Labor Market Frictions, Firm Growth, and International Trade*

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Abstract

Hiring new workers is costly and time consuming, yet many forms of investment have a fixed-cost component and can be profitably undertaken only by large producers. This paper studies how characteristics of the labor market impact income, trade and welfare via the time it takes for firms to grow large enough to justify investing in exporting or in upgrading technology. In the theory, firms make random contacts with potential employees slowly and labor market conditions determine the ease of hiring employed or unemployed workers. Firms choose an optimal time to invest in the light of their anticipated labor market experience. I use the model to examine the impact of labor market frictions on aggregate outcomes in general equilibrium. Lower frictions in job-to-job mobility strengthen firms' incentives to invest, and the economy gains from labor market policies that encourage investment in the trading partner. The model predictions are consistent with observed correlations between firm size, age and export activity, and also with observed correlations between export activity and the share of new hires attracted from other jobs and from other exporting firms. I use an extended version of the model with ex-ante differences in firm productivity to match these moments in the data, to then simulate changes in the labor market and trading environments.

JEL Classification: F16

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1 Introduction

Hiring new workers is a costly and time consuming activity. By many accounts, firms face significant costs of adjusting their labor force, both in developing and in developed countries.[1] Barriers to worker mobility influence the magnitude of these costs and delays. While labor market frictions may have diverse sources, many can be traced to institutional features of the labor market. Recent cross-country evidence and studies of policy reforms favor the view that firm-level job flows and employment adjustments are lower in more regulated labor markets.[2] Further evidence is suggestive of an association between the labor market environment and firms' employment growth. According to the estimates in Jolivet et al. (2006), job-to-job transitions are a more common occurrence in countries that feature more flexible labor markets. For example, an employed worker is five times more likely to contact a prospective employer in the United States than in Portugal, which, according to Botero et al. (2004), are respectively among the least and most regulated labor markets in the world.[3] To the extent that mobility between jobs serves to channel workers towards firms that desire to grow, labor market institutions that affect the frequency of job-to-job transitions should impact the distribution of growth rates in the economy as well as the firm size distribution.

At the same time, many forms of investment have a fixed-cost component, and can be profitably undertaken only by large producers. Participation in international trade is a paradigmatic example. Obtaining the increase in revenues associated with access to foreign markets requires spending considerable amount of resources in setting up distribution networks or developing products, among other activities. Firms need to be large to cover these costs. Other decisions of firms besides exporting respond to the presence of fixed costs, e.g. foreign direct investment (Helpman et al., 2004), technology choice (Bustos, 2011), access to imported inputs (Halpern et al., 2009), number of export destinations (Eaton et al., 2008b), and number of products (Bernard et al., 2011). In these cases, and in tune with empirical observations, the result is that larger firms select themselves into activities that enhance productivity or revenues per worker.

This dependence of investment on firm size, and of firms' growth on labor market frictions, naturally forges a link between the labor market environment and aggregate outcomes. This paper studies how labor market frictions determine income, trade and welfare through their impact on

[1] Recent papers find evidence of labor adjustment costs using different methodologies, e.g. Eslava et al. (2009) and Cosar et al. (2010) estimate convex hiring costs in a panel of Colombian firms; Cooper et al. (2007) and Yashiv and Merz (2007) detect labor adjustment costs in the United States when trying to match the distribution of growth rates across firms and the joint behavior of gross hiring and asset values, respectively; and Manning (2006) presents evidence of increasing marginal costs of recruitment in the United Kingdom.

[2] Kugler (2007) summarizes evidence based on reform episodes that affected specific groups of firms in Italy, Spain, Germany and the United States. As a general finding, a tightening in employment protection regulation tends to reduce job flows and net employment adjustments for the affected firms. Haltiwanger et al. (2008) find that hiring and firing costs reduce job flows in a firm-level dataset that includes 16 countries.

[3] Jolivet et al. (2006) report the share of employment spells ending with a job-to-job transition and estimate the probability of an on-the-job contact with a potential employer in 11 OECD countries. The correlations between each of these measures and the summary index of restrictiveness in employment laws reported in Botero et al. (2004) are -0.68 and -0.78, respectively.

the fixed-cost investment decisions of firms. I present a theory where firms expand their workforce slowly and can pay a sunk cost to enter foreign markets or upgrade their technology, and where labor market conditions determine the ease of hiring employed or unemployed workers. As it is well known, firms are usually born small and grow over time, and exporters are relatively old and large. The key aspect of the analysis is that this typical firm life cycle is shaped by barriers to worker mobility. First, I use a baseline theory to examine the impact of labor market frictions on aggregate outcomes in general equilibrium. Then, I use an extended model together with summary statistics from linked employer-employee data for a quantitative assessment of the mechanism.

The theory builds upon a standard model of a labor market with search frictions where firm size is determined by efficient job-to-job transitions, in the tradition of Burdett and Mortensen (1998). Ex-ante symmetric firms match randomly with workers, who learn about job opportunities both when unemployed and on-the-job. Rent sharing takes the same form as in Postel-Vinay and Robin (2002), where firms make take-it-or-leave-it offers and current employers can counter-offer, resulting in Bertrand competition for workers between firms. I make two main departures from this setup. First, I introduce simple firm dynamics. Firms are born and die continuously due to exogenous shocks. Those who survive contact potential employees slowly, taking into account the transition towards their long-run size to compute the present discounted value of profits. Second, firms are allowed to make a once-and-for-all investment to obtain a permanent increase in revenue per worker. This investment may represent a technology upgrade or entry into a foreign market. In the latter case, the revenue advantage of exporters derives from product differentiation and monopolistic competition, as in Krugman (1980) and Melitz (2003), and depends on the relative size of the two economies.

These elements generate a firm life cycle. Firms are born small, accumulate workers slowly, and invest when they are sufficiently large. The timing of investment is the key outcome of the model; it constitutes a summary statistic for aggregate income and the volume of trade. I show, in Section 2, that it is determined by a simple trade-off. First, firms have the natural incentives to delay the investment to save on the interest value of sunk costs, and to invest earlier to obtain greater revenues on their current workforce. Second, investing earlier generates a higher yield on firms' search for workers along their growth path until the investment. This occurs because firms may contact workers who are already employed by competitors that offer better jobs, being as such too costly to be attracted. Since the value of a job increases as the firm approaches the time of investing, by investing earlier a firm becomes a stronger competitor in the labor market and expands faster. Labor market fundamentals affect aggregate outcomes through their impact on this trade-off.

Section 3 examines the role of labor market flexibility and unemployment compensation in a single country. Characterizing the general-equilibrium effect of changes in the labor market requires keeping track of each firm's response to the distribution of competitors, and of shifts in the size distribution and in the number of firms. With more competitors, each firm grows more slowly and delays its investment. Job-to-job transitions have a central role in shaping the aggregate allocation;

in equilibrium, workers flow from younger and smaller firms who have not yet invested into older and larger firms, and lower frictions in job-to-job transitions strengthen firms' incentives to invest earlier in order to hire more often from other firms. In contrast, frictions in transitions out of unemployment have no effects on the timing of investment or in output per worker, because their impact is exactly absorbed by firm entry or exit. Transitions between jobs are not only needed for frictions to have a general equilibrium impact; as it turns out, allowing for job-to-job mobility is also necessary for the size distribution of firms to exhibit a realistic shape. Finally, I also find a distinctive role for unemployment compensation. Higher transfers to unemployed workers raise aggregate investment through a reduction in labor market competition, promoting faster firm growth towards the size that justifies the investment.

These results correspond to an economy where fixed-cost investments are key for productivity. While many types of economic activities fit this type of investment, participation in international trade is one of the most natural cases.[4] To explicitly account for the role of frictions in an open economy and to connect with exports data, Section 4 extends the analysis to a two-country setup. In this framework, as in Krugman (1980), there are no exogenous differences in productivity across firms, but as in Melitz (2003) there is selection into exporting based on firm size. In this context, the single-country results enumerated above have a natural correspondence with the volume and the income gains from trade. When countries are symmetric, policies that encourage more frequent transitions between jobs or larger unemployment benefits induce earlier entry into exporting, generating income and exports increases in both countries. When countries are asymmetric, labor market policies that favor export participation abroad induce an increase in the size of the foreign market, and this provides incentives for firms to invest earlier in the home country. Thus, the theory uncovers a complementarity between labor market policies of trading economies. An economy gains when the trading partner implements labor market reforms that encourage investment.

A salient feature of the model is that it generates predictions consistent with a range of features of micro data. In the firm life-cycle, it generates a positive correlation between firm age, size and export activity. In the labor market, it gives rise to specific patterns of job-to-job mobility by export status: exporters are more likely than non-exporters to hire workers from other firms instead of from unemployment, as well as from other exporters instead of from non-exporters. In Section 5 I verify that these patterns hold in the data. While the combination of alternative forces could account for some of these correlations, this paper provides a simple mechanism that qualitatively generates all of them. Furthermore, the prevalence of job-to-job flows in the data is informative about the magnitude of the key friction in the model to determine aggregate outcomes. I exploit these relationships to calibrate the model.

Section 6 presents the quantitative assessment. I use summary statistics from linked employer-employee data representative of formal employment in manufacturing in Argentina. For the parametrization I must extend the baseline setup in some dimensions that allow additional flexibility.

[4] The fixed-cost nature of the exporting decision has been central in the international trade literature that springs from the the facts in Bernard and Jensen (1995) and from the Krugman (1980) and Melitz (2003) models. See Das et al. (2007) for an estimation of export entry costs in the context of these theories.

Among other features, I include ex-ante heterogeneity across firms. The baseline model yields dispersion in export participation by firm size and age, albeit starkly: only firms above a threshold are exporters. However, export participation increases slowly over age and size in the data. With ex-ante heterogeneity, lower-cost firms choose to invest earlier, recruit more aggressively, and grow faster; as a cohort ages, there is a slow increase in export participation as firms progressively select into exporting.

I parametrize the extended model to match moments in the data that correspond to the central outcomes in the theory: firm age, firm size, and share of new hires entering firms from other jobs, by number of export destinations. First, I ask whether the model predicts a realistic firm life-cycle. I do so contrasting the patterns of export participation and growth over age that emerge from the calibrated model with their empirical counterparts. I find that the model fairly reproduces the joint profile of firm age, size and export status for relatively younger firms. However, it fails to match the growth rate of old exporters. Second, I use the model to measure welfare gains from changes in the labor market and international trade environments. I find lower welfare gains from trade in a more flexible labor market, suggesting a lack of complementarity between labor market and trade reforms. This contrasts with recent measurements of gains from trade and labor market reforms in different environments.

Connection with the Literature This paper characterizes in general equilibrium a model that combines firm dynamics due to slow labor adjustment, fixed-cost investments and endogenous job-to-job mobility. Previous studies that allow for endogenous productivity differences in variants of Burdett and Mortensen (1998), such as Postel-Vinay and Robin (2002) and Meghir et al. (2010), restrict their analysis to the static decisions of firms at their long-run scale. Here, the nature of the fixed-cost investment problem necessarily shifts the focus to the dynamic aspect in firms' decisions. The process determining the evolution of firm size shares some features with Klette and Kortum (2004); as in that framework, firms have linear revenue functions and expand by poaching workers (in their case, products) from other firms.[5] Recent working papers, such as Garibaldi and Moen (2010) and Acemoglu and Hawkins (2010), incorporate the idea of slow hiring as a source of firm dynamics, but do not study firms technology choice.

I study comparative statics for aggregate outcomes with respect to labor market frictions in general equilibrium. The impact of labor market characteristics on aggregate outcomes through firms' investment decisions has been explored by Acemoglu and Shimer (1999, 2000) in a directed search framework with single-worker firms, where risk aversion and wage inequality drive technology dispersion. Lagos (2006) and Mortensen and Lentz (2010) also use models with a dependence of TFP on search frictions. More broadly, a growing literature studies TFP losses generated out of misallocation, as in Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). None of these studies incorporates the slow growth of firms coupled with the fixed-cost investment problem that

[5] Two main differences are that I focus on deterministic firm growth while they allow for randomness in firm-level outcomes, and that in my case worker flows have some specific direction (from smaller to larger firms) while in their model a product lost by a firm is randomly allocated among competitors.

is the focus here.

This paper complements a growing literature that studies labor market frictions in an international trade context. The distinguishing aspect of my analysis is that frictions induce slow growth and a firm life cycle, becoming a source of dispersion in size and export status across ex-ante identical firms. Moreover, the sluggishness in the hiring technology underlying this dispersion crucially depends on job-to-job mobility. These features contrast with recent theoretical studies, such as Davis and Harrigan (2007), Davidson et al. (2008), Felbermayr et al. (2008), Eckel and Kreickemeier (2009), Ritter (2009), Helpman and Itskhoki (2010) and Helpman et al. (2010), that embed labor market imperfections in a context of heterogeneous-productivity firms. In these setups, the only force driving export participation is selection based on productivity dispersion, and job-to-job transitions are either ruled out or do not constitute an equilibrium outcome.[6] On the other hand, recent quantitative papers have assessed labor market frictions in different open-economy setups. Kambourov (2009) and Cosar (2010) simulate policy changes in two-sector models where labor market frictions slow down reallocations after a trade reform, while Cosar et al. (2010) simulate the effects of a trade liberalization in a framework with firm-specific productivity shocks and search frictions.

Finally, some recent frameworks featuring firm dynamics and exporting may be consistent with aspects of the patterns over age, size and export status that I discuss in the quantitative section. Recent theories include technology diffusion as in Ederington and McCalman (2008), accumulation of firm-specific knowledge as in Atkeson and Burstein (2010), learning about exporting as in Eaton et al. (2009) and a combination of selection based on productivity shocks and exogenous trends in demand or in productivity, as in Ruhl and Willis (2008) or Arkolakis (2009). Slow labor adjustment in a frictional labor market is a complementary force underlying firm dynamics and selection into exporting. From an empirical standpoint, it is distinguished from these models by the specific predictions regarding the composition of new hires by export status, and by the impact of labor market characteristics on firm dynamics and trade. I focus on these relationships in the quantitative section to measure frictions in job-to-job mobility, and to simulate counter-factual policy exercises.

Structure of the Paper The paper is structured as follows. The next section lays out the basic setup and characterizes the partial equilibrium problem of an individual firm. Section 3 studies the general equilibrium in a single country, where the revenue advantage of exporters is taken as given. At that stage, the model equivalently describes a closed economy where firms make a choice between technologies with different productivity. Section 4 studies international trade with two countries, where the exporter revenue premium is endogenous. In section 5 I present the basic patterns in the data concerning firm age, size, export status, and job-to-job mobility. Section 6 develops the extended version of the model and presents the quantitative exercises. Section 7 concludes. Proofs are gathered in the appendix.

[6] Also related is work by Holzner and Larch (2011), who embed search frictions with job-to-job mobility in Melitz (2003) to explain that firms do not serve all possible export destinations.

2 The Model

I develop a stylized model of the labor market in which the accumulation of employees takes time. Firms decide whether and when to expand revenues by bearing the fixed costs of entry into an export market or a more productive technology. I use this model to assess how characteristics of the labor market impact aggregate outcomes.

2.1 Preferences and Technology

There is mass of identical workers of measure one. Workers have dynastic preferences with linear utility for consumption of the final good and they discount the future at rate ρ:

$$U(c_t) = \int_0^\infty e^{-\rho t} c_t dt.$$

I focus on a steady state in which aggregate variables are constant, so that the flow value of aggregate utility equals consumption of the final good, c.

A mass of firms of measure M produce output using a constant-returns-to-scale technology with labor as the only factor of production. At any moment of time a firm employs a stock of workers of measure n that evolves according to its experience in the labor market, as I describe below. Firm productivity can take one of two values according to a firm decision that I also describe below. They can produce y_D units of output per worker with a simple technology or $y_X = \Gamma y_D$ units of output per worker using a superior technology, where $\Gamma > 1$. At first, I will speak of this as a literal choice of technology, but later I will link it to export status. A firm in an open economy that sinks the fixed cost of entry into a foreign market can earn more revenue per worker than one that sells only at home. When I focus on the role of trade, the revenue premium of exporters, Γ, will be determined endogenously.

As will become clear in the next section, the assumption of constant returns to scale is used to facilitate the introduction of job-to-job transitions. By making the marginal valuation of new workers independent of firm size, the total value of a match in a given firm will depend only on how long a firm with productivity y_D plans to wait until switching to y_X, or if it has already done so. This will imply a simple pattern of transitions between jobs, with workers moving from younger to older firms.

2.2 Labor-Market Environment

Labor markets are subject to a standard search friction whereby workers learn of jobs when unemployed or employed according to a random process. The Poisson rate at which a worker makes contact with some firm is λ_u for unemployed workers and λ_e for employed workers. In reduced form, these parameters capture institutional features of the labor market that affect worker mobility. In addition to the transitions between jobs to be described below, jobs are terminated at an exogenous rate γ and firms suffer a shock that forces them to exit at rate μ. This means that

every employee moves into the pool of unemployed workers at rate $\delta = \gamma + \mu$.[7] The steady-state rate of unemployment is $u = \delta / (\lambda_u + \delta)$. To save notation later, I define the normalized contact rate on the job $\kappa_e = \lambda_e / \delta$.

2.3 Value of Jobs

Production technologies are not constant throughout the life of firms. In equilibrium, firms with technology y_X do not switch back into y_D, but firms with technology y_D may intend to upgrade at some point in the future if they survive long enough. The decision of when to invest is examined in the next section, but, for the moment, it implies that the relevant dimension of heterogeneity across firms, in terms of the value of the jobs that they can offer to prospective workers, is how far removed they are in time from switching into the better productivity –or if they have already done so. Let x indicate this "time until switch" for a given firm. Across the economy there are (potentially) three classes of firms: $x = 0$ denotes firms that have already invested; $x \in (0, \infty)$ denotes firms that will upgrade in x periods from now; and $x = \infty$ denotes firms that will never upgrade no matter how long they survive.[8]

Let $v(x)$ represent the total value of a job held by a firm whose time until switch, if they do not suffer an exit-inducing shock before then, is x. This value reflects the joint surplus of a match shared by the firm and the worker. When a new relationship is formed, the partners divide the surplus according to the game posited by Postel-Vinay and Robin (2003): firms observe the current status of contacted workers, tender take-it-or-leave-it offers, and commit to the value promised to the worker. As a consequence, when an unemployed worker meets a firm, the offer leaves the worker indifferent between the job and the value of unemployment, w_u, and is accepted. The present discounted sum of future expected profits generated in firm x by a worker who enters the firm from unemployment equals the total value of a job held by this firm, net of the amount necessary to lure the worker, namely

$$J_u(x) = v(x) - w_u. \tag{1}$$

In contrast, when an employed worker meets a new firm, the current employer hears the job offer and makes a counter-offer. The outcome is similar to Bertrand competition: the firm offering the job of greater total value obtains the worker, offering in exchange a value equal to what the worker could obtain in the alternative employment. Transitions in this model are efficient, hence we can conjecture that workers flow from firms with higher x into firms with lower x. Therefore, when a worker moves from a firm x_0 to a firm x that is closer (in expectation) to the switching date, the firm in state x captures a present discounted value of profits

$$J(x_0, x) = v(x) - v(x_0). \tag{2}$$

[7] Firm exit is necessary to induce an invariant distribution of ages. Exogenous separations serve to bound the size of surviving firms.

[8] Since firms are homogeneous and will all choose the same outcome, the equilibrium will either feature firms who never invest ($x = \infty$) or firms who invest at some point ($0 \leq x < \infty$), but not both. In the extension with heterogeneity of Section 6 these types can coexist.

Note that both $J_u(x)$ and $J(x_0, x)$ denote present discounted sums of expected profits captured by a firm from one particular worker at the instant when the worker *enters* the firm. After that moment, the worker might leave due to an exogenous shock or make contact with another firm, triggering a renegotiation or a quit. These possible events are reflected in the computation of $J(x_0, x)$.[9]

The assumptions of the bargaining game and equation (2) can be used to derive the following.

Lemma 1 *The total value of a job held by a firm whose time until switching is x is*

$$v(x) = \frac{y_D + (y_X - y_D) e^{-(\rho+\delta)x} + \delta w_u}{\rho + \delta}. \tag{3}$$

In flow-equivalent terms, the value of a job offered by a firm that is x periods away from switching consists of the sum of the average revenue generated by the worker throughout the expected duration of the match and the value of unemployment obtained by the worker when the match is dissolved, which occurs at rate δ. This job value increases as the firm approaches the time of switching (i.e., it decreases with x), confirming our conjecture that workers move from high-x to low-x firms, but not vice-versa.[10]

2.4 Value of Firms, Stock Effect and Timing of Investment

As anticipated, firms can choose between the alternative technologies y_D and y_X. Firms enter the marketplace with no workers and grow subject to their contacts in the labor market while facing the risk of death. At birth, they are endowed with productivity y_D, but they can choose at any time to make a once-and-for-all investment to upgrade to productivity y_X. This investment entails a sunk cost with flow-equivalent value of f_X units of the final good per period. This "switching into a better technology" will be the same as "starting to export" for a firm in an open economy.

A firm has perfect foresight about the evolution of its stock of employees, facing no uncertainty beyond the exit probability.[11] As a result, firms choose an age h at which to introduce the high-productivity technology. This decision is made on the basis of the flow of workers obtained in each period and the valuation attached to each. At any moment, a firm makes contact with

$$\left(\frac{s}{\bar{s}M}\right) [\lambda_u u + \lambda_e (1 - u)] \tag{4}$$

workers, where $s/\bar{s}$ is the search effort exerted by the firm to find workers relative to average search activity in the economy, and M is the measure of firms. Until Section 6, s is assumed to be common to all firms. As a result, a worker who hears of an opening has the same probability of

[9] See Appendix A.1 for explicit formulations.

[10] In the data workers also move in the opposite direction; for example, there are transitions from exporters ($x = 0$ in the model) into non-exporters ($x > 0$). The model can be reconciled with these (relatively uncommon) flows adding heterogeneity in firm productivity or in fixed costs, as in the extension of Section 6.

[11] I.e., I treat the stock of workers in the firm as a continuous set, hence the individual contact and exit rates equal the fraction of workers who experience these shocks. Since growth is deterministic, it is equivalent to cast the firm problem in terms of fixed costs f_X per period.

being matched with any firm, and differences in the rate at which firms accumulate workers arise solely from the ability to attract workers away from other firms.

Because of our assumption of linear revenue functions, firms wish to grow as large as possible; therefore, every match with an unemployed worker results in a hire. In contrast, out of all contacts made with employed workers, a firm with time until switch of x only attracts those workers employed in firms offering jobs of lesser value, i.e. in firms at $x_0 > x$ periods from switching. Let $G(x)$ be the share of employment in firms whose time until switch is less than x; this distribution has mass points at 0 or at ∞ that measure employment in firms that have already implemented y_X or that will never do so, respectively. Out of the contacted workers in (4), the fraction of *new hires* by a firm with time until switch of x is then

$$\frac{1+\kappa_e\left[1-G(x)\right]}{1+\kappa_e} \tag{5}$$

The number of firms M in (4) and the distribution of employment across firms with different time until switch $G(x)$ in (5) reflect competition in the labor market and will be determined in general equilibrium.

The linear technology implies that the present discounted value of profits generated by all workers who *enter* a firm in state x, expressed in terms of the final good, is the sum of the values generated by each of these workers individually:

$$\pi(x)=\frac{\lambda_u u}{M}J_u(x)+\frac{\lambda_e(1-u)}{M}\int_x^{\overline{x}} J(x_0,x)\,dG(x_0)\,. \tag{6}$$

The first term in this sum is the present value of profits generated by workers attracted from the pool of unemployment and the second term corresponds to profits from workers attracted from other firms, drawn from the employment distribution G.[12] A firm whose time until switch is x attracts all workers who are contacted from firms whose time until switch is greater than x. Using this expression, we can write the value *at entry* of a firm that plans to switch into the high-productivity technology at age h as

$$\Pi(h)=\int_0^h e^{-(\rho+\mu)a}\pi(h-a)\,da+e^{-(\rho+\mu)h}\left[\frac{\pi(0)-f_X}{\rho+\mu}\right]. \tag{7}$$

A new firm starts with no workers. When it has age $a=h-x<h$, the switch lies $h-a$ periods ahead and incoming workers generate average expected profits with a present discounted value of $\pi(h-a)$; after h the firm obtains $\pi(0)$ from new workers, which is the value of incoming workers in a high-productivity firm for the rest of its expected life. To switch, it must pay the sunk cost with flow-equivalent value f_X. The effective rate of time discount, $\rho+\mu$, takes into account the probability of firm exit. This expression is written as the discounted sum of the stock value of profits generated by the flow of new hires at each age; as such, it already incorporates information

[12] The upper limit of integration $\overline{x}$ denotes firms who are furthest away from investing than any other firm.

about worker exit and on-the-job contact probabilities as part of the discounting in the future stream of profits in $\pi(\cdot)$.

Firms choose the age at which to implement the better technology. In the case with international trade, this will be the age at which they begin to export. Consider a firm that invests at h. If that firm delays the investment at that age, it incurs two types of opportunity costs. First, it has the opportunity cost of not implementing the better technology, which reduces output per worker on the stock of workers available at h. Second, it reduces the inflow of workers at each age below h, because a higher switching age h increases the time until investment, $x = h - a$, for all $a < h$. As a consequence of these two effects, $\pi(h-a)$ in (7) shifts down for all a. On the other hand, by delaying the time of investment at h, the firm has marginal savings on its costs for an amount of f_X.[13]

These marginal costs and benefits from delaying the investment are reflected in the first-order condition of the firm's problem. In any positive solution for the switching age, it satisfies:[14]

$$S(h) = f_X \text{ if } h < \infty, \tag{8}$$

$$S(h) \leq f_X \text{ if } h = \infty, \tag{9}$$

where

$$S(h) = \int_0^h e^{(\rho+\mu)x}\left[-\pi'(x)\right] dx. \tag{10}$$

I will refer to the function $S(h)$ as the *stock effect* of a delay in h. It captures the marginal opportunity costs of delaying the age of switching. As shown in (8), the firm chooses the h where these marginal costs are equal to the marginal savings in fixed costs, f_X. It is possible that the stock effect is never large enough, relative to the fixed cost, to justify the investment. This could occur, for example, if the firm never grows too large. In this case, as shown in (9), the firm chooses not to invest.

The value of a firm in (7) can also be formulated as $(\rho+\mu)\Pi(h) = \pi(h) - \Pi'(h)$. Letting $\Pi^e \equiv \max_h \Pi(h)$ be the value of the firm at entry when it chooses the switching age optimally, in an interior solution (i.e. where $\Pi'(h) = 0$) it must be that

$$\Pi^e = \frac{\pi(h)}{\rho+\mu}. \tag{11}$$

Hence, when h is chosen optimally, the value of the firm at entry is the same as if the firm obtained the value of all workers who are hired at the moment of entry (i.e., when $x = h$) in every period. This expression will be useful in the characterization of the general equilibrium.

Figure 1 illustrates the basic trade-off faced by the firm. It depicts the evolution, as the firm ages, of the value $\pi(h-a)$ generated by new workers in a firm that switches at h. After h, the figure

[13] Note that in this optimal stopping the payoffs process is not independent from the stopping time. The rates of job-to-job hires and separations depend on the timing of the investment, and therefore so it does the drift in total profits.

[14] I write $h = \infty$ to denote that the firm's optimal choice is to never invest.

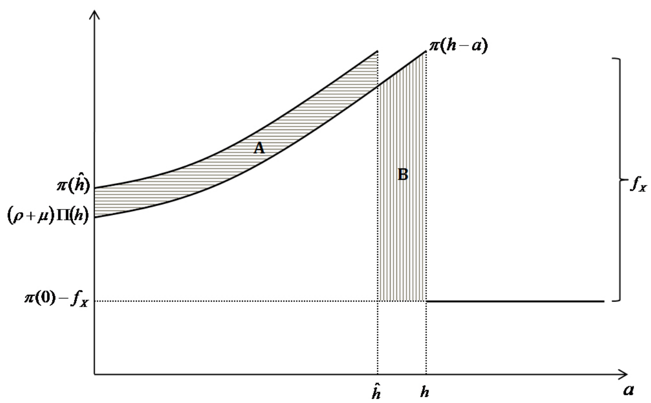

Figure 1: Value of new workers and stock effect

depicts $\pi(0) - f_X$, the value of new hires net of the flow value of sunk costs. The value of the firm at entry, $\Pi(h)$, is the discounted sum of this schedule. In present value, total profits obtained from workers accumulated before investment corresponds to the area below $\pi(h-a)$, above $\pi(0) - f_X$ and to the left of h. When a firm delays the investment from a generic age $\widehat{h}$ up to the optimal age h, it reduces the value generated by all workers attracted before $\widehat{h}$. This loss, represented by the shadowed area A, constitutes the stock effect. But it gains, thanks to lower costs, in terms of the value generated by all workers attracted after $\widehat{h}$ and before h, represented by the shadowed area B. The firm's decision balances these marginal losses and gains. When the switching age is chosen optimally, (11) holds. In this case the flow value of the firm at entry is equal to the intercept of the figure. This intercept lies above $\pi(0) - f_X$, the flow value of investing at birth, which always constitutes a feasible choice.

Since the firm grows over time, the longer it waits, the larger is the opportunity cost of not exercising the investment; this is reflected in that $S'(h) > 0$, which implies that the profit function is strictly concave. Furthermore, $S(0) = 0$, i.e. there is no stock effect at entry because there is no initial labor force; so it must be that $h > 0$ unless the sunk cost of upgrading technology is zero, in which case investment occurs right away. Finally, $S(h)$ is bounded, which implies that the firm actually intends to invest, i.e. h is a finite number, if and only if the fixed cost of investing is not too large.[15]

The switching age of a firm is affected by various parameters and aggregate variables. We can infer from Figure 1 that the stock effect is stronger, and therefore investment takes place earlier, the steeper is the rise in π with firm age. This increment reflects two margins: the rise in the number of new hires and the rise in the discounted revenues generated by the average worker as the firm ages. The latter occurs because, as the firm ages, it approaches the time of switching into the high-productivity technology. Substituting the expressions for the value of each match from (1) to (3) into the present value of profits generated by all workers who enter at x, $\pi(x)$ in (6), the

[15] This follows from firm size being bounded; if $\gamma \to 0$ (no exogenous separations) then h is necessarily finite.

integrand in the stock effect in (10) takes the form

$$e^{(\rho+\mu)x}\left[-\pi'(x)\right] = \underbrace{(\Gamma-1)\,y_D e^{-\gamma x}}_{\text{revenue}} \times \underbrace{\left(\frac{\lambda_u u}{M}\right)\{1+\kappa_e\left[1-G(x)\right]\}}_{\text{new hires}}. \tag{12}$$

Forces that generate an increase in this expression reduce the switching age, while an increase in f_X delays it. In the "revenue" margin, larger values of y_D or Γ accelerate the investment. More frequent separations, captured by a larger γ, produce the opposite effect by making it less likely that a new worker will remain in the firm until the time of investing, diminishing the value of the current stock. In the "new hires" margin, either a higher contact rate with unemployed or with employed workers leads to an increase in the number of new hires and to earlier investment.

Interactions among firms in the labor market occur through the number of firms and the employment distribution. A larger number of firms M delays investment because it increases competition for workers, shrinking the number of meetings experienced by the firm. Similarly, a first-order shift in the employment distribution $G(x)$ towards low-x firms delays the investment because it makes it more likely that a worker contacted from another job is employed in a firm that is close to investing, reducing the share of meetings that translate into new hires.

Summarizing the results from this section:[16]

Proposition 1 *In an interior solution, a firm chooses the unique h where (8) holds. The firm never invests at entry unless $f_X = 0$, but eventually invests if and only if f_X is below some finite threshold. At an interior solution, h is decreasing in y_D, Γ, λ_u and λ_e, and increasing in f_X, γ, M, and a first-order shift in $G(x)$.*

For what follows, the main implication of this proposition is that a more flexible labor market leads to earlier investment, while more competition, through either the measure of rival firms or the distribution of employment across them, delays the investment of an individual firm. These are partial-equilibrium results. To assess the full impact of labor market conditions on investment and income we need to move on to the general equilibrium, where the intensity of competition is determined endogenously, as I do next.

3 Single-Country Equilibrium

I now consider a general equilibrium in which many firms interact based on the decisions of competitors. I analyze the steady state, where aggregate variables are constant over time. Since all firms face the same problem for which, as shown in the previous section, there is a unique solution, in equilibrium they must all invest at the same time after birth, H. This common switching age induces a number of new endogenous objects: the distribution across firms of the time until switch $P(x)$, the share of high-productivity firms m_X, the share of employment in these firms e_X and

[16] Formal proofs are relegated to the appendix.

aggregate productivity y. In equilibrium, these variables must be such that a number of conditions hold. First, each individual firm, taking these variables as given, solves the problem in the previous section and optimizes over its choice of h. Second, firms must not have incentives to deviate from the common decision H. Third, the number of firms, M, must be such that the free entry condition is satisfied.

I proceed to define these aggregate variables, then I move to the definition and characterization of the equilibrium, and finally I show the comparative statics. Throughout this section the productivity gap Γ is still exogenous, so that the model describes a closed economy where firms make a choice between technologies with different productivity. Using the results from the single-country equilibrium, in the next section we will be able to characterize the impact of labor market policies on trade and income in an open economy setup.

The growth of a firm depends on where it is located relative to other firms in terms of time to invest. Across the economy, the share of firms that are less than x periods away from investing equals the fraction of firms that have survived beyond age $H - x$. Since the constant death rate μ generates an exponential distribution of ages, the share of firms that are at less than x periods from switching is

$$P\left(x; H\right) = e^{-\mu(H-x)}, \text{ for } x \in \left[0, H\right]. \tag{13}$$

Due to random matching and the common search effort for workers across firms, workers in either unemployment or employment who make contact with a potential new employer have a probability $P\left(x; H\right)$ of sampling one that is less than x periods away from switching. The pattern of transitions from high-x firms into low-x firms gives the steady-state distribution across employees of the time until switch of their employer:[17]

$$G\left(x; H\right) = \frac{\left(1 + \kappa_e\right) P\left(x; H\right)}{1 + \kappa_e P\left(x; H\right)}. \tag{14}$$

The shape of this distribution responds monotonically to first-order shifts in $P\left(\cdot\right)$; a change in the firm distribution towards stronger competitors (i.e., an increase in $P\left(\cdot\right)$ for each x) naturally translates into a rise in $G\left(\cdot\right)$.

From the firm and employment distributions evaluated at $x = 0$ we find, respectively, the share of high-productivity firms and the share of employment allocated to these firms:

$$m_X\left(H\right) \equiv P\left(0; H\right) = e^{-\mu H}, \tag{15}$$

$$e_X\left(H\right) \equiv G\left(0; H\right) = \frac{\left(1 + \kappa_e\right) m_X\left(H\right)}{1 + \kappa_e m_X\left(H\right)}. \tag{16}$$

The share of firms with productivity y_X (i.e., exporters in the open-economy setting of the next section) is simply given by the fraction of firms that has survived beyond age H. The assumption of a common search effort across firms implies that m_X represents also the probability that a worker

[17] This is obtained by setting equal to zero the expression describing the evolution of $G\left(x\right)$: $\left(1 - u\right) dG\left(x\right) = \left\{\lambda_u u + \lambda_e \left(1 - u\right)\left[1 - G\left(x\right)\right]\right\} P\left(x\right) - \delta\left(1 - u\right) G\left(x\right)$.

who learns about a job does so about one in a high-productivity firm. The fact that workers flow from type D firms into type X firms then yields the expression for e_X.

In the aggregate economy, output per employed worker is endogenous. It equals the employment-weighted average of productivity across firms:

$$y = \{[1 - e_X(H)] + e_X(H)\,\Gamma\}\, y_D. \tag{17}$$

The term in curly brackets represents the endogenous part of TFP. Hence, in the end, this theory is about the determination of e_X, the share of employment in high-productivity jobs, as a way of explaining aggregate income. The common switching age H is sufficient statistic for these aggregate variables.

The value of unemployment w_u is linked to aggregate income. The take-it-or-leave structure implies that ρw_u equals the income flow of unemployed workers. I assume that this value is chosen by the government, which levies a lump-sum tax to compensate each unemployed worker on a basis relative to income per worker in the economy:

$$\rho w_u = by. \tag{18}$$

By increasing b the government raises the transfer received by each unemployed worker as a share of income per employed worker, irrespective of the unemployment rate. Below, I consider how changes in this policy variable affect the endogenous variables.

The distributions that we have just introduced, as well as income per employee, are all functions of H. Through its effect on these variables, H impacts the decision of firms characterized in the previous section. To denote this dependency, I write now the stock effect defined in (10) as $S(h; H)$. In an interior equilibrium, the first-order condition is

$$S(h; H) = f_X. \tag{19}$$

This condition gives the age for investment h chosen by an individual firm, taking the group of aggregate variables affected by H as given. In equilibrium this decision must be consistent across firms; i.e.,

$$h = H. \tag{20}$$

Finally, firms face entry or overhead expenses with flow-equivalent value of f_D units of the final good. Using the value of a new firm Π^e from (11), the free-entry condition implies that a potential entrant must be indifferent about entering,[18]

$$(\rho + \mu)\,\Pi^e = \pi(h; H) = f_D, \tag{21}$$

[18] Since firms are continually exiting, a constant number of firms in steady state requires actual entry, so that the free-entry condition holds with equality.

where the value of firms at entry, $\pi(h; H)$, is expressed as a function of H, too.

3.1 Existence and Uniqueness of a Single-Country Equilibrium

We are now in position to define an equilibrium.

Definition 1 *A single-country equilibrium consists of labor market outcomes $\{h, H, M\}$, distributions $\{P(\cdot), G(\cdot)\}$, shares of firms and employment $\{m_X, e_X\}$, output per worker y, consumption c and unemployment value w_u such that:*
a) the first-order condition (19) from the firms' optimization problem holds;
b) the individual and the common age for switching are consistent, (20);
c) the number of firms adjusts to satisfy free entry, (21);
d) the firm and employment distributions are given, respectively, by (13) and (14);
e) the shares of high-productivity firms and of employment in these firms are given, respectively, by (15) and (16);
f) output per worker is given by (17);
g) the value of unemployment is given by (18); and
h) goods market clear.[19]

My next step is to establish equilibrium existence and uniqueness. To do so, it is useful to define the function $\Omega(h, H)$ as the ratio of the stock effect of an individual firm at its optimal choice to the common value of firms at entry. Using (19) and (21) we have that, in equilibrium, this (free entry adjusted) stock effect equals the cost of upgrading technology relative to operative costs,

$$\Omega(h, H) \equiv \frac{S(h; H)}{\pi(h; H)} = \frac{f_X}{f_D}. \tag{22}$$

Implicit in this equation is the reaction of each firm, h, to the common switching age H. This response is depicted in Figure 2, that shows condition (22) in the space of h and H for two levels of upgrading cost, $f_{X,0} < f_{X,1}$.

An equilibrium consists of an H that satisfies $\Omega(H, H) = f_X/f_D$, i.e. when the schedule depicted in Figure 2 intersects the 45° line. Since the adjusted stock effect increases with h, uniqueness can be examined based on whether the incentive to invest for each firm at the equilibrium increases when other firms delay investment. To verify this, we must take into account that $\Omega(h, H)$ simultaneously accounts for two margins, the stock effect and the value of firms at entry. Forces that increase the former lead to a lower h, while forces that increase the latter lead to more entry, increasing competition and delaying h. We must ask, then, how these two forces respond to changes in H. On the one hand, a larger H shifts the distribution of employment $G(x; H)$ towards firms that are further from investing; as we know from the previous section, this strengthens the stock effect. On the other hand, if firms take longer to invest, productivity y in (17) shrinks. The value of

[19] Total output and investment are determined by the firms' entry and investment decisions. Consumption is obtained residually from market clearing in the final good.

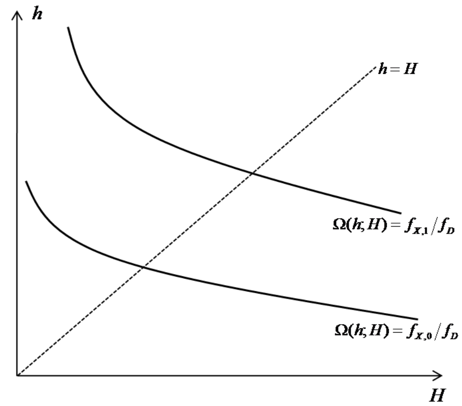

Figure 2: Investment timing h as function of H and f_X

unemployment w_u in (18) goes down as a consequence, increasing the value of a potential entrant. This induces entry and weakens the stock effect.[20]

Summing up, a larger H affects h through one negative-feedback channel (distribution of competitors) and one positive-feedback channel (worker's value of unemployment). In order to make progress, we can impose a sufficient condition on the parameters to ensure that the positive-feedback effect is weaker, namely:

$$\Gamma < \frac{1+\kappa_e/b}{1+\kappa_e}. \tag{23}$$

This condition requires that transfers to unemployed workers and the productivity differential Γ (i.e., the revenue-differential of exporters in a trade environment) are not too large relative to contacts made by employed workers.[21] When (23) holds, we can guarantee that the equilibrium H is unique.[22]

As for existence, we have, as in the partial equilibrium case, that there are no stock effects at entry, $\Omega(0, H) = 0$ for any H. Therefore, immediate investment of every firm cannot be an outcome if $f_X > 0$. An alternative candidate for an equilibrium is that firms never invest in the

[20] Later investment by competitors also implies that firms can attract more workers in every period, which is reflected in a larger upper limit of integration $\overline{x} = H$ in (6). However, the consistency condition (20) determines that this second effect disappears at the equilibrium. See the proof of Lemma 2 in Appendix A.2.

[21] The larger are Γ and b and the lower is κ_e, the greater is the increase in the value of unemployment that results from a reduction in H, in a context where firms rely relatively more on workers hired from unemployment to grow. This results in a stronger positive-feedback channel from larger H.

[22] See Appendix A.2 for proof. This condition depends on three parameters: $\{\kappa_e, b, \Gamma\}$. In the case of the exporting decision, we can impose natural restrictions on their values from readily available data to assess its validity. The share of GDP used to finance unemployment benefits in the model is $bu/(1-u)$, and from the OECD Social Expenditure Database, public spending on unemployment compensation as a fraction of GDP among OECD member countries has been on average 1% between 1980 and 2000. Jolivet et al. (2006) estimate the rate of contact on the job to be strictly lower than that from unemployment in each of eleven OECD countries since the mid-90's, implying $\lambda_e/\lambda_u < 1$. Average unemployment in the OECD since 1980 has been 7.7%. From these values, (23) determines an upper threshold for Γ of 1.8 when $\lambda_e/\lambda_u = 0.01$, and increasing in this ratio. E.g., for $\lambda_e/\lambda_u = 0.1$, approximately the value found in Jolivet et al. (2006) for France and for the UK, Γ must be smaller than 5. In turn, Mayer and Ottaviano (2010) find an exporter value-added premium of 2.7 in France and 1.3 in the UK. This raw evidence suggests that inequality (23) is not too stringent.

high-productivity technology. As in the partial equilibrium, the (free entry adjusted) stock effect $\Omega(h, H)$ is bounded, and firms invest if and only if

$$f_X/f_D < \lim_{H\to\infty} \Omega(H, H) = \frac{\rho+\delta}{\gamma} \frac{(\Gamma-1)(1+\kappa_e)}{1-b} \equiv \overline{f_X/f_D}, \tag{24}$$

i.e., whenever the sunk costs of upgrading are not too large relative to the cost of entry into the market.[23]

Summarizing the results from this section, we have

Proposition 2 *The equilibrium is unique. Firms never invest at entry if $f_X > 0$, but eventually invests if and only if $f_X/f_D < \overline{f_X/f_D}$.*

3.2 Comparative statics: Labor Market Environment and Productivity

I proceed now to show comparative statics about the impact of the labor market environment on productivity. The discussion from the previous section implies that changes in parameters that raise the adjusted stock effect $\Omega(H, H)$ for every H also result in a lower age for switching, biasing the distribution of employment towards more productive firms and raising aggregate productivity.

Proposition 3 *The switching age H and output per worker y are independent from the contact rate from unemployment λ_u. H is decreasing, and y, e_X and m_X are increasing, in contact rates on the job λ_e, transfers to unemployed workers as a fraction of income per worker b, the productivity differential Γ, and the cost of entry relative to the cost of upgrading f_D/f_X.*

The irrelevance of λ_u for the time of investment is a reflection of free entry. A higher contact rate with unemployed workers increases firms' rate of new hires from unemployment, but it also reduces the size of the unemployment pool. For an individual firm, this results in a positive proportional impact on the stock effect and on firm value. But the number of firms adjusts through free entry and competition heightens, offsetting this partial-equilibrium effect. In contrast, the frequency of contacts on the job λ_e has a non-proportional impact on the stock effect. It only strengthens the new hires margin in (12) through the higher entry rate of workers from other jobs. Variation in the number of firms cannot absorb this effect as with λ_u, and the adjustment occurs both through the number of firms and through the common age for switching. Broadly speaking, higher λ_e stretches the size distribution and benefits larger and older firms relatively more, while λ_u affects all firms in the same proportion.

Unemployment compensation b has an inverse effect relative to λ_u, in that it is irrelevant in partial equilibrium but it affects H in general equilibrium. In the partial-equilibrium analysis of Section 2, a higher value of unemployment reduces the value of firms by a constant amount, thereby leaving the trade-off between the stock effect and the fixed cost of investing unchanged. But in

[23] Note that whenever (23) holds there are values for the parameters such that (24) is not violated; e.g., if f_X/f_D is small enough.

general equilibrium, this reduction in firm value compresses the number of firms via free entry. This alleviates competition for workers in the labor market, allowing each firm to grow faster.

Qualitatively, the comparative statics on the impact of λ_e and b on productivity are similar to those in Acemoglu and Shimer (1999, 2000). Acemoglu and Shimer (1999) show that higher unemployment insurance makes risk-averse workers search for high-wage jobs, inducing firms to invest more in their own productivity. Acemoglu and Shimer (2000) show that worker search might be efficiency-enhancing when heterogeneous technologies are chosen by ex-ante identical firms. The current model has similar outcomes, but they are generated through a different mechanism; namely, the impact of search on the job and unemployment compensation on the stock effect.

3.3 Size distribution

In this economy, workers transit from young and small firms into old and large firms. Firms are continually exiting and being replaced by small entrants. This process originates a distribution of firm sizes. What does the model imply for its shape? In the data, a common feature of the size distribution of firms is a decreasing density in the upper tail. We can show that job-to-job transitions are necessary for the density not to be increasing in its entire domain, which would contradict this empirical evidence.

Proposition 4 *If the firm exit rate is lower than the rate of individual job termination ($\mu < \gamma$), then if job-to-job transitions are not allowed ($\lambda_e = 0$), the distribution of firm sizes has an increasing density in the entire domain; while if job-to-job transitions are allowed ($\lambda_e > 0$), there is a region in the distribution of firm sizes that has a decreasing density when the age of investment is sufficiently high.*

To understand this result, we can use a condition that holds whenever the density is decreasing. Let $N(h)$ be the size of firms of age h and $f(n)$ be the density of the distribution of firm sizes. Then, if $N(h) = n$,

$$f'(n) < 0 \text{ if and only if } \mu + \frac{N''(h)}{N'(h)} > 0. \tag{25}$$

This condition shows that there are two forces competing to determine the slope of $f(n)$: changes in net worker flows $N'(h)$ by firm age and the exit rate μ. Intuitively, if firm growth decelerates too fast and firms do not exit often, there is a tendency for firms to cluster at some point in the size distribution, which results in an increasing density.[24] Absent any transitions between jobs ($\lambda_e = 0$), net flows slow down at the rate of job separations, $N''(h)/N'(h) = -\gamma$. Then (25) implies that $f'(n) > 0$ if the rate of firm exit μ is below that of job separation γ. Indeed, $\mu < \gamma$ is what we see in the data. This means that, absent job-to-job transitions, there are no hopes in this model to generate a realistic distribution of firm sizes.

[24] Consider an extreme case with no firm exit where firms grow until a certain age and stop growing afterwards. The size distribution would collapse to a point at the size attained by firms at that age.

On the other hand, when $\lambda_e > 0$, there are two opposing forces: a constant number of workers is attracted in each period from unemployment at any firm size, but as firms age they attract progressively more worker from other firms. The first effect dominates at firm entry and the second dominates when firms are large enough but still do not invest. Therefore, if firms invest at a sufficiently old age, there is a region in the distribution of firm sizes where the density is decreasing.[25]

It is worth contrasting these properties with the outcome in Acemoglu and Hawkins (2010), where the slow hiring of firms in a frictional labor market also induces a distribution of firm sizes, but where there are no transitions between jobs or investment in productivity. As they note, the size distribution originating in their case has an increasing density; here, transitions from young and small firms who are far from investing into old and large firms are necessary for the distribution of firm sizes to exhibit a decreasing density in some part of its domain, as it is typically observed in the data.

4 International Trade

We can proceed now to the interaction between labor market frictions, trade, and income. Here, I show results on the impact of frictions, unemployment transfers and trade barriers on exports and income. In the next section, I will use a calibrated model to illustrate the impact of changes in the labor market and trading environments on welfare.

Suppose that there are two economies like the one described in the previous section, home and foreign. They differ, potentially, in labor market fundamentals $\{\lambda_u, \lambda_e, b\}$ and relative fixed costs f_X/f_D. Foreign country variables are denoted with an asterisk. From now on, I refer to the two types of firms that I have analyzed so far as exporting and domestic (i.e., selling only in the domestic market). The productivity advantage of type-X firms, Γ, is now a revenue advantage. All firms have the same physical productivity, but exporters generate more value for the same quantity of output. The main difference with the previous sections is that now Γ is endogenous.

4.1 Trade Environment

The trade environment shares the central features of Krugman (1980). Monopolistically competitive firms sell varieties of a differentiated good. These varieties are aggregated in each economy in the production of a final non-tradable good using a technology with constant elasticity of substitution (CES) $\sigma > 1$ across varieties. Exporters face iceberg trade costs τ, that potentially differ across economies.

[25] See proof of Proposition 4 in Appendix A.2. Since firms that have invested do not increase the value of their jobs as they grow, for $N(h) > N(H)$ the density of the size distribution is again increasing. However, this property is an artifact of allowing for just one investment opportunity. If firms chose to sequentially implement multiple investments –a case that I consider in Section 6– this region would only exist above the largest size at which firms implement an upgrading. Hence the interval with an increasing density in the upper tail shrinks as more expensive investment options are allowed into the model.

A known feature of this framework is that product differentiation leads to downward sloping demand and concave revenue-functions. As firms expand their supply, consumers derive a progressively lower marginal utility from a particular variety. In order to incorporate the linear revenue functions that I have used in the analysis so far into a trade setting, I extend this basic CES framework with a quality choice by firms. Thanks to investing in quality, firms can shift their demand curves outwards instead of necessarily sliding them down as they offer more output. In section A.3 of the appendix, I show that when workers are perfectly substitutable inside the firm between the production of quality and quantity, both effects exactly compensate as the firm expands. As a result, the willingness of consumers to pay for a variety remains constant as the firm producing that variety increases supply. I adopt this specification for analytical convenience.[26]

In this setup, the revenue premium of exporters takes the form:

$$\Gamma = \left[1 + \tau^{-(\sigma-1)} \left(\frac{p^*}{p}\right)^{\sigma}\right]^{\frac{1}{\sigma}}, \tag{26}$$

where

$$p = P\left[(1-u)\,y\right]^{\frac{1}{\sigma}}$$

reflects the size of the home economy in terms of its price index P and income per capita.[27] Therefore, from (26) we see that Γ depends on

$$\frac{p^*}{p} = \frac{P^*}{P}\left[\frac{(1-u^*)\,y^*}{(1-u)\,y}\right]^{\frac{1}{\sigma}}.$$

From the perspective of an individual firm, the relative size of the foreign economy increases due to less competition (higher P^*/P) or higher demand (higher y^*/y), resulting in a larger Γ. The revenue premium reflects that, by exporting, firms can sell to consumers who have on average a higher willingness to pay for its products. Revenues of each type of firm are measured in terms of the domestic non-tradable good.[28]

In the previous sections, we have treated Γ as parameter. Proposition 3 shows how the fraction of high-productivity firms –now, exporters– and the share of employment in these firms react to this premium. I denote these reduced-form responses from the previous section as $m_X(\Gamma)$ and

[26] Decreasing marginal revenues would constitute an additional determinant of the marginal value of jobs. This effect would come at great analytical cost without much theoretical insight. Furthermore, it is not necessary to match salient properties of the data on firm dynamics and job-to-job transitions. See the quantitative part in Section 6.

[27] In the terminology of Redding and Venables (2004), Γ is the "market access" and p is the "market capacity" of the home country. McGrattan and Prescott (2008) derive an expression similar to Γ in (26) in an open economy setting with perfectly competitive product markets and decreasing returns in production. In their case, the productivity increase results from diversification of resources across destinations by multinational firms.

[28] Revenues per worker in domestic firms, y_D, will now also react to changes in the trading environment. Due to monopolistic competition, the value of sales depends on market size, which is affected by trade. As implied by (51) in Appendix A.3, we have now that $y_D = [(1-u)\,y]^{1/\sigma}$. This introduces one minor practical difference with our previous analysis of the single-country equilibrium. Replacing the value for y_D in our expression for aggregate productivity, y, defined in (17), gives $y = (1-u)^{1/(\sigma-1)}\left[1 + (\Gamma-1)\,e_X(H)\right]^{\sigma/(\sigma-1)}$. We still have, as in the single-country case, that reductions in H raise y; now they do so in a larger magnitude due to the feedback between income and demand characteristic of monopolistic competition.

$e_X(\Gamma)$. Now, we are interested in examining how trade determines Γ, and, through this "price", the response in these aggregate variables.

4.2 Equilibrium definition and uniqueness

The solution for the aggregate variables in the home and foreign countries can be divided in two interdependent blocks of equations. A first block yields exports in each country given Γ. As shown in the appendix, exporters in the home country sell abroad at price $\tau\Gamma p$, and ship abroad a fraction of their output equal to

$$s_X(\Gamma) = 1 - \Gamma^{-\sigma}. \tag{27}$$

The total value of exports from the home country is thus

$$X(\Gamma) = (\tau\Gamma p)\, Q_X(\Gamma), \tag{28}$$

where $Q_X(\Gamma) = (1-u)\, e_X(\Gamma)\, s_X(\Gamma)$ are exported units of output.

The second block of equations concerns the relation between the two economies through the balance of payments. The revenue premia Γ and Γ^* must be such that trade balances, $X(\Gamma) = X^*(\Gamma^*)$. Using (28) and rearranging terms, we can write this balanced trade condition as

$$\frac{p^*}{p} = \left(\frac{\tau\Gamma}{\tau^*\Gamma^*}\right)\frac{Q_X(\Gamma)}{Q_X^*(\Gamma^*)}. \tag{29}$$

Note also that, from the definition of the exporter revenue premium in (26), an increase in the exporter premium in one country is associated to a reduction in the premium in the other country,

$$\frac{p^*}{p} = (\Gamma^\sigma - 1)^{\frac{1}{\sigma}}\, \tau^{1-\frac{1}{\sigma}} = \frac{1}{(\Gamma^{*\sigma} - 1)^{\frac{1}{\sigma}}\, \tau^{*1-\frac{1}{\sigma}}}. \tag{30}$$

Using these two blocks of equations, we can define an equilibrium with two countries:

Definition 2 *An international trade equilibrium consists of revenue premia* $\{\Gamma, \Gamma^*\}$*, a relative size of the foreign market* p^*/p*, and outcomes in each country* $eq \equiv \{h, H, M, P(\cdot), G(\cdot), m_X, e_X, y, c, w_u\}$ *and* $eq^* \equiv \{h^*, H^*, M^*, P^*(\cdot), G^*(\cdot), m_X^*, e_X^*, y^*, c^*, w_u^*\}$ *such that*
a) Given $\{\Gamma, \Gamma^*\}$*, home and foreign country outcomes* $\{eq, eq^*\}$ *are single-country equilibria by Definition 1; and*
b) $\{\Gamma, \Gamma^*, p^*/p\}$ *satisfy (29) and (30).*

As shown in the appendix, a larger exporter premium in the foreign economy must be met with a larger exporter premium in the trading partner for trade balance to hold. To ensure that exporters emerge in both countries I work henceforth under the assumption that the relative fixed cost of exporting f_X/f_D is sufficiently small, but positive, or that the upper bound for these costs in (24) is sufficiently large. In that way, I ensure existence and uniqueness of the equilibrium.

Proposition 5 *If $(\rho+\delta)(1+\kappa_e)/[\gamma(1-b)]$ is sufficiently large or if f_X/f_D is sufficiently small, there exists a unique trade equilibrium.*

4.3 Comparative Statics: Labor Market, Trade, and Income

I ask, next, how income and export participation are affected in each country by changes in labor market parameters and trade costs. The simplest case to analyze is an environment with symmetric countries. Since the equilibrium is unique, it must also be symmetric, implying that $p^*/p = 1$; the revenue-premium is then readily given by (26):

$$\Gamma = \Gamma^* = \left[1 + \tau^{-(\sigma-1)}\right]^{\frac{1}{\sigma}}.$$

Firms respond in the same way to Γ in both countries, hence trade balances by construction. Lower trade costs have naturally the effect of making exports more profitable through a larger Γ. The same occurs in the comparison between pairs of countries trading in industries with different degrees of product differentiation σ. The more differentiated the industry (the lower is σ), the larger the revenue advantage of exporters; comparative statics within country follow as in the single-country response to a larger Γ.[29] Meanwhile, taking τ and σ as given, changes in labor market variables or in fixed costs do not affect the revenue premium. Therefore, when we start from a symmetric configuration and changes in parameters occur simultaneously in both economies, the results in Proposition 3 imply the following.

Corollary 1 *In a trade equilibrium with symmetric countries, higher contact rates on the job λ_e or unemployment transfers b, and lower trade barriers τ, demand elasticity σ or relative costs of exporting f_X/f_D, lead to a reduction in the age for entry into exporting H and to an increase in income per worker y, in export participation m_X, and in the share of employment in exporting firms e_X in both countries.*

The central implication of this result is that trading partners gain from the joint implementation of labor market policies that facilitate transitions between jobs or more generous compensation to unemployed workers. In the quantitative exercises below, I also consider the implications on welfare.[30] I ask next how policies in the foreign country affect the domestic economy through trade. We can show that labor market policies that favor export participation in the foreign economy have a positive impact on income in the home country and in exports in both countries.

Suppose that countries are asymmetric, and consider an increase in unemployment compensation or in the rate of job-to-job transitions abroad. From Proposition 3, these changes imply a

[29] Note that trade costs τ do not affect any variable other than Γ. Product differentiation σ also affects revenues of domestic firms, but in general equilibrium this has no effects on H.

[30] Helpman and Itskhoki (2010) address similar questions in a static environment with ex-ante differences in productivity across firms and frictional labor markets. Job-to-job transitions are not present in their analysis, while here they are key to determine aggregate outcomes. On the other hand, in their framework, higher unemployment compensation may have a detrimental impact on trade when it raises labor costs enough. Here, it leads to faster switching into exporting, resulting in higher exports and income.

reduction in the common age for switching H^* given Γ^*, causing an increase in exports from the foreign country. The trade balance condition (29) then requires that the larger quantity imported by the home country is met with a higher Γ. In the new steady state there is a higher exporter premium in the domestic economy, and a lower one in the foreign country. This outcome resembles the standard adverse response in terms of trade faced by specialized countries that experience a productivity shock, common to many open-economy models. To see the final outcome in each country, we must feed back these changes in Γ and Γ^* into the single-country responses. Following similar steps we can characterize the impact of a reduction in trade barriers faced by exporters in the home country. The results are summarized as follows.

Proposition 6 *In a trade equilibrium with asymmetric countries, an increase in the rate of contacts on the job* λ_e^* *or in compensation to unemployed workers in the foreign country* b^* *leads to an increase in the share of exporting firms and in the share of employment in these firms in both countries, and to an increase in income* y *in the home country. A reduction in trade barriers faced by home exporters* τ *leads to an increase in the share of exporting firms, in the share of employment in these firms and in income* y *in the home country.*

Corollary 1 and Proposition 6 are result of a positive feedback between income per worker of trading partners. Exporting firms are high-income firms, because they generate more value than non-exporters for the same amount of output thanks to selling in two destinations. The prevalence of these firms, and their share of employment, depends on Γ, that captures the relative size of the foreign country. When the foreign country implements a policy that favors transitions between jobs, it promotes an increase in export participation. If this were the overall response, trade would not be balanced. However, at impact, this also raises income per worker in the foreign market, increasing the exporter revenue premium in the home country. As a result, firms in the home country also reduce the age for switching into exporting, and output exported by home firms adjusts up to the point that trade balances again. In the new equilibrium, both countries have a larger share of employment in the export sector.[31]

The fact that it takes time for firms to export ultimately generates these effects. As in Krugman (1980), there are no exogenous differences in productivity; but as in Melitz (2003), there is selection based on firm size. Older firms are larger and select into exporting. In Krugman (1980), in order for trade to balance, an increase in the size of an economy is met in equilibrium by an increase in the incentives to export to that country. While in that model this occurs through the appreciation of the real wage in the economy experiencing the positive shock (i.e., the home market effect) to induce entry or exit of firms (all of whom are exporters), here the adjustment to a change in conditions occurs through the revenue premia of both countries, to induce an earlier or later age

[31] The positive correlation between transfers to unemployed workers and openness echoes the empirical results of Rodrik (1998), who finds a positive link between government spending and openness. This result is interpreted as reflecting that governments increase spending to compensate workers for the risk associated with globalization. The present model suggests the reversed causality: countries that compensate unemployed workers to a greater extent turn out to be more open.

of switching. In this sense, the margin of adjustment shares the spirit of Melitz (2003), in that it derives from worker reallocations towards high-productivity firms (in our case, high-revenue firms) and from firms switching export status.

Proposition 6 establishes results about the effect of independent changes in trade barriers and in the labor market environment on income per employed worker and trade. I have not yet inquired about the impact of changes in these variables on welfare, or about the interaction between both changes in policy. I address these issues in the quantitative exercise of Section 6.

5 Patterns of Firm Dynamics and Job-to-Job Mobility in the Data

In this section I show that the model predictions are qualitatively consistent with salient patterns of firm dynamics and job-to-job mobility seen in the data. In the next section I will use these moments of the data to quantitatively evaluate the main effects in the theory.

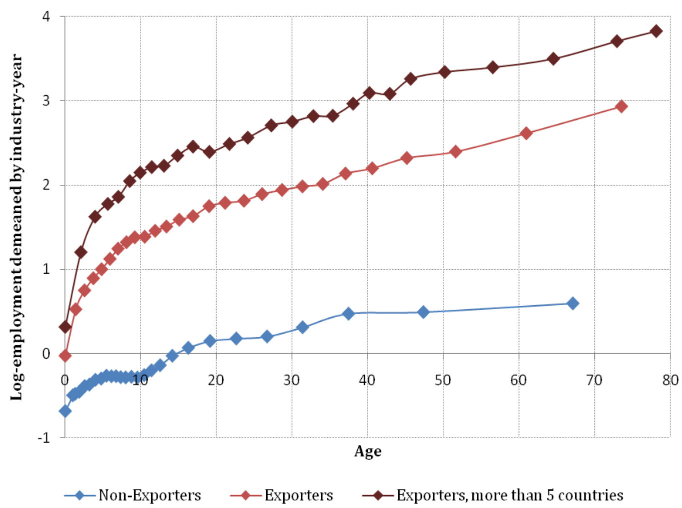

Figure 3: Firm size by age and export status

Figure 3 is based on data for the formal manufacturing sector in Argentina for the period from 1999 to 2007. In the vertical axis it shows average log-employment, demeaned by industry-year, of non exporters, exporters, and exporters to more than 5 countries. These are plotted against 25 age quantiles within industry-year-export status, plus the top 1% within these groups. Entrants appear at age zero.[32]

There are three salient patterns in this figure. First, as it is well known in the industrial organization literature, older firms are larger.[33] Second, exporters are larger than non-exporters conditioning for age, as are firms that export to many countries relative to firms that export to

[32] Figures only include continuing firms. For a description of the data see Appendix B.
[33] See for example Dunne et al. (1989).

few countries. Furthermore, the gap between types widens over time.[34] Third, the age quantiles of non-exporters are more concentrated in low ages relative to exporters, as are those of exporters to any number of countries relative to exporters to at least 5 countries. This reflects that exporters are relatively older.

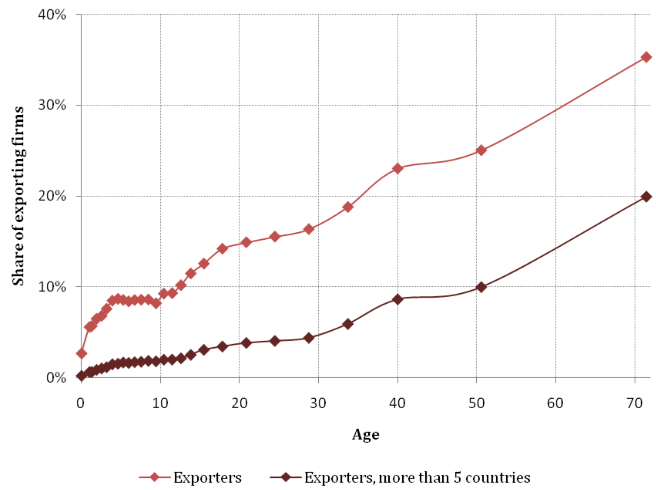

Figure 4: Export participation by firm age

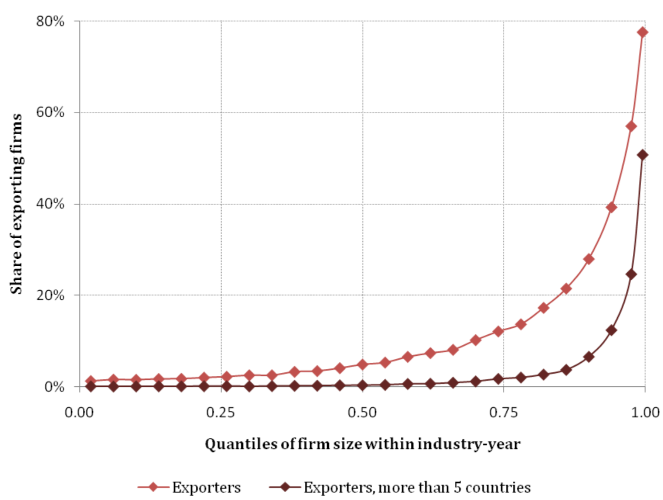

Figure 5: Export participation by firm size

The rise in export participation as firms age is depicted in Figure 4. It shows the average fraction of exporters to any number of countries, and of exporters to at least 5 countries, within 25 age quantiles within industry-year, plus the top 1%. Entrants appear again at age zero. About

[34] See Bernard and Jensen (1999) for evidence of high relative employment growth rates of U.S. exporters.

30% of firms in the oldest 5% are exporters, while on average across cohorts born between 1998 and 2003, 2% of firms are born as exporters and 10% export during their first 5 years of age.[35]

These positive correlations between age and size, and between age and export status, naturally underlie the well known fact that larger firms are more likely to export. This is shown in Figure 5. The figure presents the share of exporters, and of exporters to at least 5 destinations, for 25 groups of size (in number of workers) plus the top 1% within industry-year. Only 1% of the smallest 5% of firms export, in contrast with 60% in the largest 5%, and the share of firms exporting to more than 5 countries rises sharply towards the highest percentiles of the size distribution.[36]

The baseline theory in this paper is able to reproduce, qualitatively, these patterns of the data. The activity of finding and hiring workers is not immediate, and as such generates an adjustment cost that drives the evolution in the stock of workers. Therefore, older firms are larger. The correlation between age, size and export status arises from the waiting time of firms until they have enough workers to justify paying the fixed costs to export. On the other hand, differences in the rates at which firms of different export status accumulate workers are a reflection of random assignment of workers to firms and search on the job. By exporting, firms do not only increase their revenues, but they also become stronger competitors in the labor market and grow faster.[37]

Complementary forces other than slow labor adjustments might contribute to the patterns in figures 3 to 5. One can think, for example, of slow capital accumulation or learning about exporting.[38] A distinguishing feature of my analysis, absent from such explanations, is that it has implications for job-to-job mobility. Employed workers hold better outside options than unemployed workers, and so do workers employed in exporting firms relative to workers employed in non-exporters. By offering jobs of higher value than non-exporters, exporters are more likely to hire workers away from other firms rather than from the unemployment pool, as well as from other exporters rather than from non-exporters. As we have seen, these patterns in the composition of new hires constitute the core of the mechanism underlying the comparative statics –labor market frictions affect aggregate outcomes only when job-to-job transitions are present. In what follows, I show that these patterns in the composition of new hires are present in the data, and in the next section I use the observed rates of job-to-job hiring by export status to parametrize the model.

Consider, first, the share of new hires entering firms from other jobs. Figure 6, constructed using the same population of firms as the previous figures, presents for each year between 1999 and 2007 the average, across manufacturing firms, in the share of new hires attracted from another formal job in the economy, split in the three groups by export status.[39] The complement of this fraction

[35] See Eaton et al. (2008a) and Albornoz et al. (2010) for evidence on the probability of switching into exporting.

[36] Hallak and Sivadasan (2009) report similar patterns of export participation across the size distribution in India, U.S., Chile and Colombia.

[37] In the theory, only firms above a threshold of age or size are exporters. As a consequence, exporters and non-exporters do not overlap for the same age or size. In the quantitative section below, I include ex-ante firm heterogeneity to reconcile the model with the slow increase in export participation by firm age and size in the data.

[38] See the literature review in the introduction for papers that consider these effects. Arguably, in the presence of other dynamic factors, the role of labor market frictions in shaping firm growth depends on the complementarity between alternative resources. Eslava et al. (2009) find evidence of complementarity between labor and capital adjustment costs in Colombian firm-level data.

[39] These measures are computed from linked employer-employee data representative of the formal employment

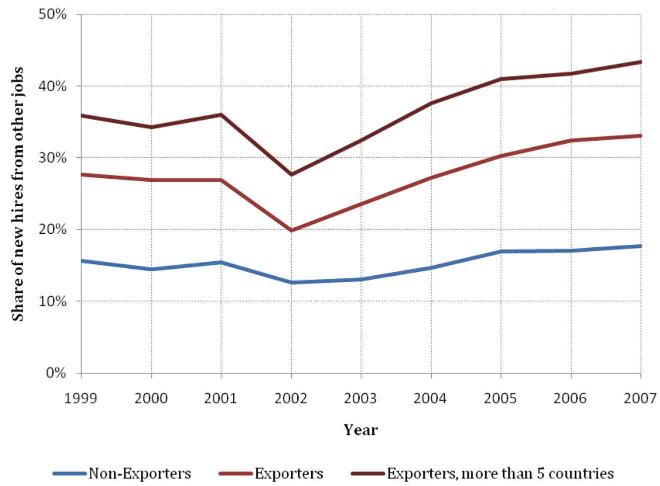

Figure 6: Share of new hires attracted from formal jobs among all new hires, by year and export status

corresponds to workers attracted from either unemployment or from the informal employment sector. On average in the entire sample period, 37% of all new hires in firms that export to more than 5 countries enter from jobs in the formal employment sector, in contrast to 25% in firms that export to no more than 5 countries and to 15% in non-exporters. Similarly, workers employed by manufacturing exporters are harder to attract than workers in manufacturing non-exporters. Figure 7 shows that the greater the intensity of export activity, the higher the chances to lure workers from exporters.[40] The ranking in figures 6 and 7 holds markedly throughout all the years in the sample, although it covers two very distinct phases of the macroeconomic cycle.[41] In Appendix B, I show that the differences in the composition of new hires by number of destinations are statistically significant when controlling for industry-year effects as well as for firm age, size, average wage and net job creation.

Arguably, the combined effect of other forces may contribute to the patterns in figures 3 to 7.[42] The theory in this paper qualitatively generates them all based on a single mechanism. In the next section, I perform a quantitative evaluation of the theory.

sector. See Appendix B for details.

[40] See also Muendler and Molina (2009) and Mion and Opromolla (2010) for evidence consistent with this pattern in Brazil and Portugal, respectively.

[41] The Argentine economy was in recession between 1999 and 2001, hit a trough in 2002, and boomed between 2003 and 2006, with a slight reduction in economic activity in 2007. The pace of the downturn and the recovery, in terms of number of firms and total employment, was similar for exporting and non-exporting firms.

[42] E.g., it is possible to envision a model incorporating alternative drivers of firm dynamics, differences in skill requirements by export status, and variation in unemployment incidence by skill group. To my knowledge, no paper investigated this route. Note that labor market frictions would also have an effect on aggregate outcomes in the presence of these forces, affecting hiring costs by skill as well as the timing skill-biased investments. Disentangling the relative importance of these alternative effects lies beyond the reach of this paper, but is interesting for future work.

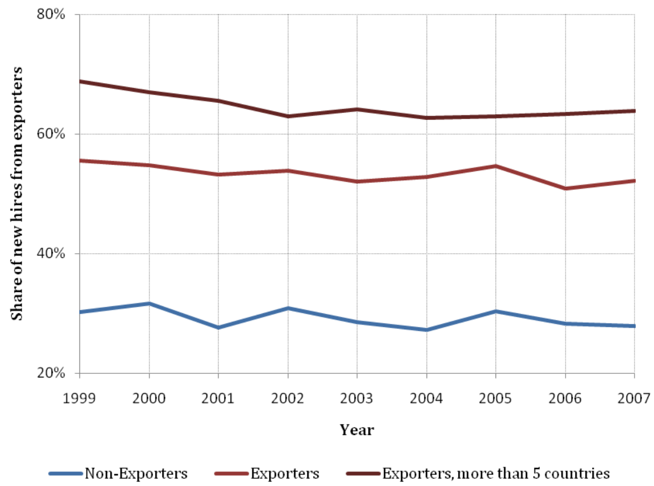

Figure 7: Share of new hires attracted from formal jobs in exporters among new hires attracted from formal jobs in manufacturing, by year and export status

6 Quantitative Analysis

The previous sections have traced the theoretical link between labor market frictions, trade and income. I have also shown that the theory is consistent with features of firm dynamics and job-to-job mobility observed in the data. Here I extend the basic model in several directions. The goal is to develop a framework that can match aggregate moments of the data so that it can be used to evaluate changes in labor market conditions and in trade barriers on income and welfare. I also want to assess how good a theory of the firm life cycle this is, based on the model's ability to reproduce some of the patterns in Section 5. To these ends, it is necessary to include additional ingredients. I introduce three extensions: two export destinations, endogenous recruitment effort, and ex-ante differences across firms.

6.1 Extensions to the basic model

I motivate each of the extensions in contrast to properties of the model developed so far, referred to as the "basic model".

Two export destinations I assume that there are two export markets, $k = 1, 2$. As figures 3 to 7 show, the patterns in the data are not just about exporting, but rather about the intensity of export activity as measured by the number of destinations. Also, as we have seen, the inclusion of more than one destination helps to generate a size distribution of firms with realistic shape. Proposition 4 says that if firms become exporters very early in life the basic model necessarily implies a size distribution with increasing density in a large interval of the domain; with more than one export destination this restriction can be relaxed.

Endogenous recruitment effort In the basic model firms are not able to choose the number of workers that they sample per period. This fixes the ratio between the average size of exporters and non-exporters at $1+\kappa_e$, restricting the share of exporting firms and the share of employment in these firms to move in the same proportion. This needs not be the case if firms can choose the recruitment effort s in (4), since in that case firms with higher valuation for meeting new workers can recruit more aggressively. Following Bertola and Caballero (1994) among others, I assume a convex cost of search, $c(s) = s^{\zeta}$ with $\zeta > 1$.[43]

Heterogeneity In the spirit of Melitz (2003), I allow for ex-ante differences across firms in productivity, ψ, and fixed costs, ϕ. The patterns of export participation over age and size in Figures 4 and 5 are reproduced starkly in the basic model, where all firms export above a threshold of age or size. With heterogeneity, these patterns result from mixing across types of firms that start to export at different ages.

6.2 Extended Firm problem

This section proceeds in parallel with Section 2. I formulate the problem of a firm, showing how the timing of entry across different export markets varies with firm productivity and relative entry costs. The equilibrium definition is relegated to Appendix A.4. Various objects are referred to with the same notation as in Section 2, but with different arguments.

Firms are now distinguished by their draws of productivity ψ and fixed cost ϕ, denoted by $\varepsilon \equiv \{\psi, \phi\}$. They also differ in their time until investing in each of the two markets. I write the next expressions under the assumption that firms enter first in market $k = 1$, and I show below a sufficient condition for this to be true in any firm. The time until entry into each of the two export markets is denoted by $x \equiv \{x_1, x_2\}$, where $x_k \in [0, \infty]$.

From the proof of Lemma 1 in Appendix A.1, the value of a new job is now

$$v(x;\psi) = \frac{\psi\left[y_0 + e^{-(\rho+\delta)x_1}(y_1 - y_0) + e^{-(\rho+\delta)x_2}(y_2 - y_1)\right] + \delta w_u}{\rho+\delta}. \tag{31}$$

With respect to the value of a job in (3), this expression adds productivity ψ as a shifter of revenues and an additional export destination. Using (53) in the appendix and normalizing the domestic price index to 1, revenues per unit of output of domestic firms and each exporter type are

$$y_0 = Y_0^{\frac{1}{\sigma}}, \tag{32}$$

$$y_1 = \Gamma_1 Y_0^{\frac{1}{\sigma}} = (Y_0 + A_1)^{\frac{1}{\sigma}}, \tag{33}$$

$$y_2 = \Gamma_{12} Y_0^{\frac{1}{\sigma}} = (Y_0 + A_1 + A_2)^{\frac{1}{\sigma}}, \tag{34}$$

[43] This cost is measured in units of the final good. Since revenues are linear, convexity is needed to bound the number of new hires. Some papers studying labor adjustment costs, like Cooper et al. (2007), also include a fixed cost of adjustment; in the present setup, where all firms want to expand and there is no firm-level uncertainty, a fixed cost of adjusting the size of the labor force would be captured in the per period operative cost.

where Y_0 is home aggregate income and $A_k \equiv \tau_k^{-(\sigma-1)} P_k^{\sigma} Y_k$ for $k = 1, 2$ reflects the size of foreign market k.

To characterize the firm problem, we must consider new aggregate variables. The basic model was structured around the observation that the time to switch into exporting was a sufficient statistic for the value of a new job. Given the employment distribution across times until switch, $G(x)$, we knew the share of workers that a firm at x could hire among all contacted workers. In the current scenario this no longer holds, but we can determine the return on search by considering the distribution of employment across firms offering jobs with different value, $G_v(v)$. Given this distribution, firms know the yield on their search effort, and subsequently decide how much to search. In the appendix, I show that from the search decision of firms we can obtain an expression, equivalent to (6), for the value of all workers who enter a firm with productivity ψ at x, now denoted as $\pi(x; \psi)$.

Using $\pi(x; \psi)$ from (57) in the appendix, we can rewrite the firm's problem (7). Firms are born as domestic producers, but they can now access two markets $k = 1, 2$ by paying entry costs with flow equivalent values of ϕf_k. Now, f_k is a component of entry costs in market k that is common across firms and ϕ is firm specific. Let h_1 be the age at which the firm chooses to enter market 1 and h_2 be the lapse before the firm enters market 2 *after* it has entered market 1. The problem of the firm is now to choose h_1 and h_2 to maximize its value at entry.

We can show that the solution to this extended firm problem has, as before, a simple structure based on the stock effect. Similarly to (10), we can define $S_1(\cdot)$ as the change, after a delay in the time of entry into market 1, in the present discounted value of all workers attracted between ages 0 and h_1; and $S_2(\cdot)$ as the change, after a delay in the age of entry into market 2, in the value of all workers attracted between ages h_1 and $h_1 + h_2$.[44] In an interior solution the firm chooses h_1 and h_2 that satisfy

$$S_1(h_1, h_2; \psi) = \phi f_1, \tag{35}$$

$$S_2(h_2; \psi) = \left(\frac{f_2}{f_1} - \frac{\Gamma_{12} - \Gamma_1}{\Gamma_1 - 1}\right) \phi f_1. \tag{36}$$

There are some novel features in these first order conditions compared to the basic model. Stock effects depend now on firm-specific productivity and fixed costs. We also have that h_2 affects the incentives to enter in market 1 through the value of a worker attracted before h_1. And the timing of entry depends on a combination of the relative fixed costs and revenue differentials in each market. This solution to the firm's problem was derived under the assumption that the firm enters first in market 1. Proposition 7 summarizes the comparative statics on the firm's decision and presents a sufficient condition such that this is indeed the outcome.

Proposition 7 *At an interior solution, (i) ages of entry into both markets are decreasing in productivity ψ and increasing in the cost shifter ϕ; (ii) the lower are f_2/f_1 or $(\Gamma_{12} - \Gamma_1)/(\Gamma_1 - 1)$,*

[44] See (60) and (61) in the appendix for explicit expressions of S_1 and S_2.

the earlier the firm enters in export market 2 *given the age of entry into market* 1*; and (iii) if the firm enters in both markets and* $f_2/f_1 > \max\left[\left(\Gamma_{12}-\Gamma_1\right)/\left(\Gamma_1-1\right),\left(\Gamma_2-1\right)/\left(\Gamma_{12}-\Gamma_2\right)\right]$*, it enters first in market* 1.

6.3 Calibration

In the appendix, I define a general equilibrium of the extended model. Using the equilibrium conditions from Definition 3 in Appendix A.4, I solve the model numerically and choose parameters to match aggregate moments for the manufacturing sector in Argentina for the period 2003 to 2007. The calibration strategy is as follows. First, some parameters are set to match their empirical counterparts. The exit rate of firms is set at $\mu = 0.075$ to fit the density of the age distribution. The exogenous job separation rate is set to $\gamma = 0.15$, to match the probability that workers employed in non-exiting firms move into the unemployment pool within the year. The unemployment rate is set at the average rate over the period of $u = 10\%$ according to the Argentine institute of statistics. The rate of time discount ρ matches an average interest rate of 6% on deposits at the fourth quarter of each year according to the Argentine Central Bank. The elasticity of demand σ equals 3, as in the estimate of Eaton et al. (2008b). I normalize the operative fixed cost to $f_0 = 10$, and I set the firm specific shifter of exporting costs ϕ to be uniformly distributed between 0 and 2.

There are 8 remaining parameters: revenue premia $\{\Gamma_1, \Gamma_2\}$, exporting costs $\{f_1, f_2\}$, labor market fundamentals $\{b, \lambda_e\}$, convexity in hiring costs ζ, and the shape parameter σ_ψ in the distribution of productivity ψ, which I assume to be Pareto. I choose their values to minimize the sum of squared residuals between the prediction of the model for ten aggregate moments and their empirical values. The two markets $k = 1, 2$ of the model correspond, in the data, to firms exporting to up to 5 countries and to more than 5 countries.[45] I match moments from the data that correspond to the main aggregate outcomes predicted by the theory: the fraction of firms exporting to different number of destinations, their shares of total employment, their average age, and their share of job-to-job transitions in new hires. The moments generated by the model as well as the targets in the data are reported in Table 1. The model matches closely all targets except for the average age of exporters to more than 5 countries.

As a result of the calibration I find: $\Gamma_1 = 1.2$, $\Gamma_2 = 1.3$, $f_1/f_0 = 1.8$, $f_2/f_0 = 6.1$, $\lambda_e/\lambda_u = 0.1$, $b = 0.04$, $\zeta = 1.9$, and $\sigma_\psi = 3$. Quantitatively, the levels of job-to-job transitions in the data are key to determine the probability of a contact on the job, λ_e. The value for λ_e relative to λ_u is line with results of Jolivet et al. (2006) in various OECD countries. The value for b implies a share of GDP spent in unemployment compensation of 0.5%; this is similar to the value in many relatively low-income OECD countries in recent years according to the OECD Social Expenditure Database.[46] The convexity in the labor adjustment cost ζ is below the finding by Yashiv and Merz (2007) for the U.S., but I do not include scale effects which are present in their estimation and tend

[45] Since most Argentine firms that export to less than five countries export to Mercosur, the two markets in the model broadly correspond to exports to this block of countries and to exports to the rest of the world.

[46] E.g., 0.1% in Turkey, 0.3% in Slovak Republic, 0.4% in Greece, and 0.5% in Hungary.

to reduce the effective cost. Finally, the average exporting cost to at least 5 countries appears to be particularly large. However, the firm-level shifter in the exporting costs ϕ implies the presence of firms with arbitrarily small costs of exporting; the estimated average would be lower in the absence of this heterogeneity.

Calibrated Moments	Model	Data
Share of firms exporting to up to 5 countries	8%	8%
Share of firms exporting to more than 5 countries	3%	3%
Share of employment in exporters to up to 5 countries	21%	20%
Share of employment in exporters to more than 5 countries	32%	33%
Average age of non-exporters	12	12
Average age of exporters to up to 5 countries	16	19
Average age of exporters to more than 5 countries	17	28
Share of job-to-job hires in total hires of non-exporters	13%	16%
Share of job-to-job hires in total hires of exporters to up to 5 countries	31%	27%
Share of job-to-job hires in total hires of exporters to more than 5 countries	39%	40%

Table 1: Matched moments, model and data

6.4 Cross-Sectional Predictions

The central prediction of the theory is the timing of firm entry into different export markets and the allocation of workers across firms with different export status. Given the (exogenous) age and productivity distributions, these outcomes give the fraction of exporting firms and their employment allocation. Since in the calibration I match these aggregate moments, a natural question is whether the model does a good job in replicating the profile of export participation over the firm age and size distributions that we see in the data.

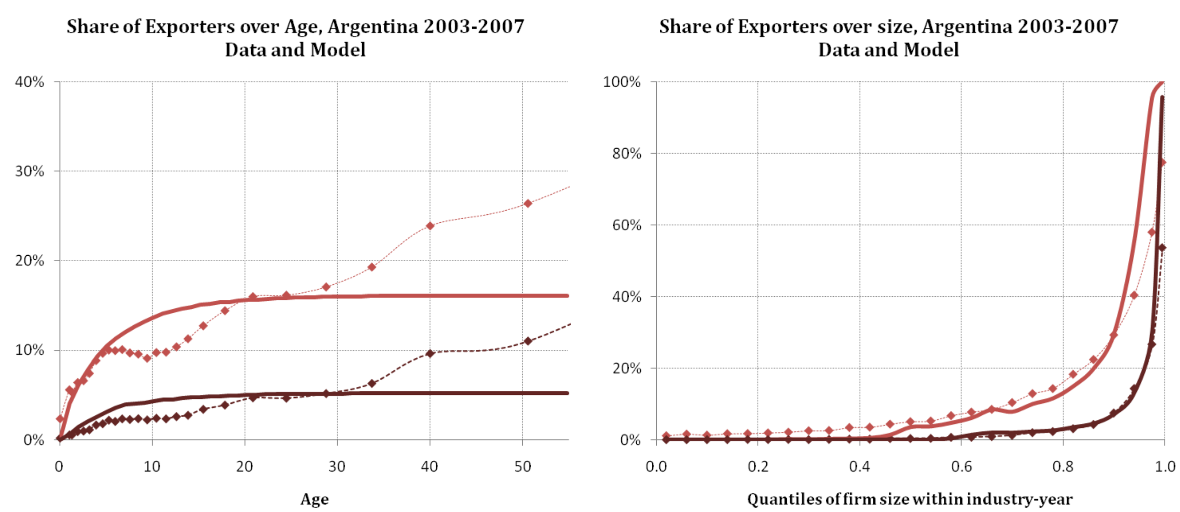

Figure 8: Export participation by firm age and size, data (dotted lines) and model (solid lines)

Figure 8 shows the parametrized model's predictions for figures 4 and 5 from the previous section. It depicts, in solid lines, the share of exporters and the share of exporters to up to 5 countries by firm age (left panel) and by firm size (right panel) generated by the model (solid lines), as well as their actual empirical values (dotted lines). In the calibration, I have matched averages in these figures, and now I ask what the calibrated model implies for the entire profiles. The model fairly reproduces the increase in export participation for firms younger than 30 years and the share of exporters to more than 5 countries across the size distribution, although it over-predicts the share of exporters among the largest firms.

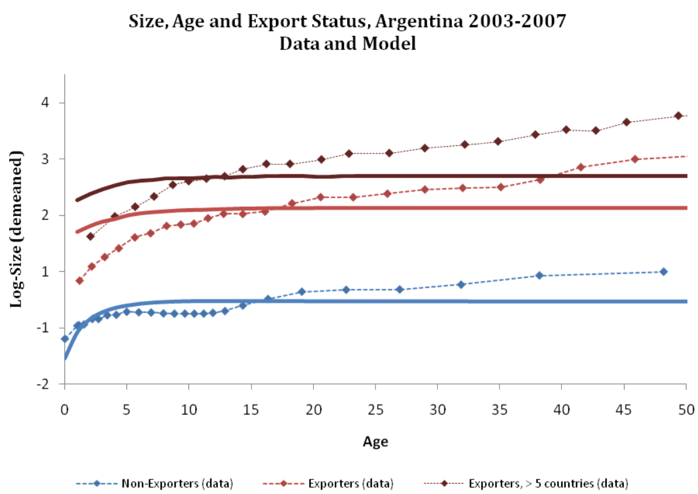

Figure 9: Firm size by age and export status, data (dotted lines) and model (solid lines)

Another question is whether the model can account for dispersion in firm size by export status. Figure 9 shows the calibrated model's prediction for Figure 3. It depicts the average (demeaned) log size of firms with different export status within groups of age. There are two effects generating the increase in size by export status in this figure: growth of continuing firms within each export group and selection of firms between exports groups over age. As implied by Proposition 7, more efficient and lower-cost firms in the group of non-exporters select themselves into exporting as they age, while, within the group of exporters, more efficient and lower-cost firms select themselves into the group of firms exporting to more destinations.

The model captures the growth rate within non-exporters that we see in the data, as well as the constant gap in size between exporters to different number of destinations. However, it fails to predict steady growth among exporters as seen in the data. This constant increase in log-size suggests the presence of forces not taken into account in the model. Natural candidates for this trend are accumulation of another resource, learning or size-dependency in the search technology.

6.5 Simulated Policies

I use the parametrized model to measure changes in welfare after changes in the trade and labor market environments. I simulate an increase in transfers to unemployed workers, and I ask about the differential impact of a reduction in trade costs in labor market environments with different flexibility. This question has been the focus of recent quantitative explorations in different setups, such as Kambourov (2009) and Cosar (2010).[47]

The overall methodology is as follows. From the calibrated values of Γ_1 and Γ_2 and the value for aggregate income Y_0 that results from the calibration, I use equations (33) and (34) to infer foreign market sizes A_1 and A_2. Then, I simulate changes on these foreign market sizes and on the calibrated labor market fundamentals b and λ_e. For each new set of parameters I recompute the equilibrium, including new values for Γ_1 and Γ_2 that result from the change in income Y_0. I make the small-country assumption that foreign market sizes A_1 and A_2 do not change in response to changes in outcomes in the home economy. Welfare equals aggregate consumption of the final good, and is found residually given the equilibrium at home from the difference between output of the final good and firms' spending in search for workers, entry, and investment in exporting.

Moments	Transfers	Barriers (dif in dif)
Employment in exporting firms	1.2%	−9.6%
Employment in exporting firms to more than 5 countries	2.0%	−13.0%
Consumption (Welfare)	1.5%	−14.9%
Income	2.0%	−14.7%

Table 2: Simulated Policies

First, I simulate a 10% increase in the rate of transfers to unemployed workers b. The percent changes in total employment in exporters and in aggregate income and consumption appear in the first column of Table 2. From the baseline theory, we know that this shock results in an increase in income for a given revenue differential of exporters. At the calibrated parameters of the extended model, it results in a 2% increase in income and in a 1.5% increase in aggregate consumption or welfare. These gains occur through a 1.2% increase in the size of the workforce allocated to exporting firms.

Second, I simulate a 10% reduction in trade costs to any destination under the calibrated value for λ_e, and also under a more flexible labor market regime, where λ_e is 10% larger.[48] In reduced form, the increase in λ_e can be interpreted as deriving from policies that encourage on-the-job search, such as lower firing costs. It can also be associated with a shift towards more decentralized bargaining schemes between firms and workers, which would encourage the type of transitions and individual bargaining highlighted by the model. The second column in Table 2 reports the percent

[47] These papers study two-sector models where the labor market friction slows down reallocations to the comparative advantage sectors after a trade reform. In the current model, in contrast, all the reallocations are within industries.

[48] Assuming that trade barriers are the same in both markets, the elasticity of demand implies that this reduction in trade costs is equivalent to a 20% increase in foreign market sizes A_1 and A_2.

difference between the change in aggregate outcomes due to the reduction in trade costs under the more and the less flexible labor market.

The impact of lower trade barriers is smaller in a more flexible environment. The increase in employment in exporting firms resulting from 10% lower trade costs is about 10% smaller in the more flexible labor market environment, while the increase in welfare is about 15% smaller. Therefore, at the calibrated values for the parameters in Argentina, the economy exhibits no complementarity between higher labor market flexibility and lower trade costs. Under a more flexible labor market, a greater share of employment is allocated in exporting firms and the revenue premium of exporters is smaller, resulting in a smaller impact of lower trade barriers.

7 Conclusion

In this paper I studied the impact of labor market frictions on aggregate outcomes in an economy where productivity upgrading or export participation require fixed-cost investments. I considered a theory where the growth of firms takes time depending on frictions in the labor market; as a result, there is a period after birth during which firms choose to use low-productivity technologies or to sell only at home. Slow growth generates dispersion in size and export activity among ex-ante identical firms; older firms are larger and more likely to export.

A critical feature of the analysis has been the existence of job-to-job transitions. Without them, there would be no role for frictions in affecting the timing of investment and the model would be unable to generate a size distribution of firms with realistic shape. At the aggregate level, the theory implies that lower barriers to job-to-job mobility and more generous unemployment compensation promote early investment, thus raising the volume of trade and income. It also shows that countries gain when these policies are implemented by trading partners.

I extended the theory to measure the impact of labor market and trade frictions on welfare. Among other features, I included ex-ante differences in productivity across firms; in this context, more productive firms enter foreign markets earlier and grow faster. I calibrated the model to match summary features of firm and labor market dynamics in Argentina. The parametrized model generates a close description of the increase in export participation by firm age and size for younger firms. In the counter-factual exercise, gains from trade are predicted to be lower in a more flexible labor market environment.

This paper represents the first step towards fully grasping the aggregate implications of labor market frictions in an economy with firm dynamics, job-to-job mobility, and fixed-cost investments that are key for productivity. The quantitative setup lends itself to extensions that are important for a complete account of the role of frictions, such as an endogenous matching rate between workers and firms or heterogeneous skill requirements by export status. In addition, the analysis was limited to comparisons between steady states, but it would be interesting to use the model to study aggregate transitions after a change in the environment. These questions are left for future research.

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A Derivations and Proofs

A.1 Appendix to Section 2

Proof of Lemma 1 The value of a job equals the sum of values obtained by the worker and the firm, $v = W + J$. At the moment in which a worker moves from firm i to firm j, the values obtained by the worker and by firm j are, respectively, $W_{i,j}$ and $J_{i,j}$. The reduced-form implication of the bargaining process in Postel-Vinay and Robin (2003) is that the splitting of the total surplus in j, v_j, only depends on the total surplus in i. In particular, it occurs as if the worker used the total value in the previous employment, v_i, as outside option in a bilateral bargaining with j in which the new firm has monopsony power:

$$W_{i,j} = W(v_i, v_j) = v_i, \tag{37}$$
$$J_{i,j} = J(v_i, v_j) = v_j - v_i. \tag{38}$$

By construction, $W_{i,j}$ satisfies

$$(\rho + \delta + \lambda_e P_{k:v_i \leq v_k}) W_{i,j} = \omega_{i,j} + \delta w_u + \lambda_e \left(\int_{k:v_i \leq v_k \leq v_j} W_{k,j} dP_k + \int_{k:v_j < v_k} W_{j,k} dP_k \right) + dW_{i,j},$$

where $\omega_{i,j}$ is the income flow obtained by the worker at the moment of the transition from i to j and δw_u is the value for the worker if the match is dissolved. The term within brackets on the right-hand side is the value in the case of a contact with firms offering jobs with value higher than i, where dP_k measures the probability of sampling firm k and $P_{k:v_i \leq v_k}$ is the measure of all firms whose value is above v_i. Using (37) and (38), changing the variable of integration to the distribution of values v_0 with associated sampling function $dP_v(v)$ and integrating by parts yields:

$$(\rho + \delta) W_{i,j} = \omega_{i,j} + \delta w_u + \lambda_e \int_{v_i}^{v_j} [1 - P_v(v_0)] dv_0 + dW_{i,j}. \tag{39}$$

On the other hand, $J_{i,j}$ is given by:

$$(\rho + \delta + \lambda_e P_{k:v_i \leq v_k}) J_{i,j} = \widetilde{y}_j - \omega_{i,j} + \lambda_e \int_{k:v_i \leq v_k \leq v_j} J_{k,j} dP_k + dJ_{i,j}, \tag{40}$$

where $\widetilde{y}_j$ is revenue per worker generated in firm j and is allowed to fluctuate over time.

Suppose that a worker employed in j meets a firm j' whose total value is the same as in j, $v_j = v_{j'}$. In this instance, (37) and (38) lead to $J_{j',j} = 0$ and $W_{j',j} = v_j$. On the other hand, $dW + dJ = dv$, the sum of the changes in value obtained by firm and worker add up to the change in value of the job. This holds when a worker from i enters j, i.e. $dW_{i,j} + dJ_{i,j} = dv_j$. The specific way how the change in total value is split pins down the evolution of the transfer flow between firm and worker. In Postel-Vinay and Robin (2003), this division results from the assumption that firms commit to a fixed wage until a renegotiation occurs. Then, evaluating (39) and (40) at $i = j'$ and summing over these equations gives

$$(\rho + \delta) v_j = \widetilde{y}_j + \delta w_u + dv_j. \tag{41}$$

Thus, the total value of a job in firm j is characterized by this differential equation that depends on the process for current revenue per worker in firm j, $\widetilde{y}_j$.

Suppose next that firms are heterogeneous in their (constant) productivity per worker ψ and that they

enter sequentially in markets $k = 1, .., K$ at ages $\{H_k\}_{k=1}^{K}$. Letting y_k be the revenues per unit of output generated by a firm who has entered in the first k markets, $\widetilde{y}_j$ depends on firm age, a, as given by the step function $\widetilde{y}(a; \{H_k\}) = \psi \sum_k 1_{(H_k \leq a < H_{k+1})} y_k$. Using this in (41), we have a linear differential equation for job value over age

$$(\rho + \delta) v (a; \{H_k\}) = \psi \widetilde{y} (a; \{H_k\}) + \delta w_u + dv (a; \{H_k\}).$$

Expressing the solution to this equation in terms of the time until switch $x_k \geq 0$ for each market k gives

$$(\rho + \delta) v (x_1, x_2, .., x_K) = \psi \left[y_0 + \sum_{k=0}^{K} e^{-(\rho+\delta)x_k} (y_{k+1} - y_k) \right] + \delta w_u.$$

In the case of $K = 1$ and $\psi = 1$ this constitutes the expression in (3); for $K = 2$ this is expression (31).

Proof of Proposition 1 The first order condition in the firm problem is:

$$\begin{aligned} \Pi'(h) &= e^{-(\rho+\mu)h} [f_X - S(h)] \leq 0 \text{ if } h = 0, \\ &= e^{-(\rho+\mu)h} [f_X - S(h)] = 0 \text{ if } h > 0. \end{aligned}$$

where, replacing the expression in (12) into (10),

$$S(h) = (\Gamma - 1) y_D (\lambda_u u/M) \int_0^h e^{-\gamma x} \{1 + \kappa_e [1 - G(x)]\} dx. \tag{42}$$

We have: (i) $S(0) = 0$; (ii) $S'(h) > 0$; and (iii) if $G(x) = 0$, $\lim_{h\to\infty} S(h) = (\Gamma - 1) y_D (\lambda_u u/M) (1 + \kappa_e)/\gamma$. From (i) and the first order condition, $f_X > 0$ implies that $h > 0$. From (ii), there is a unique interior solution to the firm problem. From (iii), if $f_X > (\Gamma - 1) y_D (\lambda_u u/M) (1 + \kappa_e)/\gamma$ then $S(h) < f_X$ for all h, and the first order condition implies that $h = \infty$. This proves the first part of the proposition. Comparative statics follow from the interior solution $S(h) = f_X$, inspection of the change in $S(h)$ with respect to each parameter and (ii) above.

A.2 Appendix to Section 3

Before proving Propositions 2 and 3 I prove the following:[49]

Lemma 2 *If (23) holds then* $\Omega_2 (H, H) > 0$.

First, write the function $\Omega(h, H)$ explicitly. Replacing for $G(x)$ from (14) in (42), and replacing for $\pi(x; H)$ from (6) evaluated at $\overline{x} = H$ we have:

$$\Omega(h, H) \equiv \frac{S(h; H)}{\pi(h; H)} = \frac{(\Gamma - 1)(1 + \kappa_e) \int_0^h e^{-\gamma x} \left[1 + \kappa_e e^{-\mu(H-x)}\right]^{-1} dx}{\widetilde{J}_u(h; H) + \kappa_e \int_h^H \widetilde{J}(x, h) dG(x)}, \tag{43}$$

where $\widetilde{J}_u(h, H) \equiv J_u(h, H)/y_D$ and $\widetilde{J}(x, h) \equiv J(x, h)/y_D$.

[49] I use the notation $\Omega_1 (h, H) \equiv \partial\Omega(h, H)/\partial h$ and $\Omega_2 (h, H) \equiv \partial\Omega(h, H)/\partial H$.

Using $J_u(x)$, $J(x_0, x)$ and $v(x)$ from (1) to (3) we can write $\Omega(h,H) = A(h,H) / [B(h,H) + C(h,H)]$, where

$$\begin{aligned} A(h,H) &= (\Gamma-1)(1+\kappa_e)\int_0^h e^{-\gamma x}\left[1+\kappa_e e^{-\mu(H-x)}\right]^{-1} dx, \\ B(h,H) &= \left[1-\rho w_u/y_D + (\Gamma-1)e^{-(\rho+\delta)h}\right]/(\rho+\delta), \\ C(h,H) &= [\kappa_e(\Gamma-1)/(\rho+\delta)]\int_h^H \left[e^{-(\rho+\delta)h} - e^{-(\rho+\delta)x}\right] dG(x), \end{aligned}$$

where, in $B(h,H)$:

$$\rho w_u/y_D = by/y_D = b\left[1+e_X(H)(\Gamma-1)\right] = b\left\{1+\left[\Gamma(1+\kappa_e)-1\right]e^{-\mu H}\right\}/\left(1+\kappa_e e^{-\mu H}\right). \tag{44}$$

The first equality above follows from (18), the second from (17) and the third from (16).

Next, note that $C(H,H) = C_2(H,H) = 0$, implying that $\Omega_2(H,H) = \widetilde{\Omega}_2(H,H)$, where $\widetilde{\Omega}(h,H) \equiv A(h,H)/B(h,H)$. Hence, $\Omega_2(H,H) > 0$ if and only if $\widetilde{\Omega}_2(H,H) > 0$. Using $A(h,H)$ and $B(h,H)$, multiplying numerator and denominator of $\widetilde{\Omega}(h,H)$ by $\left(1+\kappa_e e^{-\mu H}\right)(\rho+\delta)$, and changing the variable of integration in $A(h,H)$ to $h_0 = H - x$ gives:

$$\widetilde{\Omega}(h,H) \equiv \frac{(\rho+\delta)(\Gamma-1)(1+\kappa_e)\int_{H-h}^H \left(e^{-\gamma H}+\kappa_e e^{-\delta H}\right)\left(e^{-\gamma h_0}+\kappa_e e^{-\delta h_0}\right)^{-1} dh_0}{1-b+\left\{\kappa_e - b\left[\Gamma(1+\kappa_e)-1\right]\right\}e^{-\mu H} + (\Gamma-1)\left(1+\kappa_e e^{-\mu H}\right)e^{-(\rho+\delta)H}}. \tag{45}$$

From (23), the denominator is decreasing in H. To prove that $\widetilde{\Omega}_2(H,H) > 0$ it suffices to show that the numerator increases in H. After some manipulations, we can show that at $h = H$ this occurs if

$$LHS(H) \equiv \int_0^H \frac{1}{e^{-\gamma h_0}+\kappa_e e^{-\delta h_0}} dh_0 < \frac{1}{\gamma e^{-\gamma H} + \delta\kappa_1 e^{-\delta H}}\left(1 - \frac{e^{-\gamma H}+\kappa_e e^{-\delta H}}{1+\kappa_e}\right) \equiv RHS(H).$$

To prove this inequality it suffices to show that $LHS'(H) < RHS'(H)$ for all H. Computing these expressions and some manipulation implies that this holds if and only if $\left(e^{-\gamma H}+\kappa_e e^{-\delta H}\right)/(1+\kappa_e) < 1$, which holds for $H > 0$.

Proof of Proposition 2 Let $\Omega_0(H) \equiv \Omega(H,H)$. In an interior equilibrium, $\Omega_0(H) = f_X/f_D$. We have that $\Omega_1(h,H) > 0$ for all H by inspection of $\Omega(h,H)$, $J_u(h,H)$ and $J(x,h)$, and that $\Omega_2(H,H) > 0$ from Lemma 2. Therefore, $\Omega_0'(H) > 0$, implying that an interior equilibrium is unique. For existence of the interior equilibrium, $\Omega_0(0) = 0$ implies that $H > 0$. On the other hand, if $\lim_{H\to\infty}\Omega_0(H) \equiv \overline{f_X/f_D} \le f_X/f_D$, where $\overline{f_X/f_D}$ is defined in (24) in the text, then no interior equilibrium exists. In the other direction, if $H = \infty$ is an equilibrium, then it must be that no firm invests when no firm invests, i.e. $\lim_{h\to\infty}\lim_{H\to\infty}\Omega(h,H) \equiv \overline{f_X/f_D} \le f_X/f_D$. Therefore, $H = \infty \longleftrightarrow \overline{f_X/f_D} \le f_X/f_D$.

Proof of Proposition 3 In an interior equilibrium,

$$\Omega(H,H) = \frac{(\rho+\delta)(\Gamma-1)(1+\kappa_e)\int_0^H e^{-\gamma x}\left[1+\kappa_e e^{-\mu(H-x)}\right]^{-1} dx}{1-b\left\{1+\left[\Gamma(1+\kappa_e)-1\right]e^{-\mu H}\right\}\left(1+\kappa_e e^{-\mu H}\right)^{-1} + (\Gamma-1)e^{-(\rho+\delta)H}} = \frac{f_X}{f_D}$$

Changes in parameters that raise $\Omega(H,H)$ given H lead to lower equilibrium H. By inspection, we have $\partial\Omega(H,H)/\partial\lambda_u = 0$ and $\partial\Omega(H,H)/\partial b > 0$. Multiplying numerator and denominator of $\Omega(H,H)$ by

$\left(1+\kappa_e e^{-\mu H}\right)/(\Gamma-1)$ we see that $\partial\Omega(H,H)/\partial\Gamma>0$ if (23) holds. Finally, the numerator of $\Omega(H,H)$ is increasing in κ_e while some manipulation shows that the denominator is decreasing since $e^{-\mu H}<1$, implying $\partial\Omega(H,H)/\partial\kappa_e>0$. By inspection of (15) to (17), each of the parameter changes leading to a lower H also lead to an increase in m_X, e_X and y.

Proof of Proposition 4 I start by proving (25). Then, I use this condition to show that, if $\mu<\gamma$, then: (i) If $\lambda_e=0$, $f'(n)>0$ for all n; (ii) $f'(0)>0$; (iii) If $h<H$, then $\lim_{h\to\infty} f'(N(h))<0$; and (iv) If $h>H$, $f'(N(h))>0$. These four properties imply the proposition.

Start by considering an equilibrium where every firm switches at age H, and let $N(h)$ be the size of a firm of age h. The net flow of workers in a firm of age h is

$$N'(h)=\begin{cases}\left(\frac{\lambda_u u}{M}\right)\left[1+\kappa_e G_H(h)\right]-\left\{\gamma+\lambda_e\left[1-P_H(h)\right]\right\}N(h) & \text{if } h<H\\ \left(\frac{\lambda_u u}{M}\right)\left[1+\kappa_e G_H(H)\right]-\left\{\gamma+\varpi\lambda_e\left[1-P_H(H)\right]\right\}N(h) & \text{if } H\le h\end{cases}, \tag{46}$$

where $P_H(h)=1-e^{-\mu h}$ and $G_H(h)=P_H(h)/\left\{1+\kappa_e\left[1-P_H(h)\right]\right\}$ are the firm and employment distributions defined over age, instead of over time until investment as in (13) and (14), respectively. Workers in firms older than H who contact another firm older than H are indifferent about making a transition, in which case they move with exogenous probability $\varpi\in[0,1]$.

The rate at which workers leave the firm is weakly decreasing and the number of new hires is weakly increasing in h, so $N(h)$ is increasing. Letting $F(n)$ be the share of firms of size less than n, we have, from the exponential distribution of ages, that $F(n)=1-e^{-\mu N^{-1}(n)}$. This implies

$$f'(n)/f(n)=-\left[\mu+N''\left(N^{-1}(n)\right)/N'\left(N^{-1}(n)\right)\right]/N'\left(N^{-1}(n)\right),$$

implying (25). If $h<H$ and $\lambda_e=0$ then $N''(h)/N'(h)=-\gamma$, and if $h>H$ then $N''(h)/N'(h)=-\left\{\gamma+\varpi\lambda_e\left[1-P_H(H)\right]\right\}$, implying (i) and (iv) above. If $h<H$, from (46),

$$\frac{N''(h)}{N'(h)}+\mu=\frac{\left(\frac{\lambda_u u}{M}\right)\frac{1+\kappa_e}{1+\kappa_e e^{-\mu h}}\left(\mu-\gamma-\frac{\gamma+\lambda_e e^{-\mu h}}{\gamma+\mu+\lambda_e e^{-\mu h}}\lambda_e e^{-\mu h}\right)+\left[\left(\lambda_e e^{-\mu h}+\gamma\right)^2-\mu\gamma\right]N(h)}{\left(\frac{\lambda_u u}{M}\right)\frac{1+\kappa_e}{1+\kappa_e e^{-\mu h}}-\left(\lambda_e e^{-\mu h}+\gamma\right)N(h)}. \tag{47}$$

At $h=0$, (47) yields $N''(h)/N'(h)=-\gamma-\lambda_e(\gamma+\lambda_e)/(\gamma+\mu+\lambda_e)<0$, implying (ii). Since the denominator in the right-hand side of (47) is positive we have that, as long as $h<H$, then $\lim_{h\to\infty} N''(h)/N'(h)+\mu>0$ iff $\lambda_u u/\gamma M<\lim_{h\to\infty} N(h)=(1+\kappa_e)(\lambda_u u/\gamma M)$, implying (iii).

A.3 Appendix to Section 4

Derivation of (26) and (27) I derive the linear revenue function used in sections 4 and 6 from a standard monopolistic competition setting extended with a quality choice. Output of the final good in each country is

$$Y=\left(\int_{i\in I} z_i^{1/\sigma} q_i^{(\sigma-1)/\sigma} di\right)^{\sigma/(\sigma-1)},$$

where I is the set of available varieties from any country, while q_i and z_i denote the quantity and the quality of each variety. This good is produced in a competitive sector that uses the differentiated varieties as inputs, resulting in a demand for i of

$$q_i=z_i Y\left(\widetilde{p}_i/P\right)^{-\sigma},$$

where $\widetilde{p}_i$ is the price of variety i and $P=\left(\int z_i\widetilde{p}_i^{1-\sigma}di\right)^{1/(1-\sigma)}$ is the domestic price index.

Consider firms in the home market, indexed by 0, in a world with many countries. Given levels of quantity q and quality z, a firm selling in a set J of markets (including the domestic market) chooses the fraction s_j of total output to ship to market j. Of each unit shipped to country $j \neq 0$ only a fraction $1/\tau_j < 1$ arrives. Letting

$$p_j \equiv P_j Y_j^{1/\sigma},$$

we have that revenues of an exporter to a set J of markets are

$$\widetilde{r}_J(z,q) = \max_{s_j} \left\{ z^{1/\sigma} q^{1-1/\sigma} \sum_{j \in J} p_j \left(s_j/\tau_j\right)^{1-1/\sigma} \text{ s.t. } \sum_{j \in J} s_j = 1 \right\} = \left(\sum_{j \in J} p_j^{\sigma} \tau_j^{1-\sigma} \right)^{1/\sigma} z^{1/\sigma} q^{1-1/\sigma}, \quad (48)$$

where the second equality follows from evaluating the revenue function at the optimal shares of output directed to each destination,

$$s_j = p_j^{\sigma} \tau_j^{1-\sigma} / \sum_{j \in J} p_j^{\sigma} \tau_j^{1-\sigma}. \quad (49)$$

In each moment firms have a stock of n workers evolving over time as described in the text. This workforce can be allocated to produce final goods, either for domestic or foreign shipment, or quality. The production frontier within firms is linear in these variables: a firm with n workers and productivity ψ faces the constraint

$$(1/\sigma)\, z + (1 - 1/\sigma)\, q = \psi n.$$

The introduction of σ here serves only to save notation. Firms maximize $\widetilde{r}_J(z,q)$ in (48) subject to this constraint. As a result, the optimal allocation of workers dictates that firm quantity and quality increase linearly with the stock of workers no matter the export status:

$$z = q = \psi n. \quad (50)$$

Using (50) in the revenue function (48) and normalizing by the domestic price index, real revenues (i.e., in terms of domestic final good) in a firm with n workers exporting to a set J of markets are linear in n,

$$r_J = y_J \psi n.$$

Revenues per unit of output are

$$y_0 = Y_0^{1/\sigma}, \quad (51)$$

$$y_J = \Gamma_J y_0, \quad (52)$$

and the revenue premium of a firm exporting to a set J of markets is:

$$\Gamma_J = \left[1 + \sum_{j \in J} \left(p_j/p_0\right)^{\sigma} \tau_j^{-(\sigma-1)} \right]^{1/\sigma}. \quad (53)$$

Evaluating (49) and (53) in the case of two countries and $\psi = 1$ gives (26) and (27) in Section 4, while setting the number of markets to be at most 3 gives the revenue premia in (32) to (34) in Section 6.

Proof of Proposition 5 Using (27) for both countries and the first equality of (30) in (29) yields

$$\Gamma^* e_X^* (\Gamma^*) \, s_X^* (\Gamma^*) = \left[\tau^{1/\sigma} (1-u)\right] / \left[\tau^* (1-u^*)\right] e_X (\Gamma) \, s_X (\Gamma)^{(\sigma-1)/\sigma} . \tag{54}$$

Since $de_X/d\Gamma \geq 0$ and $ds_X/d\Gamma > 0$, if $e_X > 0$ and $e_X^* > 0$ this gives an increasing relation between Γ and Γ^*. If $(\rho+\delta)(1+\kappa_e)/[\gamma(1-b)] \to \infty$ or $f_X/f_D \to 0$, we have from Proposition 2 that $e_X(1) = s_X(1) = 0$ and that $de_X/d\Gamma > 0$ if $e_X < 1$. The same applies in the foreign country. Therefore, (54) is satisfied with both sides equal to zero at $\Gamma = \Gamma^* = 1$ and each side is strictly increasing in its respective argument if $\Gamma > 1$ and $\Gamma^* > 1$. On the other hand, the second equality in (30) gives an hyperbole in the region determined by $\Gamma > 1$ and $\Gamma^* > 1$, with the property that $\Gamma^* \to \infty$ as $\Gamma \to 1$, and vice versa. This implies that only one point in the region determined by $\Gamma > 1$ and $\Gamma^* > 1$ satisfies the equilibrium conditions.

Proof of Proposition 6 Under the conditions from Proposition 4, from (54) we can implicitly write Γ as an increasing function of Γ^*. The equilibrium values for Γ and Γ^* correspond to the intersection between this function and (30). From Proposition 3, $e_X^*(\Gamma^*)$ in (54) increases for each value of Γ^* with a rise in b^* or in λ_1^*, hence the new equilibrium must have a larger Γ and a lower Γ^*. The increase in export participation in the home country follows from Proposition 3. For the increase in export participation in the foreign economy, we have that with a rise in b^* or in λ_1^*, the increase in Γ leads to an increase in the right hand side of (54). Since Γ^* decreases in the left hand side, so does $s_X(\Gamma^*)$, meaning that $e_X^*(\Gamma^*)$ and therefore $m_X^*(\Gamma^*)$ must increase. On the other hand, consider a reduction in τ and suppose that Γ shrinks. Then, Γ^* must increase in (30), the right hand side of (54) decreases and, from Proposition 3, the left hand side increases, which can't be an equilibrium. Therefore, Γ must increase.

A.4 Appendix to Section 6

Derivation of $\pi(x;\psi)$ The present discounted value of all workers attracted by a firm with productivity ψ in state x -the equivalent to (6)- is

$$\pi(x;\psi) = \max_{s\geq 0} (\lambda_u u/M\bar{s}) \, \pi_0 (v(x;\psi)) \, s - c(s) \tag{55}$$

where $\pi_0(v) \equiv v - w_u + \kappa_e \int_{v_0 \leq v} G_v(v_0) \, dv_0$. In each period the firm solves the static problem of how many workers to attract. Higher search s entails a cost, but yields a return proportional to π_0. A firm offering jobs with value v chooses

$$s(v) = \left[(\lambda_u u/M\bar{s}) \, \pi_0(v)/\zeta\right]^{1/(\zeta-1)} . \tag{56}$$

Replacing in (55) this yields an expression equivalent to (6):

$$\pi(x;\psi) = (\zeta-1) \left[(\lambda_u u/M\bar{s}) \, \pi_0(v(x;\psi))/\zeta\right]^{\zeta/(\zeta-1)} . \tag{57}$$

Proof of Proposition 7 We can define $\Pi_0(h_1,h_2;\varepsilon)$ in parallel to $\Pi(h)$ in (7) as the value *at entry* of a firm that starts to export to markets $k = 1,2$ at ages h_1 and h_1+h_2, respectively,

$$\Pi_0(h_1,h_2;\varepsilon) = \int_0^{h_1} e^{-(\rho+\mu)a} \pi(h_1-a, h_1+h_2-a;\psi) \, da + e^{-(\rho+\mu)h_1} \left(\Pi_1(h_2;\varepsilon) - \frac{\phi f_1}{\rho+\mu}\right), \tag{58}$$

where $\Pi_1(h_2;\varepsilon)$ is the value of this firm at the moment of entry into market 1,

$$\Pi_1(h_2;\varepsilon) = \int_0^{h_2} e^{-(\rho+\mu)a}\pi(0,h_2-a;\psi)\,da_2 + e^{-(\rho+\mu)h_2}\frac{\pi(0,0;\psi)-\phi f_2}{\rho+\mu}. \tag{59}$$

The firm chooses h_1 and h_2 to maximize $\Pi_0(h_1,h_2;\varepsilon)$ in (58). In an interior solution, the first order conditions can be written as[50]

$$\begin{aligned}
[h_1] &: \int_0^{h_1} e^{(\rho+\mu)x_1}\pi_1(x_1,x_1+h_2;\psi)\,dx_1 + \phi f_1 = 0,\\
[h_2] &: \int_0^{h_1} e^{(\rho+\mu)x_1}\pi_2(x_1,x_1+h_2;\psi)\,dx_1 + e^{-(\rho+\mu)h_2}\left[\int_0^{h_2} e^{(\rho+\mu)x_2}\pi_2(0,x_2;\psi)\,dx_2 + \phi f_2\right] = 0,
\end{aligned}$$

Taking derivatives of (57) and using the resulting expression for $\pi_2(x_1,x_1+h_2;\psi)$ in these conditions gives the first-order conditions (35) and (36) in the text. Writing them explicitly we have:

$$S_1(h_1,h_2;\psi) \equiv \int_0^{h_1} e^{(\rho+\mu)x_1}\left[-\pi_1(x_1,x_1+h_2;\psi)\right]dx_1 = \phi f_1, \tag{60}$$

$$S_2(h_2;\psi) \equiv \int_0^{h_2} e^{(\rho+\mu)x_2}\left[-\pi_2(0,x_2;\psi)\right]dx_2 = \left(\frac{f_2}{f_1} - \frac{\Gamma_{12}-\Gamma_1}{\Gamma_1-1}\right)\phi f_1. \tag{61}$$

And using the expressions for $\pi_1(x_1,x_1+h_2;\psi)$ and $\pi_2(0,x_2;\psi)$ that result from (57) and the expression for G_v from (65) in these expressions gives

$$\begin{aligned}
[h_1] &: \zeta^{-1/(\zeta-1)}\left(\frac{\lambda_u u}{M\bar{s}}\right)^{\zeta/(\zeta-1)}\psi\int_0^{h_1}\frac{(1+\kappa_e)\,\pi_0(x_1,x_1+h_2;\psi)^{1/(\zeta-1)}}{1+\kappa_e\left[1-P\left(v\left(x_1,x_1+h_2;\psi\right)\right)\right]}e^{-\gamma x_1}dx_1 = \phi\frac{f_1}{y_1-y_0},\\
[h_2] &: \zeta^{-1/(\zeta-1)}\left(\frac{\lambda_u u}{M\bar{s}}\right)^{\zeta/(\zeta-1)}\psi\int_0^{h_2}\frac{(1+\kappa_e)\,\pi_0(0,x_2;\psi)^{1/(\zeta-1)}}{1+\kappa_e\left[1-P\left(v\left(0,x_2;\psi\right)\right)\right]}e^{-\gamma x_2}dx_2 = \phi\left(\frac{f_2}{y_2-y_1} - \frac{f_1}{y_1-y_0}\right).
\end{aligned}$$

The left hand side of the second equality is strictly increasing in h_2 and independent from h_1, while the first is strictly increasing in both h_1 and h_2, implying a unique interior solution to the firm problem. Since $\pi_0(x;\psi)$ and $v(x;\psi)$ are increasing in ψ, the left-hand side of both functions is increasing in ψ, while the firm specific fixed cost ϕ only appears on the right hand side, implying part (i) of the proposition. Part (ii) follows from inspection of (60) and (61). For (iii), note that the firm entering in both markets and entering first in market 1 can only be an outcome if the right-hand side of (61) is positive, i.e. if $f_2/f_1 > (\Gamma_{12}-\Gamma_1)/(\Gamma_1-1)$. Thus, if we conjecture that the firm enters first in 2 and then in 1, we have a contradiction if the right-hand side of the version of (61) under this conjecture is negative, i.e. if $f_1/f_2 < (\Gamma_{12}-\Gamma_2)/(\Gamma_2-1)$. These inequalities imply (iii).

Definition of the Equilibrium To proceed with the definition of the equilibrium, let $h_k(\varepsilon)$ denote the choice of firm ε. Then, from the first order condition of the firm problem we have, as in (11), the value of the firm at entry, now indexed by the firm type:

$$\Pi^e(\varepsilon) \equiv \pi\left(h_1(\varepsilon), h_1(\varepsilon)+h_2(\varepsilon);\psi\right)/(\rho+\mu). \tag{62}$$

Defining the equilibrium requires, first, that we identify the function $P_v(v)$ that indicates the probability

[50] I use the notation $\pi_1(a,b;\psi) = \partial\pi(x,y;\psi)/\partial x$ and $\pi_2(a,b;\psi) = \partial\pi(x,y;\psi)/\partial y$ evaluated at $(x,y)=(a,b)$.

that a worker who samples a firm finds job with value below v; this is equivalent to $P(x)$ in the basic model. To find that function define first the equilibrium value of a job offered by firm ε over age:

$$v^* (a; \varepsilon) \equiv v (\max [h_1 (\varepsilon) - a, 0], \max [h_1 (\varepsilon) + h_2 (\varepsilon) - a, 0]; \psi).$$

Notice that $v^* (h; \varepsilon)$ is strictly increasing in a, as such having a well defined inverse denoted by $a^* (v; \varepsilon)$. Using (56), define also the level of search chosen by firms of type ε and age a as:

$$s^* (a; \varepsilon) \equiv s (v^* (a; \varepsilon)). \tag{63}$$

The effective measure that a firm of age a and type ε has in the labor market is $s^* (a; \varepsilon) / \bar{s}$. Therefore, the sampling function is:

$$P_v (v) = \mathbb{E}_\varepsilon \int_0^{a^*(v;\varepsilon)} [s^* (a; \varepsilon) / \bar{s}] \mu e^{-\mu a} da, \tag{64}$$

where E_ε denotes the expectation over the distribution of firm types ε. This function yields in turn the share of employment in firms with value of jobs below v:

$$G_v (v) = P_v (v) / \{1 + \kappa_e [1 - P_v (v)]\}. \tag{65}$$

The measure of firms in the economy is determined by zero profits. Entry requires flow equivalent fixed costs of f_0 in each period, so that the free entry condition is:

$$\mathbb{E}_\varepsilon [\Pi^e (\varepsilon)] = f_0. \tag{66}$$

Aggregate income in the economy depends now on both productivity and the distribution of switching ages. Let

$$y^* (a; \varepsilon) = 1_{(a<h_1(\varepsilon))} y_0 + 1_{(h_1(\varepsilon) \le a < h_1(\varepsilon)+h_2(\varepsilon))} y_1 + 1_{(h_1(\varepsilon)+h_2(\varepsilon) \le a)} y_2$$

be the revenue per worker generated in firm ε when it has age a. Aggregate income is $Y_0 = (1 - u) y$, where output per employed worker equals

$$y = (1 - u) \mathbb{E} [y^* (a; \varepsilon)], \tag{67}$$

where the expectation is taken with respect to the equilibrium distribution of employment over states (a, ε) induced by $P_v (v)$. As before, the value of unemployment is

$$\rho w_u = by \tag{68}$$

Summarizing:

Definition 3 *A general equilibrium consists of individual rules $\{h_k (\varepsilon), s^* (a; \varepsilon)\}$, distributions $\{G_v (v), P_v (y)\}$, a number of firms M, output per worker y, consumption c and value of unemployment w_u such that:*
a) the first-order conditions from the firm problem, (35), (36) and (63), hold;
b) there is consistency between the individual decision rules and the aggregate distributions, (64) and (65);
c) the number of firms adjusts to satisfy free entry, (66);
d) output per worker is given by (67);
e) the value of unemployment is given by (68); and
f) goods market clear.

B Data appendix

B.1 Construction of Variables

The dataset used in the construction of the figures and in the regressions below is a match of two sources. Exports data is standard customs data at the firm-year level. This is linked with firm employment data from administrative records. The data is from the Employment and Business Dynamics Observatory of the Ministry of Labor and Social Security of Argentina (OEDE). All firms are required to report their formal employees on a monthly basis. Workers who are not reported are either informally employed or unemployed. In each of six two-month periods within each year between 1998 and 2008, every formal worker of age 18 to 64 is linked to the firm where he/she is reported as earning the highest wage. Workers earning below the minimum wage are excluded. Thus, the data includes the universe of firms that report employment above the minimum wage in any period in these years.

Each firm-year observation is marked as an exporter if the firm exports at least USD 10000. The age of the firm is the difference between the current year and the year of birth of the firm for tax purposes. The number of workers per firm-year is computed as the average employment over periods within year in which the firm reports positive employment. Industries are defined at the two-digit level. A worker employed in a firm is considered as a new hire if he/she is not employed in the firm in the previous period. To compute the fraction of new hires coming from other formal jobs in any sector of the economy for each firm-year, the shares are first computed for each pair of consecutive periods within year, and then averaged across periods within year for each firm. Similar steps are followed to compute the fraction of new hires from the manufacturing sector entering firms from jobs in exporting firms.

All figures are based on firms from the manufacturing sector. Exiting firms of any export status (i.e., firms present in a given year who do not report employment in the next) are excluded. Firms who do not report formal employment but who report exports are excluded, as well as industries with less than one-hundred firms in any year. The resulting sample represents on average 97% of the formal employment and 82% of all firms who either export or formally report the wages of their employees in the manufacturing sector between 1999 and 2007, with a total of 429934 firm-year observations.

B.2 Regressions

Table B.1 shows that the rankings in the share of new hires from other jobs and from exporting firms in figures 6 and 7 are statistically significant including various controls. For the regressions the sample is restricted to the period 2003-2007, but same coefficients are significant for 1999-2002. Column 1 in each set of regressions includes only industry-year fixed effects. The difference in the share of new hires from other jobs between non-exporters and exporters to no more than 5 countries is of 9 percent points, and between non-exporters and exporters to more than 5 countries is of 21 percent points. Columns 2 to 4 control for a second order polynomial in size and in age, wage, or net job creation. As we should expect from the theory, controlling for age and size shrinks the magnitude of the coefficients on the indicator for export status. To the extent that the current wage is a reflection of the higher average value of jobs offered by exporting firms, it should have a similar qualitative effect than firm size, as we observe in the regression. The number of hires and separations controls for the possibility that expanding and contracting firms exhibit different patterns in the composition of new hires. When all the controls are included, in Column 5, the differences in the composition of new hires by export status are still significant.

Table B.1: Regressions

Explained Variable->	New hires from other jobs as share of new hires					New hires from exporters as share of new hires from manufacturing				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
Exporters, up to 5 countries	0.094*	0.048*	0.035*	0.070*	0.026*	0.202*	0.128*	0.145*	0.171*	0.107*
	0.003	0.003	0.003	0.001	0.003	0.006	0.007	0.006	0.007	0.007
Exporters, more than 5 countries	0.216*	0.109*	0.099*	0.147*	0.058*	0.321*	0.208*	0.209*	0.257*	0.163*
	0.005	0.005	0.005	0.005	0.005	0.007	0.009	0.008	0.008	0.009
Size		0.038*			0.013*		0.039*			0.034*
		0.002			0.002		0.005			0.005
Size^2		0.001*			-0.003*		-0.000			-0.005*
		0.000			0.002		0.000			0.000
Age		-0.107*			-0.080*		0.083*			0.075*
		0.002			0.002		0.005			0.006
Age^2		0.022*			0.014*		-0.012*			-0.013*
		0.000			0.000		0.001			0.001
Wage			0.164*		0.180*			0.180*		0.156*
			0.002		0.002			0.006		0.006
Hires				0.092*	0.072*				-0.0312*	-0.003
				0.002	0.002				0.004	0.005
Separations				-0.028*	-0.017*				0.088*	0.048*
				0.002	0.002				0.004	0.005
Industry-Year dummies			Yes					Yes		
N			165817					37754		
R squared	0.05	0.09	0.11	0.08	0.14	0.12	0.15	0.15	0.13	0.17
Number of firms			63029					20760		

Values marked with a star are significant at 1% and standard errors clustered by firm are reported below each coefficient. The explained variables are in percent terms, the first two rows correspond to an indicator of the export status of the firm, and the remaining explanatory variables are in logs. The number of observations in the first set of regressions corresponds to the number of firm-year observations with positive number of new hires, and the number of observations in the second set is the number with positive number of new hires from the manufacturing sector.