## ONLINE APPENDIX

Labor Supply Responses to Income Taxation among Older Couples:
Evidence from a Canadian Reform

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## Online Appendix

Table A1: Excess Mass at the Public Pension and Unemployment Insurance Clawback Thresholds by Age and Benefit Receipt, 2007 to 2012 (Post-Reform)-Bunching Estimator

|  | Unmarried |  | Married |  |
| :---: | :---: | :---: | :---: | :---: |
|  | No Private | Has Private | No Private | Has Private |
|  | Pension Income <br> (1) | Pension Income <br> (2) | Pension Income <br> (3) | Pension Income <br> (4) |
| Panel A: Public Pension |  |  |  |  |
| 60 Years Old | 0.312 | -0.135 | 0.078 | 0.891*** |
|  | (0.338) | (0.425) | (0.185) | (0.293) |
| 61 Years Old | -0.047 | -0.436 | -0.259 | 0.410 |
|  | (0.288) | (0.400) | (0.203) | (0.263) |
| 62 Years Old | 0.273 | -0.437 | 0.261 | 0.249 |
|  | (0.307) | (0.459) | (0.229) | (0.305) |
| 63 Years Old | 0.189 | -0.076 | 0.051 | 0.476 |
|  | (0.421) | (0.324) | (0.253) | (0.321) |
| 64 Years Old | 0.095 | -0.117 | -0.260 | 0.488 |
|  | (0.419) | (0.417) | (0.284) | (0.356) |
| 65 Years Old | -0.525 | 0.684 | 1.138*** | $5.456^{* * *}$ |
|  | (0.393) | (0.460) | (0.317) | (0.293) |
| 66 Years Old | 0.709 | -0.099 | 1.048*** | 6.916*** |
|  | (0.513) | (0.370) | (0.303) | (0.375) |
| 67 Years Old | 1.279* | 0.553 | $0.966^{* * *}$ | 6.930*** |
|  | (0.775) | (0.401) | (0.330) | (0.388) |
| 68 Years Old | 2.398*** | 0.602 | $1.197^{* * *}$ | 8.033*** |
|  | (0.850) | (0.396) | (0.394) | (0.526) |
| 69 Years Old | 3.099*** | -0.153 | 1.095** | 8.314*** |
|  | (0.921) | (0.390) | (0.486) | (0.432) |
| Panel B: Unemployment Insurance |  |  |  |  |
| No Receipt | 0.024 | 0.036 | $-0.103^{* *}$ | -0.010 |
|  | (0.065) | (0.077) | (0.044) | (0.222) |
| Receipt | 0.530*** | 0.745 | -0.014 | $3.018^{* * *}$ |
|  | (0.188) | (0.371) | (0.126) | (0.292) |

Notes: Private pension income receipt is based on whether at least one spouse is a pensioner. The analysis is restricted to the post-reform period. Standard errors are in parentheses. ${ }^{* * *}$, ${ }^{* *}$ and ${ }^{*}$ denote significance at the $1 \%, 5 \%$ and $10 \%$ levels, respectively.

Table A2: Excess Mass at the Marginal Tax Rate Discontinuities by Year, 2001 to 2012-Bunching Estimator

|  | 2nd Federal (1) | 2nd Provincial <br> (2) | 3 rd <br> Federal <br> (3) | 3rd Provincial <br> (4) | 4th <br> Federal <br> (5) | Public Pension (6) | Unemployment Insurance (7) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2001 | $\begin{aligned} & 0.465^{* * *} \\ & (0.052) \end{aligned}$ | $\begin{aligned} & 0.467^{* * *} \\ & (0.044) \end{aligned}$ | $\begin{aligned} & 0.210^{* * *} \\ & (0.081) \end{aligned}$ | $\begin{gathered} 0.131 \\ (0.084) \end{gathered}$ | $\begin{gathered} 0.606^{* *} \\ (0.242) \end{gathered}$ | $\begin{aligned} & 0.689^{* * *} \\ & (0.183) \end{aligned}$ | $\begin{gathered} 0.452 \\ (0.301) \end{gathered}$ |
| 2002 | $\begin{aligned} & 0.293^{* * *} \\ & (0.065) \end{aligned}$ | $\begin{aligned} & 0.183^{* * *} \\ & (0.055) \end{aligned}$ | $\begin{aligned} & 0.189^{* *} \\ & (0.084) \end{aligned}$ | $\begin{gathered} -0.091 \\ (0.076) \end{gathered}$ | $\begin{gathered} -0.021 \\ (0.237) \end{gathered}$ | $\begin{gathered} 0.043 \\ (0.201) \end{gathered}$ | $\begin{gathered} 0.071 \\ (0.302) \end{gathered}$ |
| 2003 | $\begin{aligned} & 0.349^{* * *} \\ & (0.051) \end{aligned}$ | $\begin{aligned} & 0.238^{* * *} \\ & (0.052) \end{aligned}$ | $\begin{gathered} 0.072 \\ (0.101) \end{gathered}$ | $\begin{gathered} 0.096 \\ (0.083) \end{gathered}$ | $\begin{gathered} 0.216 \\ (0.253) \end{gathered}$ | $\begin{aligned} & 0.442^{* *} \\ & (0.188) \end{aligned}$ | $\begin{gathered} 0.237 \\ (0.273) \end{gathered}$ |
| 2004 | $\begin{aligned} & 0.157^{* * *} \\ & (0.051) \end{aligned}$ | $\begin{gathered} -0.005 \\ (0.052) \end{gathered}$ | $\begin{aligned} & 0.410^{* * *} \\ & (0.101) \end{aligned}$ | $\begin{gathered} -0.045 \\ (0.098) \end{gathered}$ | $\begin{gathered} -0.087 \\ (0.243) \end{gathered}$ | $\begin{aligned} & 0.818^{* * *} \\ & (0.219) \end{aligned}$ | $\begin{aligned} & 0.477^{* *} \\ & (0.241) \end{aligned}$ |
| 2005 | $\begin{aligned} & 0.421^{* * *} \\ & (0.059) \end{aligned}$ | $\begin{gathered} 0.008 \\ (0.056) \end{gathered}$ | $\begin{aligned} & 0.306^{* * *} \\ & (0.097) \end{aligned}$ | $\begin{gathered} -0.164^{*} \\ (0.093) \end{gathered}$ | $\begin{gathered} 0.319 \\ (0.231) \end{gathered}$ | $\begin{aligned} & 0.740^{* * *} \\ & (0.190) \end{aligned}$ | $\begin{gathered} 0.147 \\ (0.205) \end{gathered}$ |
| 2006 | $\begin{aligned} & 0.246^{* * *} \\ & (0.054) \end{aligned}$ | $\begin{gathered} 0.073 \\ (0.049) \end{gathered}$ | $\begin{aligned} & 0.183^{* *} \\ & (0.078) \end{aligned}$ | $\begin{gathered} -0.140^{*} \\ (0.080) \end{gathered}$ | $\begin{gathered} 0.458^{*} \\ (0.244) \end{gathered}$ | $\begin{aligned} & 0.729^{* * *} \\ & (0.205) \end{aligned}$ | $\begin{gathered} 0.274 \\ (0.242) \end{gathered}$ |
| 2007 | $\begin{aligned} & 1.519^{* * *} \\ & (0.150) \end{aligned}$ | $\begin{aligned} & 0.647^{* * *} \\ & (0.225) \end{aligned}$ | $\begin{aligned} & 0.341^{* * *} \\ & (0.115) \end{aligned}$ | $\begin{aligned} & 0.314^{* *} \\ & (0.127) \end{aligned}$ | $\begin{aligned} & 0.828^{* * *} \\ & (0.228) \end{aligned}$ | $\begin{aligned} & 3.624^{* * *} \\ & (0.200) \end{aligned}$ | $\begin{aligned} & 0.583^{* * *} \\ & (0.216) \end{aligned}$ |
| 2008 | $\begin{aligned} & 2.189^{* * *} \\ & (0.337) \end{aligned}$ | $\begin{aligned} & 1.988^{* * *} \\ & (0.248) \end{aligned}$ | $\begin{aligned} & 0.960^{* * *} \\ & (0.249) \end{aligned}$ | $\begin{aligned} & 1.040^{* * *} \\ & (0.172) \end{aligned}$ | $\begin{aligned} & 0.437^{* *} \\ & (0.174) \end{aligned}$ | $\begin{aligned} & 5.028^{* * *} \\ & (0.363) \end{aligned}$ | $\begin{aligned} & 0.750^{* * *} \\ & (0.225) \end{aligned}$ |
| 2009 | $\begin{aligned} & 3.198^{* * *} \\ & (0.329) \end{aligned}$ | $\begin{aligned} & 1.071^{* * *} \\ & (0.398) \end{aligned}$ | $\begin{aligned} & 1.008^{* * *} \\ & (0.200) \end{aligned}$ | $\begin{gathered} -0.006 \\ (0.239) \end{gathered}$ | $\begin{gathered} 0.602^{* *} \\ (0.237) \end{gathered}$ | $\begin{aligned} & 4.506^{* * *} \\ & (0.318) \end{aligned}$ | $\begin{aligned} & 0.949^{* * *} \\ & (0.191) \end{aligned}$ |
| 2010 | $\begin{aligned} & 3.606^{* * *} \\ & (0.422) \end{aligned}$ | $\begin{gathered} 0.287 \\ (0.438) \end{gathered}$ | $\begin{aligned} & 1.382^{* * *} \\ & (0.183) \end{aligned}$ | $\begin{gathered} 0.362 \\ (0.256) \end{gathered}$ | $\begin{gathered} 0.363^{*} \\ (0.195) \end{gathered}$ | $\begin{aligned} & 4.219^{* * *} \\ & (0.250) \end{aligned}$ | $\begin{aligned} & 1.285^{* * *} \\ & (0.286) \end{aligned}$ |
| 2011 | $\begin{aligned} & 3.727^{* * *} \\ & (0.416) \end{aligned}$ | $\begin{gathered} 0.009 \\ (0.462) \end{gathered}$ | $\begin{aligned} & 1.088^{* * *} \\ & (0.107) \end{aligned}$ | $\begin{gathered} 0.583^{*} \\ (0.301) \end{gathered}$ | $\begin{aligned} & 0.581^{* * *} \\ & (0.175) \end{aligned}$ | $\begin{aligned} & 4.759^{* * *} \\ & (0.330) \end{aligned}$ | $\begin{aligned} & 1.121^{* * *} \\ & (0.228) \end{aligned}$ |
| 2012 | $\begin{aligned} & 3.494^{* * *} \\ & (0.362) \end{aligned}$ | $\begin{gathered} 0.034 \\ (0.550) \end{gathered}$ | $\begin{aligned} & 1.155^{* * *} \\ & (0.138) \end{aligned}$ | $\begin{gathered} 0.650^{*} \\ (0.359) \end{gathered}$ | $\begin{aligned} & 0.709 * * * \\ & (0.204) \end{aligned}$ | $\begin{aligned} & 4.279^{* * *} \\ & (0.262) \end{aligned}$ | $\begin{aligned} & 0.946^{* * *} \\ & (0.265) \end{aligned}$ |

Notes: The bunching analysis of Figures 2a and 2b is carried out by year; see the notes in those figures for more information. Standard errors are in parentheses, clustered by individual. ${ }^{* * *}$, ${ }^{* *}$ and ${ }^{*}$ denote significance at the $1 \%, 5 \%$ and $10 \%$ levels, respectively.

Table A3: Robustness Checks of the Percent with Actual versus Predicted Pension Income Splitting Based on Amount and Pension Income Splitting Eligibility

|  | \$5,000 or More |  | \$10,000 or More |  | \$20,000 or More |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Actual <br> (1) | Predicted <br> (2) | Actual (3) | Predicted <br> (4) | Actual (5) | Predicted <br> (6) |
| Both Spouses Aged 65 or More | 29.5 | 40.4 | 17.6 | 23.3 | 5.4 | 6.3 |
| One Spouse Aged 65 or More |  |  |  |  |  |  |
| Younger Spouse Had a Pension | 28.1 | 38.2 | 17.4 | 22.1 | 5.3 | 5.2 |
| Younger Spouse Did Not Have a Pension | 20.6 | 25.5 | 12.3 | 14.5 | 3.8 | 3.9 |
| Both Spouses Below Age 65 |  |  |  |  |  |  |
| Both Spouses Had a Pension | 19.8 | 25.9 | 13.5 | 17.0 | 4.5 | 4.7 |
| One Spouse Had a Pension | 19.1 | 22.8 | 13.9 | 16.8 | 4.7 | 5.2 |
| Neither Spouse Had a Pension | 1.0 | 0.0 | 0.5 | 0.0 | 0.1 | 0.0 |

Notes: The estimates from columns 4 and 5 of Table 4 are shown here but conditional on individuals who split pension income of $\$ 5,000$ or more, $\$ 10,000$ or more, and $\$ 20,000$ or more with their spouses.

Table A4: Correlations between the Actual and Predicted Tax Variables

|  | Marginal Net-of-Tax Rate of Individual <br> (1) | Marginal Net-of-Tax Rate of Spouse (2) | After-Tax Income of Individual (3) | After-Tax Income of Spouse (4) |
| :---: | :---: | :---: | :---: | :---: |
| Panel A: Actual Tax Variables |  |  |  |  |
| Marginal Net-of-Tax Rate of Individual | 1.000 |  |  |  |
| Marginal Net-of-Tax Rate of Spouse | -0.069 | 1.000 |  |  |
| After-Tax Income of Individual | -0.340 | 0.086 | 1.000 |  |
| After-Tax Income of Spouse | 0.086 | -0.370 | -0.055 | 1.000 |
| Panel B: Predicted Tax Variables |  |  |  |  |
| Marginal Net-of-Tax Rate of Individual | 1.000 |  |  |  |
| Marginal Net-of-Tax Rate of Spouse | -0.376 | 1.000 |  |  |
| After-Tax Income of Individual | -0.448 | 0.379 | 1.000 |  |
| After-Tax Income of Spouse | 0.417 | -0.538 | $-0.374$ | 1.000 |

Notes: The correlations between changes in the log values of the tax variables are reported, since these are the tax variables used in the regression analysis.

Table A5: Robustness Checks of Extensive Margin Labor Supply Responses to Changes in the Marginal Net-of-Tax Rate and After-Tax Income, 2006 to 2007-Instrumental Variables

|  |  | Instrumental Variables |  |
| :--- | :---: | :---: | :---: |
|  | Ordinary | Reduced- | Two-Stage |
|  | Least Squares | Form | Least Squares |
|  | $(1)$ | $(2)$ | $(3)$ |
| Marginal Net-of-Tax Rate of Individual | $-0.829^{* * *}$ | $0.020^{*}$ | 0.005 |
|  | $(0.005)$ | $(0.010)$ | $(0.021)$ |
| Marginal Net-of-Tax Rate of Spouse | $-0.015^{* * *}$ | $-0.018^{*}$ | -0.021 |
|  | $(0.004)$ | $(0.011)$ | $(0.022)$ |
| After-Tax Income of Individual | $0.012^{* * *}$ | $-0.017^{* * *}$ | $-0.038^{* * *}$ |
|  | $(0.001)$ | $(0.001)$ | $(0.003)$ |
| After-Tax Income of Spouse | -0.001 | $-0.007^{* * *}$ | $-0.011^{* * *}$ |
|  | $(0.001)$ | $(0.002)$ | $(0.003)$ |
| Employment of Spouse | $0.096^{* * *}$ | $0.120^{* * *}$ | $0.124^{* * *}$ |
|  | $(0.002)$ | $(0.002)$ | $(0.004)$ |
| R-squared |  |  |  |
| Unitary Model Test | 0.264 | 0.144 | $[0.000]$ |

Notes: The model is exactly and strongly identified in the first-stage regressions ( $p<0.01$ in all cases). See the notes in Tables 5 and 6 for more information. ${ }^{* * *}$, ** and * denote significance at the $1 \%, 5 \%$ and $10 \%$ levels, respectively.

Table A6: Robustness Checks of Labor Supply Responses to Changes in the Marginal Net-of-Tax Rate and After-Tax Income without Control Variables or with Distribution Factors, 2006 to 2007-Instrumental Variables

|  | Controls |  | Distribution Factors (3) |
| :---: | :---: | :---: | :---: |
|  | No Control Variables <br> (1) | $\begin{aligned} & \text { Spousal } \\ & \text { Income Only } \\ & (2) \end{aligned}$ |  |
| Panel A: Extensive Margin |  |  |  |
| After-Tax Income of Individual | $\begin{gathered} -0.044^{* * *} \\ (0.003) \end{gathered}$ | $\begin{gathered} -0.041^{* * *} \\ (0.003) \end{gathered}$ | $\begin{gathered} -0.038^{* * *} \\ (0.003) \end{gathered}$ |
| After-Tax Income of Spouse | $\begin{gathered} -0.016^{* * *} \\ (0.003) \end{gathered}$ | $\begin{gathered} -0.009^{* * *} \\ (0.002) \end{gathered}$ | $\begin{aligned} & -0.009^{* * *} \\ & (0.002) \end{aligned}$ |
| Employment of Spouse |  | $\begin{aligned} & 0.132^{* * *} \\ & (0.002) \end{aligned}$ | $\begin{aligned} & 0.126^{* * *} \\ & (0.002) \end{aligned}$ |
| Income Ratio |  |  | $\begin{gathered} -0.055^{* * *} \\ (0.009) \end{gathered}$ |
| Sex Ratio |  |  | $\begin{gathered} -0.059^{* * *} \\ (0.021) \end{gathered}$ |
| Unitary Model Test | [0.000] | [0.000] | [0.000] |
| Panel B: Intensive Margin |  |  |  |
| Marginal Net-of-Tax Rate of Individual | $\begin{gathered} -0.009 \\ (0.193) \end{gathered}$ | $\begin{gathered} -0.090 \\ (0.186) \end{gathered}$ | $\begin{gathered} -0.146 \\ (0.183) \end{gathered}$ |
| Marginal Net-of-Tax Rate of Spouse | $\begin{gathered} 0.227 \\ (0.205) \end{gathered}$ | $\begin{gathered} 0.166 \\ (0.199) \end{gathered}$ | $\begin{gathered} 0.132 \\ (0.197) \end{gathered}$ |
| After-Tax Income of Individual | $\begin{gathered} -0.289^{* * *} \\ (0.069) \end{gathered}$ | $\begin{gathered} -0.228^{* * *} \\ (0.065) \end{gathered}$ | $\begin{gathered} -0.218^{* * *} \\ (0.064) \end{gathered}$ |
| After-Tax Income of Spouse | $\begin{gathered} -0.281^{* * *} \\ (0.062) \end{gathered}$ | $\begin{gathered} -0.230^{* * *} \\ (0.058) \end{gathered}$ | $\begin{gathered} -0.213^{* * *} \\ (0.057) \end{gathered}$ |
| Earnings of Spouse |  | $\begin{aligned} & 0.216^{* * *} \\ & (0.011) \end{aligned}$ | $\begin{aligned} & 0.204^{* * *} \\ & (0.012) \end{aligned}$ |
| Income Ratio |  |  | $\begin{gathered} -0.235^{* * *} \\ (0.057) \end{gathered}$ |
| Sex Ratio |  |  | $\begin{gathered} 0.058 \\ (0.089) \end{gathered}$ |
| Unitary Model Test | [0.922] | [0.979] | [0.947] |

Notes: Columns 1 and 2 control for income splines not the other covariates. The income ratio is the proportion of income of the individual to the couple. The sex ratio is the percent of men in the local population if the tax filer is male and the percent of women in the local population if the tax filer is female. The model is exactly and strongly identified in the first-stage regressions ( $p<0.01$ in all cases). Standard errors are in parentheses, clustered by individual. See the notes in Tables 5 and 6 for more information. ${ }^{* * *}$, ** and * denote significance at the $1 \%, 5 \%$ and $10 \%$ levels, respectively.

## Online Mathematical Appendix

Derivations of $\mathbf{d} \tilde{z}_{i} / \mathbf{d}\left(1-\tau^{i}\right)$ and $\mathbf{d} \tilde{z}_{i} / \mathbf{d}\left(1-\tau^{s}\right)$ —Unitary Model
To solve for $\mathrm{d} \tilde{z}_{i} / \mathrm{d}\left(1-\tau^{i}\right)$, totally differentiate equation (7) with respect to $\left(1-\tau^{i}\right)$ and evaluate at $\left\{\tilde{z}_{i}, z_{s}^{*}, x^{*}\right\}$ :

$$
\begin{align*}
\left(u_{c}^{i}+u_{c}^{s}\right)+\left(1-\tau^{i}\right)\left(u_{c c}^{i}+u_{c c}^{s}\right) & \frac{\mathrm{d} c}{\mathrm{~d}\left(1-\tau^{i}\right)} \\
& +\left(\left(1-\tau^{i}\right) u_{c z_{i}}^{i}+u_{z_{i} z_{i}}^{i}\right) \frac{\mathrm{d} z_{i}}{\mathrm{~d}\left(1-\tau^{i}\right)}+\left(1-\tau_{i}\right) u_{c z_{s}} \frac{\mathrm{~d} z_{s}}{\mathrm{~d}\left(1-\tau^{i}\right)}=0 \tag{19}
\end{align*}
$$

where:

$$
\begin{equation*}
\frac{\mathrm{d} c}{\mathrm{~d}\left(1-\tau^{i}\right)}=\tilde{z}_{i}+\bar{y}_{i}-x^{*}+\left(1-\tau^{i}\right) \frac{\mathrm{d} \tilde{z}_{i}}{\mathrm{~d}\left(1-\tau^{i}\right)}+\left(1-\tau^{s}\right) \frac{\mathrm{d} z_{s}^{*}}{\mathrm{~d}\left(1-\tau^{i}\right)} \tag{20}
\end{equation*}
$$

which follows from the budget constraint given by equations (2), (3), and (4). It follows that the solution can be written:

$$
\begin{equation*}
\frac{\mathrm{d} \tilde{z}_{i}}{\mathrm{~d}\left(1-\tau^{i}\right)}=\Gamma\left(u_{c}^{i}+u_{c}^{s}\right)-\Theta\left(\tilde{z}_{i}+\bar{y}_{i}-x^{\star}\right)-\Lambda \frac{\mathrm{d} z_{s}^{*}}{\mathrm{~d}\left(1-\tau^{i}\right)} \tag{21}
\end{equation*}
$$

where:

$$
\begin{array}{r}
\Gamma=-\left[\left(\left(u_{c c}^{i}+u_{c c}^{s}\right)\left(1-\tau^{i}\right)+u_{z_{i} c}^{i}\right)\left(1-\tau^{i}\right)+\left(u_{c z_{i}}\left(1-\tau^{i}\right)+u_{z_{i} z_{i}}\right)\right]^{-1} \\
\Theta=-\Gamma\left(\left(u_{c c}^{i}+u_{c c}^{s}\right)\left(1-\tau^{i}\right)+u_{z_{i} c}^{i}\right) \\
\Lambda=-\Gamma\left[\left(\left(u_{c c}^{i}+u_{c c}^{s}\right)\left(1-\tau^{i}\right)+u_{z_{i} c}^{i}\right)\left(1-\tau^{s}\right)+u_{c z_{s}}^{s}\left(1-\tau^{i}\right)\right] \tag{24}
\end{array}
$$

Multiplying both sides of equation (21) by $\mathrm{d}\left(1-\tau^{i}\right)$ expresses the solution identical to equation (10). The process of deriving $\mathrm{d} \tilde{z}_{i} / \mathrm{d}\left(1-\tau^{s}\right)$ is analogous to the one shown here, where equation (7) is totally differentiated with respect to $\left(1-\tau^{s}\right)$ and then solved for accordingly.

## Derivations of $\mathbf{d} \tilde{z}_{i} / \mathbf{d}\left(1-\tau^{i}\right)$ and $\mathbf{d} \tilde{z}_{i} / \mathbf{d}\left(1-\tau^{s}\right)$ —Collective Model

To solve for $\mathrm{d} \tilde{z}_{i} / \mathrm{d}\left(1-\tau^{i}\right)$ in the collective model, begin by noting that the first-order condition to the optimization problem in this case is:

$$
\begin{equation*}
\lambda\left(u_{c}^{i}\left(1-\tau^{i}\right)+u_{z_{i}}^{i}\right)+(1-\lambda)\left(u_{c}^{s}\left(1-\tau^{i}\right)+u_{z_{i}}^{s}\right)=0 \tag{25}
\end{equation*}
$$

Then, totally differentiate equation (25) with respect to $\left(1-\tau^{i}\right)$ and evaluate at $\left\{\tilde{z}_{i}, z_{s}^{*}, x^{*}\right\}$ :

$$
\begin{align*}
& \lambda\left\{u_{c}^{i}+\left(1-\tau^{i}\right)\left(u_{c c}^{i} \frac{\mathrm{~d} c}{\mathrm{~d}\left(1-\tau^{i}\right)}+u_{c z_{i}}^{i} \frac{\mathrm{~d} \tilde{z}_{i}}{\mathrm{~d}\left(1-\tau^{i}\right)}+u_{c z_{s}}^{i} \frac{d z_{s}^{*}}{\mathrm{~d}\left(1-\tau^{i}\right)}\right)\right. \\
& \left.\quad+u_{z_{i} c}^{i} \frac{\mathrm{~d} c}{\mathrm{~d}\left(1-\tau^{i}\right)}+u_{z_{i} z_{i}}^{i} \frac{\mathrm{~d} \tilde{z}_{i}}{\mathrm{~d}\left(1-\tau^{i}\right)}+u_{z_{i} z_{s}}^{i} \frac{\mathrm{~d} z_{s}^{*}}{\mathrm{~d}\left(1-\tau^{i}\right)}\right\} \\
& +(1-\lambda)\left\{u_{c}^{s}+\left(1-\tau^{i}\right)\left(u_{c c}^{s} \frac{\mathrm{~d} c}{\mathrm{~d}\left(1-\tau^{i}\right)}+u_{c z_{i}}^{s} \frac{\mathrm{~d} \tilde{z}_{i}}{\mathrm{~d}\left(1-\tau^{i}\right)}+u_{c z_{s}}^{s} \frac{d z_{s}^{*}}{\mathrm{~d}\left(1-\tau^{i}\right)}\right)\right. \\
& \\
& \left.+u_{z_{i} c}^{s} \frac{\mathrm{~d} c}{\mathrm{~d}\left(1-\tau^{i}\right)}+u_{z_{i} z_{i}}^{s} \frac{\mathrm{~d} \tilde{z}_{i}}{\mathrm{~d}\left(1-\tau^{i}\right)}+u_{z_{i} z_{s}}^{s} \frac{\mathrm{~d} z_{s}^{*}}{\mathrm{~d}\left(1-\tau^{i}\right)}\right\}  \tag{26}\\
& +\left(\left(u_{c}^{i}\left(1-\tau^{i}\right)+u_{z_{i}}^{i}\right)-\left(u_{c}^{s}\left(1-\tau^{i}\right)+u_{z_{i}}^{s}\right)\right) \lambda_{1-\tau^{i}}=0
\end{align*}
$$

where $\frac{\mathrm{d} c}{\mathrm{~d}\left(1-\tau^{i}\right)}$ is defined as in equation (20).
The next step is to rearrange equation (19), solving for $\mathrm{d} \tilde{z}_{i} / \mathrm{d}\left(1-\tau^{i}\right)$ by collecting like terms. For ease of notation, let:

$$
\begin{equation*}
u_{a b}=\lambda u_{a b}^{i}+(1-\lambda) u_{a b}^{s} \tag{27}
\end{equation*}
$$

for each $a, b \in\left\{c, z_{i}, z_{s}\right\}$. Equation (27) simplifies the expression for the weighted average of the second-order cross-partial derivatives of utility across the individual and spouse. It follows that the solution can be written:

$$
\left.\begin{array}{rl}
\frac{\mathrm{d} \tilde{z}_{i}}{\mathrm{~d}\left(1-\tau^{i}\right)}=\bar{\Gamma}\left(\lambda u_{c}^{i}+(1-\lambda) u_{c}^{s}\right)-\bar{\Theta}\left(\tilde{z}_{i}\right. & \left.+\bar{y}_{i}-x^{*}+\frac{\mathrm{d} R^{i}}{\mathrm{~d}\left(1-\tau^{i}\right)}\right)
\end{array}\right)-\bar{\Lambda} \frac{\mathrm{d} z_{s}^{*}}{\mathrm{~d}\left(1-\tau^{i}\right)} .
$$

where:

$$
\begin{array}{r}
\bar{\Gamma}=-\left(\left(u_{c c}\left(1-\tau^{i}\right)+u_{z_{i} c}\right)\left(1-\tau^{i}\right)+\left(u_{c z_{i}}\left(1-\tau^{i}\right)+u_{z_{i} z_{i}}\right)\right)^{-1} \\
\bar{\Theta}=-\bar{\Gamma}\left(u_{c c}\left(1-\tau^{i}\right)+u_{z_{i} c}\right) \\
\bar{\Lambda}=-\bar{\Gamma}\left(\left(u_{c c}\left(1-\tau^{i}\right)+u_{z_{i} c}\right)\left(1-\tau^{s}\right)+\left(u_{c z_{s}}\left(1-\tau^{i}\right)+u_{z_{i} z_{s}}\right)\right) \tag{31}
\end{array}
$$

Multiplying both sides of equation (28) by $\mathrm{d}\left(1-\tau^{i}\right)$ expresses the solution identical to equation (17). The process of deriving $\mathrm{d} \tilde{z}_{i} / \mathrm{d}\left(1-\tau^{s}\right)$ is analogous to the one shown here, where equation (25) is totally differentiated with respect to $\left(1-\tau^{s}\right)$ and then solved for accordingly.

