Online Appendix for "International Friends and Enemies" (Not for Publication)

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A Introduction

In this online appendix, we report the detailed derivations for the results reported in the paper and further supplementary results.

In Section B, we report additional results for the constant elasticity Armington model in Section I of the paper, including the proofs of the Propositions. Although in the paper we focus on the single-sector Armington model for expositional convenience, in Section C of this online appendix we establish a number of isomorphisms for the class of models characterized by a constant trade elasticity. In Subsection C.1, we derive our exposure measures in a version of the Eaton and Kortum (2002) model. In Subsection C.2, we derive these exposure measures in the Krugman (1980) model.

In Section D, we consider a number of extensions of our baseline friend-enemy exposure measures from Section I of the paper. In Subsection D.1, we derive the corresponding friend-enemy exposure measures allowing for both productivity and trade cost shocks. In Section D.2, we allow for trade imbalance. In Section D.3, we show that our results generalize to a multi-sector version of the constant elasticity Armington model. In Section D.4, we further generalize this multi-sector specification to allow for heterogeneous sector trade elasticities. In Section D.5, we show that we obtain analogous results to those in Section D.3 in a multi-sector version of the Eaton and Kortum (2002) model following Costinot, Donaldson and Komunjer (2012). In Section D.6, we further extend the multi-sector Armington model from Section D.3 to introduce input-output linkages following Caliendo and Parro (2015).

In Section E, we report additional descriptive evidence for Section III of the paper. In Section F, we present further empirical results on the relationship between economic and political friendship for Section IV of the paper. We demonstrate the robustness of our results to the use of either arc or point elasticities for real income exposure. We show that these two sets of elasticities are highly correlated with one another, at least for productivity shocks up to the cumulative change in the relative productivity of countries over our forty-year sample period.

Finally, in Section G, we provide further information on the data sources and definitions.

B Constant Elasticity Armington

In this section of the online appendix, we report the derivations for our baseline constant elasticity of substitution Armington model in Section I of the paper. This model falls within the class of of quantitative trade models considered by Arkolakis, Costinot and Rodriguez-Clare (2012), which satisfy the three macro restrictions of (i) a constant elasticity import demand system, (ii) profits are a constant share of income, and (iii) trade is balanced. In Section C of this online appendix, we show that our analysis also holds for other models within this class, including those of perfect competition and constant returns to scale with Ricardian technology differences as in Eaton and Kortum (2002), and those of monopolistic competition and increasing returns to scale, in which goods are differentiated by firm, as in Krugman (1980) and Melitz (2003) with an untruncated Pareto productivity distribution. The world economy consists of many countries indexed by $i, n \in \{1, \ldots, N\}$. Each country n has an exogenous supply of labor ℓ_n.

B.1 Consumer Preferences

The preferences of the representative consumer in country n are characterized by the following indirect utility function:

$$u_n = \frac{w_n}{p_n}, \qquad p_n = \left[\sum_{i=1}^{N} p_{ni}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \qquad \sigma > 1, \tag{B.1}$$

where w_n is the wage; p_n is the consumption goods price index; p_{ni} is the price in country n of the good produced by country i; and we focus on the case in which countries' goods are substitutes ($\sigma > 1$).

B.2 Production Technology

Goods are produced with labor according to a constant returns to scale production technology. These goods can be traded between countries subject to iceberg variable costs of trade, such that $\tau_{ni} \geq 1$ units must be shipped from country i to country n in order for one unit to arrive (where $\tau_{ni} > 1$ for $n \neq i$ and $\tau_{nn} = 1$). Therefore, the consumer in country n of purchasing a good ϑ from country i is:

$$p_{ni} = \frac{\tau_{ni} w_i}{z_i}, \tag{B.2}$$

where z_i captures productivity in country i and iceberg variable trade costs satisfy $\tau_{ni} > 1$ for $n \neq i$ and $\tau_{nn} = 1$.

B.3 Expenditure Shares

Using the properties of the CES demand function, country n's share of expenditure on goods produced in country i is:

$$s_{ni} = \frac{p_{ni}^{1-\sigma}}{\sum_{m=1}^{N} p_{nm}^{1-\sigma}}. \tag{B.3}$$

Totally differentiating this expenditure share equation (B.3), we get:

$$\frac{\mathrm{d}s_{ni}}{s_{ni}} = (\sigma - 1) \left(\sum_{h=1}^{N} s_{nh} \frac{\mathrm{d}p_{nh}}{p_{nh}} - \frac{\mathrm{d}p_{ni}}{p_{ni}} \right), \tag{B.4}$$

$$\mathrm{d}\ln s_{ni} = (\sigma - 1) \left(\sum_{h=1}^{N} s_{nh} \,\mathrm{d}\ln p_{nh} - \mathrm{d}\ln p_{ni} \right),$$

where, from the definition of p_{ni} in equation (B.2) above, we have:

$$\frac{\mathrm{d}p_{ni}}{p_{ni}} = \frac{\mathrm{d}\tau_{ni}}{\tau_{ni}} + \frac{\mathrm{d}w_i}{w_i} - \frac{\mathrm{d}z_i}{z_i}, \tag{B.5}$$

$$\mathrm{d}\ln p_{ni} = \mathrm{d}\ln \tau_{ni} + \mathrm{d}\ln w_i - \mathrm{d}\ln z_i.$$

B.4 Price Indices

Totally differentiating the consumption goods price index in equation (B.1), we have:

$$\frac{\mathrm{d}p_n}{p_n} = \sum_{m=1}^{N} s_{nm} \frac{\mathrm{d}p_{nm}}{p_{nm}}, \tag{B.6}$$

$$\mathrm{d}\ln p_n = \sum_{m=1}^{N} s_{nm}\, \mathrm{d}\ln p_{nm}.$$

B.5 Market Clearing

Market clearing requires that income in each country equals expenditure on the goods produced in that country:

$$w_i \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n. \tag{B.7}$$

Totally differentiating this market clearing condition (B.7), holding labor endowments constant, we have:

$$\frac{\mathrm{d}w_i}{w_i} w_i \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n \left(\frac{\mathrm{d}w_n}{w_n} + \frac{\mathrm{d}s_{ni}}{s_{ni}} \right),$$

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{n=1}^{N} \frac{s_{ni} w_n \ell_n}{w_i \ell_i} \left(\frac{\mathrm{d}w_n}{w_n} + \frac{\mathrm{d}s_{ni}}{s_{ni}} \right).$$

Using our result for the derivative of expenditure shares in equation (B.4) above, we can rewrite this as:

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{n=1}^{N} t_{in} \left(\frac{\mathrm{d}w_n}{w_n} + (\sigma - 1) \left(\sum_{h \in N} s_{nh} \frac{\mathrm{d}p_{nh}}{p_{nh}} - \frac{\mathrm{d}p_{ni}}{p_{ni}} \right) \right),$$

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in} \left(\mathrm{d}\ln w_n + (\sigma - 1) \left(\sum_{h=1}^{N} s_{nh}\, \mathrm{d}\ln p_{nh} - \mathrm{d}\ln p_{ni} \right) \right), \tag{B.8}$$

where we have defined t_{in} as the share of country i's income derived from market n:

$$t_{in} \equiv \frac{s_{ni} w_n \ell_n}{w_i \ell_i}.$$

B.6 Utility Again

Returning to our expression for indirect utility, we have:

$$u_n = \frac{w_n}{p_n}. \tag{B.9}$$

Totally differentiating indirect utility (B.9), we have:

$$\frac{\mathrm{d}u_n}{u_n} = \frac{\mathrm{d}w_n}{w_n} - \frac{\mathrm{d}p_n}{p_n}.$$

Using our total derivative of the sectoral price index in equation (B.6) above, we get:

$$\frac{\mathrm{d}u_n}{u_n} = \frac{\mathrm{d}w_n}{w_n} - \sum_{m=1}^{N} s_{nm} \frac{\mathrm{d}p_{nm}}{p_{nm}}, \tag{B.10}$$

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{m=1}^{N} s_{nm}\, \mathrm{d}\ln p_{nm}.$$

B.7 Wages and Productivity Shocks

We consider small productivity shocks, holding constant bilateral trade costs:

$$\mathrm{d}\ln \tau_{ni} \quad = 0, \quad \forall\, n, i \in N. \tag{B.11}$$

We start with our expression for the log change in wages from equation (B.8) above:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in} \left(\mathrm{d}\ln w_n + (\sigma - 1) \left(\sum_{h=1}^{N} s_{nh}\, \mathrm{d}\ln p_{nh} - \mathrm{d}\ln p_{ni} \right) \right),$$

Using the total derivative of prices (B.5) and our assumption of constant bilateral trade costs (B.11), we can write this expression for the log change in wages as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\, \mathrm{d}\ln w_n + (\sigma - 1) \left(\sum_{h=1}^{N} \sum_{n=1}^{N} t_{in} s_{nh} \left[\mathrm{d}\ln w_h - \mathrm{d}\ln z_h \right] - \left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i \right] \right), \tag{B.12}$$

which can be re-written as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\, \mathrm{d}\ln w_n + (\sigma - 1) \left(\sum_{n=1}^{N} m_{in} \left[\mathrm{d}\ln w_n - \mathrm{d}\ln z_n \right] \right),$$

$$m_{in} = \sum_{h=1}^{N} t_{ih} s_{hn} - 1_{n=i},$$

which has the following matrix representation in the paper:

$$\mathrm{d}\ln \mathbf{w} = \mathbf{T}\, \mathrm{d}\ln \mathbf{w} + \theta \mathbf{M} \left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z} \right), \tag{B.13}$$

$$\theta = \sigma - 1.$$

We solve for our friend-enemy income exposure measure by matrix inversion. Dividing both sides of equation (B.13) by $\theta + 1$, we have:

$$\frac{1}{\theta + 1}\, \mathrm{d}\ln \mathbf{w} = \frac{1}{\theta + 1} \mathbf{T}\, \mathrm{d}\ln \mathbf{w} + \frac{\theta}{\theta + 1} \mathbf{M} \left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z} \right),$$

$$\frac{1}{\theta + 1} \left(\mathbf{I} - \mathbf{T} - \theta \mathbf{M} \right) \mathrm{d}\ln \mathbf{w} = -\frac{\theta}{\theta + 1} \mathbf{M}\, \mathrm{d}\ln \mathbf{z}.$$

Now using $\mathbf{M} = \mathbf{TS} - \mathbf{I}$, we have:

$$\frac{1}{\theta+1}\left(\mathbf{I} - \mathbf{T} - \theta\mathbf{TS} + \theta\mathbf{I}\right)\, \mathrm{d}\ln\mathbf{w} = -\frac{\theta}{\theta+1}\mathbf{M}\,\mathrm{d}\ln\mathbf{z},$$

$$\left(\mathbf{I} - \frac{\mathbf{T} + \theta\mathbf{TS}}{\theta+1}\right)\, \mathrm{d}\ln\mathbf{w} = -\frac{\theta}{\theta+1}\mathbf{M}\,\mathrm{d}\ln\mathbf{z}.$$

Using our choice of world GDP as numeraire, which implies $\mathbf{Q}\,\mathrm{d}\ln\mathbf{w} = 0$, we have:

$$\left(\mathbf{I} - \frac{\mathbf{T} + \theta\mathbf{TS}}{\theta+1} + \mathbf{Q}\right)\, \mathrm{d}\ln\mathbf{w} = -\frac{\theta}{\theta+1}\mathbf{M}\,\mathrm{d}\ln\mathbf{z},$$

which can be re-written as:

$$\left(\mathbf{I} - \mathbf{V}\right)\, \mathrm{d}\ln\mathbf{w} = -\frac{\theta}{\theta+1}\mathbf{M}\,\mathrm{d}\ln\mathbf{z},$$

$$\mathbf{V} \equiv \frac{\mathbf{T} + \theta\mathbf{TS}}{\theta+1} - \mathbf{Q},$$

which yields the following solution of the change in wages in response to a productivity shock:

$$\mathrm{d}\ln\mathbf{w} = -\frac{\theta}{\theta+1}\left(\mathbf{I} - \mathbf{V}\right)^{-1}\mathbf{M}\,\mathrm{d}\ln\mathbf{z},$$

which can be re-written as:

$$\mathrm{d}\ln\mathbf{w} = \mathbf{W}\,\mathrm{d}\ln\mathbf{z},$$

where $\mathbf{W}$ is our friend-enemy income exposure measure:

$$\mathbf{W} \equiv -\frac{\theta}{\theta+1}\left(\mathbf{I} - \mathbf{V}\right)^{-1}\mathbf{M}. \tag{B.14}$$

B.8 Real Income and Productivity Shocks

From equation (B.10), the log change in real income is given by:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{m=1}^{N} s_{nm}\,\mathrm{d}\ln p_{nm},$$

Using the total derivative of prices (B.5) and our assumption of constant bilateral trade costs (B.11), we can write this log change in utility as:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{m=1}^{N} s_{nm}\left[\mathrm{d}\ln w_m - \mathrm{d}\ln z_m\right], \tag{B.15}$$

which has the following matrix representation in the paper:

$$\mathrm{d}\ln\mathbf{u} = \mathrm{d}\ln\mathbf{w} - \mathbf{S}\left(\mathrm{d}\ln\mathbf{w} - \mathrm{d}\ln\mathbf{z}\right). \tag{B.16}$$

We can re-write the above relationship as:

$$\mathrm{d}\ln\mathbf{u} = (\mathbf{I} - \mathbf{S})\ \mathrm{d}\ln\mathbf{w} + \mathbf{S}\,\mathrm{d}\ln\mathbf{z},$$

which using our solution for $\mathrm{d}\ln\mathbf{w}$ from above, can be further re-written as:

$$\mathrm{d}\ln\mathbf{u} = (\mathbf{I} - \mathbf{S})\,\mathbf{W}\,\mathrm{d}\ln\mathbf{z} + \mathbf{S}\,\mathrm{d}\ln\mathbf{z},$$

$$\mathrm{d}\ln\mathbf{u} = [(\mathbf{I} - \mathbf{S})\,\mathbf{W} + \mathbf{S}]\ \mathrm{d}\ln\mathbf{z},$$

$$\mathrm{d}\ln\mathbf{u} = \mathbf{U}\,\mathrm{d}\ln\mathbf{z},$$

where $\mathbf{U}$ is our friend-enemy real income exposure measure:

$$\mathbf{U} \equiv [(\mathbf{I} - \mathbf{S})\,\mathbf{W} + \mathbf{S}]\,. \tag{B.17}$$

B.9 Exact-Hat Algebra

We now derive the arc elasticity counterparts of our income exposure (B.14) and real income exposure (B.17) measures using exact-hat algebra. We start with the goods market clearing condition in a counterfactual equilibrium:

$$w_i'\ell_i = \sum_{n=1}^{N} \frac{(\tau_{ni}w_i'/z_i')^{-\theta}}{\sum_{m=1}^{N}(\tau_{ni}w_m'/z_m')^{-\theta}} w_n'\ell_n,$$

where we denote the counterfactual values of variables with a prime (such as x_i'); we hold labor endowments (ℓ_i) and bilateral trade costs (τ_{ni}) constant at their values in the initial equilibrium (no prime); and $\theta = \sigma - 1$ is the trade elasticity.

Denoting the relative value of variables in the counterfactual and actual equilibrium by a hat (such that $\widehat{x}_i \equiv x_i'/x_i$), we can re-write this goods market clearing condition in the counterfactual equilibrium as:

$$\widehat{w}_i\ell_i w_i = \sum_{n=1}^{N} \frac{s_{ni}\,(\widehat{w}_i/\widehat{z}_i)^{-\theta}}{\sum_{m=1}^{N} s_{nm}\,(\widehat{w}_m/\widehat{z}_m)^{-\theta}} \widehat{w}_n w_n \ell_n,$$

which can be further re-written in the form of equation (6) in the paper:

$$\ln\hat{w}_i = \left(\frac{\theta}{\theta+1}\right)\ln\hat{z}_i + \frac{1}{\theta+1}\ln\left[\sum_{n=1}^{N} t_{in}\frac{\hat{w}_n}{\sum_{m=1}^{N} s_{nm}\hat{w}_m^{-\theta}\hat{z}_m^{\theta}}\right].$$

Next, we consider real income per capita in a counterfactual equilibrium:

$$u_n' = \frac{w_n'}{\left[\sum_{i=1}^{N}(\tau_{ni}w_i'/z_i')^{-\theta}\right]^{-\frac{1}{\theta}}},$$

where we again hold bilateral trade costs (τ_{ni}) constant at their values in the initial equilibrium (no prime). We now re-write real income per capita in the counterfactual equilibrium in terms of the relative changes of variables:

$$\widehat{u}_n u_n = \frac{\widehat{w}_n w_n}{\left[\sum_{i=1}^{N}(\tau_{ni}\widehat{w}_i w_i/(\widehat{z}_i z_i))^{-\theta}\right]^{-\frac{1}{\theta}}},$$

which can be further re-written in the form of equation (7) in the paper:

$$\ln \widehat{u}_i = \ln \hat{w}_i + \frac{1}{\theta} \ln \left[\sum_{n=1}^{N} s_{in} \hat{w}_n^{-\theta} \hat{z}_n^{\theta} \right].$$

B.10 Relationship to the ACR Gains from Trade Formula

From equation (B.10), the log change in utility is given by:

$$\mathrm{d} \ln u_n = \mathrm{d} \ln w_n - \sum_{m=1}^{N} s_{nm} \, \mathrm{d} \ln p_{nm}. \tag{B.18}$$

Choosing country n's wage as the numeraire and assuming no changes in its productivity or domestic trade costs, we have:

$$\mathrm{d} \ln w_n = 0, \quad \mathrm{d} \ln z_n = 0, \quad \mathrm{d} \ln \tau_{nn} = 0, \quad \mathrm{d} \ln p_{nn} = 0.$$

The import demand system in equation (B.3) implies:

$$\mathrm{d} \ln s_{nm} - \mathrm{d} \ln s_{nn} = -(\sigma - 1) \left(\mathrm{d} \ln p_{nm} - \mathrm{d} \ln p_{nn} \right).$$

Using this result in equation (B.18), the log change in real income can be written as:

$$\mathrm{d} \ln u_n = \sum_{m=1}^{N} \frac{s_{nm} \left(\mathrm{d} \ln s_{nm} - \mathrm{d} \ln s_{nn} \right)}{(\sigma - 1)}$$

Using $\sum_{m=1}^{N} s_{nm} = 1$ and $\sum_{m=1}^{N} \mathrm{d} s_{nm} = 0$, we obtain the ACR welfare gains from trade formula for small changes:

$$\mathrm{d} \ln u_n = -\frac{\mathrm{d} \ln s_{nn}}{\sigma - 1}. \tag{B.19}$$

Integrating both sides of equation (B.19), we get:

$$\int_{u_n^0}^{u_n^1} \frac{\mathrm{d} u_n}{u_n} = -\frac{1}{\sigma - 1} \int_{s_{nn}^0}^{s_{nn}^1} \frac{\mathrm{d} s_{nn}}{s_{nn}},$$

$$\ln \left(\frac{u_n^1}{u_n^0} \right) = -\frac{1}{\sigma - 1} \ln \left(\frac{s_{nn}^1}{s_{nn}^0} \right),$$

$$\left(\frac{s_n^1}{s_n^0} \right) = \left(\frac{s_{nn}^1}{s_{nn}^0} \right)^{-\frac{1}{\sigma - 1}}, \tag{B.20}$$

which corresponds to the ACR welfare gains from trade formula for large changes.

C Single-Sector Isomorphisms

While for convenience of exposition we focus on the single-sector Armington model in the paper, we obtain the same income and real income exposure measures for all models with a constant trade elasticity in the class considered by Arkolakis, Costinot and Rodriguez-Clare (2012). In Subsection C.1, we derive our exposure measures in a version of the Eaton and Kortum (2002) model. In Subsection C.2, we derive these measures in the Krugman (1980) model.

The Armington model in the paper and the Eaton and Kortum (2002) model assume perfect competition and constant returns to scale, whereas the Krugman (1980) model assumes monopolistic competition and increasing returns to scale. In the Eaton and Kortum (2002) model, the trade elasticity corresponds to the Fréchet shape parameter. In contrast, in the Armington model and the Krugman (1980) model, the trade elasticity corresponds to the elasticity of substitution between varieties. Nevertheless, our income and real income exposure measures hold across all three models, because they all feature a constant trade elasticity.

In Subsection C.2, we focus on the representative firm Krugman (1980) model of monopolistic competition for simplicity, but our income and real income exposure measures also hold in the heterogeneous firm model of Melitz (2003) with an untruncated Pareto productivity distribution. In Subsection C.2, we also show that our income and real income exposure measures with monopolistic competition and increasing returns to scale take a similar form whether we consider shocks to the variable or fixed components of production costs.

C.1 Eaton and Kortum (2002)

We consider a version of Eaton and Kortum (2002) with labor as the sole factor of production. Trade arises because of Ricardian technology differences; production technologies are constant returns to scale; and markets are perfectly competitive. The world economy consists of a set of countries indexed by $i, n \in \{1, \dots, N\}$. Each country n has an exogenous supply of labor ℓ_n.

C.1.1 Consumer Preferences

The preferences of the representative consumer in country n are characterized by the following indirect utility function:

$$u_n = \frac{w_n}{p_n}, \tag{C.1}$$

where w_n is the wage and p_n is the consumption goods price index, which is defined over consumption of a fixed continuum of goods according to the constant elasticity of substitution (CES) functional form:

$$p_n = \left[\int_0^1 p_n(\vartheta)^{1-\sigma} \, d\vartheta \right]^{\frac{1}{1-\sigma}}, \qquad \sigma > 1, \tag{C.2}$$

where $p_n(\vartheta)$ denotes the price of good ϑ in country n.

C.1.2 Production Technology

Goods are produced with labor according to a constant returns to scale production technology. These goods can be traded between countries subject to iceberg variable costs of trade, such that

$\tau_{ni} \geq 1$ units must be shipped from country i to country n in order for one unit to arrive (where $\tau_{ni} > 1$ for $n \neq i$ and $\tau_{nn} = 1$). Therefore, the price for consumers in country n of purchasing a good ϑ from country i is:

$$p_{ni}(\vartheta) = \frac{\tau_{ni} w_i}{z_i a_i(\vartheta)}, \tag{C.3}$$

where z_i captures common determinants of productivity across goods within country i and $a_i(\vartheta)$ captures idiosyncratic determinants of productivity for each good ϑ within that country. Iceberg variable trade costs satisfy $\tau_{ni} > 1$ for $n \neq i$ and $\tau_{nn} = 1$. Productivity for each good ϑ in each sector k and each country i is drawn independently from the following Fréchet distribution:

$$F_i(a) = \exp\left(-a^{-\theta}\right), \qquad \theta > 1, \tag{C.4}$$

where we normalize the Fréchet scale parameter to one, because it enters the model isomorphically to z_i.

C.1.3 Expenditure Shares

Using the properties of this Fréchet distribution, country n's share of expenditure on goods produced in country i is:

$$s_{ni} = \frac{\left(\tau_{ni} w_i / z_i\right)^{-\theta}}{\sum_{m=1}^{N} \left(\tau_{nm} w_m / z_m\right)^{-\theta}} = \frac{\left(\rho_{ni}\right)^{-\theta}}{\sum_{m=1}^{N} \left(\rho_{nm}\right)^{-\theta}}, \tag{C.5}$$

where we have defined the following price inclusive of trade costs term:

$$\rho_{ni} \equiv \frac{\tau_{ni} w_i}{z_i}. \tag{C.6}$$

Totally differentiating this expenditure share equation (C.5) we get:

$$\frac{\mathrm{d}s_{ni}}{s_{ni}} = \theta \left(\sum_{h=1}^{N} s_{nh} \frac{\mathrm{d}\rho_{nh}}{\rho_{nh}} - \frac{\mathrm{d}\rho_{ni}}{\rho_{ni}} \right), \tag{C.7}$$

$$\mathrm{d}\ln s_{ni} = \theta \left(\sum_{h=1}^{N} s_{nh} \, \mathrm{d}\ln \rho_{nh} - \mathrm{d}\ln \rho_{ni} \right),$$

where, from the definition of ρ_{ni} in equation (C.6) above, we have:

$$\frac{\mathrm{d}\rho_{ni}}{\rho_{ni}} = \frac{\mathrm{d}\tau_{ni}}{\tau_{ni}} + \frac{\mathrm{d}w_i}{w_i} - \frac{\mathrm{d}z_i}{z_i}, \tag{C.8}$$

$$\mathrm{d}\ln \rho_{ni} = \mathrm{d}\ln \tau_{ni} + \mathrm{d}\ln w_i - \mathrm{d}\ln z_i.$$

C.1.4 Price Indices

Using the properties of the Fréchet distribution (C.4), the consumption goods price index is given by:

$$p_n = \gamma \left[\sum_{m=1}^{N} (\rho_{nm})^{-\theta} \right]^{-\frac{1}{\theta}}, \tag{C.9}$$

where

$$\gamma \equiv \left[\Gamma \left(\frac{\theta + 1 - \sigma}{\theta} \right) \right]^{\frac{1}{1-\sigma}},$$

and $\Gamma(\cdot)$ is the Gamma function. Totally differentiating this price index (C.9), we have:

$$\frac{\mathrm{d}p_n}{p_n} = \sum_{m=1}^{N} s_{nm} \frac{\mathrm{d}\rho_{nm}}{\rho_{nm}}, \tag{C.10}$$

$$\mathrm{d}\ln p_n = \sum_{m=1}^{N} s_{nm}\, \mathrm{d}\ln \rho_{nm}.$$

C.1.5 Market Clearing

Market clearing requires that income in each country equals expenditure on goods produced in that country:

$$w_i \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n. \tag{C.11}$$

Totally differentiating this market clearing condition (C.11), holding labor endowments constant, we have:

$$\frac{\mathrm{d}w_i}{w_i} w_i \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n \left(\frac{\mathrm{d}w_n}{w_n} + \frac{\mathrm{d}s_{ni}}{s_{ni}} \right),$$

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{n=1}^{N} \frac{s_{ni} w_n \ell_n}{w_i \ell_i} \left(\frac{\mathrm{d}w_n}{w_n} + \frac{\mathrm{d}s_{ni}}{s_{ni}} \right).$$

Using our result for the derivative of expenditure shares in equation (C.7) above, we can rewrite this as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in} \left(\mathrm{d}\ln w_n + \theta \left(\sum_{h=1}^{N} s_{nh}\, \mathrm{d}\ln \rho_{nh} - \mathrm{d}\ln \rho_{ni} \right) \right), \tag{C.12}$$

where we have defined t_{in} as country n's expenditure on country i as a share of country i's income:

$$t_{in} \equiv \frac{s_{ni} w_n \ell_n}{w_i \ell_i}.$$

C.1.6 Utility Again

Returning to our expression for indirect utility, we have:

$$u_n = \frac{w_n}{p_n}. \tag{C.13}$$

Totally differentiating indirect utility (C.13), we have:

$$\frac{\mathrm{d}u_n}{u_n} = \frac{\mathrm{d}w_n}{w_n} - \frac{\mathrm{d}p_n}{p_n}.$$

Using our total derivative of the sectoral price index in equation (C.10) above, we get:

$$\frac{\mathrm{d}u_n}{u_n} = \frac{\mathrm{d}w_n}{w_n} - \sum_{m=1}^{N} s_{nm} \frac{\mathrm{d}\rho_{nm}}{\rho_{nm}}, \tag{C.14}$$

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{m=1}^{N} s_{nm}\, \mathrm{d}\ln \rho_{nm}.$$

C.1.7 Wages and Common Productivity Shocks

We consider small productivity shocks, holding constant bilateral trade costs:

$$\mathrm{d}\ln \tau_{ni} \quad = 0, \quad \forall\, n, i \in N. \tag{C.15}$$

We start with our expression for the log change in wages from equation (C.12) above:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in} \left(\mathrm{d}\ln w_n + \theta \left(\sum_{h=1}^{N} s_{nh}\, \mathrm{d}\ln \rho_{nh} - \mathrm{d}\ln \rho_{ni} \right) \right),$$

Using the total derivative of the price term (C.8) and our assumption of constant bilateral trade costs (C.15), we can write this expression for the log change in wages as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\, \mathrm{d}\ln w_n + \theta \left(\sum_{h \in N}^{N} \sum_{n=1}^{N} t_{in} s_{nh} \left(\mathrm{d}\ln w_h - \mathrm{d}\ln z_h \right) - \left(\mathrm{d}\ln w_i - \mathrm{d}\ln z_i \right) \right). \tag{C.16}$$

which can be re-written as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\, \mathrm{d}\ln w_n + \theta \left(\sum_{n=1}^{N} m_{in} \left[\mathrm{d}\ln w_n - \mathrm{d}\ln z_n \right] \right),$$

$$m_{in} = \sum_{h=1}^{N} t_{ih} s_{hn} - 1_{n=i},$$

which has the same matrix representation as in the paper:

$$\mathrm{d}\ln \mathbf{w} = \mathbf{T}\, \mathrm{d}\ln \mathbf{w} + \theta \mathbf{M} \left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z} \right). \tag{C.17}$$

C.1.8 Utility and Common Productivity Shocks

From equation (C.14), the log change in utility is given by:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{m=1}^{N} s_{nm}\,\mathrm{d}\ln \rho_{nm}.$$

Using the total derivative of the price term (C.8) and our assumption of constant bilateral trade costs (C.15), we can write this log change in utility as:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{m=1}^{N} s_{nm}\left[\mathrm{d}\ln w_m - \mathrm{d}\ln z_m\right],$$

which has the same matrix representation as in the paper:

$$\mathrm{d}\ln \mathbf{u} = \mathrm{d}\ln \mathbf{w} - \mathbf{S}\left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z}\right). \tag{C.18}$$

C.2 Krugman (1980)

We consider a version of Krugman (1980) with labor as the sole factor of production, in which markets are monopolistically competitive, and trade arises from love of variety and increasing returns to scale. The world economy consists of a set of countries indexed by $i, n \in \{1, \ldots, N\}$. Each country n has an exogenous supply of labor ℓ_n.

C.2.1 Consumer Preferences

The preferences of the representative consumer in country n are characterized by the following indirect utility function:

$$u_n = \frac{w_n}{p_n}, \qquad p_n = \left[\sum_{i=1}^{N} \int_0^{M_i} p_{ni}(j)^{1-\sigma}\, dj\right]^{\frac{1}{1-\sigma}}, \qquad \sigma > 1, \tag{C.19}$$

where w_n is the wage; p_n is the consumption goods price index; $p_{ni}(j)$ is the price in country n of a variety j produced in country i; M_i is the endogenous mass of varieties; and varieties are substitutes ($\sigma > 1$).

C.2.2 Production Technology

Varieties are produced under conditions of monopolistic competition and increasing returns to scale. To produce a variety, a firm must incur a fixed cost of F units of labor and a constant variable cost in terms of labor that depends on a country's productivity z_i. Therefore the total amount of labor ($l_i(j)$) required to produce $x_i(j)$ units of variety j in country i is:

$$l_i(j) = F_i + \frac{x_i(j)}{z_i}. \tag{C.20}$$

Varieties can be traded between countries subject to iceberg variable costs of trade, such that $\tau_{ni} \geq 1$ units must be shipped from country i to country n in order for one unit to arrive (where $\tau_{ni} > 1$ for $n \neq i$ and $\tau_{nn} = 1$). Profit maximization and zero profits imply that equilibrium prices are a constant markup over marginal cost:

$$p_{ni}\left(j\right) = \left(\frac{\sigma}{\sigma - 1}\right)\rho_{ni}, \qquad \rho_{ni} \equiv \frac{\tau_{ni} w_i}{z_i}, \tag{C.21}$$

and equilibrium employment for each variety is equal to a constant:

$$l_i\left(j\right) = \bar{l} = \sigma F_i. \tag{C.22}$$

Given this constant equilibrium employment for each variety, labor market clearing implies that the total mass of varieties supplied by each country is proportional to its labor endowment:

$$M_i = \frac{\ell_i}{\sigma F_i}. \tag{C.23}$$

From the definition of ρ_{ni} in equation (C.21) above, we have:

$$\frac{\mathrm{d}\rho_{ni}}{\rho_{ni}} = \frac{\mathrm{d}\tau_{ni}}{\tau_{ni}} + \frac{\mathrm{d}w_i}{w_i} - \frac{\mathrm{d}z_i}{z_i}, \tag{C.24}$$

$$\mathrm{d}\ln\rho_{ni} = \mathrm{d}\ln\tau_{ni} + \mathrm{d}\ln w_i - \mathrm{d}\ln z_i.$$

Totally differentiating in the labor market clearing condition (C.23), holding country endowments constant, we have:

$$\frac{\mathrm{d}M_i}{M_i} = -\frac{\mathrm{d}F_i}{F_i}, \tag{C.25}$$

$$\mathrm{d}\ln M_i = -\,\mathrm{d}\ln F_i.$$

C.2.3 Expenditure Shares

Using the symmetry of equilibrium prices and the properties of the CES demand function, country n's share of expenditure on goods produced in country i is:

$$s_{ni} = \frac{M_i p_{ni}^{1-\sigma}}{\sum_{m=1}^{N} M_m p_{nm}^{1-\sigma}} = \frac{\left(\ell_i/F_i\right)\rho_{ni}^{1-\sigma}}{\sum_{m=1}^{N}\left(\ell_m/F_m\right)\rho_{nm}^{1-\sigma}}. \tag{C.26}$$

Totally differentiating this expenditure share equation (C.26) we get:

$$\frac{\mathrm{d}s_{ni}}{s_{ni}} = -\left[\frac{\mathrm{d}F_i}{F_i} + \left(\sigma - 1\right)\frac{\mathrm{d}\rho_{ni}}{\rho_{ni}}\right] + \sum_{h=1}^{N} s_{nh}\left[\frac{\mathrm{d}F_h}{F_h} + \left(\sigma - 1\right)\frac{\mathrm{d}\rho_{nh}}{\rho_{nh}}\right], \tag{C.27}$$

$$\mathrm{d}\ln s_{ni} = -\left[\mathrm{d}\ln F_i + \left(\sigma - 1\right)\,\mathrm{d}\ln\rho_{ni}\right] + \sum_{h=1}^{N} s_{nh}\left[\mathrm{d}\ln F_h + \left(\sigma - 1\right)\,\mathrm{d}\ln\rho_{nh}\right].$$

C.2.4 Price Indices

Using the symmetry of equilibrium prices, the price index (C.19) can be re-written as:

$$p_n = \left[\sum_{i=1}^{N} M_i p_{ni}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}.$$

Totally differentiating this price index, we have:

$$\frac{\mathrm{d}p_n}{p_n} = \sum_{i=1}^{N} s_{ni}\left[\frac{1}{1-\sigma}\frac{\mathrm{d}M_i}{M_i} + \frac{\mathrm{d}p_{ni}}{p_{ni}}\right],$$

which using the equilibrium pricing rule (C.21) and the labor market clearing condition (C.23) can be written as:

$$\frac{\mathrm{d}p_n}{p_n} = \sum_{i=1}^{N} s_{ni}\left[\frac{1}{\sigma-1}\frac{\mathrm{d}F_i}{F_i} + \frac{\mathrm{d}\rho_{ni}}{\rho_{ni}}\right], \tag{C.28}$$

$$\mathrm{d}\ln p_n = \sum_{i=1}^{N} s_{ni}\left[\frac{1}{\sigma-1}\mathrm{d}\ln F_i + \mathrm{d}\ln\rho_{ni}\right].$$

C.2.5 Market Clearing

Market clearing requires that income in each country equals expenditure on goods produced in that country:

$$w_i\ell_i = \sum_{n=1}^{N} s_{ni}w_n\ell_n. \tag{C.29}$$

Totally differentiating this market clearing condition (C.29), holding labor endowments constant, we have:

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{n=1}^{N}\frac{s_{ni}w_n\ell_n}{w_i\ell_i}\left(\frac{\mathrm{d}w_n}{w_n} + \frac{\mathrm{d}s_{ni}}{s_{ni}}\right).$$

Using our result for the derivative of expenditure shares in equation (C.27) above, we can rewrite this as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\left(\begin{array}{c}\mathrm{d}\ln w_n + \left(\sum_{h=1}^{N} s_{nh}\,\mathrm{d}\ln F_h - \mathrm{d}\ln F_i\right)\\ +(\sigma-1)\left(\sum_{h=1}^{N} s_{nh}\,\mathrm{d}\ln\rho_{nh} - \mathrm{d}\ln\rho_{ni}\right)\end{array}\right), \tag{C.30}$$

where we have defined t_{in} as the share of country i's income derived from market n:

$$t_{in} \equiv \frac{s_{ni}w_n\ell_n}{w_i\ell_i}.$$

C.2.6 Utility Again

Returning to our expression for indirect utility, we have:

$$u_n = \frac{w_n}{p_n}. \tag{C.31}$$

Totally differentiating indirect utility (C.31), we have:

$$\frac{\mathrm{d}u_n}{u_n} = \frac{\mathrm{d}w_n}{w_n} - \frac{\mathrm{d}p_n}{p_n}.$$

Using our total derivative of the sectoral price index in equation (C.28) above, we get:

$$\frac{\mathrm{d}u_n}{u_n} = \frac{\mathrm{d}w_n}{w_n} - \sum_{m=1}^{N} s_{nm} \left[\frac{1}{\sigma - 1} \frac{\mathrm{d}F_i}{F_i} + \frac{\mathrm{d}\rho_{ni}}{\rho_{ni}} \right], \tag{C.32}$$

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{m=1}^{N} s_{nm} \left[\frac{1}{\sigma - 1} \mathrm{d}\ln F_i + \mathrm{d}\ln \rho_{ni} \right].$$

C.2.7 Wages and Productivity Shocks

We consider small shocks to productivity, holding constant bilateral trade costs and fixed costs:

$$\mathrm{d}\ln \tau_{ni} = 0, \qquad \mathrm{d}\ln F_i = 0, \qquad \forall\, n, i \in N. \tag{C.33}$$

We start with our expression for the log change in wages from equation (C.30) above:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in} \left(\begin{array}{c} \mathrm{d}\ln w_n + \left(\sum_{h=1}^{N} s_{nh}\, \mathrm{d}\ln F_h - \mathrm{d}\ln F_i \right) \\ + (\sigma - 1) \left(\sum_{h=1}^{N} s_{nh}\, \mathrm{d}\ln \rho_{nh} - \mathrm{d}\ln \rho_{ni} \right) \end{array} \right),$$

Using the total derivative of the price term (C.24) and our assumptions of constant bilateral trade costs and constant fixed costs (C.33), we can write this expression for the log change in wages as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\, \mathrm{d}\ln w_n + (\sigma - 1) \left(\sum_{h=1}^{N} \sum_{n=1}^{N} t_{in} s_{nh} \left[\mathrm{d}\ln w_h - \mathrm{d}\ln z_h \right] - \left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i \right] \right), \tag{C.34}$$

which can be re-written as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\, \mathrm{d}\ln w_n + (\sigma - 1) \left(\sum_{n=1}^{N} m_{in} \left[\mathrm{d}\ln w_n - \mathrm{d}\ln z_n \right] \right),$$

$$m_{in} = \sum_{h=1}^{N} t_{ih} s_{hn} - 1_{n=i},$$

which has the same matrix representation as in the paper:

$$\mathrm{d}\ln \mathbf{w} = \mathbf{T}\, \mathrm{d}\ln \mathbf{w} + \theta \mathbf{M} \left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z} \right), \tag{C.35}$$

$$\theta = \sigma - 1.$$

C.2.8 Real Income and Productivity Shocks

From equation (C.32), the log change in utility is given by:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{i=1}^{N} s_{ni}\left[\frac{1}{\sigma-1}\,\mathrm{d}\ln F_i + \mathrm{d}\ln \rho_{ni}\right],$$

Using the total derivative of the price term (C.24) and our assumptions of constant bilateral trade costs and constant fixed costs (C.33), we can write this log change in utility as:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{i=1}^{N} s_{ni}\left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right], \tag{C.36}$$

which has the same matrix representation as in the paper:

$$\mathrm{d}\ln \mathbf{u} = \mathrm{d}\ln \mathbf{w} - \mathbf{S}\left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z}\right). \tag{C.37}$$

C.2.9 Wages and Fixed Cost Shocks

We next consider small shocks to fixed costs, holding constant bilateral trade costs and productivity:

$$\mathrm{d}\ln \tau_{ni} = 0, \qquad \mathrm{d}\ln z_i = 0, \qquad \forall\, n, i \in N. \tag{C.38}$$

We start with our expression for the log change in wages from equation (C.30) above:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\left(\begin{array}{c} \mathrm{d}\ln w_n + \left(\sum_{h=1}^{N} s_{nh}\,\mathrm{d}\ln F_h - \mathrm{d}\ln F_i\right) \\ +(\sigma-1)\left(\sum_{h=1}^{N} s_{nh}\,\mathrm{d}\ln \rho_{nh} - \mathrm{d}\ln \rho_{ni}\right)\end{array}\right),$$

Using the total derivative of the price term (C.24) and our assumptions of constant bilateral trade costs and constant productivity (C.38), we can write this expression for the log change in wages as:

$$\mathrm{d}\ln w_i = \left[\begin{array}{c} \sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln w_n + \left(\sum_{n=1}^{N} t_{in}\sum_{h=1}^{N} s_{nh}\,\mathrm{d}\ln F_h - \mathrm{d}\ln F_i\right) \\ +(\sigma-1)\left(\sum_{h=1}^{N}\sum_{n=1}^{N} t_{in}s_{nh}\,\mathrm{d}\ln w_h - \mathrm{d}\ln w_i\right)\end{array}\right], \tag{C.39}$$

which can be re-written as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln w_n + \sum_{n=1}^{N} m_{in}\,\mathrm{d}\ln F_n + (\sigma-1)\left(\sum_{n=1}^{N} m_{in}\,\mathrm{d}\ln w_n\right),$$

$$m_{in} = \sum_{h=1}^{N} t_{ih}s_{hn} - 1_{n=i},$$

which has a similar matrix representation to that for productivity shocks above:

$$\mathrm{d}\ln \mathbf{w} = \mathbf{T}\,\mathrm{d}\ln \mathbf{w} + \theta\mathbf{M}\left(\mathrm{d}\ln \mathbf{w} + \frac{1}{\theta}\,\mathrm{d}\ln \mathbf{F}\right), \tag{C.40}$$

$$\theta = \sigma - 1.$$

C.2.10 Real Income and Fixed Cost Shocks

From equation (C.32), the log change in utility is given by:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{i=1}^{N} s_{ni}\left[\frac{1}{\sigma-1}\,\mathrm{d}\ln F_i + \mathrm{d}\ln \rho_{ni}\right],$$

Using the total derivative of the price term (C.24) and our assumptions of constant bilateral trade costs and constant productivity (C.38), we can write this log change in utility as:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{i=1}^{N} s_{ni}\left[\frac{1}{\sigma-1}\,\mathrm{d}\ln F_i + \mathrm{d}\ln w_i\right], \tag{C.41}$$

which has a similar matrix representation to that for productivity shocks above:

$$\mathrm{d}\ln \mathbf{u} = \mathrm{d}\ln \mathbf{w} - \mathbf{S}\left(\mathrm{d}\ln \mathbf{w} + \frac{1}{\theta}\,\mathrm{d}\ln \mathbf{F}\right). \tag{C.42}$$

D Extensions

In Subsection D.1, we derive the corresponding friend-enemy matrix representations with both productivity and trade cost shocks, as discussed in Section I of the paper. In Section D.2, we relax one of the ACR macro restrictions to allow for trade imbalance, as discussed in Section I of the paper. In Section D.3, we show that our results generalize to a multi-sector Armington model with a single constant trade elasticity, as discussed in Section I of the paper. In Section D.4, we further generalize this specification to allow for heterogeneous sector trade elasticities. In Section D.5, we show that our results also hold in a multi-sector version of the Eaton and Kortum (2002) model following Costinot, Donaldson and Komunjer (2012). In Section D.6, we further extend the multi-sector specification to introduce input-output linkages following Caliendo and Parro (2015), as discussed in Section I of the paper.

D.1 Trade Cost Reductions

We now show that we obtain similar results incorporating trade cost reductions. We start with our expression for the log change in wages from equation (B.8) above:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\left(\mathrm{d}\ln w_n + (\sigma-1)\left(\sum_{h=1}^{N} s_{nh}\,\mathrm{d}\ln p_{nh} - \mathrm{d}\ln p_{ni}\right)\right),$$

which can be re-written as follows:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln w_n + (\sigma-1)\left(\begin{array}{c} \sum_{h=1}^{N}\sum_{n=1}^{N} t_{in}s_{nh}\left[\mathrm{d}\ln w_h + \mathrm{d}\ln \tau_{nh} - \mathrm{d}\ln z_h\right] \\ -\sum_{n=1}^{N} t_{in}\left[\mathrm{d}\ln w_i + \mathrm{d}\ln \tau_{ni} - \mathrm{d}\ln z_i\right] \end{array}\right).$$

We now define inward and outward measures of trade costs as:

$$\mathrm{d}\ln \tau_n^{\mathbf{in}} \equiv \sum_{i} s_{ni}\,\mathrm{d}\ln \tau_{ni}, \tag{D.1}$$

$$\mathrm{d}\ln \tau_i^{\mathbf{out}} \equiv \sum_n t_{in}\,\mathrm{d}\ln \tau_{ni}. \tag{D.2}$$

Using these definitions of inward and outward trade costs, we can rewrite the above proportional change in wages as follows:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln w_n + (\sigma-1)\left(\begin{array}{c} \sum_{h=1}^{N}\sum_{n=1}^{N} t_{in}s_{nh}\left[\mathrm{d}\ln w_h - \mathrm{d}\ln z_h\right] - \left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right] \\ +\sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln \tau_n^{\mathbf{in}} - \mathrm{d}\ln \tau_i^{\mathbf{out}} \end{array}\right), \tag{D.3}$$

which can be re-written as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln w_n + (\sigma-1)\left(\begin{array}{c} \sum_{n=1}^{N} m_{in}\left[\mathrm{d}\ln w_n - \mathrm{d}\ln z_n\right] \\ +\sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln \tau_n^{\mathbf{in}} - \mathrm{d}\ln \tau_i^{\mathbf{out}} \end{array}\right),$$

$$m_{in} = \sum_{h=1}^{N} t_{ih}s_{hn} - 1_{n=i},$$

which has the following matrix representation:

$$\mathrm{d}\ln \mathbf{w} = \mathbf{T}\,\mathrm{d}\ln \mathbf{w} + \theta\left[\mathbf{M}\left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z}\right) + \mathbf{T}\,\mathrm{d}\ln \boldsymbol{\tau}^{\mathbf{in}} - \mathrm{d}\ln \boldsymbol{\tau}^{\mathbf{out}}\right], \tag{D.4}$$

$$\theta = \sigma - 1.$$

We next consider our expression for the log change in utility from equation (B.10) above:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{m=1}^{N} s_{nm}\,\mathrm{d}\ln p_{nm},$$

which can be re-written as follows:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{m=1}^{N} s_{nm}\left[\mathrm{d}\ln w_m + \mathrm{d}\ln \tau_{nm} - \mathrm{d}\ln z_m\right].$$

Using our definition of inward trade costs from equation (D.1), we can re-write this proportional change in real income as:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{m=1}^{N} s_{nm}\left(\mathrm{d}\ln w_m - \mathrm{d}\ln z_m\right) - \mathrm{d}\ln \tau_n^{\mathbf{in}},$$

which has the following matrix representation:

$$\mathrm{d}\ln \mathbf{u} = \mathrm{d}\ln \mathbf{w} - \mathbf{S}\left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z}\right) - \mathrm{d}\ln \boldsymbol{\tau}^{\mathbf{in}}. \tag{D.5}$$

D.2 Trade Imbalance

In this section of the online appendix, we relax another of the ACR macro restrictions to allow for trade imbalance. In particular, we consider the constant elasticity Armington model from Section I of the paper, but allow expenditure to differ from income.

D.2.1 Preferences and Expenditure Shares

We measure the instantaneous real expenditure of the representative agent:

$$u_n = \frac{w_n \ell_n + \bar{d}_n}{p_n}, \qquad p_n = \left[\sum_{i=1}^{N} p_{ni}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \qquad \sigma > 1, \tag{D.6}$$

where $\bar{d}_n$ is the nominal trade deficit. Expenditure shares take the same form as in equation (B.3) in Section B of this online appendix.

D.2.2 Market Clearing

Market clearing requires that income in each country equals expenditure on goods produced in that country:

$$w_i \ell_i = \sum_{n=1}^{N} s_{ni} \left[w_n \ell_n + \bar{d}_n \right]. \tag{D.7}$$

D.2.3 Trade Matrices

We now establish some properties our trade matrices under trade imbalance. We continue to use $q_i \equiv w_i \ell_i / \left(\sum_n w_n \ell_n \right)$ to denote country i's share of world income. Let $e_i \equiv \left(w_i \ell_i + \bar{d}_i \right) / \left(\sum_n w_n \ell_n \right)$ denote country i's share of world expenditures, where we use the fact that the aggregate deficit for the world as a whole is equal to zero. Let $d_i \equiv q_i / e_i$ denote country i's income-to-expenditure ratio, which is equal to one divided by one plus its nominal trade deficit relative to income. Let $\mathbf{D} \equiv \mathrm{Diag}\left(\mathbf{d} \right)$ be the diagonalization of the vector $\mathbf{d}$; note $\mathbf{q}' = \mathbf{e}'\mathbf{D}$. Under trade balance, $q_i = e_i$ for all i, and $\mathbf{D} = \mathbf{I}$.

We continue to use $\mathbf{S}$ to denote the expenditure share matrix and $\mathbf{T}$ to denote the income share matrix: s_{ni} captures the expenditure share of importer n on exporter i and t_{in} captures the share of exporter i's income derived from selling to importer n. Under trade balance, $q_i t_{in} = q_n s_{ni}$, but this is no longer the case under trade imbalance. Instead, we have the following results.

Lemma A.1. *Under trade imbalance,* $\mathbf{q}' = \mathbf{e}'\mathbf{S}$, $\mathbf{e}' = \mathbf{q}'\mathbf{T}$. *Moreover,*

1. $\mathbf{q}'$ *is the unique left-eigenvector of* $\mathbf{D}^{-1}\mathbf{S}$ *with all positive entries summing to one; the corresponding eigenvalue is one.* $\mathbf{q}'$ *is also the unique left-eigenvector of* $\mathbf{TD}$ *and* $\mathbf{TS}$ *with eigenvalue equal to one.*

2. $\mathbf{e}'$ *is the unique left-eigenvector of* $\mathbf{SD}^{-1}$ *with all positive entries summing to one; the corresponding eigenvalue is one.* $\mathbf{e}'$ *is also the unique left-eigenvector of* $\mathbf{DT}$ *and* $\mathbf{ST}$ *with eigenvalue equal to one.*

Proof. Dividing the market clearing condition by $\left(\sum_j w_j \ell_j \right)$, we have

$$\frac{w_i \ell_i}{\sum_j w_j \ell_j} = \sum_n \left[s_{ni} \frac{w_n \ell_n + \bar{d}_n}{\sum_j w_j \ell_j} \right] \iff q_i = \sum_n s_{ni} e_n \iff \mathbf{q}' = \mathbf{e}'\mathbf{S}.$$

Let $d_i \equiv q_i/e_i$ and $\mathbf{D} \equiv \text{Diag}(\mathbf{d})$. Note $\mathbf{q}' = \mathbf{e}'\mathbf{D}$ and $\mathbf{q}'\mathbf{D}^{-1} = \mathbf{e}'$; thus the market clearing condition can be re-written as

$$\mathbf{e}'\mathbf{D} = \mathbf{e}'\mathbf{S} \iff \mathbf{e}' = \mathbf{e}'\mathbf{S}\mathbf{D}^{-1},$$

and

$$\mathbf{q}' = \mathbf{e}'\mathbf{S} \iff \mathbf{q}' = \mathbf{q}'\mathbf{D}^{-1}\mathbf{S}.$$

$\mathbf{q}$ is therefore the unique positive left-eigenvector of $\mathbf{D}^{-1}\mathbf{S}$ with eigenvalue 1, and $\mathbf{e}'$ is the unique positive left-eigenvector of $\mathbf{S}\mathbf{D}^{-1}$ with eigenvalue one. The remaining claims about $\mathbf{T}$ follow analogously. □

D.2.4 Income Comparative Statics

Using these properties of the trade matrices, we now derive countries' income and real income exposure to productivity shocks under trade imbalance. As the model does not generate predictions for how trade imbalances respond to shocks, we follow the common approach in the quantitative international trade literature of treating them as exogenous. In particular, we assume that trade imbalances are constant as a share of world GDP, which given our choice of world GDP as the numeraire, corresponds to holding the nominal trade deficits $\bar{d}_n$ fixed for all countries n. Totally differentiating the market clearing condition (D.7), holding labor endowments constant, we have:

$$\frac{\mathrm{d}w_i}{w_i} w_i \ell_i = \sum_{n=1}^{N} s_{ni} \left(w_n \ell_n + \bar{d}_n\right) \frac{w_n \ell_n}{\left(w_n \ell_n + \bar{d}_n\right)} \left(\frac{\mathrm{d}s_{ni}}{s_{ni}} + \frac{\mathrm{d}w_n}{w_n}\right) + \sum_{n=1}^{N} s_{ni} \left(w_n \ell_n + \bar{d}_n\right) \frac{\bar{d}_n}{\left(w_n \ell_n + \bar{d}_n\right)} \left(\frac{ds_{ni}}{s_{ni}} + \frac{\mathrm{d}\bar{d}_n}{\bar{d}_n}\right),$$

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{n=1}^{N} t_{in} d_n \left(\frac{\mathrm{d}s_{ni}}{s_{ni}} + \frac{\mathrm{d}w_n}{w_n}\right) + \sum_{n=1}^{N} t_{in} \left(1 - d_n\right) \left(\frac{\mathrm{d}s_{ni}}{s_{ni}} + \frac{\mathrm{d}\bar{d}_n}{\bar{d}_n}\right),$$

where we have defined t_{in} as the share of country i's income from market n:

$$t_{in} \equiv \frac{s_{ni} \left(w_n \ell_n + \bar{d}_n\right)}{w_i \ell_i},$$

and d_n as country n's ratio of income to expenditure:

$$d_n \equiv \frac{w_n \ell_n}{w_n \ell_n + \bar{d}_n}.$$

Using our result for the derivative of expenditure shares above, we can rewrite this as:

$$\begin{aligned} \frac{\mathrm{d}w_i}{w_i} &= \sum_{n=1}^{N} t_{in} d_n \left(\frac{\mathrm{d}w_n}{w_n} + (\sigma - 1) \left(\sum_{h=1}^{N} s_{nh} \frac{\mathrm{d}p_{nh}}{p_{nh}} - \frac{\mathrm{d}p_{ni}}{p_{ni}}\right)\right) \\ &\quad + \sum_{n=1}^{N} t_{in} \left(1 - d_n\right) \left(\frac{\mathrm{d}\bar{d}_n}{\bar{d}_n} + (\sigma - 1) \left(\sum_{h=1}^{N} s_{nh} \frac{\mathrm{d}p_{nh}}{p_{nh}} - \frac{\mathrm{d}p_{ni}}{p_{ni}}\right)\right). \end{aligned} \tag{D.8}$$

We can re-write this expression as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in} \left(d_n \,\mathrm{d}\ln w_n + (\sigma - 1) \left(\sum_{h=1}^{N} s_{nh} \,\mathrm{d}\ln p_{nh} - \mathrm{d}\ln p_{ni}\right)\right) + \sum_{n=1}^{N} t_{in} \left(1 - d_n\right) \,\mathrm{d}\ln \bar{d}_n.$$

D.2.5 Real Expenditure Comparative Statics

Returning to our expression for real expenditure (D.6), we have:

$$u_n = \frac{w_n \ell_n + \bar{d}_n}{p_n}.$$

Totally differentiating real expenditure, holding labor endowments constant, we have:

$$\frac{\mathrm{d}u_n}{u_n} = d_n \frac{\mathrm{d}w_n}{w_n} + (1 - d_n) \frac{\mathrm{d}\bar{d}_n}{\bar{d}_n} - \frac{dp_n}{p_n}.$$

Using the total derivative of the sectoral price index, we get:

$$\frac{\mathrm{d}u_n}{u_n} = d_n \frac{dw_n}{w_n} + (1 - d_n) \frac{\mathrm{d}\bar{d}_n}{\bar{d}_n} - \sum_{m=1}^{N} s_{nm} \frac{dp_{nm}}{p_{nm}}, \tag{D.9}$$

which can be re-written as:

$$\mathrm{d}\ln u_n = d_n \,\mathrm{d}\ln w_n + (1 - d_n)\ \mathrm{d}\ln \bar{d}_n - \sum_{m=1}^{N} s_{nm} d\ln p_{nm}.$$

D.2.6 Wages and Productivity Shocks

We consider small productivity shocks, holding constant bilateral trade costs:

$$\mathrm{d}\ln \tau_{ni} \;\; = 0, \qquad\quad \forall\, n, i \in N, \tag{D.10}$$

and we assume that trade deficits remain constant in terms of our numeraire of world GDP:

$$\mathrm{d}\ln \bar{d}_n = 0.$$

Under these assumptions, the change in prices is:

$$\mathrm{d}\ln p_{ni} = \ \mathrm{d}\ln w_i - \ \mathrm{d}\ln z_i.$$

We start with our expression for the log change in wages in equation (D.8) above. With constant trade deficits ($\mathrm{d}\ln \bar{d}_n = 0$), this expression simplifies to:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in} \left(d_n \,\mathrm{d}\ln w_n + (\sigma - 1) \left(\sum_{h=1}^{N} s_{nh} \,\mathrm{d}\ln p_{nh} - \ \mathrm{d}\ln p_{ni} \right) \right).$$

Using the total derivative of prices and our assumption of constant bilateral trade costs (D.10), this further simplifies to:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} d_n t_{in} \,\mathrm{d}\ln w_n + (\sigma - 1) \left(\sum_{h=1}^{N} \sum_{n=1}^{N} t_{in} s_{nh} \left[\mathrm{d}\ln w_h - \ \mathrm{d}\ln z_h \right] - \left[\mathrm{d}\ln w_i - \ \mathrm{d}\ln z_i \right] \right). \tag{D.11}$$

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} d_n t_{in} \,\mathrm{d}\ln w_n + (\sigma - 1) \left(\sum_{n=1}^{N} m_{in} \left[\mathrm{d}\ln w_n - \ \mathrm{d}\ln z_n \right] \right),$$

$$m_{in} = \sum_{h=1}^{N} t_{ih} s_{hn} - 1_{n=i},$$

which has the following matrix representation:

$$\mathrm{d} \ln \mathbf{w} = \mathbf{TD} \, \mathrm{d} \ln \mathbf{w} + \theta \mathbf{M} \left(\mathrm{d} \ln \mathbf{w} - \mathrm{d} \ln \mathbf{z} \right), \tag{D.12}$$

$$\theta = \sigma - 1$$

where the matrices $\mathbf{T}$ and $\mathbf{M}$ are defined in Section B of this online appendix. The diagonal matrix $\mathbf{D}$ captures trade deficits through the ratio of income to expenditure:

$$\mathbf{D} = \underbrace{\begin{pmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & d_N \end{pmatrix}}_{N \times N}, \qquad d_n = \frac{w_n \ell_n}{w_n \ell_n + \bar{d}_n}.$$

D.2.7 Real Expenditure and Productivity Shocks

We start with our expression for the log change in real expenditure in equation (D.9) above. With constant trade deficits ($\mathrm{d} \ln \bar{d}_n = 0$), this simplifies to:

$$\mathrm{d} \ln u_n = d_n \, \mathrm{d} \ln w_n - \sum_{m=1}^{N} s_{nm} \, \mathrm{d} \ln p_{nm}.$$

Using the total derivative of prices and our assumption of constant bilateral trade costs (D.10), we can write this proportional change in real expenditure as:

$$\mathrm{d} \ln u_n = d_n \, \mathrm{d} \ln w_n - \sum_{m=1}^{N} s_{nm} \left(\mathrm{d} \ln w_m - d \ln z_m \right), \tag{D.13}$$

which has the following matrix representation:

$$\mathrm{d} \ln \mathbf{u} = \mathbf{D} \, \mathrm{d} \ln \mathbf{w} - \mathbf{S} \left(\mathrm{d} \ln \mathbf{w} - \mathrm{d} \ln \mathbf{z} \right), \tag{D.14}$$

where the matrix $\mathbf{S}$ is defined in Section B of this online appendix and the diagonal matrix $\mathbf{D}$ is defined in the previous subsection of this online appendix.

D.3 Multiple Sectors

In this section of the online appendix, we show that our friends-and-enemies exposure measures extend naturally to a multi-sector version of the constant elasticity Armington model, as discussed in Section I of the paper. The world economy consists of many countries indexed by $i, n \in \{1, \ldots, N\}$ and a set of sectors indexed by $k \in \{1, \ldots, K\}$. Each country n has an exogenous supply of labor ℓ_n.

D.3.1 Consumer Preferences

Consumer preferences are defined across sectors according to the following Cobb-Douglas indirect utility function:

$$u_n = \frac{w_n}{\prod_{k=1}^{K} \left(p_n^k\right)^{\alpha_n^k}}, \qquad \sum_{k=1}^{K} \alpha_n^k = 1. \tag{D.15}$$

Each sector is characterized by constant elasticity of substitution preferences across country varieties:

$$p_n^k = \left[\sum_{i=1}^{N} \left(p_{ni}^k\right)^{-\theta}\right]^{-\frac{1}{\theta}}, \qquad \theta = \sigma - 1, \qquad \sigma > 1. \tag{D.16}$$

D.3.2 Production Technology

Goods are produced with labor under conditions of perfect competition, such that the cost to a consumer in country n of purchasing the variety of country i within sector k is:

$$p_{ni}^k = \frac{\tau_{ni}^k w_i}{z_i^k}, \tag{D.17}$$

where z_i^k captures productivity and iceberg trade costs satisfy $\tau_{ni}^k > 1$ for $n \neq i$ and $\tau_{nn}^k = 1$.

D.3.3 Expenditure Shares

Using the properties of CES demand, country n's share of expenditure on goods produced in country i within sector k is given by:

$$s_{ni}^k = \frac{\left(p_{ni}^k\right)^{-\theta}}{\sum_{m=1}^{N} \left(p_{nm}^k\right)^{-\theta}}. \tag{D.18}$$

Totally differentiating the expenditure share equation (D.18), we get:

$$\frac{\mathrm{d}s_{ni}^k}{s_{ni}^k} = \theta \left(\sum_{h=1}^{N} s_{nh}^k \frac{\mathrm{d}p_{nh}^k}{p_{nh}^k} - \frac{\mathrm{d}p_{ni}^k}{p_{ni}^k}\right), \tag{D.19}$$

$$\mathrm{d}\ln s_{ni}^k = \theta \left(\sum_{h=1}^{N} s_{nh}^k \,\mathrm{d}\ln p_{nh}^k - \mathrm{d}\ln p_{ni}^k\right),$$

where, from equilibrium prices in equation (D.17), we have:

$$\frac{\mathrm{d}p_{ni}^k}{p_{ni}^k} = \frac{\mathrm{d}\tau_{ni}^k}{\tau_{ni}^k} + \frac{\mathrm{d}w_i}{w_i} - \frac{\mathrm{d}z_i^k}{z_i^k}, \tag{D.20}$$

$$\mathrm{d}\ln p_{ni}^k = \mathrm{d}\ln \tau_{ni}^k + \mathrm{d}\ln w_i - \mathrm{d}\ln z_i^k.$$

D.3.4 Price Indices

Totally differentiating the sectoral price index (D.16), we have:

$$\frac{\mathrm{d}p_n^k}{p_n^k} = \sum_{m=1}^{N} s_{nm}^k \frac{\mathrm{d}p_{nm}^k}{p_{nm}^k}, \tag{D.21}$$

$$\mathrm{d}\ln p_n^k = \sum_{m=1}^{N} s_{nm}^k \, \mathrm{d}\ln p_{nm}^k.$$

D.3.5 Market Clearing

Market clearing requires that income in each country equals expenditure on goods produced in that country:

$$w_i \ell_i = \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{ni}^k w_n \ell_n. \tag{D.22}$$

Totally differentiating this market clearing condition, holding labor endowments constant, we have:

$$\frac{\mathrm{d}w_i}{w_i} w_i \ell_i = \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{ni}^k w_n \ell_n \left(\frac{\mathrm{d}w_n}{w_n} + \frac{\mathrm{d}s_{ni}^k}{s_{ni}^k} \right),$$

Using our result for the derivative of expenditure shares in equation (D.19) above, we can rewrite this as:

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \left(\frac{\mathrm{d}w_n}{w_n} + \theta \left(\sum_{h=1}^{N} s_{nh}^k \frac{\mathrm{d}p_{nh}^k}{p_{nh}^k} - \frac{\mathrm{d}p_{ni}^k}{p_{ni}^k} \right) \right), \tag{D.23}$$

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \left(\mathrm{d}\ln w_n + \theta \left(\sum_{h=1}^{N} s_{nh}^k d\ln p_{nh}^k - d\ln p_{ni}^k \right) \right),$$

where we have defined t_{in}^k as the share of country i's income derived from market n and industry k:

$$t_{in}^k \equiv \frac{\alpha_n^k s_{ni}^k w_n \ell_n}{w_i \ell_i}.$$

D.3.6 Utility Again

Returning to our expression for indirect utility in equation (D.15), we have:

$$u_n = \frac{w_n}{\prod_{k=1}^{K} (p_n^k)^{\alpha_n^k}}.$$

Totally differentiating indirect utility, we have:

$$\frac{\mathrm{d}u_n}{u_n} = \frac{\mathrm{d}w_n}{w_n} - \sum_{k=1}^{K} \alpha_n^k \frac{\mathrm{d}p_n^k}{p_n^k}.$$

Using our total derivative of the sectoral price index in equation (D.21) above, we get:

$$\frac{\mathrm{d}u_n}{u_n} = \frac{\mathrm{d}w_n}{w_n} - \sum_{k=1}^{K} \alpha_n^k \sum_{m=1}^{N} s_{nm}^k \frac{\mathrm{d}p_{nm}^k}{p_{nm}^k}, \tag{D.24}$$

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{k=1}^{K} \alpha_n^k \sum_{m=1}^{N} s_{nm}^k \,\mathrm{d}\ln p_{nm}^k.$$

D.3.7 Wages and Common Productivity Shocks

We consider small productivity shocks for each country that are common across sectors, holding constant bilateral trade costs:

$$\begin{array}{ll} \mathrm{d}\ln z_i^k = \mathrm{d}\ln z_i, & \forall = k \in K,\ i \in N, \\ \mathrm{d}\ln \tau_{ni}^k = 0, & \forall\, n, i\ \in N. \end{array} \tag{D.25}$$

We start with our expression for the log change in wages from equation (D.23) above:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N}\sum_{k=1}^{K} t_{in}^k \left(\mathrm{d}\ln w_n + \theta \left(\sum_{h=1}^{N} s_{nh}^k \,\mathrm{d}\ln p_{nh}^k - \mathrm{d}\ln p_{ni}^k \right)\right).$$

Using the total derivative of prices (D.20) and our assumptions of common productivity shocks and no change in bilateral trade costs in equation (D.25), we can write this expression for the log change in wages as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N}\sum_{k=1}^{K} t_{in}^k \,\mathrm{d}\ln w_n + \theta \left(\sum_{h=1}^{N}\sum_{n=1}^{N}\sum_{k=1}^{K} t_{in}^k s_{nh}^k \left[\mathrm{d}\ln w_h - \mathrm{d}\ln z_h\right] - \left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right] \right),$$

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in} \,\mathrm{d}\ln w_n + \theta \left(\sum_{n=1}^{N} m_{in} \left[\mathrm{d}\ln w_n - \mathrm{d}\ln z_n\right] \right),$$

$$t_{in} = \sum_{k=1}^{K} t_{in}^k,$$

$$m_{in} = \sum_{h=1}^{N}\sum_{k=1}^{K} t_{ih}^k s_{hn}^k - 1_{n=i},$$

which has the following matrix representation in the paper:

$$\mathrm{d}\ln \mathbf{w} = \mathbf{T}\,\mathrm{d}\ln \mathbf{w} + \theta \mathbf{M}\left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z}\right). \tag{D.26}$$

We again solve for our friend-enemy income exposure measure by matrix inversion. Dividing both sides of equation (D.26) by $\theta + 1$, we have:

$$\frac{1}{\theta+1}\,\mathrm{d}\ln \mathbf{w} = \frac{1}{\theta+1}\mathbf{T}\,\mathrm{d}\ln \mathbf{w} + \frac{\theta}{\theta+1}\mathbf{M}\left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z}\right),$$

$$\frac{1}{\theta+1}\left(\mathbf{I}-\mathbf{T}-\theta\mathbf{M}\right)\,\mathrm{d}\ln\mathbf{w}=-\frac{\theta}{\theta+1}\mathbf{M}\,\mathrm{d}\ln\mathbf{z}.$$

Now note that $\mathbf{M}=\mathbf{TS}-\mathbf{I}$, where the $\mathbf{S}$ matrix is defined with elements $\mathbf{S}_{ni}=\sum_{k=1}^{K}\alpha_n^k s_{ni}^k$. Using this relationship, we have:

$$\frac{1}{\theta+1}\left(\mathbf{I}-\mathbf{T}-\theta\mathbf{TS}+\theta\mathbf{I}\right)\,\mathrm{d}\ln\mathbf{w}=-\frac{\theta}{\theta+1}\mathbf{M}\,\mathrm{d}\ln\mathbf{z},$$

$$\left(\mathbf{I}-\frac{\mathbf{T}+\theta\mathbf{TS}}{\theta+1}\right)\,\mathrm{d}\ln\mathbf{w}=-\frac{\theta}{\theta+1}\mathbf{M}\,\mathrm{d}\ln\mathbf{z}.$$

Using our choice of world GDP as numeraire, which implies $\mathbf{Q}\,\mathrm{d}\ln\mathbf{w}=0$, we have:

$$\left(\mathbf{I}-\frac{\mathbf{T}+\theta\mathbf{TS}}{\theta+1}+\mathbf{Q}\right)\,\mathrm{d}\ln\mathbf{w}=-\frac{\theta}{\theta+1}\mathbf{M}\,\mathrm{d}\ln\mathbf{z},$$

which can be re-written as:

$$\left(\mathbf{I}-\mathbf{V}\right)\,\mathrm{d}\ln\mathbf{w}=-\frac{\theta}{\theta+1}\mathbf{M}\,\mathrm{d}\ln\mathbf{z},$$

$$\mathbf{V}\equiv\frac{\mathbf{T}+\theta\mathbf{TS}}{\theta+1}-\mathbf{Q},$$

which yields the following solution of the change in wages in response to a productivity shock:

$$\mathrm{d}\ln\mathbf{w}=-\frac{\theta}{\theta+1}\left(\mathbf{I}-\mathbf{V}\right)^{-1}\mathbf{M}\,\mathrm{d}\ln\mathbf{z},$$

which can be re-written as:

$$\mathrm{d}\ln\mathbf{w}=\mathbf{W}\,\mathrm{d}\ln\mathbf{z}, \tag{D.27}$$

where $\mathbf{W}$ is our friend-enemy income exposure measure:

$$\mathbf{W}\equiv-\frac{\theta}{\theta+1}\left(\mathbf{I}-\mathbf{V}\right)^{-1}\mathbf{M}. \tag{D.28}$$

D.3.8 Real Income and Common Productivity Shocks

We start with our expression for the log change in utility in equation (D.24) above:

$$\mathrm{d}\ln u_n=\,\mathrm{d}\ln w_n-\sum_{k=1}^{K}\alpha_n^k\sum_{m=1}^{N}s_{nm}^k\,\mathrm{d}\ln p_{nm}^k,$$

or equivalently:

$$\mathrm{d}\ln u_n=\,\mathrm{d}\ln w_n-\sum_{m=1}^{N}\sum_{k=1}^{K}\alpha_n^k s_{nm}^k\,\mathrm{d}\ln p_{nm}^k.$$

Using the total derivative of prices (D.20) and our assumptions of common productivity shocks and no change in bilateral trade costs in equation (D.25), we can write this change in log utility as:

$$\mathrm{d}\ln u_n=\,\mathrm{d}\ln w_n-\sum_{m=1}^{N}\sum_{k=1}^{K}\alpha_n^k s_{nm}^k\left(\,\mathrm{d}\ln w_m-\,\mathrm{d}\ln z_m\right), \tag{D.29}$$

which has the following matrix representation in the paper:

$$\mathrm{d}\ln\mathbf{u} = \mathrm{d}\ln\mathbf{w} - \mathbf{S}\left(\mathrm{d}\ln\mathbf{w} - \mathrm{d}\ln\mathbf{z}\right). \tag{D.30}$$

We can re-write the above relationship as:

$$\mathrm{d}\ln\mathbf{u} = (\mathbf{I} - \mathbf{S})\ \mathrm{d}\ln\mathbf{w} + \mathbf{S}\,\mathrm{d}\ln\mathbf{z},$$

which, using our solution for $\mathrm{d}\ln\mathbf{w}$ from equation (D.27), can be further re-written as:

$$\mathrm{d}\ln\mathbf{u} = (\mathbf{I} - \mathbf{S})\,\mathbf{W}\,\mathrm{d}\ln\mathbf{z} + \mathbf{S}\,\mathrm{d}\ln\mathbf{z},$$

$$\mathrm{d}\ln\mathbf{u} = \left[(\mathbf{I} - \mathbf{S})\,\mathbf{W} + \mathbf{S}\right]\ \mathrm{d}\ln\mathbf{z},$$

$$\mathrm{d}\ln\mathbf{u} = \mathbf{U}\,\mathrm{d}\ln\mathbf{z}, \tag{D.31}$$

where $\mathbf{U}$ is our friend-enemy real income exposure measure:

$$\mathbf{U} \equiv \left[(\mathbf{I} - \mathbf{S})\,\mathbf{W} + \mathbf{S}\right]. \tag{D.32}$$

D.3.9 Industry-Level Sales Exposure

In this multi-sector model, our approach also yields bilateral friend-enemy measures of income exposure to global productivity shocks for each sector. Labor income in each sector and country equals value-added, which in turn equals expenditure on goods produced in that sector and country:

$$w_i \ell_i^k = y_i^k = \sum_{n=1}^{N} \alpha_n^k s_{ni}^k w_n \ell_n.$$

Totally differentiating this industry market clearing condition, we have:

$$\frac{\mathrm{d}y_i^k}{y_i^k} w_i \ell_i = \sum_{n=1}^{N} \alpha_n^k s_{ni}^k w_n \ell_n \left(\frac{\mathrm{d}s_{ni}^k}{s_{ni}^k} + \frac{\mathrm{d}w_n}{w_n}\right),$$

$$\frac{\mathrm{d}y_i^k}{y_i^k} = \sum_{n=1}^{N} t_{in}^k \left(\frac{\mathrm{d}w_n}{w_n} + \frac{\mathrm{d}s_{ni}^k}{s_{ni}^k}\right).$$

Using the total derivative of expenditure shares in equation (D.19), we can rewrite this as:

$$\frac{\mathrm{d}y_i^k}{y_i^k} = \sum_{n=1}^{N} t_{in}^k \left(\frac{\mathrm{d}w_n}{w_n} + \theta\left(\sum_{h=1}^{N} s_{nh}^k \frac{\mathrm{d}p_{nh}^k}{p_{nh}^k} - \frac{\mathrm{d}p_{ni}^k}{p_{ni}^k}\right)\right),$$

which can be re-written as:

$$\mathrm{d}\ln y_i^k = \sum_{n=1}^{N} t_{in}^k \left(\mathrm{d}\ln w_n + \theta\left(\sum_{h=1}^{N} s_{nh}^k\,\mathrm{d}\ln p_{nh}^k - \mathrm{d}\ln p_{ni}^k\right)\right).$$

Using the total derivative of prices (D.20) and our assumptions of common productivity shocks and no change in bilateral trade costs in equation (D.25), we can re-write this change in sector value-added as:

$$\mathrm{d}\ln y_i^k = \sum_{n=1}^{N} t_{in}^k \left(\mathrm{d}\ln w_n + \theta \left(\sum_{h=1}^{N} s_{nh}^k \left(\mathrm{d}\ln w_h - \mathrm{d}\ln z_h \right) - \left(\mathrm{d}\ln w_i - \mathrm{d}\ln z_i \right) \right) \right),$$

which has the following matrix representation:

$$\mathrm{d}\ln \mathbf{y^k} = \mathbf{T^k}\,\mathrm{d}\ln \mathbf{w} + \theta \mathbf{M^k} \left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z} \right). \tag{D.33}$$

Using our solution for changes in wages as a function of productivity shocks in equation (D.27) above, we have:

$$\mathrm{d}\ln \mathbf{y^k} = \mathbf{T^k}\mathbf{W}\,\mathrm{d}\ln \mathbf{z} + \theta \mathbf{M^k} \left(\mathbf{W} - \mathbf{I} \right) \mathrm{d}\ln \mathbf{z},$$

$$\mathrm{d}\ln \mathbf{y^k} = \left[\mathbf{T^k}\mathbf{W} + \theta \mathbf{M^k} \left(\mathbf{W} - \mathbf{I} \right) \right] \mathrm{d}\ln \mathbf{z},$$

which can be re-written as:

$$\mathrm{d}\ln \mathbf{y^k} = \mathbf{W^k}\,\mathrm{d}\ln \mathbf{z}, \tag{D.34}$$

$$\mathbf{W^k} \equiv \mathbf{T^k}\mathbf{W} + \theta \mathbf{M^k} \left(\mathbf{W} - \mathbf{I} \right), \tag{D.35}$$

which corresponds to our friends-and-enemies measure of sector value-added exposure to productivity shocks.

We now show that our aggregate friends-and-enemies measure of income exposure ($\mathbf{W}$) in equation (D.28) is a weighted average of our industry friends-and-enemies measures of sector value-added exposure in equation (D.35). Note that aggregate income equals aggregate value-added:

$$w_i \ell_i = \sum_{k=1}^{K} y_i^k.$$

Totally differentiating, holding endowments constant, we have:

$$\frac{\mathrm{d}w_i}{w_i} w_i \ell_i = \sum_{k=1}^{K} \frac{\mathrm{d}y_i^k}{y_i^k} y_i^k.$$

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{k=1}^{K} r_i^k \frac{\mathrm{d}y_i^k}{y_i^k},$$

$$\mathrm{d}\ln w_i = \sum_{k=1}^{K} r_i^k\,\mathrm{d}\ln y_i^k, \tag{D.36}$$

where $r_i^k \equiv \frac{y_i^k}{w_i \ell_i}$ is the value-added share of industry k. Together, equations (D.27), (D.34) and (D.36) imply:

$$\mathbf{W}_i\,\mathrm{d}\ln \mathbf{z} = \sum_{k=1}^{K} r_i^k \mathbf{W}_i^k\,\mathrm{d}\ln \mathbf{z},$$

where $\mathbf{W}_i$ is the income exposure vector for country i with respect to productivity shocks in its trade partners and $\mathbf{W}_i^k$ is the sector value-added exposure vector for country i and sector k with respect to productivity shocks in those trade partners. It follows that our aggregate friends-and-enemies measure of income exposure ($\mathbf{W}_i$) is a weighted average of our industry friends-and-enemies measures of sector income exposure ($\mathbf{W}_i^k$):

$$\mathbf{W}_i = \sum_k r_i^k \mathbf{W}_i^k.$$

Therefore, we can decompose our aggregate income exposure measure into the contributions of the income exposure measures of particular industries, and how much of that income exposure of particular industries is explained by various terms (market-size, cross-substitution etc within the industry).

D.4 Heterogeneous Sector Trade Elasticities

For simplicity, we have so far assumed a common elasticity of substitution (σ) and trade elasticity (θ) across sectors. We now further generalize our analysis to allow for heterogeneous trade elasticities across sectors. The specification remains as in the previous subsection, except that the elasticity of substitution (σ^k) and trade elasticity (θ^k) now vary across sectors k. Market clearing requires that income in each country equals expenditure on goods produced in that country:

$$w_i \ell_i = \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{ni}^k w_n \ell_n. \tag{D.37}$$

Totally differentiating this market clearing condition, holding labor endowments constant, we have:

$$\frac{\mathrm{d}w_i}{w_i} w_i \ell_i = \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{ni}^k w_n \ell_n \left(\frac{\mathrm{d}w_n}{w_n} + \frac{\mathrm{d}s_{ni}^k}{s_{ni}^k} \right),$$

Using the derivative of expenditure shares, we can rewrite this as:

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \left(\frac{\mathrm{d}w_n}{w_n} + \theta^k \left(\sum_{h=1}^{N} s_{nh}^k \frac{\mathrm{d}p_{nh}^k}{p_{nh}^k} - \frac{\mathrm{d}p_{ni}^k}{p_{ni}^k} \right) \right), \tag{D.38}$$

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \left(\mathrm{d}\ln w_n + \theta^k \left(\sum_{h=1}^{N} s_{nh}^k d\ln p_{nh}^k - d\ln p_{ni}^k \right) \right),$$

where we have defined t_{in}^k as the share of country i's income derived from market n and industry k:

$$t_{in}^k \equiv \frac{\alpha_n^k s_{ni}^k w_n \ell_n}{w_i \ell_i}.$$

Now consider a shock to productivities, holding all else constant:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \left(\mathrm{d}\ln w_n + \theta^k \left(\sum_{h=1}^{N} s_{nh}^k \left[\mathrm{d}\ln w_h - \mathrm{d}\ln z_h \right] - \left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i \right] \right) \right).$$

We can equivalently re-write this expression as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N}\sum_{k=1}^{K} t_{in}^k \,\mathrm{d}\ln w_n + \theta \sum_{n=1}^{N}\sum_{k=1}^{K} \frac{\theta^k}{\theta} t_{in}^k \left(\sum_{h=1}^{N} s_{nh}^k \left[\mathrm{d}\ln w_h - \mathrm{d}\ln z_h \right] - \left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i \right] \right),$$

where we have multiplied and divided by θ. We can further rewrite this expression as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N}\sum_{k=1}^{K} t_{in}^k \,\mathrm{d}\ln w_n + \theta \sum_{n=1}^{N} \left(\sum_{h=1}^{N}\sum_{k=1}^{K} \frac{\theta^k}{\theta} t_{ih}^k s_{hn}^k - \left(\frac{\sum_{k=1}^{K} \theta^k t_{ih}^k}{\theta} \right) 1_{n=i} \right) \left(\mathrm{d}\ln w_n - \mathrm{d}\ln z_n \right),$$

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in} \,\mathrm{d}\ln w_n + \theta \sum_{n=1}^{N} m_{in} \left(\mathrm{d}\ln w_n - \mathrm{d}\ln z_n \right), \tag{D.39}$$

$$t_{in} = \sum_{k=1}^{K} t_{in}^k,$$

$$m_{in} = \sum_{h=1}^{N}\sum_{k=1}^{K} \frac{\theta^k}{\theta} t_{ih}^k s_{hn}^k - \left(\sum_{h=1}^{N}\sum_{k=1}^{K} \frac{\theta^k}{\theta} t_{ih}^k \right) 1_{n=i},$$

where the definition of m_{in} now depends on the ratio of the sectoral trade elasticity (θ^k) to the common parameter (θ). Equation (D.39) has the following matrix representation as in the paper:

$$\mathrm{d}\ln \mathbf{w} = \mathbf{T}\,\mathrm{d}\ln \mathbf{w} + \theta \mathbf{M} \left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z} \right), \tag{D.40}$$

where only the definitions and interpretation of θ and $\mathbf{M}$ differ. Totally differentiating the indirect utility function, we again have:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{m=1}^{N}\sum_{k=1}^{K} \alpha_n^k s_{nm}^k \,\mathrm{d}\ln p_{nm}^k.$$

Now consider a shock to productivities, holding all else constant:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{m=1}^{N}\sum_{k=1}^{K} \alpha_n^k s_{nm}^k \left(\mathrm{d}\ln w_m - \mathrm{d}\ln z_m \right), \tag{D.41}$$

which has the following matrix representation as in the paper:

$$\mathrm{d}\ln \mathbf{u} = \mathrm{d}\ln \mathbf{w} - \mathbf{S} \left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z} \right). \tag{D.42}$$

D.5 Multi-Sector Isomorphism

In Section D.3 of this online appendix, we generalized our results to a multi-sector version of the Armington model. In this section of the online appendix, we show that the same results hold in a multi-sector version of the Eaton and Kortum (2002) model following Costinot, Donaldson and Komunjer (2012). The world economy consists of a set of countries indexed by $i, n \in \{1, \ldots, N\}$ and a set of sectors indexed by $k \in \{1, \ldots, K\}$. Each country n has an exogenous supply of labor ℓ_n.

D.5.1 Consumer Preferences

Consumer preferences are defined across sectors according to the following Cobb-Douglas indirect utility function:

$$u_n = \frac{w_n}{\prod_{k=1}^{K} \left(p_n^k\right)^{\alpha_n^k}}, \qquad \sum_{k=1}^{K} \alpha_n^k = 1. \tag{D.43}$$

Each sector contains a fixed continuum of goods that enter the sectoral price index according to the following CES functional form:

$$p_n^k = \left[\int_0^1 p_n^k \left(\vartheta\right)^{1-\sigma^k} d\vartheta\right]^{\frac{1}{1-\sigma^k}}, \qquad \sigma^k > 1. \tag{D.44}$$

D.5.2 Production Technology

Goods are produced with labor and can be traded subject to iceberg variable trade costs, such that the cost to a consumer in country n of purchasing a good ϑ from country i is:

$$p_{ni}^k \left(\vartheta\right) = \frac{\tau_{ni}^k w_i}{z_i^k a_i^k \left(\vartheta\right)}, \tag{D.45}$$

where z_i^k captures determinants of productivity that are common across all goods within a country i and sector k and $a_i^k \left(\vartheta\right)$ captures idiosyncratic determinants of productivity for each good within that country and sector. Iceberg trade costs satisfy $\tau_{ni}^k > 1$ for $n \neq i$ and $\tau_{nn}^k = 1$. Productivity for each good ϑ in each sector k and each country i is drawn independently from the following Fréchet distribution:

$$F_i^k \left(a\right) = \exp\left(-a^{-\theta}\right), \qquad \theta > 1, \tag{D.46}$$

where we normalize the Fréchet scale parameter to one, because it enters the model isomorphically to z_i^k.

D.5.3 Expenditure Shares

Using the properties of this Fréchet distribution, country n's share of expenditure on goods produced in country i within sector k is given by:

$$s_{ni}^k = \frac{\left(\tau_{ni}^k w_i / z_i^k\right)^{-\theta}}{\sum_{m=1}^{N} \left(\tau_{nm}^k w_m / z_m^k\right)^{-\theta}} = \frac{\left(\rho_{ni}^k\right)^{-\theta}}{\sum_{m=1}^{N} \left(\rho_{nm}^k\right)^{-\theta}}, \tag{D.47}$$

where we have defined the following price term:

$$\rho_{ni}^k \equiv \frac{\tau_{ni}^k w_i}{z_i^k}. \tag{D.48}$$

Totally differentiating the expenditure share equation (D.47), we get:

$$\frac{\mathrm{d}s_{ni}^k}{s_{ni}^k} = \theta \left(\sum_{h=1}^{N} s_{nh}^k \frac{\mathrm{d}\rho_{nh}^k}{\rho_{nh}^k} - \frac{\mathrm{d}\rho_{ni}^k}{\rho_{ni}^k}\right), \tag{D.49}$$

$$\mathrm{d}\ln s_{ni}^k = \theta\left(\sum_{h=1}^{N} s_{nh}^k \,\mathrm{d}\ln\rho_{nh}^k - \,\mathrm{d}\ln\rho_{ni}^k\right),$$

where from the definition of ρ_{ni}^k above we have:

$$\frac{\mathrm{d}\rho_{ni}^k}{\rho_{ni}^k} = \frac{\mathrm{d}\tau_{ni}^k}{\tau_{ni}^k} + \frac{\mathrm{d}w_i}{w_i} - \frac{\mathrm{d}z_i^k}{z_i^k}, \tag{D.50}$$

$$\mathrm{d}\ln\rho_{ni}^k = \,\mathrm{d}\ln\tau_{ni}^k + \,\mathrm{d}\ln w_i - \,\mathrm{d}\ln z_i^k.$$

D.5.4 Price Indices

Using the properties of the Fréchet distribution (D.46), the sectoral price index is given by:

$$p_n^k = \gamma^k\left[\sum_{m=1}^{N}\left(\rho_{nm}^k\right)^{-\theta}\right]^{-\frac{1}{\theta}}, \tag{D.51}$$

where

$$\gamma^k \equiv \left[\Gamma\left(\frac{\theta+1-\sigma^k}{\theta}\right)\right]^{\frac{1}{1-\sigma^k}},$$

and $\Gamma(\cdot)$ denotes the Gamma function. Totally differentiating this sectoral price index (D.51), we have:

$$\frac{\mathrm{d}p_n^k}{p_n^k} = \sum_{m=1}^{N} s_{nm}^k \frac{\mathrm{d}\rho_{nm}^k}{\rho_{nm}^k}, \tag{D.52}$$

$$\mathrm{d}\ln p_n^k = \sum_{m=1}^{N} s_{nm}^k \,\mathrm{d}\ln\rho_{nm}^k.$$

D.5.5 Market Clearing

Market clearing requires that income in each country equals expenditure on goods produced in that country:

$$w_i\ell_i = \sum_{n=1}^{N}\sum_{k=1}^{K}\alpha_n^k s_{ni}^k w_n\ell_n. \tag{D.53}$$

Totally differentiating this market clearing condition, holding labor endowments constant, we have:

$$\frac{\mathrm{d}w_i}{w_i}w_i\ell_i = \sum_{n=1}^{N}\sum_{k=1}^{K}\alpha_n^k s_{ni}^k w_n\ell_n\left(\frac{\mathrm{d}w_n}{w_n} + \frac{\mathrm{d}s_{ni}^k}{s_{ni}^k}\right),$$

Using our result for the derivative of expenditure shares in equation (D.49) above, we can rewrite this as:

$$\frac{\mathrm{d}w_i}{w_i} = \sum_{n=1}^{N}\sum_{k=1}^{K} t_{in}^k\left(\frac{\mathrm{d}w_n}{w_n} + \theta\left(\sum_{h\in N} s_{nh}^k\frac{\mathrm{d}\rho_{nh}^k}{\rho_{nh}^k} - \frac{\mathrm{d}\rho_{ni}^k}{\rho_{ni}^k}\right)\right), \tag{D.54}$$

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N}\sum_{k=1}^{K} t_{in}^k \left(\mathrm{d}\ln w_n + \theta \left(\sum_{h=1}^{N} s_{nh}^k \, \mathrm{d}\ln \rho_{nh}^k - \mathrm{d}\ln \rho_{ni}^k \right)\right),$$

where we have defined t_{in}^k as country n's expenditure on country i in industry k as a share of country i's income:

$$t_{in}^k \equiv \frac{\alpha_n^k s_{ni}^k w_n \ell_n}{w_i \ell_i}.$$

D.5.6 Utility Again

Returning to our expression for indirect utility in equation (D.43), we have:

$$u_n = \frac{w_n}{\prod_{k=1}^{K} (p_n^k)^{\alpha_n^k}}.$$

Totally differentiating indirect utility, we have:

$$\frac{\mathrm{d}u_n}{u_n} = \frac{\mathrm{d}w_n}{w_n} - \sum_{k=1}^{K} \alpha_n^k \frac{\mathrm{d}p_n^k}{p_n^k}.$$

Using our total derivative of the sectoral price index in equation (D.52) above, we get:

$$\frac{\mathrm{d}u_n}{u_n} = \frac{\mathrm{d}w_n}{w_n} - \sum_{k=1}^{K} \alpha_n^k \sum_{m=1}^{N} s_{nm}^k \frac{\mathrm{d}\rho_{nm}^k}{\rho_{nm}^k}, \tag{D.55}$$

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{k=1}^{K} \alpha_n^k \sum_{m=1}^{N} s_{nm}^k \, \mathrm{d}\ln \rho_{nm}^k.$$

D.5.7 Wages and Common Productivity Shocks

We consider small productivity shocks for each country that are common across sectors, holding constant bilateral trade costs:

$$\begin{array}{ll} \mathrm{d}\ln z_i^k = \mathrm{d}\ln z_i, & \forall\, k \in K,\ i \in N, \\ \mathrm{d}\ln \tau_{ni}^k = 0, & \forall\, n, i \in N. \end{array} \tag{D.56}$$

We start with our expression for the log change in wages from equation (D.54) above:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N}\sum_{k=1}^{K} t_{in}^k \left(\mathrm{d}\ln w_n + \theta \left(\sum_{h=1}^{N} s_{nh}^k \, \mathrm{d}\ln \rho_{nh}^k - \mathrm{d}\ln \rho_{ni}^k \right)\right).$$

Using the total derivative of prices (D.50) and our assumption of constant bilateral trade costs (D.56), we can write this log change in wages as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N}\sum_{k=1}^{K} t_{in}^k \, \mathrm{d}\ln w_n + \theta \left(\sum_{h=1}^{N}\sum_{n=1}^{N}\sum_{k=1}^{K} t_{in}^k s_{nh}^k \left(\mathrm{d}\ln w_h - \mathrm{d}\ln z_h \right) - \left(\mathrm{d}\ln w_i - \mathrm{d}\ln z_i \right) \right), \tag{D.57}$$

which can be re-written as:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln w_n + \theta\left(\sum_{n=1}^{N} m_{in}\left[\,\mathrm{d}\ln w_n - \,\mathrm{d}\ln z_n\right]\right),$$

$$t_{in} = \sum_{k=1}^{K} t_{in}^{k},$$

$$m_{in} = \sum_{h=1}^{N}\sum_{k=1}^{K} t_{ih}^{k} s_{hn}^{k} - 1_{n=i},$$

which has the following matrix representation:

$$\mathrm{d}\ln \mathbf{w} = \mathbf{T}\,\mathrm{d}\ln \mathbf{w} + \theta\mathbf{M}\left(\,\mathrm{d}\ln \mathbf{w} - \,\mathrm{d}\ln \mathbf{z}\right). \tag{D.58}$$

D.5.8 Utility and Common Productivity Shocks

We start with our expression for the log change in utility in equation (D.55) above:

$$\mathrm{d}\ln u_n = \,\mathrm{d}\ln w_n - \sum_{k=1}^{K} \alpha_n^k \sum_{m=1}^{N} s_{nm}^k\,\mathrm{d}\ln \rho_{nm}^k,$$

or equivalently:

$$\mathrm{d}\ln u_n = \,\mathrm{d}\ln w_n - \sum_{m=1}^{N}\sum_{k=1}^{K} \alpha_n^k s_{nm}^k\,\mathrm{d}\ln \rho_{nm}^k.$$

Using the total derivative of prices (D.50) and our assumption of constant bilateral trade costs (D.56), we can write this change in utility as:

$$\mathrm{d}\ln u_n = \,\mathrm{d}\ln w_n - \sum_{m=1}^{N}\sum_{k=1}^{K} \alpha_n^k s_{nm}^k\left(\,\mathrm{d}\ln w_m - \,\mathrm{d}\ln z_m\right),$$

which has the following matrix representation:

$$\mathrm{d}\ln \mathbf{u} = \,\mathrm{d}\ln \mathbf{w} - \mathbf{S}\left(\,\mathrm{d}\ln \mathbf{w} - \,\mathrm{d}\ln \mathbf{z}\right). \tag{D.59}$$

D.6 Multiple Sectors and Input-Output Linkages

In this section of the online appendix, we report the derivations for a specification with multiple sectors and input-output linkages following Caliendo and Parro (2015), henceforth CP. In particular, we consider a generalization of the multi-sector Armington model in Section D.3 of this online appendix to incorporate input-output linkages, as discussed in Section I of the paper. The world economy consists of a set of countries indexed by $i, n \in \{1, \ldots, N\}$ and a set of sectors referenced by $k \in \{1, \ldots, K\}$. Each country n has an exogenous supply of labor ℓ_n.

D.6.1 Notations

We use i, n, o, r to index for countries and j, k, l for industries. We refer to the varieties in industry k produced in country i as "goods ik". We use subscripts to denote countries and superscripts to denote industries. Let $I_{(NK)}$ denote the identity matrix with dimension $NK \times NK$.

D.6.2 Consumer Preferences

Consumer preferences are defined across sectors according to the following Cobb-Douglas indirect utility function:

$$u_n = \frac{w_n}{\prod_{k=1}^{K} \left(p_n^k\right)^{\alpha_n^k}}, \qquad \sum_{k=1}^{K} \alpha_n^k = 1. \tag{D.60}$$

Each sector is characterized by constant elasticity of substitution preferences across country varieties:

$$p_n^k = \left[\sum_{i=1}^{N} \left(p_{ni}^k\right)^{-\theta}\right]^{-\frac{1}{\theta}}, \qquad \theta = \sigma - 1, \qquad \sigma > 1. \tag{D.61}$$

D.6.3 Production Technology

Goods are produced with labor and can be traded subject to iceberg trade costs, such that the cost to a consumer in country n of purchasing country i's variety within sector k is:

$$p_{ni}^k\left(\omega\right) = \tau_{ni}^k c_i^k, \qquad c_i^k = \left(\frac{w_i}{z_i^k}\right)^{\gamma_i^k} \prod_{j=1}^{K} \left(p_i^j\right)^{\gamma_i^{k,j}}, \qquad \sum_{k=1}^{K} \gamma_i^{k,j} = 1 - \gamma_i^k, \tag{D.62}$$

where c_i^k denotes the unit cost function within that country and sector; γ_i^k is the share of labor in production costs; $\gamma_i^{k,j}$ is the share of materials from sector j used in sector k; z_i^k captures determinants of productivity that are common across all goods within a country i and sector k; and it proves convenient to define this common component of productivity in value-added terms (such that it augments labor). Iceberg variable trade costs satisfy $\tau_{ni}^k > 1$ for $n \neq i$ and $\tau_{nn}^k = 1$.

D.6.4 Expenditure Shares

Using the properties of CES demand, country n's share of expenditure on goods produced in country i within sector k is given by:

$$s_{ni}^k = \frac{\left(p_{ni}^k\right)^{-\theta}}{\sum_{m=1}^{N} \left(p_{nm}^k\right)^{-\theta}}. \tag{D.63}$$

Totally differentiating this expenditure share equation, we get:

$$\frac{\mathrm{d}s_{ni}^k}{s_{ni}^k} = \theta \left(\sum_{h=1}^{N} s_{nh}^k \frac{\mathrm{d}p_{nh}^k}{p_{nh}^k} - \frac{\mathrm{d}p_{ni}^k}{p_{ni}^k}\right).$$

so that

$$\mathrm{d}\ln s_{ni}^{k} = \theta\left(\sum_{h=1}^{N} s_{nh}^{k}\,\mathrm{d}\ln p_{nh}^{k} - \mathrm{d}\ln p_{ni}^{k}\right), \tag{D.64}$$

where, from equilibrium prices in equation (D.62), we have:

$$\frac{\mathrm{d}p_{ni}^{k}}{p_{ni}^{k}} = \frac{\mathrm{d}\tau_{ni}^{k}}{\tau_{ni}^{k}} + \gamma_i^k\left(\frac{\mathrm{d}w_i}{w_i} - \frac{\mathrm{d}z_i^k}{z_i^k}\right) + \sum_{j=1}^{K}\gamma_i^{k,j}\frac{\mathrm{d}p_i^j}{p_i^j}, \tag{D.65}$$

$$\mathrm{d}\ln p_{ni}^{k} = \mathrm{d}\ln \tau_{ni}^{k} + \gamma_i^k\left(\mathrm{d}\ln w_i - \mathrm{d}\ln z_i^k\right) + \sum_{j=1}^{K}\gamma_i^{k,j}\,\mathrm{d}\ln p_i^j.$$

D.6.5 Price Indices

Totally differentiating the sectoral price index (D.61), we have:

$$\frac{\mathrm{d}p_{n}^{k}}{p_{n}^{k}} = \sum_{m=1}^{N} s_{nm}^{k}\frac{\mathrm{d}p_{nm}^{k}}{p_{nm}^{k}}, \tag{D.66}$$

$$\mathrm{d}\ln p_{n}^{k} = \sum_{m=1}^{N} s_{nm}^{k}\,\mathrm{d}\ln p_{nm}^{k}.$$

D.6.6 Labor Market Clearing

The labor market clearing condition is:

$$w_n\ell_n = \sum_{j=1}^{K}\gamma_n^j y_n^j. \tag{D.67}$$

where y_n^j is total sales by country n's industry j. Totally differentiating this labor market clearing condition, holding endowments constant, we have:

$$\frac{\mathrm{d}w_n}{w_n}w_n\ell_n = \sum_{j=1}^{K}\gamma_n^j\frac{\mathrm{d}y_n^j}{y_n^j}y_n^j,$$

$$\frac{\mathrm{d}w_n}{w_n} = \sum_{j=1}^{K}\xi_n^j\frac{\mathrm{d}y_n^j}{y_n^j}, \tag{D.68}$$

where ξ_n^j is the share of sector j in country n's total income:

$$\xi_n^j \equiv \frac{\gamma_n^j y_n^j}{w_n\ell_n}.$$

D.6.7 Goods Market Clearing

Goods market clearing requires that income in each country and sector equals expenditure on goods produced in that country and sector:

$$y_i^k = \sum_{n=1}^{N} s_{ni}^k x_n^k, \tag{D.69}$$

where expenditure in country n in sector k is:

$$x_n^k = \alpha_n^k w_n \ell_n + \sum_{j=1}^{K} \gamma_n^{j,k} y_n^j, \tag{D.70}$$

and recall that $\gamma_n^{j,k}$ is the share of materials from sector k used in sector j. Combining these two relationships and the labor market clearing (D.67), we obtain the following market clearing condition:

$$\begin{aligned} y_i^k &= \sum_{n=1}^{N} s_{ni}^k \left[\alpha_n^k w_n \ell_n + \sum_{j=1}^{K} \gamma_n^{k,j} y_n^j \right], \\ &= \sum_{n=1}^{N} s_{ni}^k \left[\alpha_n^k \sum_{j=1}^{K} \gamma_n^j y_n^j + \sum_{j=1}^{K} \gamma_n^{k,j} y_n^j \right], \\ &= \sum_{n=1}^{N} \sum_{j=1}^{K} s_{ni}^k \left[\alpha_n^k \gamma_n^j + \gamma_n^{k,j} \right] y_n^j. \end{aligned}$$

Totally differentiating this market clearing condition, we have:

$$\begin{aligned} \frac{\mathrm{d}y_i^k}{y_i^k} y_i^k &= \sum_{n=1}^{N} s_{ni}^k \left[\alpha_n^k w_n \ell_n + \sum_{j=1}^{K} \gamma_n^{k,j} y_n^j \right], \\ &= \sum_{n=1}^{N} s_{ni}^k \left[\alpha_n^k \sum_{j=1}^{K} \gamma_n^j y_n^j + \sum_{j=1}^{K} \gamma_n^{k,j} y_n^j \right], \\ &= \sum_{n=1}^{N} \sum_{j=1}^{K} s_{ni}^k \left[\alpha_n^k \gamma_n^j + \gamma_n^{k,j} \right] y_n^j. \end{aligned}$$

$$\frac{\mathrm{d}y_i^k}{y_i^k} y_i^k = \sum_{n=1}^{N} \sum_{j=1}^{K} \frac{\mathrm{d}s_{ni}^k}{s_{ni}^k} s_{ni}^k \left[\alpha_n^k \gamma_n^j + \gamma_n^{k,j} \right] y_n^j + \sum_{n=1}^{N} \sum_{j=1}^{K} s_{ni}^k \left[\alpha_n^k \gamma_n^j + \gamma_n^{k,j} \right] \frac{\mathrm{d}y_n^j}{y_n^j} y_n^j.$$

Let ϑ_{in}^k denote the fraction of ik's revenue derived from selling to consumers in country n; $\boldsymbol{\Theta}$ denote an $NK \times NK$ matrix with entries Θ_{in}^{kj} capturing the fraction of ik's revenue derived from selling to producers in country n industry j; and $\boldsymbol{\Delta}$ denote the Leontief-inverse of Θ, such that $\boldsymbol{\Delta} \equiv \left(\mathbf{I}_{(NK)} - \boldsymbol{\Theta}\right)^{-1}$, with the (ik, nj)-th entry, Δ_{in}^{kj}, capturing the network-adjusted fraction of

ik's revenue derived from market nj, either directly or indirectly through customers of customers, ad infinitum. The above market clearing can be re-written as:

$$
\begin{aligned}
\mathrm{d}\ln y_i^k &= \sum_{n=1}^{N} \mathrm{d}\ln s_{ni}^k \frac{s_{ni}^k \alpha_n^k \sum_{j=1}^{K} \gamma_n^j y_n^j}{y_i^k} + \sum_{n=1}^{N}\sum_{j=1}^{K} \mathrm{d}\ln s_{ni}^k \frac{s_{ni}^k \gamma_n^{k,j}}{y_i^k} y_n^j \\
&\quad + \sum_{n=1}^{N} \frac{s_{ni}^k \alpha_n^k w_n \ell_n}{y_i^k} \sum_{j=1}^{K} \frac{\gamma_n^j y_n^j}{w_n \ell_n} \mathrm{d}\ln y_n^j + \sum_{n=1}^{N}\sum_{j=1}^{K} \frac{s_{ni}^k \gamma_n^{k,j}}{y_i^k} y_n^j \,\mathrm{d}\ln y_n^j, \\
&= \sum_{n=1}^{N} \vartheta_{in}^k \,\mathrm{d}\ln w_n + \sum_{n=1}^{N} \left(\vartheta_{in}^k + \sum_{j=1}^{K} \Theta_{in}^{kj} \right) \mathrm{d}\ln s_{ni}^k + \sum_{n=1}^{N}\sum_{j=1}^{K} \Theta_{in}^{kj} \,\mathrm{d}\ln y_n^j,
\end{aligned}
$$

where in the second equality we used equations (D.67) and (D.68). Subtracting the last term on the right hand side from both sides of the equations and taking the Leontief-inverse of Θ_{nm}^{ik}, we obtain:

$$
\mathrm{d}\ln y_i^k = \sum_{o=1}^{N}\sum_{l=1}^{K} \Delta_{io}^{kl} \left[\sum_{n=1}^{N} \vartheta_{on}^l \,\mathrm{d}\ln w_n + \sum_{n=1}^{N} \left(\vartheta_{on}^l + \sum_{j=1}^{K} \Theta_{on}^{lj} \right) \mathrm{d}\ln s_{no}^l \right]. \tag{D.71}
$$

Combining this result with (D.68), we get

$$
\mathrm{d}\ln w_i = \sum_{k} \xi_i^k \sum_{o=1}^{N}\sum_{l=1}^{K} \Delta_{io}^{kl} \left[\sum_{n=1}^{N} \vartheta_{on}^l \,\mathrm{d}\ln w_n + \sum_{n=1}^{N} \left(\vartheta_{on}^l + \sum_{j=1}^{K} \Theta_{on}^{lj} \right) \mathrm{d}\ln s_{no}^l \right]. \tag{D.72}
$$

D.6.8 Wages and Common Productivity Shocks

We consider small productivity shocks for each country that are common across sectors, holding constant bilateral trade costs:

$$
\begin{aligned}
&\mathrm{d}\ln z_i^k = \mathrm{d}\ln z_i, && \forall k \in K,\ i \in N, \\
&\mathrm{d}\ln \tau_{ni}^k = 0, && \forall\, n, i \in N.
\end{aligned}
$$

We start with our expression for the change in prices above:

$$
\begin{aligned}
\mathrm{d}\ln p_{ni}^k &= \mathrm{d}\ln \tau_{ni}^k + \gamma_i^k \left(\mathrm{d}\ln w_i - \mathrm{d}\ln z_i^k \right) + \sum_{j=1}^{K} \gamma_i^{k,j} \,\mathrm{d}\ln p_i^j, \\
&= \gamma_i^k \left(\mathrm{d}\ln w_i - \mathrm{d}\ln z_i \right) + \sum_{j=1}^{K} \gamma_i^{k,j} \,\mathrm{d}\ln p_i^j.
\end{aligned}
$$

Using our result for the total derivative of price indices (D.66), we can rewrite this expression for the change in prices as:

$$
\mathrm{d}\ln p_{ni}^k = \gamma_i^k \left(\mathrm{d}\ln w_i - \mathrm{d}\ln z_i \right) + \sum_{j=1}^{K} \gamma_i^{k,j} \sum_{m=1}^{N} s_{im}^j \,\mathrm{d}\ln p_{im}^j.
$$

We use $\Sigma_{im}^{kj} = \gamma_i^{k,j} s_{im}^{j}$ to denote expenditure in country i and sector k on the goods produced by country m and sector j as a share of revenue in country i and sector k. Using this notation, we can rewrite the above expression for the change in prices as:

$$\mathrm{d}\ln p_{ni}^k - \mathrm{d}\ln \tau_{ni} = \gamma_i^k \left(\mathrm{d}\ln w_i - \mathrm{d}\ln z_i \right) + \sum_{j=1}^{K}\sum_{m=1}^{N} \Sigma_{im}^{kj} \left(\mathrm{d}\ln p_{im}^j - \mathrm{d}\ln \tau_{im} \right) + \sum_{j=1}^{K}\sum_{m=1}^{N} \Sigma_{im}^{kj}\, \mathrm{d}\ln \tau_{im}. \tag{D.73}$$

Let $\mathbf{\Sigma}$ denote the $NK \times NK$ matrix with entries Σ_{im}^{kj} capturing the input cost share (relative to revenue) on goods mj by producer ik. Let us also define the Leontief inverse $\mathbf{\Gamma} \equiv \left(\mathbf{I}_{(NK)} - \mathbf{\Sigma} \right)^{-1}$, with the (nj, ik)-th entry, Γ_{ni}^{jk}, capturing the network-adjusted share of nj's revenue spent on inputs ik, either directly or indirectly through suppliers and suppliers of suppliers, ad infinitum. Finally, let $\Lambda_{ni}^{j} \equiv \sum_{k=1}^{K} \gamma_i^k \Gamma_{ni}^{jk}$ denote the network-adjusted input cost share of nj's revenue on value-added (labor) in country i; note that $\sum_{i=1}^{N} \Lambda_{ni}^{j} = 1$ for all nj due to constant returns to scale. Equation (D.73) can be re-written as:

$$\mathrm{d}\ln p_{ni}^k = \sum_{l=1}^{N} \Lambda_{il}^{k} \left(\mathrm{d}\ln w_l - \mathrm{d}\ln z_l \right). \tag{D.74}$$

We can now use (D.74) to re-write the linearized expenditure shares from equation (D.64) as:

$$\begin{aligned} \mathrm{d}\ln s_{ni}^k &= \theta \left(\sum_{h=1}^{N} s_{nh}^k \sum_{r=1}^{N} \Lambda_{hr}^k \left(\mathrm{d}\ln w_r - \mathrm{d}\ln z_r \right) - \sum_{r} \Lambda_{ir}^k \left(\mathrm{d}\ln w_r - \mathrm{d}\ln z_r \right) \right), \\ &= \theta \sum_{r=1}^{N} \left(\sum_{h=1}^{N} s_{nh}^k \Lambda_{hr}^k - \Lambda_{ir}^k \right) \left(\mathrm{d}\ln w_r - \mathrm{d}\ln z_r \right). \end{aligned} \tag{D.75}$$

Substitute this into (D.72), we get:

$$\begin{aligned} \mathrm{d}\ln w_i = &\sum_{k=1}^{K} \xi_i^k \sum_{o=1}^{N}\sum_{l=1}^{K} \Delta_{io}^{kl} \sum_{n=1}^{N} \vartheta_{on}^l\, \mathrm{d}\ln w_n \\ &+ \sum_{k=1}^{K} \xi_i^k \sum_{o=1}^{N}\sum_{l=1}^{K} \Delta_{io}^{kl} \theta \sum_{n=1}^{N} \left(\vartheta_{on}^l + \sum_{j=1}^{K} \Theta_{on}^{lj} \right) \left(\sum_{r} \left(\sum_{h=1}^{N} s_{nh}^l \Lambda_{hr}^l - \Lambda_{or}^l \right) \left(\mathrm{d}\ln w_r - \mathrm{d}\ln z_r \right) \right). \end{aligned}$$

To simplify notation, let us now define $\Pi_{io}^l \equiv \sum_{k=1}^{K} \xi_i^k \Delta_{io}^{kl}$ to be the network-adjusted share of income in country i derived from selling to country o industry l. Also denote $\Upsilon_{nor}^l \equiv \sum_{h=1}^{N} s_{nh}^l \Lambda_{hr}^l - \Lambda_{or}^l$; $\theta \Upsilon_{nor}^l$ is the elasticity of n's expenditure on goods lo with respect to r's factor cost. Then the above expression can be re-written as:

$$\begin{aligned} \mathrm{d}\ln w_i = &\sum_{n=1}^{N} \underbrace{\left(\sum_{o=1}^{N}\sum_{l=1}^{K} \Pi_{io}^l \vartheta_{on}^l \right)}_{\equiv \mathbf{T}_{in}} \mathrm{d}\ln w_n \\ &+ \theta \sum_{n=1}^{N} \underbrace{\left(\sum_{r=1}^{N}\sum_{o=1}^{N}\sum_{l=1}^{K} \Pi_{io}^l \left(\vartheta_{or}^l + \sum_{j=1}^{K} \Theta_{or}^{lj} \right) \Upsilon_{ron}^l \right)}_{\equiv \mathbf{M}_{in}} \left(\mathrm{d}\ln w_n - \mathrm{d}\ln z_n \right). \end{aligned} \tag{D.76}$$

Finally, we define $\mathbf{T}$ as an $N \times N$ matrix with entries $\mathbf{T}_{in} \equiv \sum_{o=1}^{N} \sum_{l=1}^{K} \Pi_{io}^{l} \vartheta_{on}^{l}$. Element $\mathbf{T}_{in}$ captures the network-adjusted share of i's income derived from selling to consumers in country n; it sums across the network-adjusted income share that country i derives from selling to country-industry ol, times the revenue share that ol derives from selling to consumers in country n. We define $\mathbf{M}$ as an $N \times N$ matrix with entries $\mathbf{M}_{in}$:

$$\mathbf{M}_{in} \equiv \sum_{r=1}^{N} \sum_{o=1}^{N} \sum_{l=1}^{K} \Pi_{io}^{l} \left(\vartheta_{or}^{l} + \sum_{j=1}^{K} \Theta_{or}^{lj} \right) \Upsilon_{ron}^{l}.$$

To interpret, $\theta \Upsilon_{ron}^{l}$ captures how r's expenditure on goods ol responds to factor cost in n. Element $\mathbf{M}_{in}$ sums the cross-substitution effects across i's exposure to all markets through network linkages: Π_{io}^{l} is i's network-adjusted income share derived from selling to producers in country o industry l; goods ol are then exposed to substitution due to changes in n's factor costs through markets that ol supplies to, including consumers (ϑ_{or}^{l}) and producers $\left(\sum_{j=1}^{K} \Theta_{or}^{lj} \right)$ in all countries (r).

We have thus obtained the same matrix representation as in the paper:

$$\mathrm{d} \ln \mathbf{w} = \mathbf{T} \, \mathrm{d} \ln \mathbf{w} + \theta \mathbf{M} \left(\mathrm{d} \ln \mathbf{w} - \mathrm{d} \ln \mathbf{z} \right). \tag{D.77}$$

We again solve for our friend-enemy income exposure measure by matrix inversion:

$$\mathrm{d} \ln \mathbf{w} = \mathbf{W} \, \mathrm{d} \ln \mathbf{z}, \tag{D.78}$$

where

$$\mathbf{W} \equiv -\frac{\theta}{\theta + 1} \left(\mathbf{I} - \mathbf{V} \right)^{-1} \mathbf{M}, \tag{D.79}$$

and, using our choice of world GDP as numeraire, which implies $\mathbf{Q} \, \mathrm{d} \ln \mathbf{w} = 0$,

$$\mathbf{V} \equiv \frac{\mathbf{T} + \theta \mathbf{T} \mathbf{S}}{\theta + 1} - \mathbf{Q}.$$

D.6.9 Real Income

Returning to our expression for real income per capita (D.60), we have:

$$u_n = \frac{w_n}{\prod_{k=1}^{K} (p_n^k)^{\alpha_n^k}}.$$

Totally differentiating this expression for real income per capita, we have:

$$\frac{\mathrm{d} u_n}{u_n} = \frac{\mathrm{d} w_n}{w_n} - \sum_{k=1}^{K} \alpha_n^k \frac{\mathrm{d} p_n^k}{p_n^k}.$$

Using our total derivative of the sectoral price index above, we get:

$$\frac{\mathrm{d} u_n}{u_n} = \frac{\mathrm{d} w_n}{w_n} - \sum_{k=1}^{K} \alpha_n^k \sum_{m=1}^{N} s_{nm}^k \frac{\mathrm{d} p_{nm}^k}{p_{nm}^k}, \tag{D.80}$$

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{k=1}^{K} \alpha_n^k \sum_{m=1}^{N} s_{nm}^k \,\mathrm{d}\ln p_{nm}^k.$$

Plugging (D.74) into the above, we get:

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{i=1}^{N} \underbrace{\left(\sum_{k=1}^{K} \sum_{m=1}^{N} \alpha_n^k s_{nm}^k \Lambda_{mi}^k \right)}_{\mathbf{S}_{ni}} \left(\mathrm{d}\ln w_i - \mathrm{d}\ln z_i \right).$$

We define $\mathbf{S}$ as an $N \times N$ matrix with entries $\mathbf{S}_{ni} \equiv \sum_{k=1}^{K} \sum_{m=1}^{N} \alpha_n^k s_{nm}^k \Lambda_{mi}^k$. Element $\mathbf{S}_{ni}$ captures the network-adjusted expenditure share of consumer n on value-added by country i; it sums across the expenditure share of consumer n on goods mk, times the network-adjusted input cost share of mk on factor i, captured by Λ_{mi}^k. We have thus obtained the same matrix representation for real income as in the paper:

$$\mathrm{d}\ln \mathbf{u} = \mathrm{d}\ln \mathbf{w} - \mathbf{S} \left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z} \right). \tag{D.81}$$

We can re-write the above relationship as:

$$\mathrm{d}\ln \mathbf{u} = (\mathbf{I} - \mathbf{S})\, \mathrm{d}\ln \mathbf{w} + \mathbf{S}\, \mathrm{d}\ln \mathbf{z},$$

which, using our solution for $\mathrm{d}\ln \mathbf{w}$ from equation (D.78), can be further re-written as:

$$\begin{aligned} \mathrm{d}\ln \mathbf{u} &= (\mathbf{I} - \mathbf{S})\, \mathbf{W}\, \mathrm{d}\ln \mathbf{z} + \mathbf{S}\, \mathrm{d}\ln \mathbf{z}, \\ &= \left[(\mathbf{I} - \mathbf{S})\, \mathbf{W} + \mathbf{S} \right] \mathrm{d}\ln \mathbf{z}, \\ &= \mathbf{U}\, \mathrm{d}\ln \mathbf{z}, \end{aligned}$$

where $\mathbf{U}$ is our friend-enemy real income exposure measure:

$$\mathbf{U} \equiv \left[(\mathbf{I} - \mathbf{S})\, \mathbf{W} + \mathbf{S} \right]. \tag{D.82}$$

D.6.10 Industry-Level Sales Exposure

Similarly to the multi-sector model without input linkages, our approach also yields bilateral friend-enemy measures of sales exposure to global productivity shocks for each sector. From equations (D.71) and (D.75) we obtain the following expression for changes in the sales of industry k in country i:

$$\begin{aligned} \mathrm{d}\ln y_i^k = &\sum_{n=1}^{N} \underbrace{\left(\sum_{o=1}^{N} \sum_{l=1}^{K} \Delta_{io}^{kl} \vartheta_{on}^l \right)}_{\equiv \mathbf{T}_{in}^k} \mathrm{d}\ln w_n, \\ &+ \theta \sum_{n=1}^{N} \underbrace{\left(\sum_{r=1}^{N} \sum_{o=1}^{N} \sum_{l=1}^{K} \Delta_{io}^{kl} \left(\vartheta_{or}^l + \sum_{j=1}^{K} \Theta_{or}^{lj} \right) \Upsilon_{ron}^l \right)}_{\equiv \mathbf{M}_{in}^k} \left(\mathrm{d}\ln w_n - \mathrm{d}\ln z_n \right). \end{aligned}$$

We define $\mathbf{T}^k$ as an $N \times N$ matrix with entries $\mathbf{T}^k_{in} \equiv \sum_{o=1}^{N} \sum_{l=1}^{K} \Delta^{kl}_{io} \vartheta^l_{on}$. Element $\mathbf{T}^k_{in}$ captures the network-adjusted share of sector ik's income derived from selling to consumers in country n; it sums across the network-adjusted income share that sector ik derives from selling to country-industry ol, times the revenue share that ol derives from selling to consumers in country n. We define $\mathbf{M}^k$ as an $N \times N$ matrix with entries $\mathbf{M}^k_{in}$:

$$\mathbf{M}^k_{in} \equiv \sum_{r=1}^{N} \sum_{o=1}^{N} \sum_{l=1}^{K} \Delta^{kl}_{io} \left(\vartheta^l_{or} + \sum_{j=1}^{K} \Theta^{lj}_{or} \right) \Upsilon^l_{ron}.$$

To interpret, $\theta \Upsilon^l_{ron}$ captures how r's expenditure on goods ol responds to factor cost in n. Element $\mathbf{M}^k_{in}$ sums the cross-substitution effects across sector ik's exposure to all markets through network linkages: Δ^{kl}_{io} is ik's network-adjusted income share derived from selling to producers in country o industry l; goods ol are then exposed to substitution due to changes in n's factor costs through markets that ol supplies to.

We get the following matrix representation:

$$\begin{aligned} \mathrm{d}\ln \mathbf{y^k} &= \mathbf{T^k} \,\mathrm{d}\ln \mathbf{w} + \theta \mathbf{M^k} \left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z} \right), \\ &= \left[\mathbf{T^k W} + \theta \mathbf{M^k} \left(\mathbf{W} - \mathbf{I} \right) \right] \mathrm{d}\ln \mathbf{z}, \end{aligned} \tag{D.83}$$

which can be re-written as:

$$\mathrm{d}\ln \mathbf{y^k} = \mathbf{W^k} \,\mathrm{d}\ln \mathbf{z}, \tag{D.84}$$

$$\mathbf{W^k} \equiv \mathbf{T^k W} + \theta \mathbf{M^k} \left(\mathbf{W} - \mathbf{I} \right), \tag{D.85}$$

which corresponds to our friends-and-enemies measure of sector sales exposure to productivity shocks. Note that $\mathbf{W^k}$ also captures sector value-added exposure to productivity shocks, as value added is a constant share of revenues in this model. Finally, recall from equation (D.68) that

$$\mathrm{d}\ln w_n = \sum_{k=1}^{K} \xi^k_n \,\mathrm{d}\ln y^k_n.$$

where ξ^k_n is the value-added share of industry k in country n's total income. Together, this relationship and equations (D.78) and (D.84) imply that:

$$\mathbf{W}_i \,\mathrm{d}\ln \mathbf{z} = \sum_{k=1}^{K} \xi^k_n \mathbf{W}^k_i \,\mathrm{d}\ln \mathbf{z},$$

where $\mathbf{W}_i$ is the income exposure vector for country i with respect to productivity shocks in its trade partners and $\mathbf{W}^k_i$ is the sector value-added exposure vector for country i and sector k with respect to productivity shocks in those trade partners. It follows that our aggregate friends-and-enemies measure of income exposure ($\mathbf{W}_i$) is a weighted average of our industry friends-and-enemies measures of sector value-added exposure ($\mathbf{W}^k_i$):

$$\mathbf{W}_i = \sum_{k=1}^{K} \xi^k_n \mathbf{W}^k_i.$$

Therefore, we can decompose how much of our aggregate income exposure measure is driven by the value-added exposure of particular industries, and how much of that value-added exposure of particular industries is explained by various terms (market-size, cross-substitution, etc. within the industry).

D.6.11 Isomorphisms

Although for expositional convenience we focus on an extension of the constant elasticity Armington model with multiple sectors and input-output linkages, the same results hold in an extension of the Eaton and Kortum (2002) model to incorporate both multiple sectors and input-output linkages as in Caliendo and Parro (2015).

D.6.12 Income and Real Income Exposure to Common Productivity and Trade Cost Shocks

We now derive our income and real income exposure measures in the multi-sector model with input-output linkages allowing for both productivity and trade cost shocks. We consider small productivity and trade cost shocks for each country that are common across sectors k:

$$\begin{aligned} \mathrm{d}\ln z_i^k &= \mathrm{d}\ln z_i, && \forall k \in K,\ i \in N, \\ \mathrm{d}\ln \tau_{ni}^k &= \mathrm{d}\ln \tau_{ni}, && \forall\, n, i \in N. \end{aligned}$$

We start with our expression for the change in prices above:

$$\mathrm{d}\ln p_{ni}^k = \mathrm{d}\ln \tau_{ni}^k + \gamma_i^k \left(\mathrm{d}\ln w_i - \mathrm{d}\ln z_i^k \right) + \sum_{j=1}^{K} \gamma_i^{k,j}\, \mathrm{d}\ln p_i^j,$$

Using our result for the total derivative of price indices (D.66), we can rewrite this expression for the change in prices as:

$$\mathrm{d}\ln p_{ni}^k - \mathrm{d}\ln \tau_{ni} = \gamma_i^k \left(\mathrm{d}\ln w_i - \mathrm{d}\ln z_i \right) + \sum_{j=1}^{K} \gamma_i^{k,j} \sum_{m=1}^{N} s_{im}^j\, \mathrm{d}\ln p_{im}^j.$$

We use $\Sigma_{im}^{kj} = \gamma_i^{k,j} s_{im}^j$ to denote expenditure in country i and sector k on the goods produced by country m and sector j as a share of revenue in country i and sector k. Using this notation, we can rewrite the above expression for the change in prices as:

$$\mathrm{d}\ln p_{ni}^k - \mathrm{d}\ln \tau_{ni} = \gamma_i^k \left(\mathrm{d}\ln w_i - \mathrm{d}\ln z_i \right) + \sum_{j=1}^{K} \sum_{m=1}^{N} \Sigma_{im}^{kj} \left(\mathrm{d}\ln p_{im}^j - \mathrm{d}\ln \tau_{im} \right) + \sum_{j=1}^{K} \sum_{m=1}^{N} \Sigma_{im}^{kj}\, \mathrm{d}\ln \tau_{im}. \quad \text{(D.86)}$$

Recall $\boldsymbol{\Sigma}$ is the $NK \times NK$ matrix with entries Σ_{im}^{kj} capturing the input cost share (relative to revenue) on goods mj by producer ik, and $\boldsymbol{\Gamma} \equiv \left(\mathbf{I}_{(NK)} - \boldsymbol{\Sigma} \right)^{-1}$ is the Leontief inverse, with the (nj, ik)-th entry, Γ_{ni}^{jk}, capturing the network-adjusted share of nj's revenue spent on inputs ik, either directly or indirectly through suppliers and suppliers of suppliers, ad infinitum. Equation (D.86) can be re-written as:

$$\mathrm{d}\ln p_{ni}^k - \mathrm{d}\ln \tau_{ni} = \sum_{l=1}^{N} \left(\sum_{j=1}^{K} \Gamma_{il}^{kj} \sum_{m=1}^{N} \sum_{x=1}^{K} \Sigma_{lm}^{jx} d\ln \tau_{lm} + \sum_{j=1}^{K} \Gamma_{il}^{kj} \gamma_l^j \left(\mathrm{d}\ln w_l - \mathrm{d}\ln z_l \right) \right). \quad \text{(D.87)}$$

Recall $\Lambda_{ni}^{j} \equiv \sum_{k=1}^{K} \gamma_i^k \Gamma_{ni}^{jk}$ is the network-adjusted input cost share of nj's revenue on value-added (labor) in country i. Also let

$$\mathrm{d}\ln c_i^{k,\tau} \equiv \sum_{l=1}^{N} \sum_{j=1}^{K} \Gamma_{il}^{kj} \sum_{m=1}^{N} \sum_{x=1}^{K} \Sigma_{lm}^{jx} \,\mathrm{d}\ln \tau_{lm},$$

which captures the log changes in the cost of good ik due to changes in trade costs, holding factor costs constant. Equation (D.87) can be re-written as:

$$\mathrm{d}\ln p_{ni}^{k} = \mathrm{d}\ln \tau_{ni} + \mathrm{d}\ln c_i^{k,\tau} + \sum_{l} \Lambda_{il}^{k} \left(\mathrm{d}\ln w_l - \mathrm{d}\ln z_l \right). \tag{D.88}$$

We can now use (D.88) to re-write the linearized expenditure shares from equation (D.64) as:

$$\begin{aligned}
\mathrm{d}\ln s_{ni}^{k} &= \theta\Bigg(\sum_{h=1}^{N} s_{nh}^{k} \left\{ \mathrm{d}\ln \tau_{nh} + \mathrm{d}\ln c_h^{k,\tau} + \sum_{r=1}^{N} \Lambda_{hr}^{k} \left(\mathrm{d}\ln w_r - \mathrm{d}\ln z_r \right) \right\} && \text{(D.89)}\\
&\quad - \left\{ \mathrm{d}\ln \tau_{ni} + \mathrm{d}\ln c_i^{k,\tau} + \sum_{r} \Lambda_{ir}^{k} \left(\mathrm{d}\ln w_r - \mathrm{d}\ln z_r \right) \right\} \Bigg), && \text{(D.90)}\\
&= \theta \sum_{r=1}^{N} \left(\sum_{h=1}^{N} s_{nh}^{k} \Lambda_{hr}^{k} - \Lambda_{ir}^{k} \right) \left(\mathrm{d}\ln w_r - \mathrm{d}\ln z_r \right) + \theta \sum_{h=1}^{N} \left(s_{nh}^{k} - \mathbf{1}_{h=i} \right) \left\{ \mathrm{d}\ln \tau_{nh} + \mathrm{d}\ln c_h^{k,\tau} \right\}.
\end{aligned}$$

Substitute this into (D.72), we get

$$\begin{aligned}
\mathrm{d}\ln w_i &= \sum_{k=1}^{K} \xi_i^k \sum_{o=1}^{N} \sum_{l=1}^{K} \Delta_{io}^{kl} \sum_{n=1}^{N} \vartheta_{on}^{l} \,\mathrm{d}\ln w_n \\
&\quad + \theta \sum_{k=1}^{K} \xi_i^k \sum_{o=1}^{N} \sum_{l=1}^{K} \Delta_{io}^{kl} \sum_{n=1}^{N} \left(\vartheta_{on}^{l} + \sum_{j=1}^{K} \Theta_{on}^{lj} \right) \left(\sum_{r} \left(\sum_{h=1}^{N} s_{nh}^{l} \Lambda_{hr}^{l} - \Lambda_{or}^{l} \right) \left(\mathrm{d}\ln w_r - \mathrm{d}\ln z_r \right) \right) \\
&\quad + \theta \sum_{k=1}^{K} \xi_i^k \sum_{o=1}^{N} \sum_{l=1}^{K} \Delta_{io}^{kl} \sum_{n=1}^{N} \left(\vartheta_{on}^{l} + \sum_{j=1}^{K} \Theta_{on}^{lj} \right) \left(\sum_{h=1}^{N} \left(s_{nh}^{l} - \mathbf{1}_{h=i} \right) \left\{ \mathrm{d}\ln \tau_{nh} + \mathrm{d}\ln c_h^{l,\tau} \right\} \right).
\end{aligned}$$

Recall we have defined $\Pi_{io}^{l} \equiv \sum_{k=1}^{K} \xi_i^k \Delta_{io}^{kl}$ as the network-adjusted share of income in country i derived from selling to country o industry l. Also recall $\Upsilon_{nor}^{l} \equiv \sum_{h=1}^{N} s_{nh}^{l} \Lambda_{hr}^{l} - \Lambda_{or}^{l}$; $\theta \Upsilon_{nor}^{l}$ is the elasticity of n's expenditure on goods lo with respect to r's factor cost. Using these results, we can rewrite income exposure to productivity and trade cost shocks as:

$$\begin{aligned}
\mathrm{d}\ln w_i &= \sum_{n=1}^{N} \underbrace{\left(\sum_{o=1}^{N} \sum_{l=1}^{K} \Pi_{io}^{l} \vartheta_{on}^{l} \right)}_{\equiv \mathbf{T}_{in}} \mathrm{d}\ln w_n \\
&\quad + \theta \sum_{n=1}^{N} \underbrace{\left(\sum_{r=1}^{N} \sum_{o=1}^{N} \sum_{l=1}^{K} \Pi_{io}^{l} \left(\vartheta_{or}^{l} + \sum_{j=1}^{K} \Theta_{or}^{lj} \right) \Upsilon_{ron}^{l} \right)}_{\equiv \mathbf{M}_{in}} \left(\mathrm{d}\ln w_n - \mathrm{d}\ln z_n \right) && \text{(D.91)}\\
&\quad + \theta \sum_{n=1}^{N} \sum_{r=1}^{N} \sum_{o=1}^{N} \sum_{l=1}^{K} \Pi_{io}^{l} \left(\vartheta_{on}^{l} + \sum_{j=1}^{K} \Theta_{on}^{lj} \right) \left(s_{nr}^{l} - \mathbf{1}_{r=i} \right) \left\{ \mathrm{d}\ln \tau_{nr} + \mathrm{d}\ln c_r^{l,\tau} \right\}.
\end{aligned}$$

For real income, note

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{k=1}^{K} \alpha_n^k \sum_{m=1}^{N} s_{nm}^k \,\mathrm{d}\ln p_{nm}^k.$$

Plugging equation (D.88) into the above expression, we can write real income exposure to productivity and trade cost shocks as:

$$\begin{aligned}
\mathrm{d}\ln u_n \quad = \quad & \mathrm{d}\ln w_n - \sum_{i=1}^{N} \underbrace{\left(\sum_{k=1}^{K} \sum_{m=1}^{N} \alpha_n^k s_{nm}^k \Lambda_{mi}^k \right)}_{\mathbf{S}_{ni}} (\,\mathrm{d}\ln w_i - \,\mathrm{d}\ln z_i) \\
& - \sum_{k=1}^{K} \alpha_n^k \sum_{m=1}^{N} s_{nm}^k \left(\mathrm{d}\ln \tau_{nm} + \,\mathrm{d}\ln c_m^{k;\tau} \right).
\end{aligned}$$

E Descriptive Evidence

In Section III of the paper, we provide evidence on our economic exposure measures for our balanced panel of 143 countries over the 43 years from 1970-2012. In this section of the online appendix, we report additional empirical results for these exposure measures.

E.1 Real Income Exposure

We begin by showing that our exposure measures are not only theoretically-consistent measures of sensitivity to foreign productivity growth, but are also hard to proxy with simpler measures of trading relationships between countries. In Table E.1, we regress our input-output real income exposure ($\boldsymbol{U}^{IO}$) and income exposure ($\boldsymbol{W}^{IO}$) measures on a number of simpler measures of trading relationships between countries: (i) log value of bilateral trade; (ii) aggregate import shares (the expenditure share matrix from our single-sector model ($\boldsymbol{S}^{SSM}$)); (iii) the expenditure share matrix from our input-output model ($\boldsymbol{S}^{IO}$); (iv) the income share matrix from our input-output model ($\boldsymbol{T}^{IO}$); and (v) the cross-substitution matrix from our input-output model ($\boldsymbol{M}^{IO}$). In the first panel, we report results for income exposure. In the second panel, we report results for real income exposure. In the remaining panels, we report results for the three separate components of the market-size effect, cross-substitution effect and cost of living effect. In the interests of brevity, we report results for the year 2000, but find the same pattern for all years.

Our income and real income exposure measures have statistically significant correlations with all of these proxies. For income exposure in the first panel, we find that the regression R-squared is always less than than 0.26. For real income exposure in the second panel, we find that the expenditure share matrix from the single-sector model ($\boldsymbol{S}^{SSM}$) has substantial explanatory power in Column (2), although around one third of the variation in real income exposure remains unexplained. We find an even stronger relationship with the expenditure share matrix from the input-output model ($\boldsymbol{S}^{IO}$) in Column (3), with the R-squared rising further to more than 0.80, highlighting the additional information in the input-output structure. We find a similar high R-squared for the income share ($\boldsymbol{T}^{IO}$) and cross-substitution ($\boldsymbol{M}^{IO}$) matrices from the input-output model in Columns (4) and (5), which is consistent with the close relationship between all three matrices in the input-output model.

In the remaining panels of the table, we demonstrate a similar pattern of results for the market-size, cross-substitution and cost of living components of our exposure measures. For both the market-size and cost of living effects, we find regression R-squared of less than 0.30. For the cross-substitution effect, we find higher R-squared for the income share matrix ($\boldsymbol{T}^{IO}$) and the cross-substitution matrix ($\boldsymbol{M}^{IO}$), which is consistent with the central role played by these matrices in determining the cross-substitution effect.

Table E.1: Correlations of Income ($\boldsymbol{W}^{IO}$) and Real Income ($\boldsymbol{U}^{IO}$) Exposure and their Components with Other Measures of Trading Relationships Between Countries

	(1) log value	(2) $\boldsymbol{S}^{SSM}$	(3) $\boldsymbol{S}^{IO}$	(4) $\boldsymbol{T}^{IO}$	(5) $\boldsymbol{M}^{IO}$
$\boldsymbol{W}^{IO}$	-0.000339***	-0.349***	-2.532***	-2.260***	-2.723***
	(0.0000104)	(0.0278)	(0.389)	(0.293)	(0.242)
Observations	20592	20592	20592	20592	20592
R-squared	0.0435	0.170	0.152	0.132	0.259
Year	2000	2000	2000	2000	2000

	(1) log value	(2) $\boldsymbol{S}^{SSM}$	(3) $\boldsymbol{S}^{IO}$	(4) $\boldsymbol{T}^{IO}$	(5) $\boldsymbol{M}^{IO}$
$\boldsymbol{U}^{IO}$	0.00000571***	0.0137***	0.123***	0.113***	0.0975***
	(0.000000231)	(0.000851)	(0.00359)	(0.00620)	(0.00449)
Observations	20592	20592	20592	20592	20592
R-squared	0.0292	0.613	0.845	0.780	0.783
Year	2000	2000	2000	2000	2000

	(1) log value	(2) $\boldsymbol{S}^{SSM}$	(3) $\boldsymbol{S}^{IO}$	(4) $\boldsymbol{T}^{IO}$	(5) $\boldsymbol{M}^{IO}$
Market Size^{IO}	-0.000302***	-0.276***	-1.892***	-1.410***	-2.035***
	(0.00000995)	(0.0273)	(0.379)	(0.291)	(0.252)
Observations	20592	20592	20592	20592	20592
R-squared	0.0377	0.116	0.0929	0.0564	0.158
Year	2000	2000	2000	2000	2000

	(1) log value	(2) $\boldsymbol{S}^{SSM}$	(3) $\boldsymbol{S}^{IO}$	(4) $\boldsymbol{T}^{IO}$	(5) $\boldsymbol{M}^{IO}$
Cross Substitution^{IO}	-0.0000370***	-0.0731***	-0.640***	-0.850***	-0.687***
	(0.00000147)	(0.00513)	(0.0391)	(0.00308)	(0.0202)
Observations	20592	20592	20592	20592	20592
R-squared	0.0276	0.396	0.519	0.998	0.879
Year	2000	2000	2000	2000	2000

	(1) log value	(2) $\boldsymbol{S}^{SSM}$	(3) $\boldsymbol{S}^{IO}$	(4) $\boldsymbol{T}^{IO}$	(5) $\boldsymbol{M}^{IO}$
Price Index^{IO}	0.000344***	0.363***	2.655***	2.373***	2.820***
	(0.0000105)	(0.0277)	(0.388)	(0.293)	(0.239)
Observations	20592	20592	20592	20592	20592
R-squared	0.0444	0.181	0.165	0.144	0.274
Year	2000	2000	2000	2000	2000

Note: Observations are a cross-section of exporting and importing countries in the year 2000; $\boldsymbol{W}^{IO}$ is our income exposure measure for the input-output model; $\boldsymbol{U}^{IO}$ is our real income exposure measure for the input-output model; log value is the log of one plus the value of bilateral trade; $\boldsymbol{S}^{SSM}$ is the share of each exporter in aggregate importer expenditure (the expenditure share matrix in the single-sector model); $\boldsymbol{S}^{IO}$ is the expenditure share matrix in the input-output model; $\boldsymbol{T}^{IO}$ is the income share matrix in the input-output model; $\boldsymbol{M}^{IO}$ is the cross-substitution matrix in the input-output model; table reports the regressions of $\boldsymbol{W}^{IO}$ and $\boldsymbol{U}^{IO}$ (rows) on alternative measures of trading relationships between countries (columns); standard errors in parentheses are heteroskedasticity robust; *** denotes significance at the 1 percent level; ** denotes significance at the 5 percent level; * denotes significance at the 10 percent level.

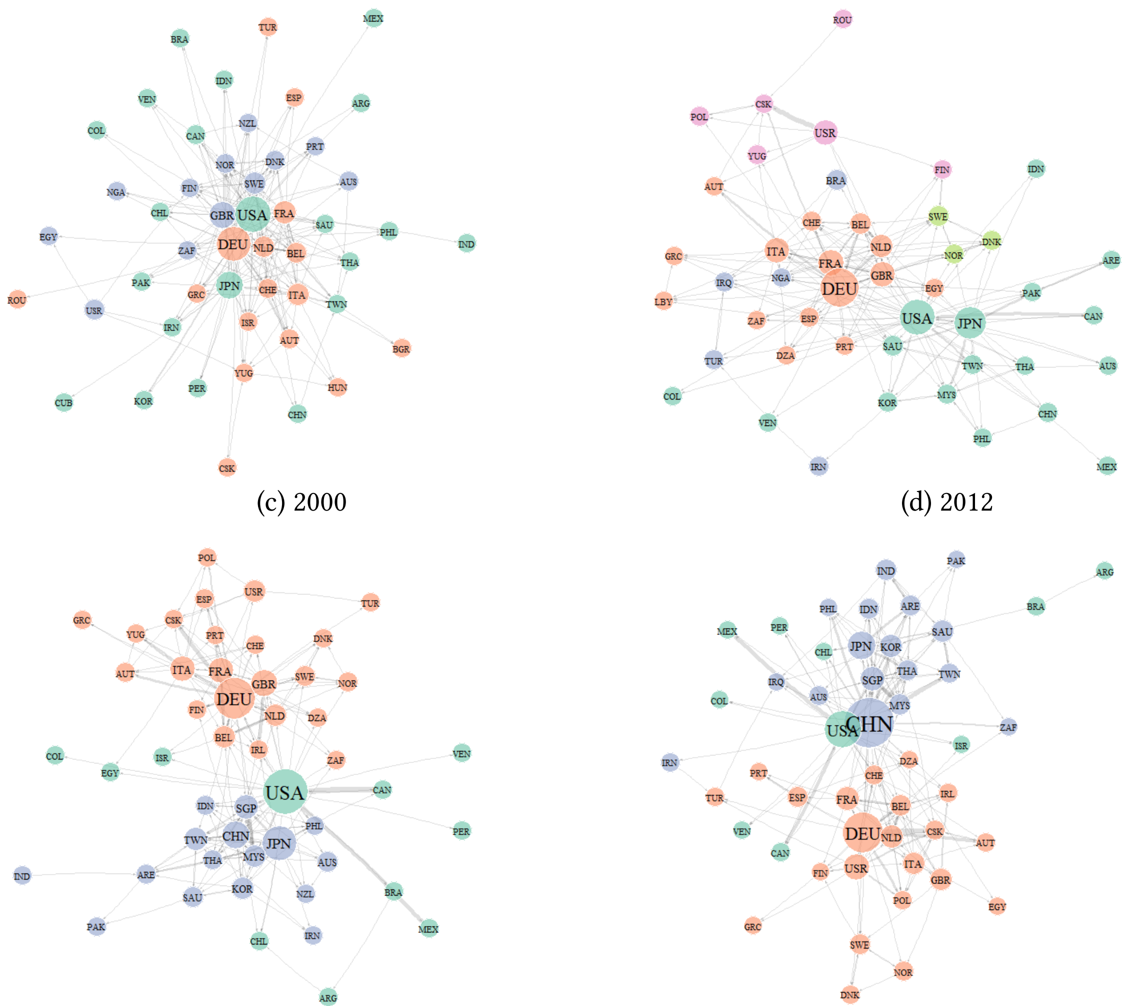
Across all of these specifications, we find that none of these proxies fully captures our theoretically-consistent measures of income and real income exposure. Furthermore, it would be hard to infer income and real income exposure without our measures, since the coefficients in these regressions do not have a structural interpretation, and could not be estimated without our measures.

In the remainder of this section, we show that our real income exposure measure captures the large-scale changes in the network of trade relationships that occurred over our sample period. In Figure E.1 we illustrate global real income exposure to productivity shocks in 1970, 1985, 2000 and 2012 using a network graph, where the nodes are countries and the edges capture bilateral real income exposure. For legibility, we display the 50 largest countries in terms of GDP and the 200 edges with the largest absolute values of bilateral real income exposure.[1] The size of each node captures the importance of each country as a source of productivity shocks (as a source of real income exposure for other countries); the arrow for each edge shows the direction of bilateral real income exposure (from the source of the productivity shock to the exposed country); and the thickness of each edge shows the absolute magnitude of the bilateral real income exposure. Countries are grouped to maximize modularity (the fraction of edges within the groups minus the expected fraction if the edges were distributed at random).

At the beginning of our sample period in 1970, the global network of real income exposure is dominated by the U.S., Germany and other Western industrialized countries (top-left panel). Moving forward to 1985, we see the emergence of Japan and a cluster of Newly Industrialized Countries (NICs) in Asia, and we observe Western Europe increasingly emerging as a separate cluster of interdependent nations. By the time we reach 2000, the separate clusters of countries in Asia and Western Europe become even more apparent, with China beginning to displace Japan at the center of the Asian cluster. By the end of our sample period in 2012, China replaces the U.S. at the center of the global network of real income exposure, with the US more tightly connected to China and other Asian countries than to the cluster of Western European countries. Therefore, we find substantial changes over our sample period, not only in the mean and dispersion of real income exposure, but also in the network of bilateral interdependencies between countries.

[1] All of the bilateral real income exposure links shown in the figure are positive.

Figure E.1: Global Real Income Exposure, 1970, 1985, 2000 and 2012



(a) 1970

(b) 1985

(c) 2000

(d) 2012

Source: NBER World Trade Database and authors' calculations using our multi-sector input-output model.

E.2 Preferential Trade Agreements (PTAs)

In Subsection III.C of the paper, provide a validation check on our real income exposure measure using separate data on PTAs not used in its construction. In this section of the online appendix, we report a number of robustness checks for this specification.

In particular, if our real income exposure measure correctly captures economic interdependence between countries, we would expect to observe systematic increases in real income exposure between member countries following the formation of a PTA. To examine this hypothesis empirically, we consider the following conventional event-study "difference-in-differences" specification:

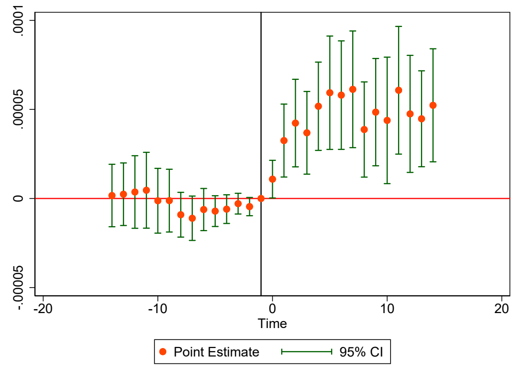
$$\boldsymbol{U}_{nit}^{IO} = \sum_{s \in \{S_-, S_+\}} \beta_s (\mathbb{I}_{ni}^{PTA} \times \mathbb{I}_s) + \xi_{ni} + d_{ct} + h_{nit}, \tag{E.1}$$

where $\mathbb{I}_{ni}^{PTA}$ is a dummy variable that equals one if an importer-exporter pair ever signs a trade agreement during our sample period; s is a treatment year index, which equals zero in the year an importer-exporter pair joins a PTA, such that negative values of s indicate years before joining a PTA, and zero or positive values represent years after joining a PTA; $\mathbb{I}_s$ is a dummy variable that equals one in treatment year s and zero otherwise; we choose treatment year minus one as the excluded category; S_- and S_+ are the minimum and maximum values of treatment years, respectively; ξ_{ni} are importer-exporter pair fixed effects, which control for time-invariant factors that affect both bilateral real income exposure and whether an importer-exporter joins a PTA, such as bilateral distance, contiguity, etc; d_{ct} are continent-year dummies, which control for secular changes over time in real income exposure and the propensity to join PTAs; and h_{nit} is a stochastic error.

The key coefficients of interest are β_s on the treatment-year interactions, which capture the impact of the PTA on real income exposure in treatment year s, relative to the excluded category of treatment year minus one. The exporter-importer fixed effects control for the non-random selection of exporter-importer pairs into PTAs based on time-invariant factors. Therefore, if exporter-importer pairs with high levels of real income exposure are more likely to form PTAs in all years, this is controlled for in the exporter-importer fixed effect. The key identifying assumption in equation (E.1) is parallel trends between the treatment and control group within continents. As a check on this identifying assumption, we include the treatment-year interactions for years both before and after joining a PTA, which allows us to provide evidence on whether treated exporter-importer pairs exhibit different trends from untreated pairs even before joining a PTA.

Robustness to Controlling for Bilateral Trade In Panel (a) of Figure 3 in the paper, we report our baseline estimation results using the two-way fixed effects estimator. In Figure E.2 of this online appendix, we show that we find a similar pattern of results if we additionally control for the log value of bilateral trade between an importer-exporter in each year. Therefore, we again find that our exposure measures cannot be fully captured by simpler measures of trading relationships between countries.

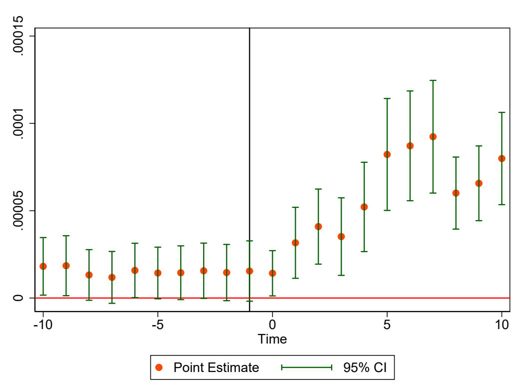
Figure E.2: Estimated Treatment Effects of Preferential Trade Agreements (PTAs) on Real Income Exposure (Controlling for Trade Flows)



Note: Estimated treatment-year interactions (β_s) from the event-study specification in equation (E.1) using the two-way fixed effects estimator and real income exposure of importer n to productivity growth in exporter i at time t ($\boldsymbol{U}^{IO}_{nit}$) in our input-output model; specification includes the log of one plus the value of bilateral trade between an exporter-importer pair in each year, importer-exporter fixed effects and continent-year fixed effects; standard errors clustered by importer-exporter pair to allow for serial correlation in the error term over time.

Alternative Event-Study Estimators In Figure 3 in the paper, we report the estimation results using both the two-way fixed effects estimator (panel (a)) and the Chaisemartin and D'Haultfloeuille (2020) estimator (panel (b)). In Figure E.3, we show that we also find the same pattern of results using the alternative event-study estimator of Borusyak et al. (2021). Therefore, our findings using our exposure measures of an increase in economic interdependence between countries following trade integration are robust to the use of alternative event-study estimators to control for the variable timing of the treatment and potential treatment heterogeneity.

Figure E.3: Estimated Treatment Effects of Preferential Trade Agreements (PTAs) on Real Income Exposure (Borusyak et al. (2021) Estimator)



Note: Estimated treatment-year interactions (β_s) from the event-study specification in equation (E.1) using the Borusyak et al. (2021) estimator and real income exposure of importer n to productivity growth in exporter i at time t ($\boldsymbol{U}^{IO}_{nit}$) in our input-output model; specification includes importer-exporter fixed effects and continent-year fixed effects; standard errors are clustered by importer-exporter pair.

F Economic and Political Friends and Enemies (Robustness)

In Section F.1 of the paper, we provide evidence on the relationship between economic and political friendship. We use two different sources of quasi-experimental variation. First, a large empirical literature following Autor et al. (2013) argues that China's rapid economic growth was driven its domestic supply-side reforms in 1978. Therefore, we use an exogenous increase in China's productivity from domestic reform as a source of exogenous variation in other countries' real income exposure.

Second, we use the large-scale reductions in the cost of air travel that occurred over our sample period as a source of exogenous variation in bilateral trade costs following Feyrer (2019). The key idea underlying this approach is that the position of land masses around the globe generates large differences between bilateral distances by sea and the great circle distances that are more typical of air travel. As a result, countries with long sea routes relative to air routes benefit disproportionately from reductions in the relative cost of air travel, giving rise to uneven changes in bilateral trade costs over time.

In our baseline specification, we measure the elasticity of real income to foreign productivity growth using the point elasticity from the linearization of the conditions for general equilibrium in the input-output model. In this section of the online appendix, we show that we find the same qualitative and quantitative pattern of results using arc elasticities from exact-hat algebra counterfactuals in the input-output model.

In Subsection F.1, we replicate our baseline results for the relationship between economic and political friendship using arc elasticities from exact-hat algebra counterfactuals for 10 percent productivity shocks. In the interests of brevity, we focus on our first source of quasi-experimental variation from an exogenous increase in China's productivity.

F.1 Chinese Productivity Growth (Arc Elasticities)

In this subsection, we replicate our baseline results for an exogenous increase in China's productivity using arc elasticities from exact-hat algebra counterfactuals in the input-output model, instead of the point elasticities from our linearization that are used in the paper. We undertake a counterfactual in which China experiences an exogenous 10 percent increase in productivity in a given year and compute the resulting percentage changes in real income in all other countries. Taking the ratio of these percentage changes in real income to the 10 percent productivity shock, we compute the arc elasticity of real income in each other country in response to this 10 percent exogenous increase in China's productivity ($\boldsymbol{U}_{nct}^{IO}$). We repeat this process for each year of our sample period.

Using these arc elasticities, we replicate our baseline estimation results for Chinese productivity growth in Tables 2 and 3 in the paper. In Table F.1, we report the results from estimating equation (14) in the paper using OLS. We find a similar qualitative and quantitative pattern of results using arc elasticities as using point elasticities in the paper. We find a strong positive and statistically significant relationship between changes in political alignment and changes in real income exposure. The point estimates using arc elasticities are close to those using point elasticities, typically only differing by a few decimal places. Again we find that this relationship between changes in political alignment and changes in real income exposure is robust to controlling for simpler measures of trading relationships such as log bilateral trade.

Table F.1: Changes in Political Alignment towards China and Changes in Initial Real Income Exposure towards China from 1980-2010 (OLS Specification and Arc Elasticities)

	(1)	(2)	(3)	(4)	(5)	(6)
	$\Delta \boldsymbol{A}^{\kappa}_{nct}$	$\Delta \boldsymbol{A}^{S}_{nct}$	$\Delta \boldsymbol{A}^{\pi}_{nct}$	$\Delta \boldsymbol{A}^{\kappa}_{nct}$	$\Delta \boldsymbol{A}^{S}_{nct}$	$\Delta \boldsymbol{A}^{\pi}_{nct}$
$\Delta \boldsymbol{U}^{IO}_{nct}$	49.63***	24.88***	52.40***	54.92***	27.21***	60.17***
	(14.47)	(6.523)	(15.01)	(14.95)	(6.827)	(15.81)
$\Delta \ln \boldsymbol{X}_{nct}$				-0.0251*	-0.0110*	-0.0369***
				(0.0136)	(0.00580)	(0.0139)
Estimation	OLS	OLS	OLS	OLS	OLS	OLS
Observations	119	119	119	119	119	119
R-squared	0.0671	0.0647	0.0603	0.0942	0.0848	0.107

Note: This table replicates Table 2 from the paper using arc elasticities for 10 percent productivity shocks from exact-hat algebra counterfactuals instead of the point elasticities from our linearization; Long-differences specification from 1980-2010 for a cross-section of countries excluding China; each column corresponds to a separate regression, with the left-hand side variable reported at the top of the column and the right-hand side variables listed in the rows; $\Delta \boldsymbol{A}^{\kappa}_{nct}$ is the 30-year change in our preferred κ-score measure of country n's bilateral political alignment towards China that controls for the empirical distribution with which each country votes yes, no and abstain; $\Delta \boldsymbol{A}^{S}_{nct}$ is the 30-year change in the S-score measure of country n's bilateral political alignment towards China; $\Delta \boldsymbol{A}^{\pi}_{nct}$ is the 30-year change in the π-score measure of country n's bilateral political alignment towards China; $\Delta \boldsymbol{U}^{IO}_{nct}$ is the 30-year change in our input-output measure of country n's real income exposure to China; $\Delta \ln \boldsymbol{X}_{nct}$ is the 30-year change in the log of one plus country n's bilateral trade with China; standard errors in parentheses are heteroskedasticity robust; *** denotes significance at the 1 percent level; ** denotes significance at the 5 percent level; * denotes significance at the 10 percent level.

In Table F.2, we report the results from estimating equation (14) in the paper using 2SLS. We instrument the change in real income exposure to China ($\Delta \boldsymbol{U}^{IO}_{nct}$) with our model-based instrument, namely the counterfactual change in real income exposure to China in response to an exogenous 100 percent increase in Chinese productivity. While our model implies that real income exposure drives bilateral political alignment, it does not have a clear prediction for the impact of log trade conditional on real income exposure. Therefore, we treat log trade as a reduced-form control, and develop a reduced-form instrument for it. In particular, we instrument the change in bilateral trade with China from 1980-2010 ($\Delta \ln \boldsymbol{X}_{nct}$) with its initial level in 1980, using the shift-share type insight that the countries that experienced the largest increases in bilateral trade with China following its domestic productivity growth are likely to be those with the highest initial levels of bilateral trade with China. We continue to find a positive and statistically significant relationship between changes in bilateral political alignment and changes in real income exposure predicted by our instruments, consistent with exogenous increases in economic dependence on China causing political realignment towards China. The point estimates using arc elasticities are again close to those using point elasticities, typically only differing by a few decimal places. We again find that our results are robust to controlling for simpler measures of trading relationships such as log bilateral trade.

Table F.2: Changes in Political Alignment towards China and Changes in Initial Real Income Exposure towards China from 1980-2010 (IV Specification and Arc Elasticities)

	(1)	(2)	(3)	(4)	(5)	(6)
	$\Delta\boldsymbol{A}^{\kappa}_{nct}$	$\Delta\boldsymbol{A}^{S}_{nct}$	$\Delta\boldsymbol{A}^{\pi}_{nct}$	$\Delta\boldsymbol{A}^{\kappa}_{nct}$	$\Delta\boldsymbol{A}^{S}_{nct}$	$\Delta\boldsymbol{A}^{\pi}_{nct}$
$\Delta\boldsymbol{U}^{IO}_{nct}$	96.32***	28.04***	100.3***	81.21***	25.97**	66.96***
	(26.27)	(10.61)	(28.43)	(22.30)	(11.60)	(24.57)
$\Delta \ln \boldsymbol{X}_{nct}$				-0.0137	-0.00187	-0.0303*
				(0.0159)	(0.00888)	(0.0177)
Estimation	IV	IV	IV	IV	IV	IV
Observations	119	119	119	119	119	119
Kleibergen-Paap F	20.41	20.41	20.41	10.85	10.85	10.85
Anderson-Rubin p-value	<0.01	<0.01	<0.01	<0.01	0.02	<0.01

Note: This table replicates Table 3 from the paper using arc elasticities for 10 percent productivity shocks from exact-hat algebra counterfactuals instead of point elasticities from our linearization; Long-differences specification from 1980-2010 for a cross-section of countries excluding China; each column corresponds to a separate regression, with the left-hand side variable reported at the top of the column and the right-hand side variables listed in the rows; $\Delta\boldsymbol{A}^{\kappa}_{nct}$ is the 30-year change in our preferred κ-score measure of country n's bilateral political alignment towards China that controls for the empirical distribution with which each country votes yes, no and abstain; $\Delta\boldsymbol{A}^{S}_{nct}$ is the 30-year change in the S-score measure of country n's bilateral political alignment towards China; $\Delta\boldsymbol{A}^{\pi}_{nct}$ is the 30-year change in the π-score measure of country n's bilateral political alignment towards China; $\Delta\boldsymbol{U}^{IO}_{nct}$ is the 30-year change in our input-output measure of country n's real income exposure to China; $\Delta \ln \boldsymbol{X}_{nct}$ is the 30-year change in the log of one plus country n's bilateral trade with China; Columns (1)-(3) instrument changes in real income exposure $\Delta\boldsymbol{U}^{IO}_{nct}$ with our model-based instrument, namely the counterfactual change in real income exposure to China in response to a 100 percent increase in productivity in China; Columns (4)-(6) instrument both $\Delta\boldsymbol{U}^{IO}_{nct}$ and $\Delta \ln \boldsymbol{X}_{nct}$ using our model-based instrument and the initial level of the log of one plus bilateral trade with China in 1980; Kleibergen-Paap is the Kleibergen-Paap rk Wald F-statistic; Anderson-Rubin p-value is for the Anderson-Rubin χ^2-statistic; the second-stage R-squared is not reported for these IV specifications, because it does not have a meaningful interpretation; standard errors in parentheses are heteroskedasticity robust; *** denotes significance at the 1 percent level; ** denotes significance at the 5 percent level; * denotes significance at the 10 percent level.

Taking the results of this section as a whole, we find the same qualitative and quantitative pattern of results for the relationship between bilateral political alignment and bilateral real income exposure, whether we use point elasticities from our linearization (as in the paper), or arc elasticities from exact-hat algebra counterfactuals (as in this section of the online appendix).

G Data Appendix

In this section of the online appendix, we provide additional information on our data sources and definitions, as summarized in Section II of the paper. In Section G.1, we report further information on our economic data. In Section G.2, we give additional details for our political data and measures of bilateral political alignment.

G.1 Economic Data

NBER World Trade Database Our data on international trade are from the NBER World Trade Database, which reports values of bilateral trade between countries for around 1,500 4-digit Standard International Trade Classification (SITC) codes. The ultimate source for these data is the

United Nations COMTRADE database and we use an updated version of the dataset from Feenstra et al. (2005) for the time period 1970-2012.[2] We augment these trade data with information on countries' gross domestic product (GDP), population and bilateral distances from the GEODIST and GRAVITY datasets from CEPII.[3] Note that the NBER World Trade Database lacks direct data on a country's expenditure on domestic goods (X_{nnt}). Therefore, we compute this domestic expenditure as gross output minus exports. To measure gross output for each country, we multiply its GDP (value added) by 2.2, which is the mean ratio of gross output to GDP in the EU-KLEMS database (which includes the USA and Japan).

In our multi-sector models, we report results aggregating the products in the NBER World Trade Database to the 20 International Standard Industrial Classification (ISIC) industries listed below. In our baseline input-output specification, we use a common input-output matrix for all countries, based on the median input-output coefficients across the country sample in Caliendo and Parro (2015). We make this assumption of a common input-output matrix to expand the country coverage to countries for which IO matrices are unavailable and because the IO matrices reported for some countries involve substantial imputation. We allow some variation in self-expenditure shares across sectors using the following procedure. First, we compute global sector-level expenditure shares that take into account both the final-demand coefficients and the input-output matrix. We then multiply these shares by the imputed gross output of each country to get sectoral expenditure levels for each country. Finally, we allocate these sectoral expenditure levels to various origin countries using the imports data. The residual is treated as sectoral self-expenditure. We show below that this procedure for measuring sectoral self-expenditure generates income and real income exposure measures that are strongly correlated with those from the EORA database that reports sectoral self-expenditure shares.

[2]See https://cid.econ.ucdavis.edu/wix.html.

[3]See http://www.cepii.fr/cepii/en/bdd_modele/bdd.asp.

Table G.1: International Standard Industrial Classification (ISIC) Tradeable Sectors

Industry Code	Short Name	Long Description
1	Agriculture	Agriculture Agriculture forestry and Fishing
2	Mining	Mining and quarrying
3	Food	Food products, beverages and tobacco
4	Textile	Textiles, textile products, leather and footwear
5	Wood	Wood and products of wood and cork
6	Paper	Pulp, paper, paper products, printing and publishing
7	Petroleum	Coke, refined petroleum and nuclear fuel
8	Chemicals	Chemicals
9	Plastic	Rubber and plastics products
10	Minerals	Other nonmetallic mineral products
11	Basic Metals	Basic metals
12	Metal Products	Fabricated metal products, except machinery and equipment
13	Machinery nec	Machinery and equipment n.e.c
14	Office	Office, accounting and computing machinery
15	Electrical	Electrical machinery and apparatus, n.e.c.
16	Communication	Radio, television and communication equipment
17	Medical	Medical, precision and optical instruments, watches and clocks
18	Auto	Motor vehicles trailers and semi-trailers
19	Other Transport	Other transport equipment
20	Other	Manufacturing n.e.c and recycling

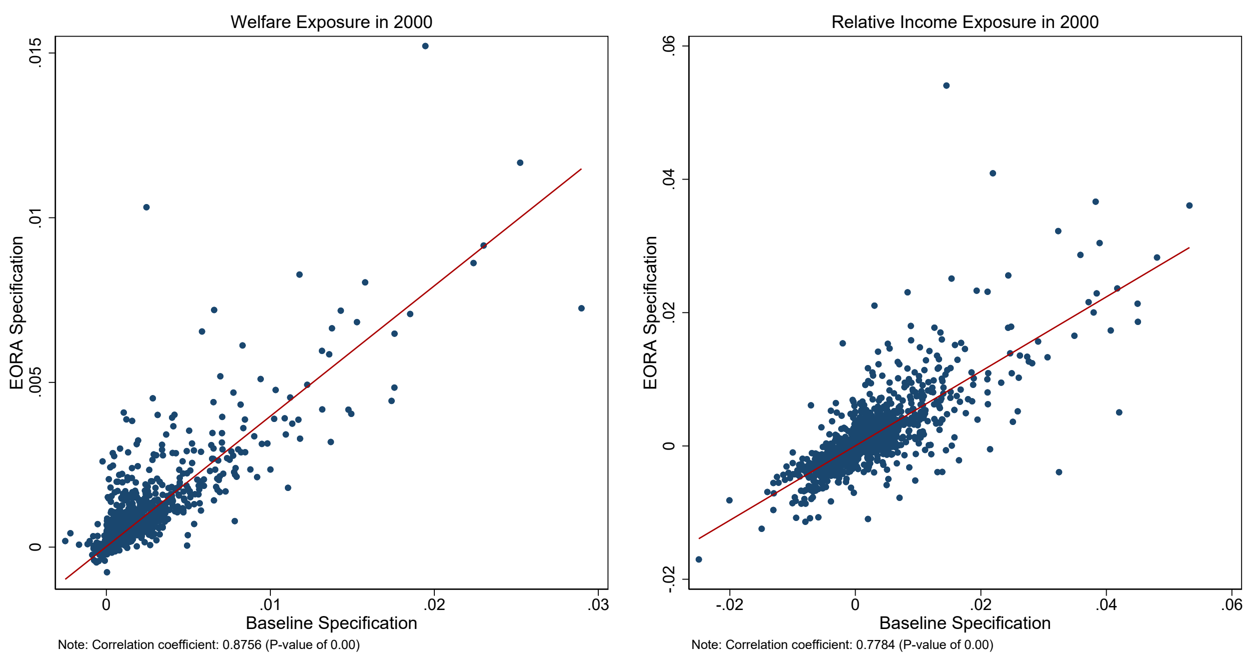
EORA Database Our baseline specification using the NBER World Trade Database has the advantage of including a large number of more than 140 countries countries over a long time period of more than 40 years and with a relatively disaggregated industry classification of 40 sectors (out of which 20 are tradable sectors). A limitation is that we need to measure self-expenditure shares using the procedure discussed in the previous paragraph, and assume the same input-output matrix for all countries, based on the median input-output coefficients across the country sample in Caliendo and Parro (2015).

To assess the sensitivity of our results to these assumptions, we replicated our analysis using the EORA Global Supply Chain Database (https://www.worldmrio.com/), which contains country-specific input-output tables and can be used to directly measure self-expenditure shares in each sector. Limitations of this dataset are that it considers a much shorter time period from 1990-2015, a more aggregated industry classification of 26 sectors (out of which 11 are tradable sectors), and is constructed with substantial imputation.

We replicate our input-output analysis using the EORA data for this shorter time period and more aggregated industry classification. For comparability with our baseline specification, we use a country-specific but time invariant input-output table for each country in the EORA data. We compare results using the two datasets for the 126 countries in common to both datasets over the time period from 1990-2015 for which both data are available. Recall that real income exposure ($\boldsymbol{U}^{IO}$) is invariant to our choice of numeraire. To ensure that our results for income exposure ($\boldsymbol{W}^{IO}$) are not sensitive to our choice of numeraire, we normalize income exposure by the income-weighted average for the OECD (excluding the country experiencing the productivity shock).

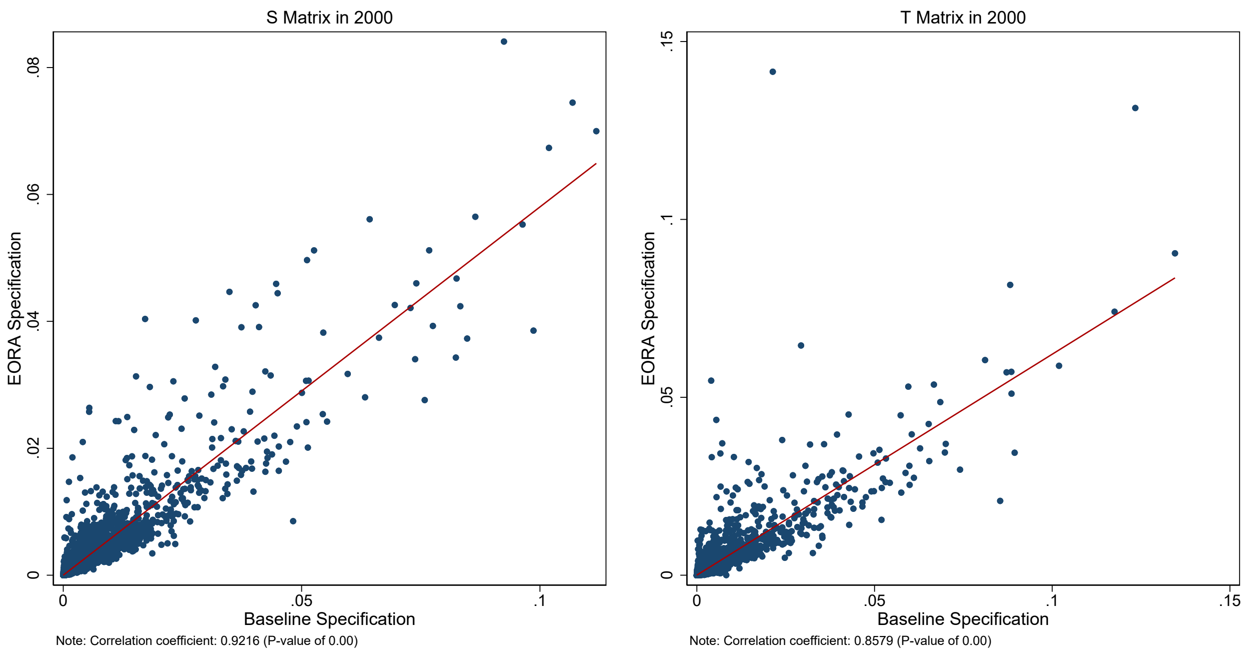
As shown in Figure G.1, we find a strong, positive and statistically significant correlation between our input-output exposure measures computed using each dataset, which is equal to 0.78 for relative income exposure ($\boldsymbol{W}^{IO}$) and 0.87 for real income exposure ($\boldsymbol{U}^{IO}$) in the year 2000. As shown in Figure G.2, we also find a strong, positive and statistically significant correlation between the underlying trade share matrices, which is equal to 0.92 for expenditure shares ($\boldsymbol{S}^{IO}$) and 0.86 for income shares ($\boldsymbol{T}^{IO}$) in the year 2000. Although both figures show results for the year 2000, we find the same high correlation for all years from 1990-2015 for which both data are available. Taken together, this strong correlation between results using the COMTRADE and EORA databases provides evidence that our findings are robust to different assumptions about self-expenditure shares and input-output matrices.

Figure G.1: Comparison of Our Baseline Exposure Measures using the NBER World Trade Database to those using the EORA Database in 2000 (Input-Output Model)



Note: Baseline specification uses the NBER World Trade Database; EORA specification uses the EORA Database; Real income exposure to productivity growth ($\boldsymbol{U}^{IO}$) is measured using our multi-sector input-output model and is invariant to our choice of numeraire; Relative income exposure equals income exposure to productivity growth ($\boldsymbol{W}^{IO}$) measured using our multi-sector input-output model and normalized by its income-weighted mean across OECD countries (excluding the shocked country), to ensure that results for income exposure are not sensitive to our choice of numeraire.

Figure G.2: Comparison of Our Baseline Expenditure and Income Share Matrices using the NBER World Trade Database to those using the EORA Database in 2000 (Input-Output Model)



Note: Baseline specification uses the NBER World Trade Database; EORA specification uses the EORA Database; Expenditure share matrix ($\boldsymbol{S}$) is the expenditure share matrix ($\boldsymbol{S}^{IO}$) from our multi-sector input-output model; Income share matrix ($\boldsymbol{T}$) is the income share matrix ($\boldsymbol{T}^{IO}$) from our multi-sector input-output model.

G.2 Political Data

We next provide further details on the data sources and definitions for our political alignment measures. We begin by discussing our three main sources of data on bilateral political alignment. First, we use data on observed voting behavior in the United Nations General Assembly (UNGA) to reveal the bilateral similarity of countries' foreign policies. Second, we use measures of strategic rivalries, as classified by political scientists, based on contemporary perceptions by political decision makers of whether countries regard one another as competitors, a source of actual or latent threats, or enemies. Third, we use information on formal alliances, including mutual defense pacts, neutrality and non-aggression treaties and ententes. A key advantage of each of these measures relative to data on military conflict is that much international political influence does not involve open hostilities, including international treaties, other supra-national agreements, international institutions, and back-room diplomacy.

United Nations Voting Behavior We follow a large literature in political science in measuring countries' bilateral political attitudes towards one another using the similarity of their votes in the United Nations (UN). The ultimate source for our UN voting data is Voeten (2013), which reports non-unanimous plenary votes in the United Nations General Assembly (UNGA) from 1962-2012, and includes on average around 128 votes each year. Countries are recorded as either voting "no" (coded 1), "abstain" (coded 2) or "yes" (coded 3). In particular, we use the bilateral measures of voting similarity constructed using these data in the Chance-Corrected Measures of Foreign Policy Similarity (FPSIM) database, as reported in Häge (2011).[4] We denote the outcome of vote v for country i by $O_i(v)$:

$$O_i(v) \in \{1, 2, 3\} \qquad v \in \{1, \ldots, V\}. \tag{G.1}$$

Building on a large literature in international relations, we consider a number of different measures of bilateral voting similarity. Our first and simplest measure is the S-score of Signorino and Ritter (1999), which measures the extent of agreement between the votes of countries n and i as one minus the sum of the squared actual deviation between their votes scaled by the sum of the squared maximum possible deviations between their votes:

$$\mathbb{S}_{ni}^{S} = 1 - \frac{\sum_{v=1}^{V} (O_n(v) - O_i(v))^2}{\frac{1}{2}\sum_{v=1}^{V} (d_{\max}(v))^2}, \tag{G.2}$$

where $(d_{\max}(v))^2 = (\sup\{O_n(v) - O_i(v)\})^2$ represents the maximum possible disagreement for each vote and this measure is bounded between minus one (maximum possible disagreement) and one (maximum possible agreement).

A limitation of this S-score measure is that it does not control for properties of the empirical distribution function of country votes. In particular, country votes may align by chance, such that the frequency with which countries agree on a "yes" depends on the frequency with which countries vote "yes." Similarly, the frequency with which they agree on each of the other voting outcomes ("no" and "abstain") depends on the frequencies with they choose these other voting outcomes. Therefore, we also consider two alternative measures of countries' bilateral similarity in voting patterns that control in different ways for properties of the empirical distribution of voting outcomes. First, the π-score of Scott (1955) adjusts the observed variability of the countries' bilateral voting outcomes with the variability of each country's own voting outcomes around average outcomes across the two countries taken together:

$$\mathbb{S}_{ni}^{\pi} = 1 - \frac{\sum_{v=1}^{V} (O_n(v) - O_i(v))^2}{\sum_{n=1}^{V} \left(O_n(v) - \frac{\bar{O}_n + \bar{O}_i}{2}\right)^2 + \sum_{v=1}^{V} \left(O_i(v) - \frac{\bar{O}_n + \bar{O}_i}{2}\right)^2}, \tag{G.3}$$

where $\bar{O}_i = (1/V)\sum_{v=1}^{V} O_i(v)$ is the average outcome for country i.

Second, the κ-score of Cohen (1960) adjusts the observed variability of the countries' bilateral voting outcomes with the variability of each country's own voting outcomes around its own average outcome and the difference between the two countries' average outcomes:

$$\mathbb{S}_{ni}^{\kappa} = 1 - \frac{\sum_{v=1}^{V} (O_n(v) - O_i(v))^2}{\sum_{v=1}^{V} \left(O_n(v) - \bar{O}_n\right)^2 + \sum_{v=1}^{V} \left(O_i(v) - \bar{O}_i\right)^2 + \sum_{v=1}^{V} \left(\bar{O}_n - \bar{O}_i\right)^2}. \tag{G.4}$$

[4] See https://dataverse.harvard.edu/dataset.xhtml?persistentId=doi:10.7910/DVN/ALVXLM

Both the π-score and κ-score have an attractive statistical interpretation, as discussed further in Krippendorf (1970), Fay (2005) and Häge (2011). In the case of binary (0,1) voting outcomes, these indices reduce to the form of $1-(D_o/D_e)$, where D_o is the observed frequency of agreement and D_e is the expected frequency of agreement. The key difference between the two indices is in their assumptions about the expected frequency of agreement. The π-score estimates the expected frequency of agreement using the average of the two countries marginal distributions of voting outcomes. In contrast, the κ-score estimates the expected frequency of agreement using each country's own individual marginal distribution of voting outcomes.

Finally, as Bailey et al. (2017) point out, measures based on dyadic similarity of vote choices—such as the S, π, and κ scores—do not account for the heterogeneity in resolutions being voted on. As a result, these measures could incorrectly attribute changes in agenda as changes in state preferences. To resolve this issue, Bailey et al. (2017) apply spatial voting models from the roll call literature to estimate each country's political preferences embedded in its UN votes. The outcome of this statistical procedure is a time-varying, one-dimensional measure called "ideal points", which reflects each country's preference.[5] Bailey et al. (2017) show that ideal points consistently capture the position of states vis-à-vis a US-led liberal order. We derive a measure of bilateral distance by taking the absolute difference between the ideal points of countries i and j in year t.

Strategic Rivalries Our second set of measures of countries' bilateral alignment are indicator variables that pick up whether country i is a strategic rival of j in year t, as classified by Thompson (2001) and Colaresi et al. (2010). These rivalry measures capture the risk of conflict with a country of significant relative size and military strength, based on contemporary perceptions by political decision makers, gathered from historical sources on foreign policy and diplomacy. Specifically, rivalries are identified by whether two countries regard each other as competitors, a source of actual or latent threats that pose some possibility of becoming militarized, or enemies.

Colaresi et al. (2010) further refine the data to distinguish between three types of rivalries: spatial, where rivals contest the exclusive control of a territory; positional, where rivals contest relative shares of influence over activities and prestige within a system or subsystem; and ideological, where rivals contest the relative virtues of different belief systems relating to political, economic or religious activities.

Strategic rivalry is much more prevalent than military conflict, as shown in Aghion et al. (2018). In our sample from 1970-2012, we find that a total of 42 countries have had at least one strategic rival; 74 country-pairs have been strategic rivals at some point; and the total number of country-pair-years that exhibit strategic rivalry is 2,452. For example, China is classified as a strategic rival of the U.S. (1970–1972 and 1996–present), India (the entire sample period), Japan (1996–present), the former Soviet Union (1970–1989), and Vietnam (1973–1991). By comparison, the United States is coded as a strategic rival of China (1970-72 and 1996-2012), Cuba (1970-2012), and the former Soviet Union (1970-89 and 2007-2012).

[5] Specifically, Bailey et al. (2017)'s methodology identifies preference change over time by exploiting duplicate resolutions that are voted repeatedly in consecutive sessions. This methodology also weights resolutions based on how much they reflect the main policy preference dimension in order to ensure that ideal points are not heavily influenced by resolutions that reflect idiosyncratic factors.

Formal Alliances Our third set of political alignment measures are indicator variables for whether country i is in a formal alliance with country j in year t from the Correlates of War Formal Alliances v4.1 (Gibler 2008). This dataset records all formal alliances among states between 1816 and 2012, including mutual defense pacts, neutrality and non-aggression treaties, and ententes. A defense pact is the highest level of military commitment, requiring alliance members to come to each other's aid militarily if attacked by a third party. Neutrality and non-aggression pacts pledge signatories to either remain neutral in case of conflict or not use force against the other alliance members. Ententes obligate members to consult in times of crisis or armed attack. Over our entire sample period from 1970-2012, 1,946 country-pairs are in a formal alliance, and 117 countries have at least one formal ally. In the year 2010, China had four allies: Iran, North Korea, Russia, and Pakistan. In contrast, the United States was in alliance with 49 nations in the same year, a significantly greater number than the median country, which has 10 allies.

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