>

Since taxes continue to respond to the increases in government debt after they surpass the post-fiscal limit tax rate,  $\tau^{FL}$  (gray line), agents face persistently positive innovations in taxes. With the expectation that taxes will stop increasing and remain fixed at  $\tau^{FL}$ , the expected after-tax return on capital and the prospect of reneging rise. Both of these expectational effects increase incentives

<sup>27</sup>The strength of the expectational effects is heavily dependent on the slope and intercept of the logistic function specified in (19). Appendix E conducts sensitivity analysis on these parameters.

<sup>28</sup>Cochrane (2011) also argues that stagflation is a likely outcome of looming fiscal stress.

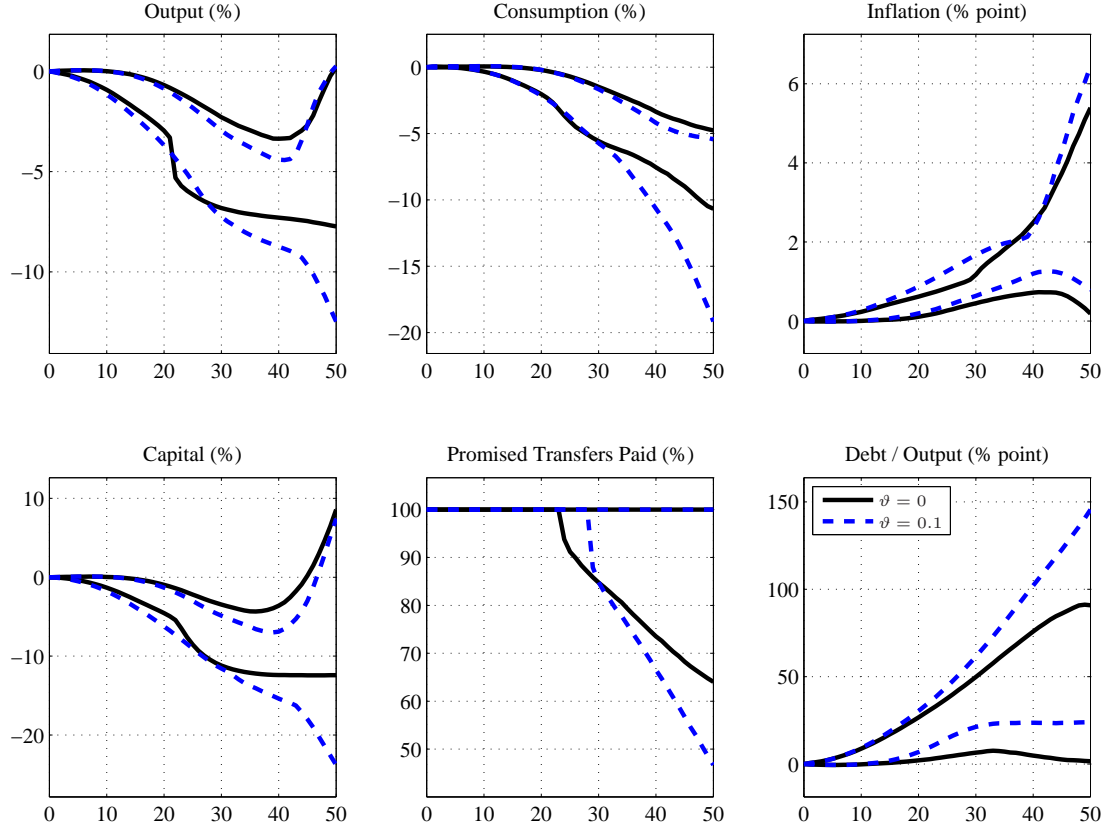


Figure 12: Inner 80th percentile range of time paths based on Monte Carlo simulations of the model. Simulations are distinguished by the magnitude of the probability of death. Reported values are in deviations from the stochastic steady state and based on 20,000 simulations.

to invest and lead to steady increases in the capital stock even before the fiscal limit is reached. In the Perpetual Youth model, these forces are partially offset by the negative wealth effects imposed by higher inflation. Thus, as the probability of death rises, the duration of the contractionary period increases and the expectational effects from lower taxes become operative at a later date. Nevertheless, lower marginal costs eventually dominate the expectational effect of moving to a regime where debt is revalued and inflation falls. Although the tax base expands, falling inflation and growing transfers continue to push real debt higher.

The outcomes in this counterfactual make clear the devastating consequences that long-term fiscal stress can impose on economies. Regardless of the probability of death, output falls and the monetary authority loses control of inflation for over four decades, even though it is aggressively targeted. Finite planning horizons lead to further reductions in output and make it more difficult for the monetary authority to meet its inflation target. When agents make decisions based on a ten year planning horizon ( $\vartheta = 0.1$ ), the total loss in output exceeds 8 percent while, at the same time, inflation rises by over 2 percent. Only during the mid 1970s and early 1980s did the U.S. experience prolonged periods of inflation and large reductions in output. Moreover, in the post-World War II era, declines in output of this magnitude are unprecedented. Even during the Great Recession (2007-2009), output fell by just over 5 percent.

Figure 11 is useful for understanding how fiscal uncertainty impacts equilibrium outcomes.

However, it is based upon policy regimes that take effect at specific dates and is not useful for understanding the *range* of macroeconomic outcomes that are possible under a more diverse set of policy scenarios. To account for the range of possible outcomes and fully characterize equilibrium, I conduct 20,000 Monte Carlo simulations of the model by drawing sequences of regimes and shocks to transfers, starting from the initial policy mix ( $S_P = 1, S_Z = 1$ ).

To highlight the extreme outcomes that are possible in each model, [figure 12](#) plots the 10th and 90th percentile bands of time paths for each variable under the representative agent ( $\vartheta = 0$ ) and Perpetual Youth ( $\vartheta = 0.1$ ) models. For roughly the first decade, the deviation from the stationary distribution is quite small regardless of the probability of death. Given the initially low probability of hitting the fiscal limit, agents expect policy changes to occur far into the future and heavily discount these outcomes. Eventually, an increasing probability of hitting the fiscal limit implies a broad range of outcomes that could include any outcome from a very severe contractionary period with high growth rates of debt and inflation to a very modest contractionary period with low debt and virtually no inflation.<sup>29</sup> These outcomes reflect that the fiscal limit can be hit and government transfers can become non-stationary at any point, or not at all.

Due to the feedback effects between real variables and inflation, a higher probability of death increases the likelihood of a deep contractionary period and rapidly rising debt and inflation. Nevertheless, with agents conditioning on policy adjustments that ensure its stability, the debt/output ratio never climbs to the extreme levels the CBO projects ([figure 1](#)). The presence of intergenerational transfers of wealth does however increase the likelihood of higher degrees of reneging. Although the degree of reneging is heavily dependent on the characteristics and probability distributions of the model, these simulations reaffirm the basic message from [figure 2](#)—there is a strong probability that large reductions in entitlement benefits will be required to stabilize the growth rate of government debt without reform that permanently places transfers on a stationary path.

Aside from passing reform, [section 3.2](#) makes clear that there is little the monetary and fiscal authorities can do to prevent stagflation. A fiscal authority that raises taxes more aggressively in response to rising debt (a higher  $\gamma$ ), causes deeper and more persistent stagflation for two reasons. First, it decreases incentives to work and invest, which leads to sharper reductions in both aggregate supply and demand. Second, it increases the probability of hitting the fiscal limit, which increases the likelihood of debt revaluation, and drives up the growth rate of inflation. The presence of a fiscal limit also ties the hands of the monetary authority. A more aggressive response to inflation (a higher  $\phi$ ) has no impact on the trajectory of government debt. Thus, tax rates rise at the same speed, and the expectational effects of the fiscal limit continue to exert stagflationary pressures.

**6.2 IMPACT OF LONGER-TERM GOVERNMENT DEBT** [Figure 13](#) shows how the transition paths in [figure 11](#) change when a more general portfolio of government debt is added to the baseline model and the monetary authority no longer pegs the nominal interest rate after the fiscal limit.<sup>30</sup>

<sup>29</sup>The large jumps in real variables are caused by the tax rate jumping to its exogenously specified value,  $\tau^{FL}$ , when the fiscal limit is hit. An alternative approach is to specify  $\tau^{FL}$  endogenously so that it equals the prevailing tax rate,  $\tau_{t-1}$ , when the fiscal limit is hit. Making  $\tau^{FL}$  endogenous alleviates these unnatural features, but as [appendix F](#) shows, the qualitative results are similar across these two specifications of  $\tau^{FL}$ .

<sup>30</sup>To accommodate a debt-maturity structure, the bond payment parameter,  $\rho$ , is set so the average maturity of government debt is three years. This changes the calculated steady state of the model. To remain consistent with the baseline calibration,  $\bar{\tau} = 0.22925$  is set to ensure a constant debt/output ratio across maturity lengths and  $\tau^{FL} = 0.27$  remains four percentage points above steady state. Monetary and fiscal policy coefficients are also held constant across maturity lengths. To ensure stability, the response of taxes to government debt,  $\gamma$ , is increased to 0.20. To allow for

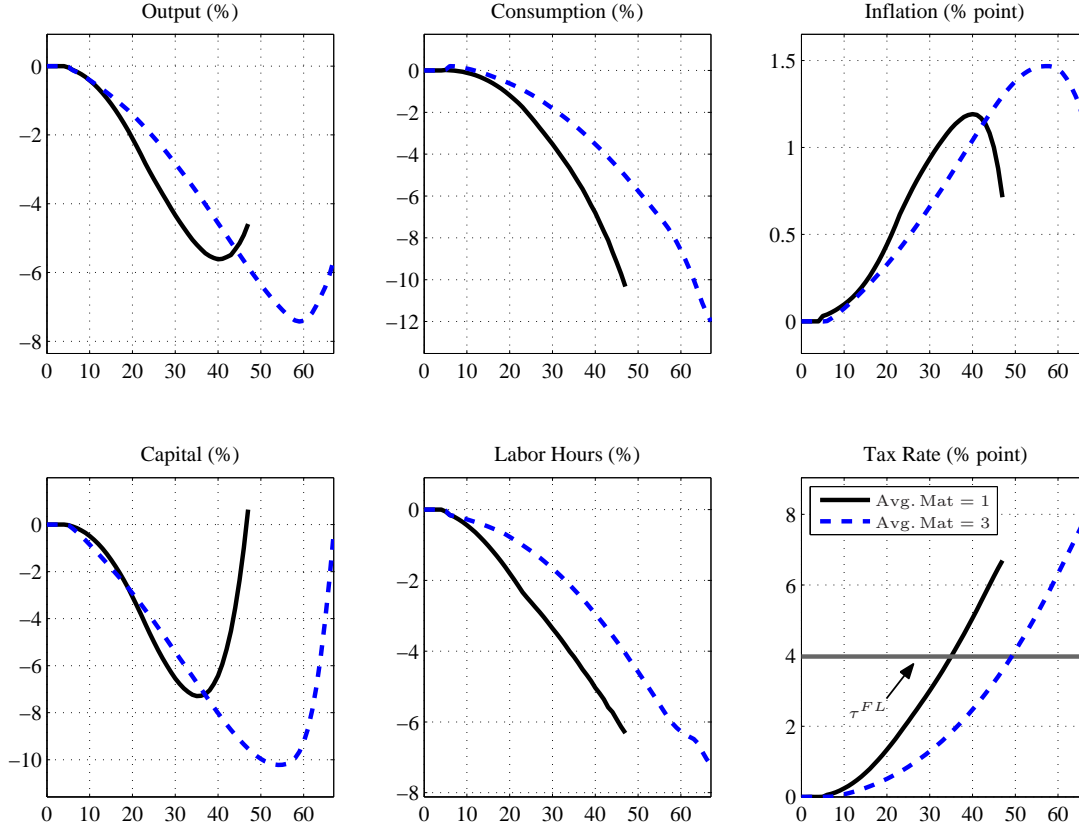


Figure 13: Responses to government transfers switching to a non-stationary path ( $S_Z = 2$ ) in period 5, conditional on active monetary/passive tax/active transfers policy remaining in place even after the fiscal limit is hit. The probability of death,  $\vartheta$ , is set to zero. Responses are distinguished by the average maturity of government debt. Reported values are in deviations from the stochastic steady state.

The presence of longer-term government bonds stretches out the financing of government liabilities over several years. Since taxes are pushed further into the future, agents react to the higher after-tax return on capital by increasing investment. Although this effect is partially offset by steadily falling bond prices and an initially lower probability of reneging, the path of capital is initially higher when the average maturity of government debt is increased to three years. Labor supply unambiguously increases due to a higher after-tax real wage rate and lower real wealth.

Longer-term government debt also pushes the expectational effects of the fiscal limit further into the future, as inflation peaks roughly two decades after its peak in the model with only one-period nominal debt. Although a growing prospect of debt revaluation imposes steadily rising inflationary pressures, the presence of longer-term debt weakens this effect, since changes to the yield curve serve as a shock absorber. Growing transfers are now met with a steadily declining price of the long bond, which increases the slope of the yield curve and pushes debt and inflation into the future. Thus, the fiscal authority is able to respond to growing government debt by in-

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movements in the bond price, the monetary authority no longer pegs the nominal interest rate after the fiscal limit. Instead,  $\phi = 0.2$  so that the nominal interest rate weakly responds to changes in inflation. Finally, to isolate the differences in the equilibrium paths across maturity lengths, the probability of death is temporarily set to zero. All remaining parameters are set to their baseline values.

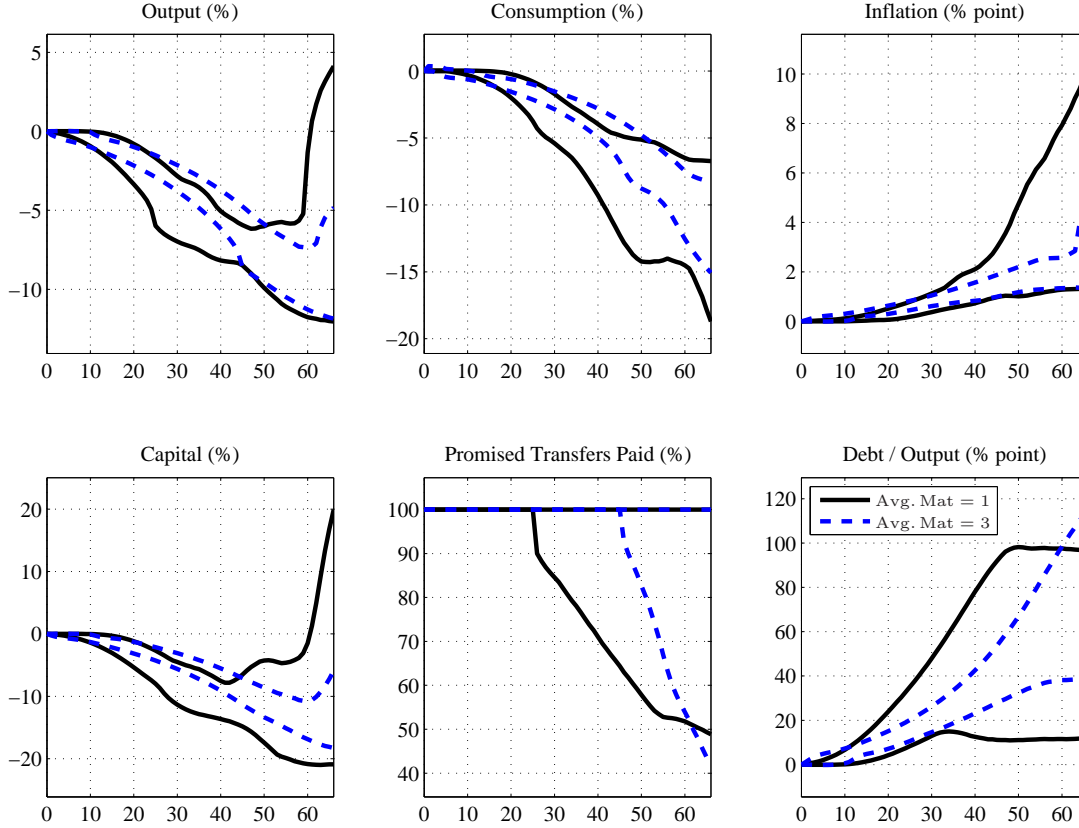


Figure 14: Inner 80th percentile range of time paths based on Monte Carlo simulations of the model. The probability of death is 6 percent. Simulations are distinguished by the average maturity of government debt. Reported values are in deviations from the stationary distribution and based on 20,000 simulations.

creasing taxes for a longer period. Incentives to work and invest steadily erode and cause larger decreases in the capital stock when compared to the model with only short-term government debt. This produces sharp increases in marginal costs, which eventually translate into larger increases in inflation. Figure 13 reiterates one of the main points of section 3—longer-term government debt pushes current inflation into the future, but still leads to severe consequences in the long-run.

To assess the quantitative features of adding a debt maturity structure to the model, I return the probability of death parameter to its baseline value ( $\vartheta = 0.06$ ) and conduct 20,000 Monte Carlo simulations of the model. Figure 14 plots the 10th and 90th percentile bands of time paths for the models with one period nominal debt and a three year average maturity of government debt.

With short-term debt, the 90th percentile shows the fiscal limit is consistently hit 45 years into the future. Even though transfers continue to grow at 1 percent, this flattens the trajectory of debt, since both of the post-fiscal limit regimes stabilize debt. With a 50 percent chance of debt revaluation, the 90th percentile of inflation quickly rises to levels not seen since the 1970s. However, sharper increases in inflation—and many of the dire scenarios the CBO and others project [Kotlikoff and Burns (2005)]—are prevented by reneging on government transfers. Lower expected transfers induce precautionary savings. As agents try to offset the negative wealth effect from reneging by increasing savings on capital, real marginal costs fall, which prevents further inflation.

The presence of longer-term debt reduces the expectational effects of hitting the fiscal limit.

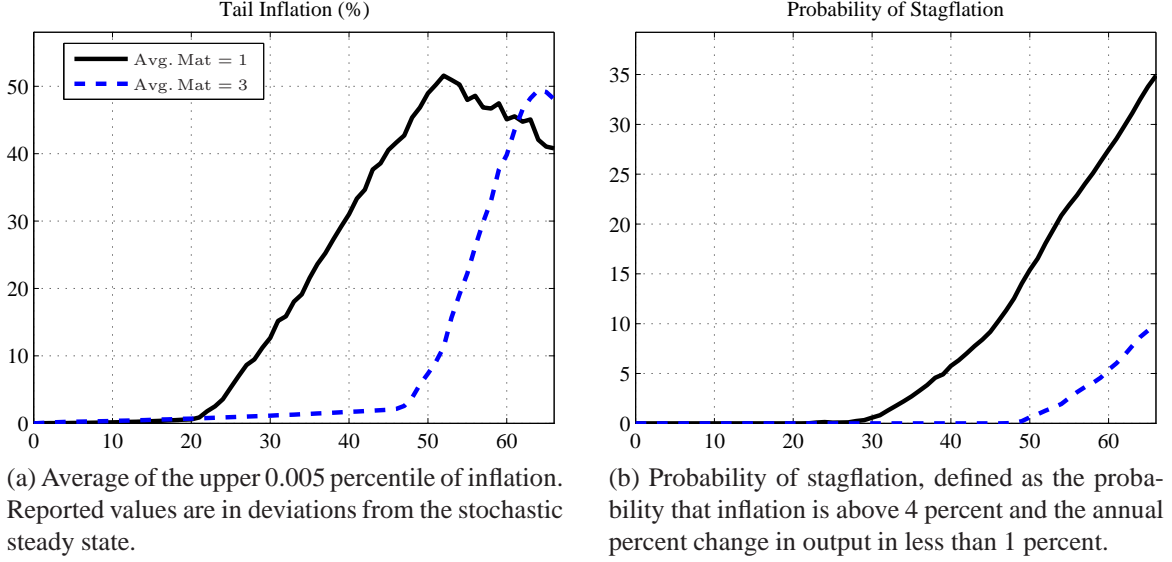


Figure 15: Simulated impact of lengthening the debt maturity structure. The probability of death is 6 percent. Simulations are distinguished by the average maturity of government debt. Reported values are based on 20,000 simulations.

Although rising tax rates immediately produce contractionary outcomes, stronger reductions in real variables do not take effect until 15 years after the model with only short-term debt. Thus, the short-run impact on nominal variables is reduced—over the next fifty years, the 90th percentile of inflation never exceeds 3 percent and real debt remains below World War II levels. Moreover, the likelihood of significant reneging is reduced in the short-run. With 90 percent confidence, the fiscal authority meets at least 80 percent of its transfers commitments fifty years into the future.<sup>31</sup>

The probability of the fiscal limit remains low for several decades, since longer-term debt delays the financing of government liabilities. This keeps the initial policy mix ( $S_P = 1$ ) open longer, but pushes the distribution of debt beyond the levels seen when only short-term debt is present. As debt levels grow, the probability of hitting the fiscal limit rises and inflation mounts, but the upper distributions of inflation and output never reach the extreme levels seen when only one-period government bonds are issued.

To obtain a better sense for the tail risk of inflation across maturity lengths, I follow Davig et al. (2011) and compute the average of the upper 0.005 percentile of inflation. To compute this statistic, order the  $\pi_t^{(n)}$ ,  $n = 1, 2, \dots, N$  realizations of inflation from  $N$  simulations and average the  $N \cdot \mathcal{T}$  outcomes, where  $\mathcal{T}$  is the desired percentile. The conditional tail expectation is given by

$$E[\pi_t | \pi_t > \pi^{\mathcal{T}}] = \frac{1}{N \cdot \mathcal{T}} \sum_{n=1}^N \pi_t^{(n)} \mathcal{I}_{[\mathcal{T}, \infty)},$$

where  $\pi^{\mathcal{T}}$  is the value of inflation corresponding to the  $\mathcal{T}^{\text{th}}$  percentile and  $\mathcal{I}_{[\mathcal{T}, \infty)}$  is an indicator function that signifies a value of inflation greater than the  $\mathcal{T}^{\text{th}}$  percentile.

Figure 15a makes clear that longer-term debt reduces the risk of growing inflation and virtually eliminates the prospect of hyperinflation over the next several decades. When only short-term debt

<sup>31</sup>Bohn (2010) and Cochrane (2011) also contend that lengthening the average maturity structure of government debt can reduce the short-run effects of fiscal stress, but without developing a fully specified model.



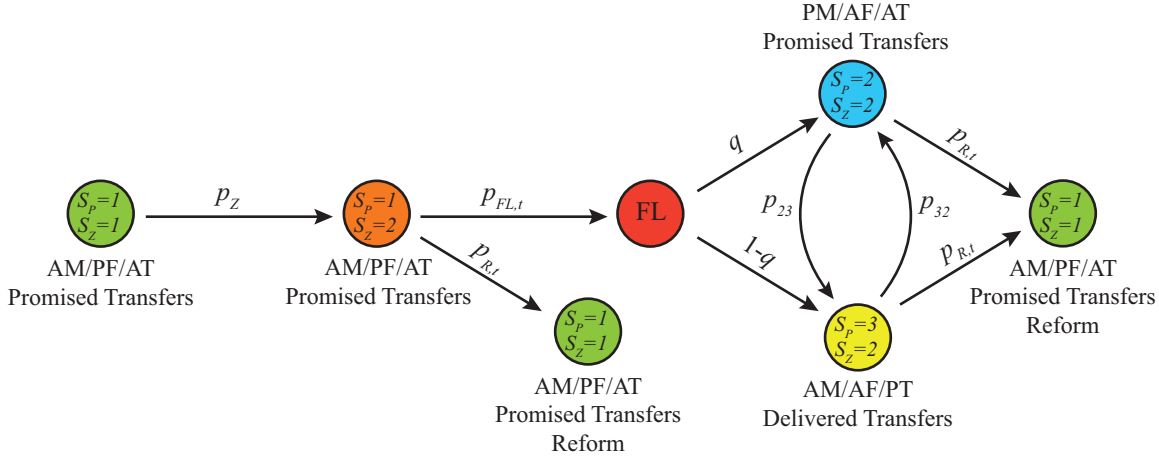


Figure 16: Possible evolution of monetary and fiscal policy regimes with reform

is present, tail outcomes of inflation start to increase in period 20 and surpass 50 percent inflation rates by period 50. In sharp contrast, when the average maturity of debt is increased to three years, inflation only poses a serious risk more than 50 years into the future. When taxes come due and the probability of the fiscal limit rises, tail inflation rises sharply, but *never* surpasses the levels seen when only one-period bonds are issued. Eventually tail inflation peaks, as agents place a higher weight on the fiscal authority reneging on transfers, instead of debt revaluation.

These results confirm that high levels of inflation are tail events that agents discount more heavily as the average maturity of debt rises. Regardless of the maturity structure of debt, discounting helps to suppress current inflation but also serves as a warning signal to policymakers—the short-run consequences of growing transfers may seem insignificant, but if agents’ expectations of inflation become unhinged, the consequences can be quite severe.

The Monte Carlo simulations considered thus far are sometimes misleading because they report percentiles across all simulations instead of looking at the characteristics of each simulation. To illustrate the worst-case scenarios, [figure 15b](#) plots the probability of stagflation across maturity lengths, where stagflation is defined as any outcome where inflation is above 4 percent and the annual percent change in output in less than 1 percent. The main finding of [figures 14](#) and [15a](#) carries over—a longer average maturity of debt reduces the short-run risk of stagflation.<sup>32</sup> However, a new finding also emerges—whereas the probability of stagflation rises in period 30 and surpasses 30 percent by period 60 when only one-period bonds are issued, the probability of stagflation remains below 10 percent when the average maturity of debt is increased to 3 years. This result shows that a longer debt maturity structure not only delays the risk of stagflation, but also prevents heightened long-run risk. Fifteen years after the probability of stagflation starts to rise, there is a 10 percent probability that stagflation occurs, regardless of the average maturity of debt.

**6.3 COSTS OF DELAYING ENTITLEMENT REFORM** Much of the recent fiscal policy debate centers around government spending cuts and how they can be used to help stave off the dire scenarios the CBO projects ([figure 1](#)).<sup>33</sup> The bleak economic outlook facing entitlement programs

<sup>32</sup>The qualitative results of [figure 15b](#) are identical for alternative definitions of stagflation.

<sup>33</sup>For additional discussion on fiscal retrenchment, see Leeper (2010b), Bi et al. (2011), and Corsetti et al. (2010).

(figure 2) makes drastic reform measures such as adopting a single-payer health care system, privatizing Social Security, reducing Social Security benefits (for example via changes to indexing or by raising the retirement age), and amending Medicare and Medicaid reimbursement rates realistic policy outcomes. By extending the baseline model to allow for the possibility of reform—legislation that indefinitely places government transfers back on a sustainable path—this section assesses the economic consequences of delaying comprehensive reform.

Figure 16 illustrates how policy evolves. When government transfers switch to a non-stationary path ( $S_Z = 2$ ), the fiscal authority considers passing reform, hoping to use taxes to only *temporarily* finance the rise in government debt. To capture the increasing political pressure for reform associated with rising debt, the probability of reform is endogenously determined by

$$p_{R,t} = 1 - \frac{\exp(\eta_0^R - \eta_1^R(b_{t-1}^M - (b^M)^*))}{1 + \exp(\eta_0^R - \eta_1^R(b_{t-1}^M - (b^M)^*))}, \quad (40)$$

where  $\eta_0^R$  and  $\eta_1^R > 0$  are the intercept and slope of the logistic function.<sup>34</sup> Entitlement reform places transfers back on a stable trajectory, regardless of whether it is passed before or after the fiscal limit is hit. However, given the economic consequences associated with the fiscal limit, the fiscal authority finds itself in a horse race to pass reform prior to the fiscal limit. When the fiscal limit is hit, either monetary or fiscal policy adjusts as described in section 2, but there remains a positive probability of reform that continues to increase with government debt.<sup>35</sup>

Figure 17 illustrates the economic consequences of delaying entitlement reform by 10, 20, and 30 years after government transfers become non-stationary, conditional on the initial policy mix ( $S_P = 1$ ) holding even after the fiscal is hit. Once government transfers switch to an explosive trajectory in period 5, stagflationary outcomes quickly ensue, and each year Congress fails to pass entitlement reform, the economic situation steadily deteriorates.

Regardless of when entitlement reform is passed, accumulated debt service obligations and a persistently high probability of hitting the fiscal limit continues to drive up debt and inflation even after the date of reform. As taxes respond, incentives to work and invest further deteriorate. Eventually, transfers decay to a level that is sufficient for current tax policy to reduce government debt and propel output and inflation back to their stationary levels, but these results serve as another warning sign to policymakers—the number of years it takes the economy to rebound from a period of growing transfers increases exponentially with the number of years it takes to pass reform.

## 7 CONCLUSION

As the CBO makes clear, the U.S. is entering a period of heightened fiscal uncertainty. With little or no indication from policymakers about how future policy will adjust, this paper explores alternative scenarios for the evolution of policy while taking seriously the reality that there exists a limit—political or economic—to the revenues that can be generated from taxes. The possibility of hitting the fiscal limit has always existed, but with Tea Party resistance to higher taxes and explosive entitlement projections, this outcome is becoming increasingly relevant.

Recent work has been aimed at understanding the macroeconomic implications of this uncertainty but within the strict confines of a representative agent model, which is unable to account

<sup>34</sup>The logistic function parameters,  $\eta_0^R$  and  $\eta_1^R$  are calibrated so that there is a 4 percent chance of reform in the stationary equilibrium and a 20 percent chance of reform when the fiscal limit is hit.

<sup>35</sup>Passing reform is different than reneging on government transfers because it changes agents' expectations.

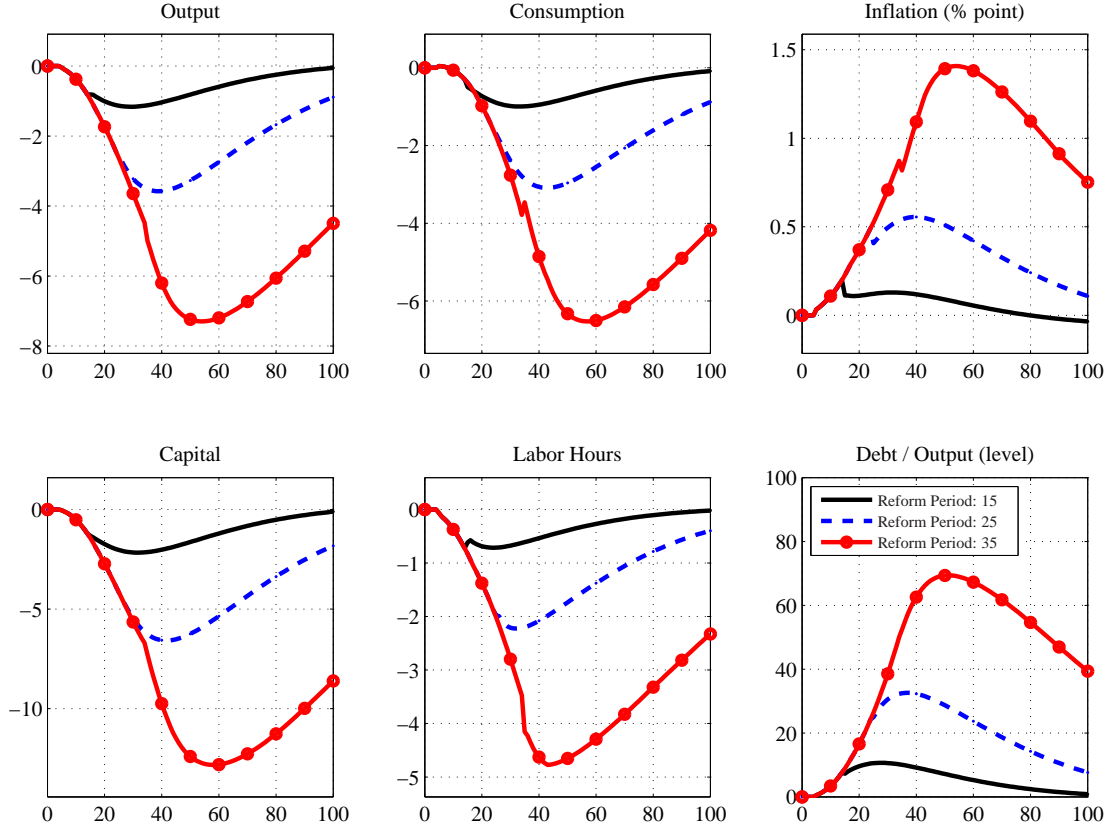


Figure 17: Impact of passing entitlement reform 10 (solid line), 20 (dashed line), and 30 (circles) year after government transfers switch to a non-stationary process (period 5), conditional on conditional on active monetary/passive tax/active transfers policy remaining in place even after the fiscal limit is hit. The probability of death is 6 percent. Reported values are in deviations from the stochastic steady state.

for key intergenerational redistributions of wealth. This paper introduces a fiscal limit into a Perpetual Youth model to assess the impact of intergenerational transfers of wealth on equilibrium outcomes. Another critical component that is commonly left out of most policy analysis is a debt maturity structure. This paper examines how the presence of longer-term government debt impacts the expectational effects of the fiscal limit. Finally, this paper investigates the impact of delaying entitlement reform—legislation that indefinitely places government transfers back on a stable trajectory and removes the probability of hitting the fiscal limit. Four key findings emerge:

1. When government liabilities are seen by agents as net wealth, the expectational effects of policy uncertainty are substantially stronger. As a consequence, growing government transfers impose a deeper and more persistent contractionary period, which produces heightened inflation risk and further hinders the central bank's inflation targeting policy. Although current levels of inflation remain low, the extreme tails of the distribution show that higher levels of inflation can quickly strike as agents' expectations adjust to rising government debt. The dire scenarios the CBO and others project never transpire, but these results still serve as a warning sign for policymakers—without comprehensive reform that ensures the sustainability of government entitlement programs, the central bank's ability to combat inflation becomes increasingly difficult, the risk of a painful and protracted recession rises, and sub-

stantial reductions in entitlement benefits become increasingly likely.

2. The presence of longer-term government debt drastically reduces the short/medium-run impacts of policy uncertainty. For the next fifty years, inflation only poses mild risk, even in the upper-tail of its distribution. Moreover, contractionary outcomes are less likely and much less severe. These results suggest that the fiscal authority can temporarily reduce the aggregate effects of fiscal stress by increasing the average maturity of government debt. Such a policy will buy policymakers additional time to resolve the looming fiscal crisis, but it must be approached with extreme caution. Without reform, the underlying problem persists and the long-run risk of stagflation steadily mounts.
3. Explosive government transfers bring economies toward the fiscal limit and force agents to condition on a broad set of possible outcomes. When only one period nominal debt is included in the model, fiscal uncertainty produces *immediate* and steadily rising inflation. This, however, is inconsistent with current inflation expectations, which remain stable and low. Many economists contend that large fiscal imbalances and growing debt imply looming inflation [Feldstein (2009); Ferguson (2009)]. The presence of longer-term government debt delays the expectational effects of the fiscal limit and reconciles these points.
4. If entitlement reform is passed well before the fiscal limit is hit, policymakers can drastically reduce the severity and duration of the stagflationary period caused by exponentially rising government transfers obligations. If, however, reform is delayed, the economic consequences of hitting the fiscal limit become stark, as the monetary authority loses control of inflation and contractionary outcomes persist for several decades after reform passes. Thus, economic outcomes may steadily improve shortly after reform passes, but the consequences of delaying reform may hinder economic performance well into the future.

## APPENDIX A DETAILS ABOUT FIGURES 1 AND 2

Figure 1a plots the actual and projected debt-to-GDP ratio from 1900-2035 [Congressional Budget Office (2011)].<sup>36</sup> CBO projections are based on two scenarios—the Alternative Fiscal Financing (AF) and Extended-Baseline scenario (EB)—that reflect different assumptions about future federal government revenues and spending. The EB projection (dashed line) assumes current law will remain in effect. This means that in the EB scenario, the Bush Tax cuts of 2001 and 2003, which were recently extended in 2010, will sunset, the alternative minimum tax (AMT) tax base will continue to expand, and the tax provisions of the recent health care act (HR 3590) will take effect. Each of these policies projects higher revenues, which offset much of the growth in entitlement spending and keep the growth rate of debt relatively low. The AF projection (dash-dotted line) assumes that routine adjustments to current law will continue to be enacted in the future. Some of the adjustments include: (1) All tax provisions currently set to expire will be extended through 2021, including provisions related to the AMT; (2) Medicare’s reimbursement rates for physicians will continue to grow at the same rate as the Medicare Economic Index; (3) Smaller decreases in discretionary spending. These assumptions contribute to a much bleaker budgetary outlook and portend an unsustainable path for the growth rate of government debt.

Figure 1b gives a breakdown of historical and projected non-interest federal government spending as a percentage of GDP. According to the EB scenario, total spending on entitlement programs will rise from 10.4 to 15.5 percent of GDP by 2035. At the same time, other non-interest spending will fall from 12.2 to 7.8 percent of GDP. The AF scenario paints an even more grim picture—entitlement spending rises further to 16.4 percent of GDP, while other non-interest spending only falls to 8.5 percent of GDP. The AF scenario also projects a much different path for revenues. In the EB scenario revenue increase to 23.2 percent of GDP by 2035, but in the AF scenario, revenues remain constant at about 18 percent of GDP.

Figure 2a plots the actual and projected trust fund ratio, defined as assets as a percentage of annual expenditures, from 1970-2040 [Social Security Administration (2011)]. SSA projections paint a bleak outlook for the short-run solvency of government entitlement programs. When the trust fund ratio falls below 100 percent, projected costs exceed income, but entitlement programs would still be able to pay out full benefits until the trust fund ratio falls to zero. The disability insurance (DI) trust fund ratio falls below 100 percent in 2013 and assets are completely exhausted by 2018. The outlook for the Hospital Insurance (HI, Medicare Part A) trust fund is also bleak. The trust fund ratio falls below 100 percent by 2012 and assets are fully depleted by 2024. The Old Age Survivors Insurance (OASI) trust fund is in better shape—the trust fund ratio remains above 100 percent until 2035 and assets are not exhausted until 2039.

Under the EB scenario, figure 2b shows how the effects of aging and excess cost growth contribute to the projected growth in entitlement spending as a percentage of GDP. Since 1975, Medicare costs have grown at an average of 2.3 percentage points faster than per capita GDP. The CBO defines this difference as *excess* cost growth. Over the next 25 years, the CBO projects the effect of aging will account for 64 percent of the growth in entitlement expenditures. By 2085, the effect of cost growth becomes the dominant factor, explaining 71 percent of the spending growth.

<sup>36</sup>As of the 2011 Long Term Budget Outlook, the CBO only publishes projections 25 years into the future due to the high degree of uncertainty surrounding fiscal policy. These projections were made prior to the Budget Control Act of 2011.

## APPENDIX B DERIVATIONS

**B.1 INDIVIDUAL LAW OF MOTION FOR CONSUMPTION** To derive the individual consumption function, note that the first order conditions are given by

$$\frac{m_{s,t}}{P_t} = \kappa \frac{R_t}{R_t - 1} c_{s,t}, \quad (\text{A1})$$

$$\frac{w_{s,t}}{P_t} = \frac{\chi}{1 - \tau_{s,t}} \frac{c_{s,t}}{1 - n_{s,t}}, \quad (\text{A2})$$

$$1 = E_t \left\{ q_{t,t+1}(s) \frac{P_t}{P_{t+1}} R_t \right\}, \quad (\text{A3})$$

$$1 = E_t \left\{ q_{t,t+1}(s) \left[ (1 - \tau_{s,t+1}) \frac{R_{t+1}^k}{P_{t+1}} + 1 - \delta \right] \right\}, \quad (\text{A4})$$

$$1 = E_t \left\{ q_{t,t+1}(s) \frac{1 + \rho P_{t+1}^M}{P_t^M} \frac{P_t}{P_{t+1}} \right\}. \quad (\text{A5})$$

We can use (A3)-(A5) to obtain the following identity:

$$\frac{P_t^S b_{s,t}^S}{P_t} + \frac{P_t^M b_{s,t}^M}{P_t} + k_{s,t} = E_t \left\{ q_{t,t+1}(s) \left[ \frac{P_t}{P_{t+1}} R_t k_{s,t} + \frac{b_{s,t}^S}{P_{t+1}} + \frac{(1 + \rho P_{t+1}^M) b_{s,t}^M}{P_{t+1}} \right] \right\}.$$

Given (A1), further manipulations then imply

$$\begin{aligned} & \frac{P_t^S b_{s,t}^S}{P_t} + \frac{P_t^M b_{s,t}^M}{P_t} + \frac{m_{s,t}}{P_t} + k_{s,t} \\ &= E_t \left\{ q_{t,t+1}(s) \left[ \frac{P_t}{P_{t+1}} R_t k_{s,t} + \frac{m_{s,t} + b_{s,t}^S}{P_{t+1}} + \frac{(1 + \rho P_{t+1}^M) b_{s,t}^M}{P_{t+1}} \right] \right\} + \frac{R_t - 1}{R_t} \frac{m_{s,t}}{P_t} \\ &= E_t \left\{ q_{t,t+1}(s) \left[ \left( (1 - \tau_{s,t+1}) \frac{R_{t+1}^k}{P_{t+1}} + 1 - \delta \right) k_{s,t} + \frac{m_{s,t} + b_{s,t}^S}{P_{t+1}} + \frac{(1 + \rho P_{t+1}^M) b_{s,t}^M}{P_{t+1}} \right] \right\} + \frac{R_t - 1}{R_t} \frac{m_{s,t}}{P_t} \\ &= E_t \{ q_{t,t+1}(s) a_{s,t+1} \} + \frac{R_t - 1}{R_t} \frac{m_{s,t}}{P_t}. \end{aligned}$$

Thus, the budget constraint can be rewritten as

$$c_{s,t} + E_t \{ q_{t,t+1}(s) a_{s,t+1} \} + \frac{R_t - 1}{R_t} \frac{m_{s,t}}{P_t} = \omega_{s,t} + (1 - \vartheta)^{-1} a_{s,t}.$$

Note that (A1) implies

$$c_{s,t} + \frac{R_t - 1}{R_t} \frac{m_{s,t}}{P_t} = (1 + \kappa) c_{s,t}. \quad (\text{A6})$$

Plugging (A6) into the budget constraint then yields

$$a_{s,t} = (1 - \vartheta) E_t \{ q_{t,t+1}(s) a_{s,t+1} \} + (1 - \vartheta) (1 + \kappa) c_{s,t} - (1 - \vartheta) \omega_{s,t},$$

which can be solved forward to obtain

$$a_{s,t} = (1 - \vartheta)^{T-t} E_t \{ q_{t,T}(s) a_{s,T} \} + (1 - \vartheta) \sum_{k=0}^{T-t-1} (1 - \vartheta)^k E_t \{ q_{t,T}(s) [(1 + \kappa) c_{s,T} - \omega_{s,T}] \}.$$



Applying limits, imposing the transversality condition, (5), and re-indexing then implies

$$a_{s,t} = (1 - \vartheta) \sum_{T=t}^{\infty} (1 - \vartheta)^{T-t} E_t \{ q_{t,T}(s) [(1 + \kappa) c_{s,T} - \omega_{s,T}] \}. \quad (\text{A7})$$

Using the fact that  $\beta^{T-t} c_{s,t} = q_{t,T}(s) c_{s,T}$ , (A7) reduces to

$$c_{s,t} = \frac{1 - \beta(1 - \vartheta)}{1 + \kappa} \left[ \frac{a_{s,t}}{1 - \vartheta} + \sum_{T=t}^{\infty} (1 - \vartheta)^{T-t} E_t \{ q_{t,T}(s) \omega_{s,T} \} \right], \quad (\text{A8})$$

which is identical to (6) in the main text.

**B.2 AGGREGATE LAW OF MOTION FOR CONSUMPTION** Equation (6) in the main text implies

$$C_t = \xi \left[ A_t + \sum_{T=t}^{\infty} (1 - \vartheta)^{T-t} E_t \{ Q_{t,T} \Omega_T \} \right]. \quad (\text{A9})$$

Advancing (A9) one period and multiplying by  $(1 - \vartheta) Q_{t,t+1}$  gives

$$(1 - \vartheta) E_t \{ Q_{t,t+1} C_{t+1} \} = \xi \left[ (1 - \vartheta) E_t \{ Q_{t,t+1} A_{t+1} \} + E_t \sum_{T=t+1}^{\infty} (1 - \vartheta)^{T-t} \{ Q_{t,T} \Omega_T \} \right]. \quad (\text{A10})$$

Following a procedure similar to the individual case, there is a unique aggregate SDF that yields the following intertemporal relationship for a portfolio with random return  $A_{t+1}$  at time  $t + 1$ :

$$\frac{P_t^S B_t^S}{P_t} + \frac{P_t^M B_t^M}{P_t} + \frac{M_t}{P_t} + K_t = E_t [Q_{t,t+1} A_{t+1}] + \frac{R_t - 1}{R_t} \frac{M_t}{P_t}.$$

Thus, the aggregate budget constraint can be written as

$$C_t + E_t \{ Q_{t,t+1} A_{t+1} \} + \frac{R_t - 1}{R_t} \frac{M_t}{P_t} = \Omega_t + A_t. \quad (\text{A11})$$

Imposing the aggregate counterpart of (A1), (A11) is given by

$$(1 + \kappa) C_t + E_t \{ Q_{t,t+1} A_{t+1} \} = \Omega_t + A_t,$$

which can be substituted into (A9) to obtain

$$C_t = \xi \left[ (1 + \kappa) C_t + E_t \{ Q_{t,t+1} A_{t+1} \} + \sum_{T=t+1}^{\infty} (1 - \vartheta)^{T-t} E_t \{ Q_{t,T} \Omega_T \} \right]. \quad (\text{A12})$$

Combining (A10) and (A12) implies

$$\begin{aligned} C_t &= \xi [(1 + \kappa) C_t + E_t \{ Q_{t,t+1} A_{t+1} \}] + (1 - \vartheta) E_t \{ Q_{t,t+1} C_{t+1} \} - \xi (1 - \vartheta) E_t \{ Q_{t,t+1} A_{t+1} \} \\ &= \xi [(1 + \kappa) C_t + \vartheta E_t \{ Q_{t,t+1} A_{t+1} \}] + (1 - \vartheta) E_t \{ Q_{t,t+1} C_{t+1} \}. \end{aligned}$$

Solving for aggregate consumption then yields (10) in the main text.

**B.3 EQUILIBRIUM DEBT UNDER A FISCAL LIMIT** To solve for the equilibrium market value of debt, first use the intertemporal equilibrium condition, (34), at  $t = 0$  and the pre-fiscal limit tax rule, (30), to write the initial market value of debt as

$$\begin{aligned}\hat{P}_0^M + \hat{b}_0^M &= \tilde{\beta} E_0[\tilde{\tau} \hat{\tau}_1 - \tilde{Z} \hat{Z}_1] + \tilde{\beta} \sum_{k=1}^{\infty} \tilde{\beta}^k E_0[\tilde{\tau} \hat{\tau}_{1+k} - \tilde{Z} \hat{Z}_{1+k}] \\ &= \tilde{\beta} E_0[\tilde{\tau} \hat{\tau}_1 - \tilde{Z} \hat{Z}_1] + \tilde{\beta} E_0[\hat{P}_1^M + \hat{b}_1^M] \\ &= \tilde{\beta}[\gamma \tilde{\tau}(\hat{P}_0^M + \hat{b}_0^M) - \tilde{Z} E_0 \hat{Z}_1] + \tilde{\beta} E_0[\hat{P}_1^M + \hat{b}_1^M].\end{aligned}$$

Then solve for  $\hat{P}_0^M + \hat{b}_0^M$  and iterate forward to obtain

$$\hat{P}_0^M + \hat{b}_0^M = \left( \frac{\tilde{\beta}}{1 - \tilde{\beta} \tilde{\tau} \gamma} \right)^{T-1} E_0[\hat{P}_{T-1}^M + \hat{b}_{T-1}^M] - \tilde{Z} E_0 \sum_{k=1}^{T-1} \left( \frac{\tilde{\beta}}{1 - \tilde{\beta} \tilde{\tau} \gamma} \right)^k \hat{Z}_k. \quad (\text{A13})$$

From (34), we can write the market value of debt at time  $T - 1$  as

$$\hat{P}_{T-1}^M + \hat{b}_{T-1}^M = \tilde{\beta}^{1-T} \sum_{k=T}^{\infty} \tilde{\beta}^k E_{T-1}[\tilde{\tau} \hat{\tau}_k - \tilde{Z} \hat{Z}_k].$$

Imposing active fiscal policy after time  $T$ , (A13) can be written as

$$\hat{P}_0^M + \hat{b}_0^M = - \left[ \left( \frac{1}{1 - \tilde{\beta} \tilde{\tau} \gamma} \right)^{T-1} \sum_{k=T}^{\infty} \tilde{\beta}^k E_0 \hat{Z}_k + \sum_{k=1}^{T-1} \left( \frac{\tilde{\beta}}{1 - \tilde{\beta} \tilde{\tau} \gamma} \right)^k E_0 \hat{Z}_k \right] \tilde{Z}.$$

Substituting for expected transfers ( $E_0 \hat{Z}_k = \rho^k \hat{Z}_0$ ) and updating by  $t$  periods yields (37) in the main text, which is a function of the monetary and fiscal policy targets,  $\{R^*, \pi^*, (b^M)^*, z^*\}$ .

## APPENDIX C NUMERICAL ALGORITHM

Policy function iteration is implemented with the following algorithm:

1. Perform the following initializations:
  - Define the parameters of the model according to their calibrated values (table 1) and specify a convergence criterion for the policy functions (a value no greater than  $1 \times 10^{-6}$ ). Set the variance of the transfers shock,  $\sigma_Z^2$ , to  $4 \times 10^{-6}$ .
  - Calculate the deterministic steady state of the model, conditional on the initial policy mix ( $S_P = 1, S_Z = 1$ ).
  - Discretize the state space in a manner that ensures adequate coverage over the simulation horizon. The minimum state is given by  $\{m_{t-1}, b_{t-1}^M, K_{t-1}, Z_t, S_{P,t}, S_{Z,t}\}$ .
2. Obtain initial conjectures for the policy functions, given by  $\Theta_t = \{N_t, \pi_t, K_t, P_t^M, Q_{t,t+1}\}$ . One approach is to first solve the linear model under both the initial (AM/PF/AT) and debt revaluation (PM/AT/AT) regimes using Sims (2002) gensys algorithm. Then use these linear solutions as initial guesses for the corresponding fixed-regime non-linear models, assuming there is no probability of hitting the fiscal limit. Finally, use weighted averages of these non-linear solutions to obtain initial conjectures for each regime combination (3 policy states and 2 transfers states form 6 combinations) of the model described in section 2.

3. Given values for each node (points of the discretized state space), find the updated policy functions that satisfy the equilibrium conditions of the model. Using the original policy function guess or the solution from the previous iteration, first calculate updated (time  $t + 1$ ) values for each of the policy variables using piecewise linear interpolation/extrapolation. Then solve for the variables necessary to calculate expectations and apply numerical integration (figure 18 illustrates the range of outcomes that must to be considered) using the Trapezoid rule outlined in (Judd, 1998, chap. 7). Using Chris Sims' root finder, `csolve`, solve for the zeros of the equations with embedded expectations, subject to each of the remaining equilibrium conditions. The output of `csolve.m` on each node are policy values that satisfy the equilibrium system of equations to a specified tolerance level. This set of values characterizes the updated policy functions for the next iteration.
4. If the maximum improvement for all policies over all nodes in the discretized state space is less than the convergence criterion, then the policy functions have converged to their equilibrium values at all nodes. Otherwise, repeat step 3 using the updated policy functions as the initial policy functions until convergence is achieved. When the algorithm is running properly, the policy functions will monotonically converge to the specified tolerance level. To provide evidence that the solution is locally unique, perturb the converged policy functions in several dimensions and check that the algorithm converges back to the same solution.<sup>37</sup>

## APPENDIX D DIFFERENTIATED CAPITAL AND LABOR TAX RATES

In the baseline model, identical tax rates are levied against capital and labor income. This section grants the fiscal authority the flexibility to differentiate between these rates.

Figure 19 describes how policy evolves. The economy begins in “normal times” when the monetary authority aggressively targets inflation (AM) and the fiscal authority passively adjusts capital (PKT) and labor (PNT) tax rates to stabilize debt and meet its (stationary) transfers commitments (AT). Specifically, the monetary authority sets the short-term nominal interest rate according to

$$R_t = \begin{cases} \bar{R}(\pi_t/\pi^*)^\phi, & \text{for } S_{P,t} \in \{1, 2, 3, 5\}, \\ \bar{R}, & \text{for } S_{P,t} = 4, \end{cases} \quad (\text{A14})$$

and the fiscal authority sets capital and labor tax rates according to

$$\tau_t^N = \begin{cases} \bar{\tau}^N (b_{t-1}^M / (b^M)^*)^{\gamma_N}, & \text{for } S_{P,t} \in \{1, 3\} \text{ (if the labor tax limit does *not* bind),} \\ \tau_N^{FL}, & \text{for } S_{P,t} \in \{2, 4, 5\} \text{ (if the labor tax limit binds),} \end{cases} \quad (\text{A15})$$

and

$$\tau_t^K = \begin{cases} \bar{\tau}^K (b_{t-1}^M / (b^M)^*)^{\gamma_K}, & \text{for } S_{P,t} \in \{1, 2\} \text{ (if the capital tax limit does *not* bind);} \\ \tau_K^{FL}, & \text{for } S_{P,t} \in \{3, 4, 5\} \text{ (if the capital tax limit binds),} \end{cases} \quad (\text{A16})$$

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<sup>37</sup>One concern is whether this solution method consistently satisfies the transversality conditions, since it only iterates on the policy functions and has no formal mechanism for imposing these restrictions. As a safeguard, however, I simulate the model for thousands of periods and check that its average asset levels (i.e. capital, bonds) are convergent. Moreover, it is easy to show that simulated paths in models that explicitly violate the transversality condition will typically diverge even if the algorithm converges. Although these exercises do not provide proof, they do provide reasonable confidence that transversality conditions are met.

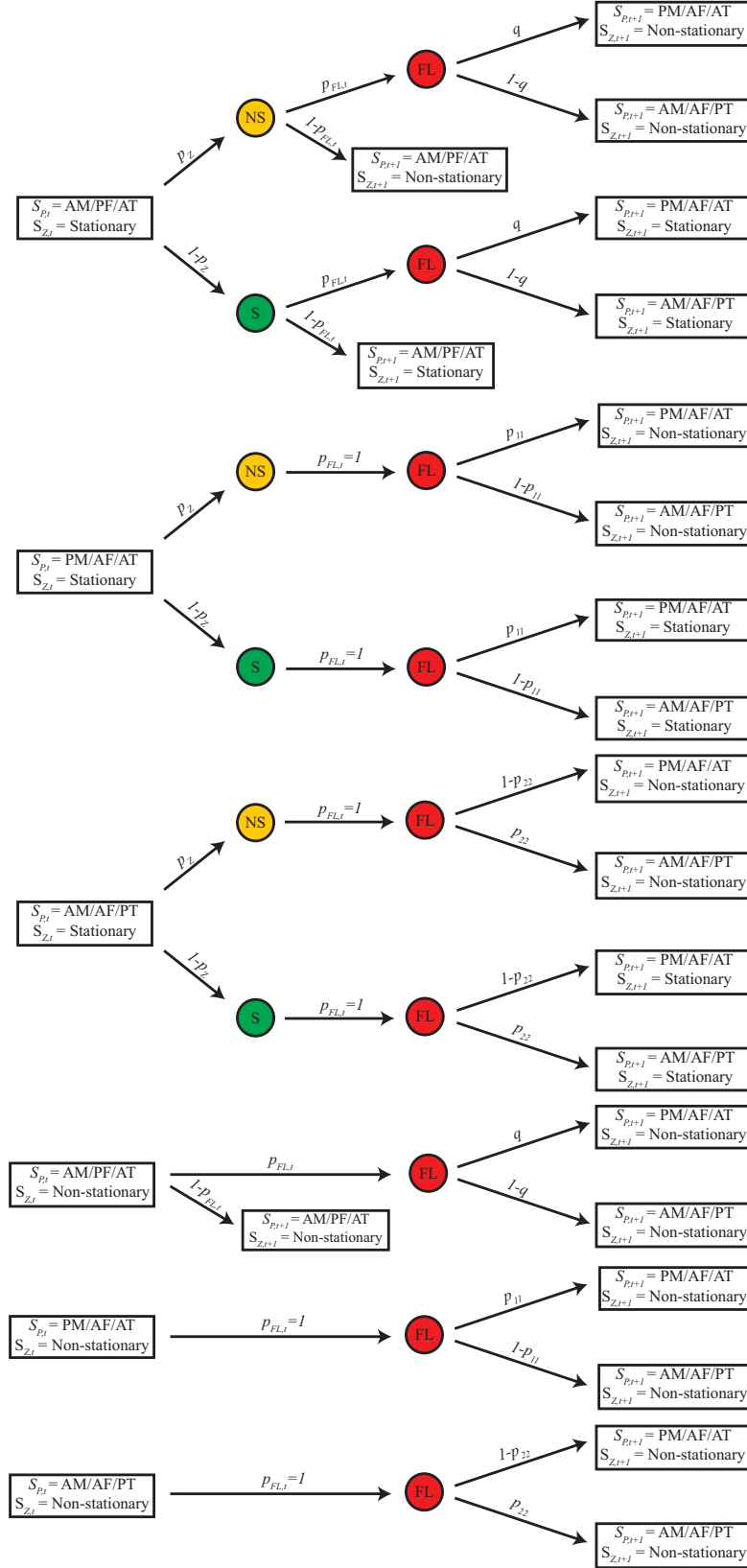


Figure 18: The range of outcomes accounted for during numerical integration (single tax rate).

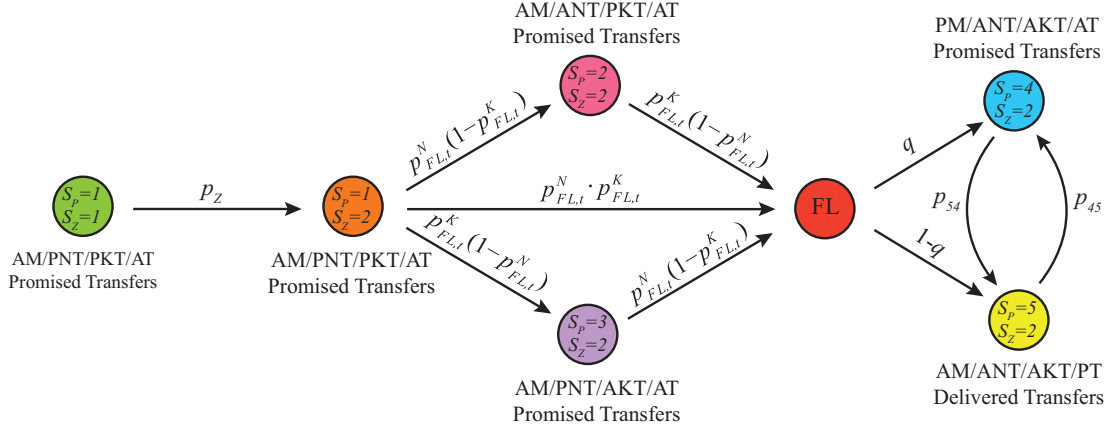


Figure 19: Possible evolution of monetary and fiscal policy regimes with differentiated tax rates

where  $\bar{\tau}^K$  and  $\bar{\tau}^N$  are the steady state capital and labor tax rates.<sup>38</sup> Government transfers continue to evolve according to (16). Once transfers begin to follow an explosive trajectory, capital and labor taxes steadily rise. As political resistance mounts, the probabilities of capital and labor tax rates hitting their respective fiscal limits rise according to

$$p_{FL,t}^N = 1 - \frac{\exp(\eta_0^N - \eta_1^N(\tau_{t-1}^N - \bar{\tau}^N))}{1 + \exp(\eta_0^N - \eta_1^N(\tau_{t-1}^N - \bar{\tau}^N))} \quad (\text{A17})$$

and

$$p_{FL,t}^K = 1 - \frac{\exp(\eta_0^K - \eta_1^K(\tau_{t-1}^K - \bar{\tau}^K))}{1 + \exp(\eta_0^K - \eta_1^K(\tau_{t-1}^K - \bar{\tau}^K))}, \quad (\text{A18})$$

where  $\eta_0^i$  and  $\eta_1^i > 0$ ,  $i \in \{K, N\}$ , are the intercept and slope of the logistic functions.<sup>39</sup> This setup adds a layer complexity to the evolution of fiscal policy, since capital and labor taxes can hit their respective fiscal limits at different dates. The capital [labor] tax rate hits its limit prior to the labor [capital] tax rate with probability  $p_{FL,t}^K(1 - p_{FL,t}^N)$  [ $p_{FL,t}^N(1 - p_{FL,t}^K)$ ]. In this event, the fiscal authority continues to passively adjust the labor [capital] tax rate with the size of government debt. As government debt continues to grow, opposition to this policy quickly mounts and eventually the labor [capital] tax rate hits its fiscal limit. Capital and labor tax rates reach their limits at the same time with probability  $p_{FL,t}^K \cdot p_{FL,t}^N$ . Regardless of the timing, both tax rates eventually hit their fiscal limits and either monetary or fiscal policy adjusts as described in section 2.<sup>40</sup>

Figure 20 shows how the transition paths in figure 11 change when capital and labor tax limits bind at different dates. The possibility that the fiscal authority only has capital or labor taxes available as a debt financing mechanism prior to the fiscal limit increases the growth rate of government debt. More rapidly rising debt increases tax rates, which further distorts capital and labor markets

<sup>38</sup>Following Leeper et al (2010), steady state capital and labor taxes are set to  $\bar{\tau}^K = 0.184$  and  $\bar{\tau}^N = 0.223$ . When capital and labor taxes do not bind, the fiscal authority responds to government debt with reaction coefficients  $\gamma_K = 0.20$  and  $\gamma_N = 0.15$ . The post-fiscal limit capital and labor tax rates are  $\tau_K^{FL} = 0.225$  and  $\tau_N^{FL} = 0.265$ .

<sup>39</sup>The logistic functions are calibrated so that there is a 2 percent probability of both tax rates hitting their limits (14 percent probability of either rate hitting its limit) in the deterministic steady state and a 5 percent probability when  $\tau^i = \tau_i^{FL}$  (22 percent probability of either rate hitting its limit), which is consistent with the baseline calibration.

<sup>40</sup>The expected duration of the debt revaluation and reneging regimes remains the same ( $p_{44} = 0.9$  and  $p_{55} = 0.99$ ).

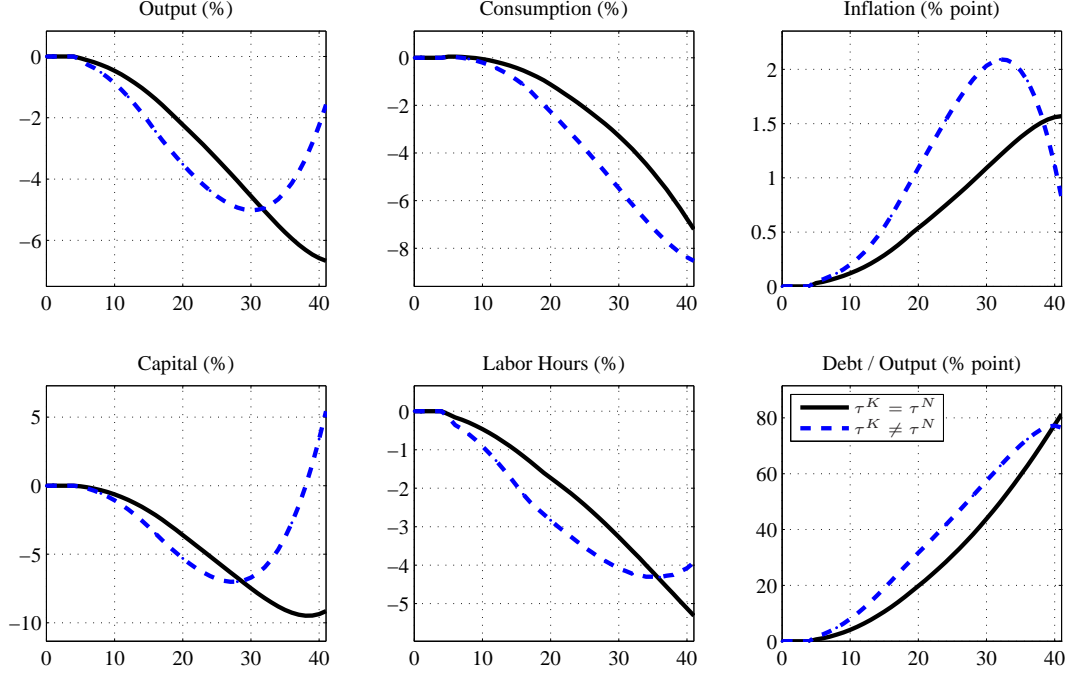


Figure 20: Responses to government transfers switching to a non-stationary path ( $S_Z = 2$ ) in period 5, conditional on active monetary/passive capital tax/passive labor tax/active transfers policy remaining in place even after the fiscal limit is hit. Responses are distinguished by the specification of capital and labor taxes. Reported values are in deviations from the stochastic steady state.

and increases the probability of reaching the fiscal limit—the point at which *both* capital and labor tax rates hit their limits. This has two primary implications. First, it creates a greater likelihood of debt revaluation, which increases the inflation growth. Second, it increases the weight that agents place on the capital tax rate hitting its limit, which increases the expected after-tax return on capital. Once this effect dominates the effect of rising capital tax rates, investment steadily rises. Rising aggregate supply reduces marginal costs and propels inflation on a downward trajectory.

Overall, the expectational effects of the fiscal limit are magnified and brought closer to the present when capital and labor tax rates bind at different dates. Although proportional taxes distort capital and labor markets, these results make clear that any expectation that Congress is unwilling to use both taxes to finance debt increases the negative economic effects of looming fiscal stress.

## APPENDIX E PROBABILITY OF THE FISCAL LIMIT

The probability of reaching the fiscal limit is governed by the logistic function specified in (19). Without being able to point to historical episodes to calibrate the intercept,  $\eta_0$ , or slope,  $\eta_1$ , of the logistic function, it is worthwhile to examine how these parameters affect the qualitative results. Under the baseline calibration there is a 2% probability of reaching the fiscal limit when  $\tau = \bar{\tau}$  and a 5% probability when  $\tau = \tau^{FL}$ . Figure 21 shows how equilibrium outcomes are affected when these probabilities change to  $\{1\%, 4\%\}$ ,  $\{3\%, 6\%\}$ , and  $\{4\%, 7\%\}$ , respectively.

Higher probabilities of reaching the fiscal limit are associated with greater inflationary pressures due to the increased likelihood of moving to a regime where debt becomes revalued. This has two immediate effects. First, it forces nominal interest rates higher and drives up real debt,



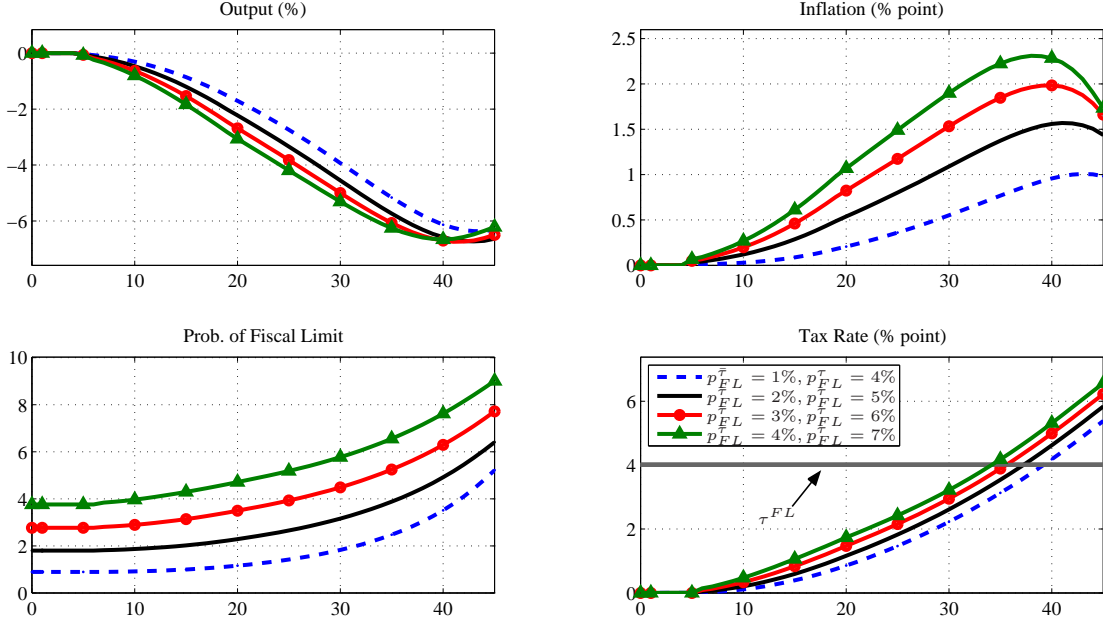


Figure 21: Responses to government transfers switching to a non-stationary path ( $S_Z = 2$ ) in period 5, conditional on active monetary/passive capital tax/passive labor tax/active transfers policy remaining in place even after the fiscal limit is hit. Responses are distinguished by the probability of hitting the fiscal limit. There is a  $\{1\%, 2\%, 3\%, 4\%\}$  probability of reaching the fiscal limit when  $\tau = \bar{\tau}$  and a  $\{4\%, 5\%, 6\%, 7\%\}$  probability when  $\tau = \tau^{FL}$ . The grey line represents the post-fiscal limit tax rate. Reported values are in deviations from the stochastic steady state.

since the monetary authority aggressively targets inflation prior to the fiscal limit. Second, when agents are finitely lived, it causes a shift in real wealth from current to future generations. Both of these effects place additional downward pressure on investment and labor supply, which leads to more drastic reductions in aggregate supply.

As discussed in the main text, the expectation of lower tax rates after the fiscal limit increases the after-tax return on capital, which reverses the trajectories of the capital stock, output, and inflation. With a greater prospect of reaching the fiscal limit, this expectational effect is much stronger and quicker to take effect. Overall, a higher probability of hitting the fiscal limit increases the volatility of aggregate outcomes—stagflation is more severe, but less persistent.

## APPENDIX F ENDOGENOUS POST-FISCAL LIMIT TAX RATE

In the baseline model, agents have perfect foresight over the post fiscal limit tax rate,  $\tau^{FL}$ . This simplifying assumption imposes the undesirable feature that taxes exogenously jump to  $\tau^{FL}$  when the economy hits the fiscal limit. A more reasonable approach is to set  $\tau^{FL}$  to the current tax rate when the fiscal limit is actually hit. Since agents face uncertainty over the timing of the fiscal limit, this approach forces agents to condition on a broad set of post-fiscal limit tax rates.

Figure 22, shows how the counterfactual in figure 11 changes when agents face uncertainty about  $\tau^{FL}$ .<sup>41</sup> When  $\tau^{FL}$  is endogenous, agents no longer place positive probability on taxes jumping to the post-fiscal limit tax rate. Instead, when government transfers switch to a non-stationary path, agents place positive probability on the fiscal limit being hit when the tax rate is relatively

<sup>41</sup>I temporarily set the probability of death to zero to isolate the effect of adding uncertainty over  $\tau^{FL}$ .

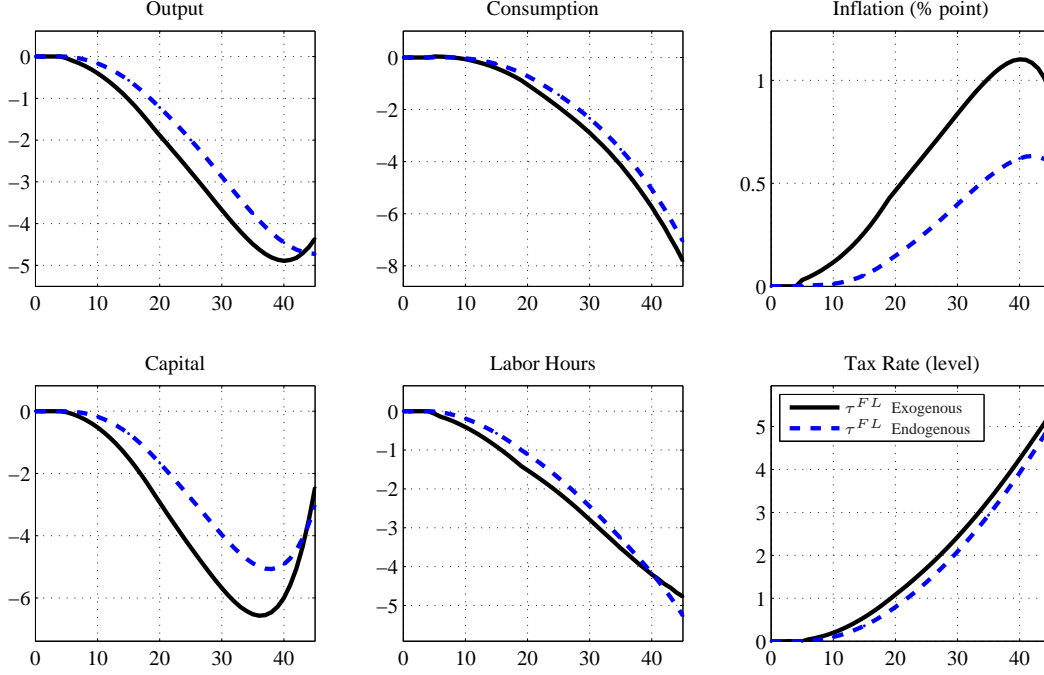


Figure 22: Responses to government transfers switching to a non-stationary path ( $S_Z = 2$ ) in period 5, conditional on active monetary/passive tax/active transfers policy remaining in place even after the fiscal limit is hit. The probability of death,  $\vartheta$ , is set to zero. Responses are distinguished by the treatment of the post fiscal limit tax rate. Reported values are in deviations from the stochastic steady state.

low. This lowers agents' expected future tax burden and increases incentives to invest. A relatively higher capital stock increases aggregate supply and keeps inflationary pressures initially lower.

Investment depends on the expected present values of capital tax rates. At each date  $t$ , this present value is a number that comes from weighting all future tax rates by their probability. These weighted tax rates are then discounted, with the heaviest discount placed on the more distant future periods. When the probability of the fiscal limit is small, only the discounting matters because agents are expecting to remain in the passive tax policy regime for the relevant horizon and switching to the fiscal limit occurs so far in the future that the post-fiscal limit tax rate is heavily discounted. As the probability of hitting the fiscal limit increases, agents believe it is more likely that the tax rate will be fixed at its current ( $t - 1$ ) level and, as a consequence, much higher future rates get discounted by both the small probability of not hitting the fiscal limit and by the discount factor. Once agents beliefs about the fiscal limit dominate the effects of discounting, agents expect a lower tax burden and investment tilts upward. Steadily rising investment eventually dominates the falling labor supply, propelling output upward and reducing inflation. Thus, the qualitative expectational effects of the fiscal limit are identical to the case where  $\tau^{FL}$  is exogenous.

Although the treatment of the post-fiscal limit tax rates quantitatively alters the equilibrium paths in the counterfactual, it has very little impact on the range of time paths that occur when simulating the model. Moreover, the degree of renegeing and the tail risk of inflation are nearly identical. This is because the average date that the fiscal limit is hit, and therefore the average post-fiscal limit tax rate, is consistent with the exogenously specified post-fiscal limit tax rate. Considering its computational simplicity, these findings make the baseline specification very attractive.

## REFERENCES

- ANNICCHIARICO, B., N. GIAMMARIOLI, AND A. PIERGALLINI (2006): “Fiscal Policy in a Monetary Economy with Capital and Finite Lifetime,” *European Central Bank Working Paper* 661.
- ANNICCHIARICO, B., G. MARINI, AND A. PIERGALLINI (2008): “Monetary Policy and Fiscal Rules,” *The B.E. Journal of Macroeconomics*, 8.
- ANNICCHIARICO, B. AND A. PIERGALLINI (2007): “Monetary Rules and Deficit Shocks,” *Spanish Economic Review*, 9, 39–57.
- AUERBACH, A. J. AND L. J. KOTLIKOFF (1987): *Dynamic Fiscal Policy*, New York, NY: Cambridge University Press.
- BI, H. (2011): “Sovereign Default Risk Premia, Fiscal Limits and Fiscal Policy,” Bank of Canada Working Paper 2011-10.
- BI, H. AND E. M. LEEPER (2010): “Sovereign Debt Risk Premia and Fiscal Policy in Sweden,” NBER Working Paper 15810.
- BI, H., E. M. LEEPER, AND C. LEITH (2011): “Uncertain Fiscal Consolidations,” Manuscript, Indiana University.
- BLANCHARD, O. J. (1985): “Debt, Deficits, and Finite Horizons,” *Journal of Political Economy*, 93, 223–47.
- BOHN, H. (2010): “The Economic Consequences of Rising U.S. Government Debt: Privileges at Risk,” CESifo Working Paper 3079.
- COCHRANE, J. H. (2001): “Long-Term Debt and Optimal Policy in the Fiscal Theory of the Price Level,” *Econometrica*, 69, 69–116.
- (2011): “Understanding Policy in the Great Recession: Some Unpleasant Fiscal Arithmetic,” *European Economic Review, Special Issue on Monetary and Fiscal Interactions in Times of Economic Stress*, 55, 2–30.
- COLEMAN, II, W. J. (1991): “Equilibrium in a Production Economy with an Income Tax,” *Econometrica*, 59, 1091–1104.
- CONGRESSIONAL BUDGET OFFICE (2009): “The Long-Term Budget Outlook,” CBO, Washington, D.C.
- (2011): “The Long-Term Budget Outlook,” CBO, Washington, D.C.
- CORSETTI, G., K. KUESTER, A. MEIER, AND G. MUELLER (2010): “Debt Consolidation and Fiscal Stabilization of Deep Recessions,” *American Economic Review*, 100, 41–45.
- CUSHING, M. J. (1999): “The Indeterminacy of Prices Under Interest Rate Pegging: The non-Ricardian case,” *Journal of Monetary Economics*, 44, 131–148.

- DANIEL, B. C. AND C. SHIAMPTANIS (2010): “Fiscal Policy in the European Monetary Union,” Manuscript, SUNY Albany.
- DAVIG, T. (2004): “Regime-switching Debt and Taxation,” *Journal of Monetary Economics*, 51, 837–859.
- DAVIG, T. AND E. M. LEEPER (2006): “Fluctuating Macro Policies and the Fiscal Theory,” in *NBER Macroeconomics Annual 2006, Volume 21*, ed. by D. Acemoglu, K. Rogoff, and M. Woodford, MIT Press, Cambridge, 247–298.
- (2010): “Temporarily Unstable Government Debt and Inflation,” NBER Working Paper 16799.
- (2011): “Monetary-Fiscal Policy Interactions and Fiscal Stimulus,” *European Economic Review*, 55, 211–227.
- DAVIG, T., E. M. LEEPER, AND T. B. WALKER (2010): “‘Unfunded Liabilities’ and Uncertain Fiscal Financing,” *Journal of Monetary Economics, Carnegie-Rochester Conference Series on Public Policy*, 57, 600–619.
- (2011): “Inflation and the Fiscal Limit,” *European Economic Review, Special Issue on Monetary and Fiscal Interactions in Times of Economic Stress*, 55, 31–47.
- DE NARDI, M., S. IMROHOROGLU, AND T. J. SARGENT (1999): “Projected U.S. Demographics and Social Security,” *Review of Economic Dynamics*, 2, 575–615.
- DIXIT, A. K. AND J. E. STIGLITZ (1977): “Monopolistic Competition and Optimum Product Diversity,” *American Economic Review*, 67, 297–308.
- EUSEPI, S. AND B. PRESTON (2010a): “Debt, Policy Uncertainty and Expectations Stabilization,” Forthcoming, Journal of the European Economic Association.
- (2010b): “The Maturity Structure of Debt, Monetary Policy and Expectations Stabilization,” Manuscript, Columbia University.
- FELDSTEIN, M. (2009): “Inflation is looming on America’s horizon,” The Financial Times, April 19. URL: [http://belfercenter.ksg.harvard.edu/publication/18990/inflation\\_is\\_looming\\_on\\_americas\\_horizon.html](http://belfercenter.ksg.harvard.edu/publication/18990/inflation_is_looming_on_americas_horizon.html).
- FERGUSON, N. (2009): “History lesson for economists in thrall to Keynes,” The Financial Times, May 29. URL: <http://www.ft.com/intl/cms/s/0/a635d12c-4c7c-11de-a6c5-00144feabdc0.html#axzz1XkTy6nz0>.
- FERNÁNDEZ-VILLAYERDE, J., P. GUERRÓN-QUINTANA, K. KUESTER, AND J. RUBIO-RAMÍREZ (2011): “Fiscal Uncertainty and Economic Activity,” Manuscript, Duke University.
- FREEDMAN, C., M. KUMHOF, D. LAXTON, D. MUIRB, AND S. MURSULA (2010): “Global Effects of Fiscal Stimulus During the Crisis,” *Journal of Monetary Economics*, 57, 506–526.

- FRENKEL, J. A. AND A. RAZIN (1986): “Fiscal Policies in the World Economy,” *Journal of Political Economy*, 94, 564–94.
- GALLANT, A. R., P. E. ROSSI, AND G. TAUCHEN (1993): “Nonlinear Dynamic Structures,” *Econometrica*, 61, 871–907.
- HUGGETT, M. AND G. VENTURA (1999): “On the Distributional Effects of Social Security Reform,” *Review of Economic Dynamics*, 2, 498–531.
- İMROHOROĞLU, A., S. İMROHOROĞLU, AND D. H. JOINES (1995): “A Life Cycle Analysis of Social Security,” *Economic Theory*, 6, 83–114.
- INTERNATIONAL MONETARY FUND (2009): “Fiscal Implications of the Global Economic and Financial Crisis,” IMF Staff Position Note 09/13.
- IRELAND, P. N. (1997): “A Small, Structural, Quarterly Model for Monetary Policy Evaluation,” *Carnegie-Rochester Conference Series on Public Policy*, 47, 83–108.
- JUDD, K. L. (1998): *Numerical Methods in Economics*, Cambridge, Massachusetts: The MIT Press.
- KOTLIKOFF, L. J. AND S. BURNS (2005): *The Coming Generational Storm: What You Need to Know about America’s Economic Future*, Cambridge, Massachusetts: The MIT Press.
- KOTLIKOFF, L. J., K. SMETTERS, AND J. WALLISER (1998): “The Economic Impact of Transitioning to a Privatized Social Security System,” in *Redesigning Social Security*, ed. by H. Siebert, Kiel University, 327–356.
- (2007): “Mitigating America’s Demographic Dilemma by Pre-funding Social Security,” *Journal of Monetary Economics*, 54, 247–266.
- LEEPER, E. M. (1991): “Equilibria Under ‘Active’ and ‘Passive’ Monetary and Fiscal Policies,” *Journal of Monetary Economics*, 27, 129–147.
- (2010a): “Anchors Aweigh: How Fiscal Policy Can Undermine ‘Good’ Monetary Policy,” in *Monetary Policy under Financial Turbulence*, ed. by L. F. Céspedes, R. Chang, and D. Saravia, Santiago, Chile.
- (2010b): “Monetary Science, Fiscal Alchemy,” NBER Working Paper 16510.
- LEITH, C. AND S. WREN-LEWIS (2000): “Interactions between Monetary and Fiscal Policy Rules,” *Economic Journal*, 110, 93–108.
- PIERGALLINI, A. (2006): “Real Balance Effects and Monetary Policy,” *Economic Inquiry*, 44, 497–511.
- RICHTER, A. W., N. A. THROCKMORTON, AND T. B. WALKER (2011): “Accuracy, Speed and Robustness of Policy Function Iteration,” Manuscript, Indiana University.

- ROTEMBERG, J. J. (1982): “Sticky Prices in the United States,” *Journal of Political Economy*, 90, 1187–1211.
- ROTEMBERG, J. J. AND M. WOODFORD (1997): “An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy,” in *NBER Macroeconomics Annual 1997, Volume 12*, ed. by B. S. Bernanke and J. J. Rotemberg, MIT Press, Cambridge, 297–361.
- SBORDONE, A. M. (2002): “Prices and unit labor costs: a new test of price stickiness,” *Journal of Monetary Economics*, 49, 265–292.
- SIMS, C. A. (1994): “A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy,” *Economic Theory*, 4, 381–99.
- (2002): “Solving Linear Rational Expectations Models,” *Computational Economics*, 20, 1–20.
- (2011): “Stepping on a rake: The role of fiscal policy in the inflation of the 1970s,” *European Economic Review*, 55, 48–56.
- SMETTERS, K. AND J. WALLISER (2004): “Opting out of social security,” *Journal of Public Economics*, 88, 1295–1306.
- SOCIAL SECURITY ADMINISTRATION (2011): “The 2011 Annual Report of the Board of Trustees of the Federal OASDI Trust Funds,” SSA, Woodlawn, MD.
- TRABANDT, M. AND H. UHLIG (2010): “How Far Are We From The Slippery Slope? The Laffer Curve Revisited,” European Central Bank Working Paper 1174.
- VAN DER PLOEG, F. AND G. MARINI (1988): “Finite Horizons and the non-Neutrality of Money,” *Economic Letters*, 26, 57–61.
- WOODFORD, M. (2001): “Fiscal Requirements for Price Stability,” *Journal of Money, Credit and Banking*, 33, 669–728.
- (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton, New Jersey: Princeton University Press.
- YAARI, M. E. (1965): “Uncertain Lifetime, Life Insurance, and the Theory of the Consumer,” *Review of Economic Studies*, 32, 137–50.